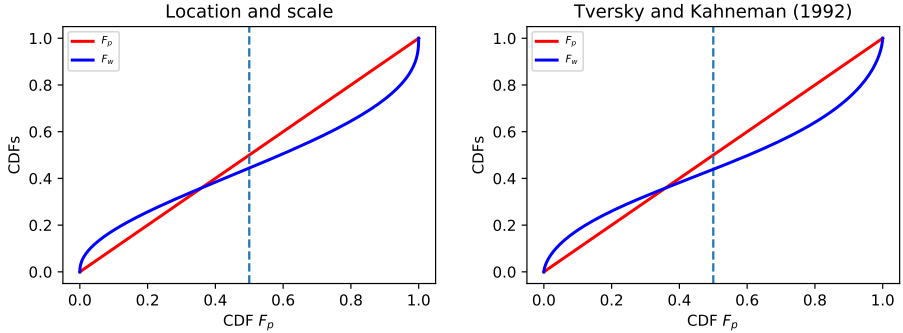
What are we weighting for?

A mechanistic model for probability weighting

Ole Peters Alexander Adamou Yonatan Berman Mark Kirstein

D-TEA 2020, 16 June 2020

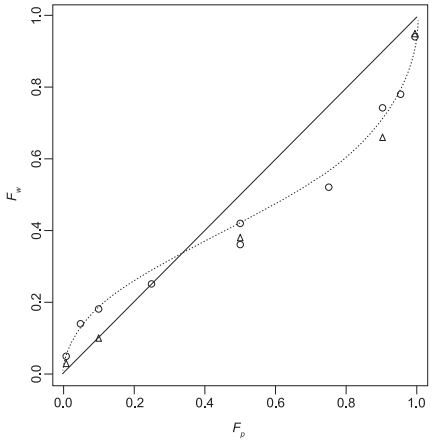
Main results



1. inverse-S shape can be explained by difference in uncertainty
2. cautious estimation of probabilities generates such differences

▸ PW K&T 1979

Definition of Probability Weighting (PW)



(Tversky and Kahneman 1992, p. 310, Fig. 1, relabelled axes)

- low probabilities treated as higher → high probabilities treated as lower
- stable empirical pattern: inverse-S shape

Received wisdom:

- PW = maladaptive irrational cognitive bias

In search of a mechanism

↪ How does this pattern emerge?

↪ Can we derive a functional form (rather than fit a function)?

Setup

Task: model payout, x, of a gamble as a random variable.

Disinterested Observer (DO)

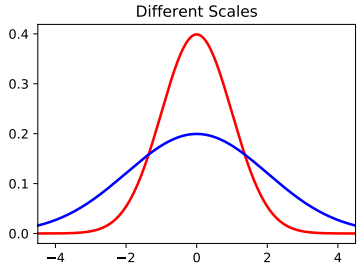
DO assigns
probabilities $p(x)$
CDF $F_p(x)$

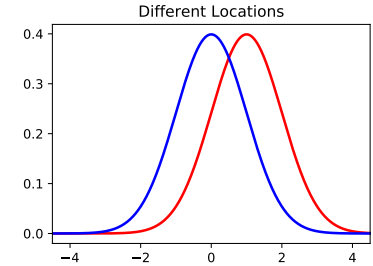
Decision Maker (DM)

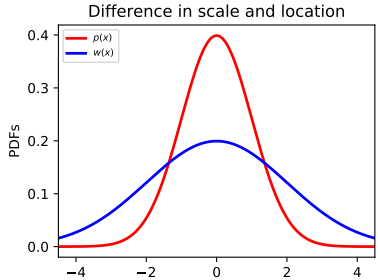
DM assigns different
probabilities $w(x)$ (decision weights)
CDF $F_w(x)$

