What are we weighting for?

A mechanistic model for probability weighting

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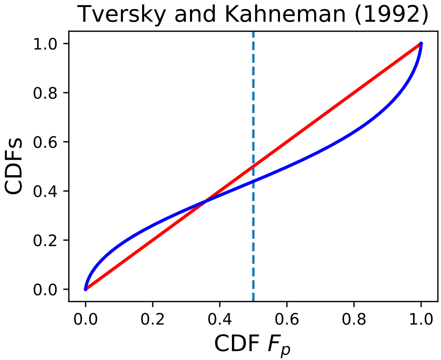
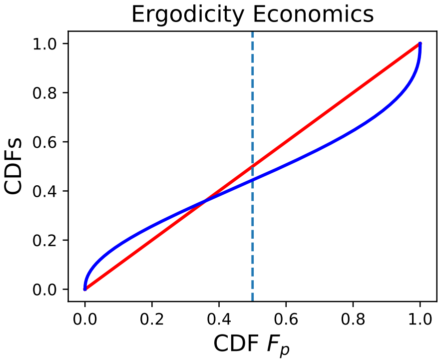
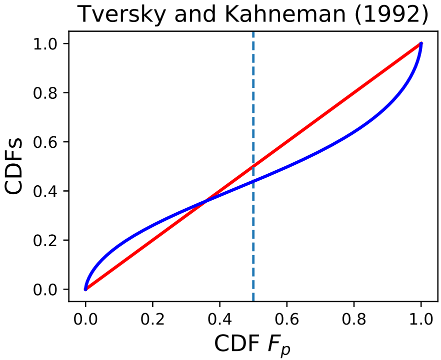
Main Results
Probability Weighting
Ergodicity Question
Estimation
Conclusion

Main results

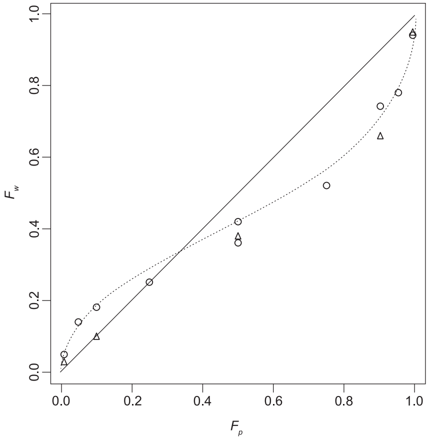


Main results





Definition of Probability Weighting (PW)



(Tversky and Kahneman 1992, p. 310, Fig. 1, relabelled axes)

- low probabilities treated as higher; high probabilities treated as lower
- stable empirical pattern: inverse-S shape
- Cumulative Prospect Theory (CPT)

Classical interpretation of PW:

- maladaptive irrational cognitive bias

In search of a mechanism

$\hookrightarrow$ How does this pattern emerge?

$\hookrightarrow$ Can we derive a functional form (rather than fit a function)?

Setup

Task: model payout, x, of a gamble as a random variable.

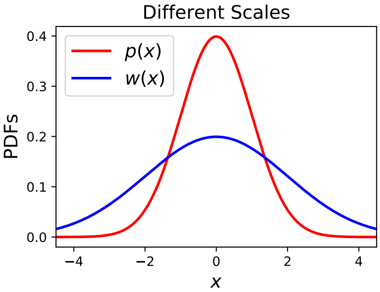
Disinterested Observer (DO)

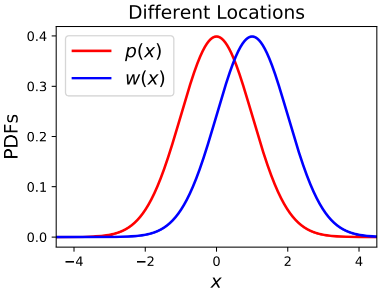
DO assigns
PDF $p(x)$
CDF $F_p(x)$

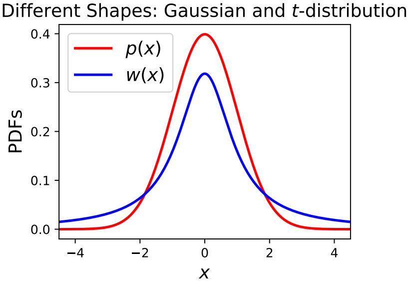
Decision Maker (DM)