Possible model differences

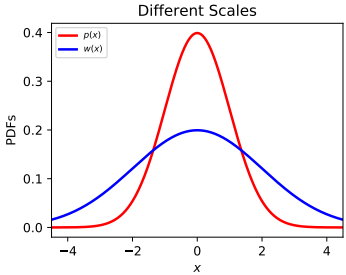
Locations, Scales, Shapes







Thought experiment: DM assumes greater scale



Numerically easy for any pair of distributions (models):

1. list values of DO's CDF, $F_p(x)$, at set x_i
2. list values of DM's CDF, $F_w(x)$, at same x_i
3. plot $F_w(x)$ *vs.* $F_p(x)$

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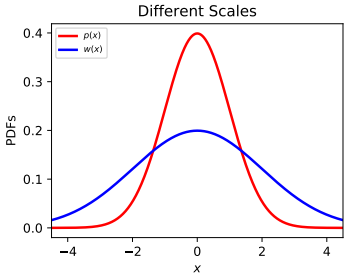

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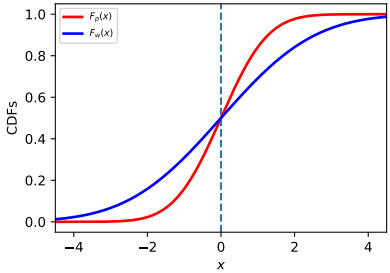

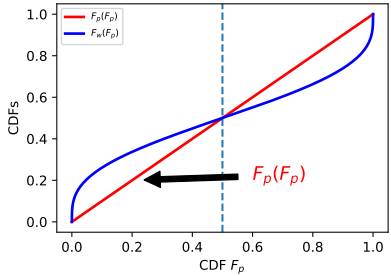

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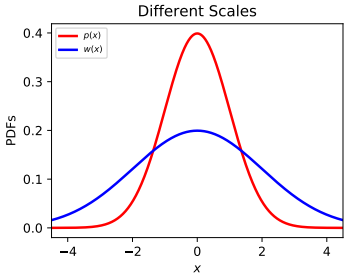

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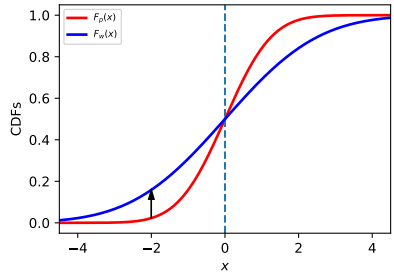

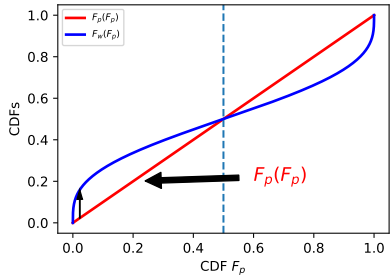

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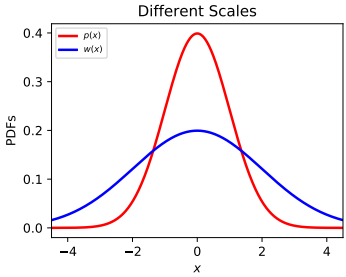

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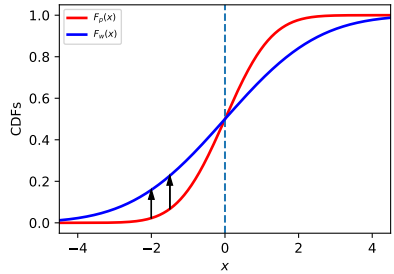

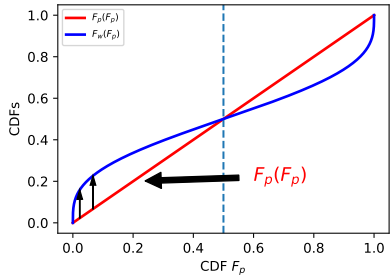

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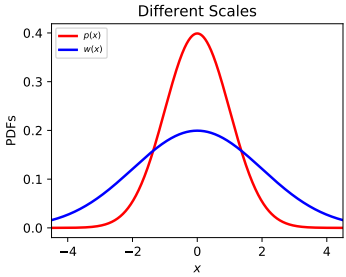

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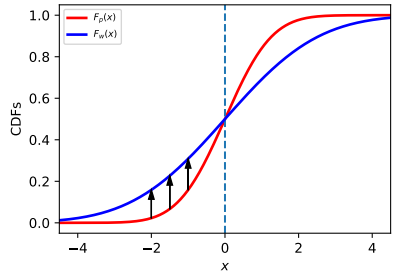

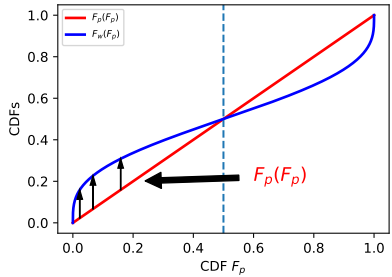

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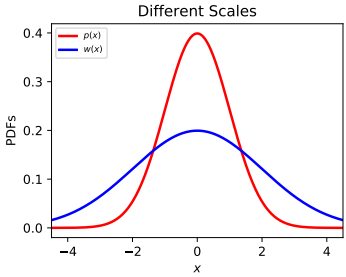

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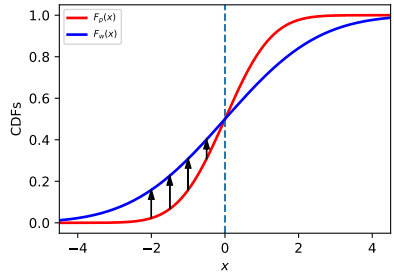

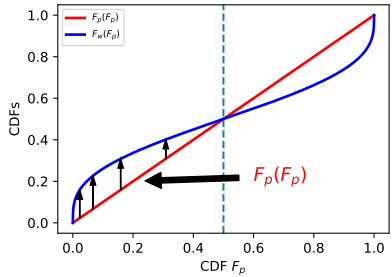

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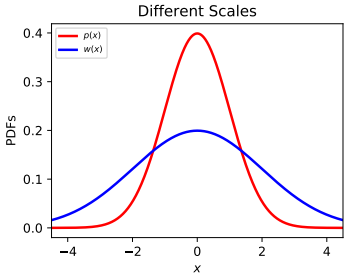

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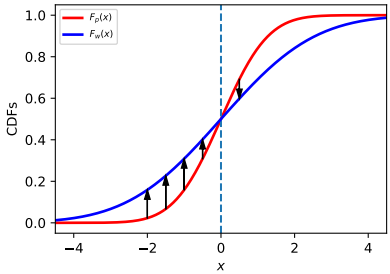

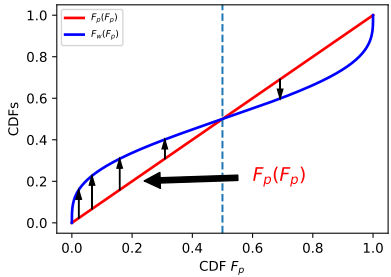

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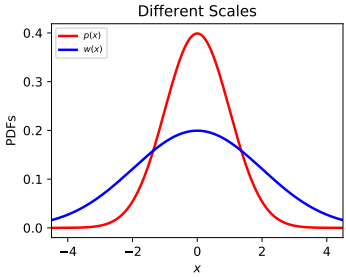

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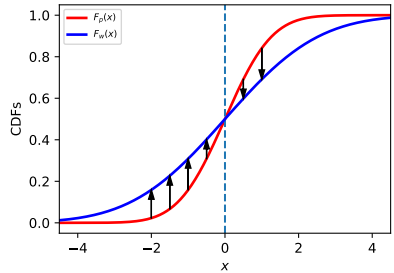

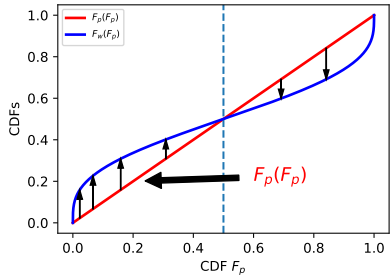

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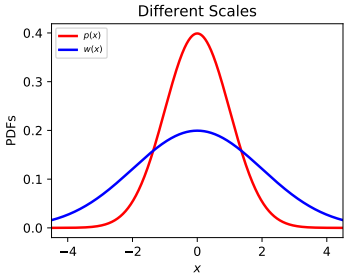

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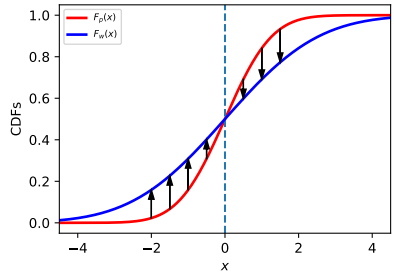

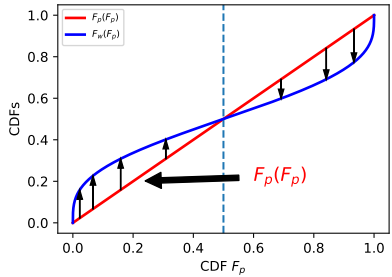

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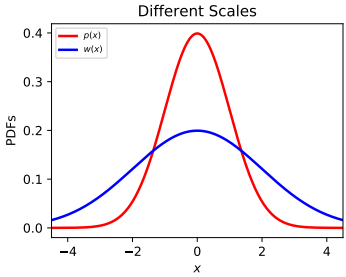

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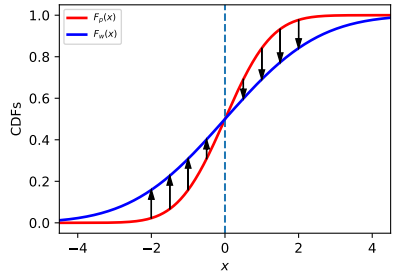
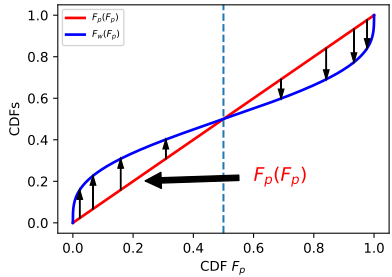
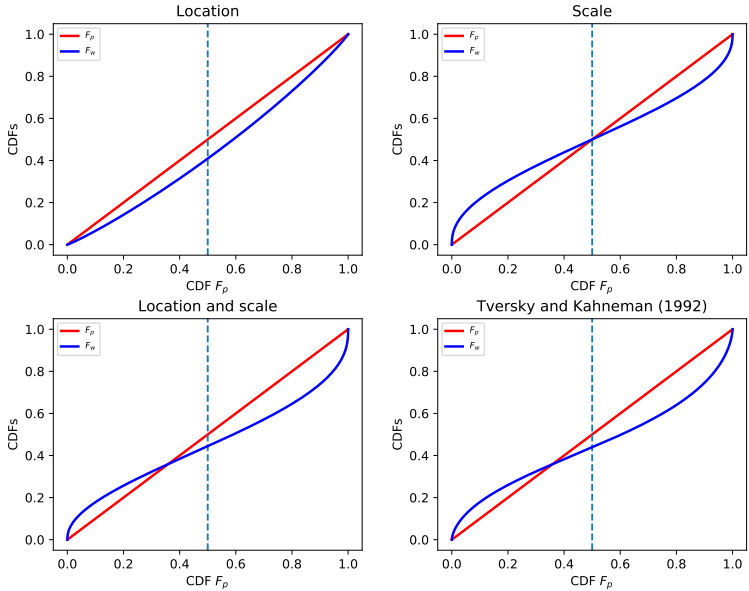




Applying the Procedure to the Uncertainty Types

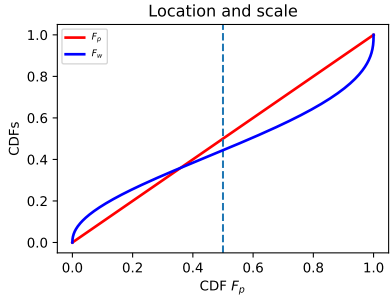


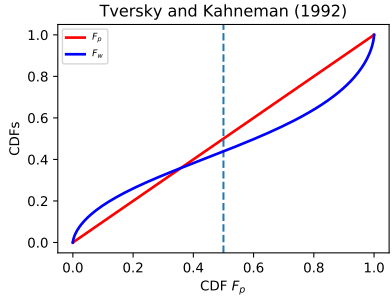
Interim conclusion

- DM's greater scale gives inverse-S shape (unimodal distributions)
- difference in locations gives asymmetry
- reproduces observations of probability weighting

Job done. Thank you for your attention ;)

Functional Forms





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Main Result
Probability Weighting
Ergodicity Question
Estimation
Conclusion

The Ergodicity Question

Typical DO concern

What happens on average to the ensemble of subjects?

$\neq$

Typical DM concern

What happens to me on average over time?

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Main Result
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Why DM's greater scale?

- DM has no control over experiment
- experiment may be unclear to DM
- DM may not trust DO
- ...

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Experiencing probabilities

- probabilities are not observable
- DO encounters probabilities as known frequency in ensemble of experiments
- DM encounters probabilities as frequencies estimated over time

↪ **estimates have uncertainties – a cautious DM will account for these**

- typical situation: rarer event $\longrightarrow$ larger relative error estimated probability

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Estimating probabilities

Rare Event

- $p(x) = 0.0001$
- 10 000 observations
- $\sim$ 99.5% of such time series will contain 0 or 1 events
- Naïve estimation: $\hat{p}(x) = 0$ or $\hat{p}(x) = 0.0001$

$\hookrightarrow$ either impossible or 1000% (over)estimation

Common Event

- $p(x) = 0.1$
- 10 000 observations
- $\sim$ 99.5% of time series would contain between 50 and 150 events,
- Naïve estimation: $0.05 < \hat{p}(x) < 0.15$

$\hookrightarrow$ only $\approx$ 50% error in $\hat{p}(x)$

$\hookrightarrow$ small $p(x)$, small count
$\hookrightarrow$ small count, big uncertainty

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Relative estimation error is large for rare events

Asymptotic probability	Most likely count	Standard error in count	Standard error in probability	Relative error in probability
0.1	1000	32	0.003	3%
0.01	100	10	0.001	10%
0.001	10	3	0.0003	30%
0.0001	1	1	0.0001	100%

Table: $T = 10\,000$, assuming Poisson statistics, relative estimation errors $\sim 1/\sqrt{\text{count}}$

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DMs don't like surprises

To avoid surprises, let's say DMs add estimation uncertainty *err* to every estimated probability, then normalize, s.t.

$$w(x) = \frac{p(x)+\varepsilon\left[p(x)\right]}{\int\left(p(s)+\varepsilon\left[p(s)\right]\right)ds}$$

This allows us to derive a functional form, *e.g.* for the Gaussian case ... (find in manuscript)

...visually similar to function chosen by Kahneman and Tversky.

Not sure we need much more. I'd just have one figure that gives a nice inverse S, for a Gaussian, say, based on estimation error.

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Conclusion

Ergodicity Economics and probability weighting

- inverse-S shape appears as neutral indicator of a difference in opinion
- reported observations consistent with DM's extra uncertainty
- may arise from DM estimating probabilities over time

↪ Probability weighting is rational cautious behaviour under uncertainty over time

- See full paper at bit.ly/lml-pw-r1
- links to play with the code are inside

Thank you for your attention!

I'm looking forward to the discussion
Comments & questions are very welcome, here or to

✉ m.kirstein@lml.org.uk
@nonergodicMark

Submit an open peer review to this paper on
bit.ly/lml-pw-r1

RESEARCHERS.ONE

BACK UP

Probability Weighting as an Estimation Issue

"It is important to distinguish overweighting, which refers to a property of decision weights, from the overestimation that is commonly found in the assessment of the probability of rare events. [...] In many real-life situations, overestimation and overweighting may both operate to increase the impact of rare events." (Kahneman and Tversky 1979, p. 281)

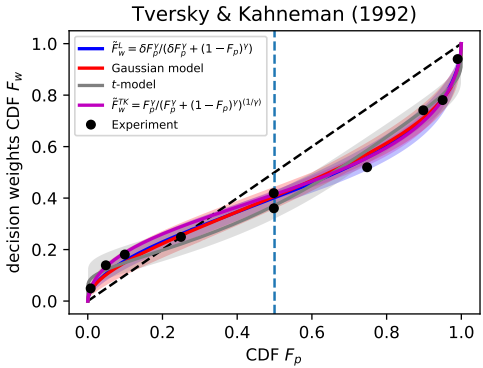
↪ distinguish between
- uncertainty estimation and
- "weighting"

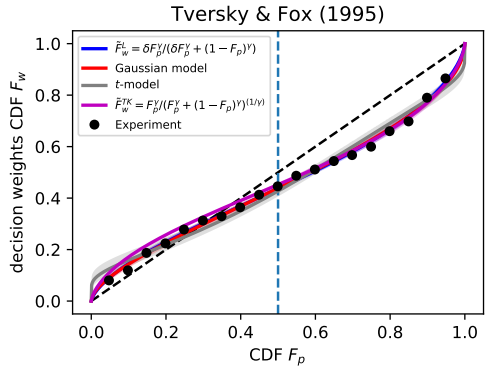
we analyse the former and find very good agreement with the empirical inverse-S pattern

↪ How big is the residual "probability weighting" after accounting for uncertainty estimation?

◂ Main Results

Estimation Error Explains 99% of Probability Weighting





- similar fits of Gaussian & t-distributed model

↪ How big is the residual "probability weighting" after accounting for estimation errors?

Main Results

Functional Forms

Gaussian

Tversky and Kahneman (1992, $\gamma = 0.68$)

$$\tilde{F}_w^{TK}\left(F_p;\gamma\right) = \left(F_p\right)^{\gamma} \frac{1}{\left[\left(F_p\right)^{\gamma} + \left(1-F_p\right)^{\gamma}\right]^{1/\gamma}} \tag{1}$$

Lattimore, Baker, and Witte (1992)

$$\tilde{F}_w^{L}\left(F_p;\delta,\gamma\right) = \frac{\delta F_p^{\gamma}}{\delta F_p^{\gamma} + \left(1-F_p\right)^{\gamma}} \tag{2}$$

We derive decision weight as a function of probability with $(\alpha\sigma)^2$ as the DM's scale

$$w(p) = p^{\frac{1}{\alpha^2}} \frac{\left(2\pi\sigma^2\right)^{\frac{1-\alpha^2}{2\alpha^2}}}{\alpha}\ , \tag{3}$$

which is a power law in p with a pre-factor to ensure normalisation

Linking Probability Weighting to Relative Uncertainties

Decision weight w is the normalised sum of the probability $p(x)$ and its uncertainty $\varepsilon\left[p(x)\right]$

$$w(x) = \frac{p(x) + \varepsilon\left[p(x)\right]}{\int_{-\infty}^{\infty}\left(p(s) + \varepsilon\left[p(s)\right]\right) ds} \,. \tag{4}$$

This can be expressed as

$$w(x) = p(x)\left(\frac{1 + \frac{\varepsilon\left[p(x)\right]}{p(x)}}{\int_{-\infty}^{\infty} p(s)\left\{1 + \frac{\varepsilon\left[p(s)\right]}{p(s)}\right\} ds}\right) \,, \tag{5}$$

where $\frac{\varepsilon\left[p(x)\right]}{p(x)}$ is the relative error, which is large (small) for small (large) probabilities
In the long-time limit $w(x) \to p(x)$

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