DM assigns different
PDF $w(x)$
CDF $F_w(x)$

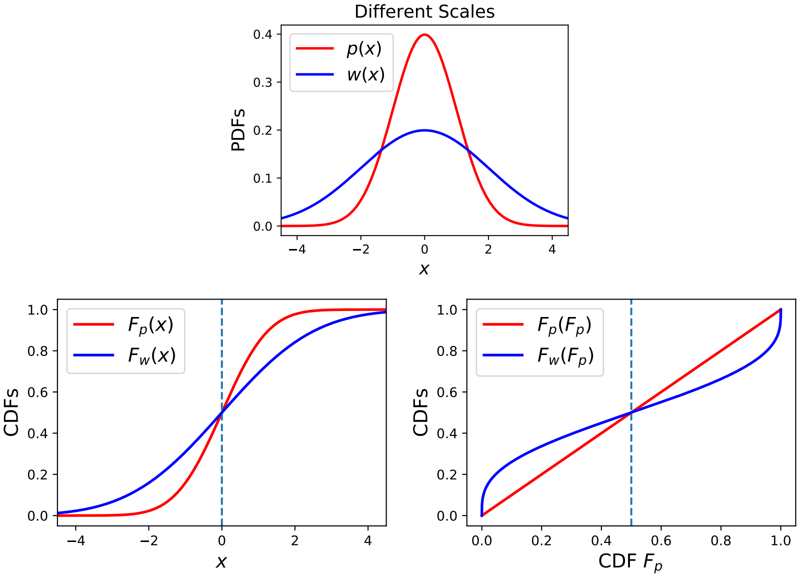
Scales, Locations, Shapes







Thought experiment: DM assumes greater scale



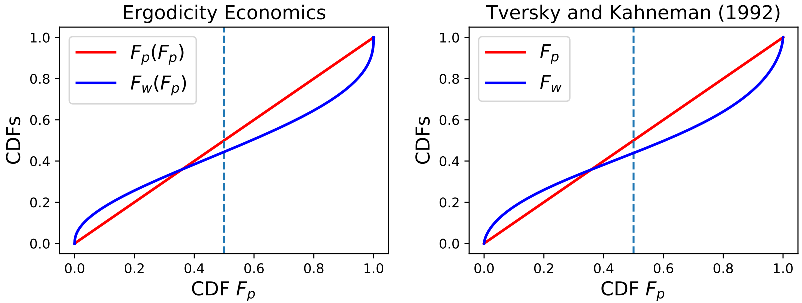
For the case of two Gaussians with different scale we derive a functional form

$$w(p) = p^{\frac{1}{\alpha^2}} \underbrace{\frac{\left(2\pi\sigma^2\right)^{\frac{1-\alpha^2}{2\alpha^2}}}{\alpha}}_{\text{normalisation factor}}, \tag{1}$$

where

- DO's scale is σ
- DM's scale is $\alpha\sigma$
- $\alpha < 1 \rightarrow$ S shape
- $\alpha > 1 \rightarrow$ inverse-S shape

Interim conclusion



- DM's greater scale gives inverse-S shape (unimodal distributions)
- difference in locations gives asymmetry
- reproduces observations of probability weighting

Job done. Thank you for your attention ;)

The Ergodicity Question

Typical DO concern

What happens on average to the ensemble of subjects?

$\neq$

Typical DM concern

What happens to me on average over time?

Why DM's greater scale?

- DM has no control over experiment
- experiment may be unclear to DM
- DM may not trust DO
- ...

Experiencing probabilities

- probabilities are not observable
- probabilities encountered as
  - known frequency in ensemble of experiments (DO)
  - frequencies estimated over time (DM)

$\hookrightarrow$ **estimates have uncertainties – cautious DM accounts for these**

Estimating probabilities

Rare Event

- $p(x) = 0.001$
- 100 observations
- $\sim 99.5\%$ get 0 or 1 events
- $\hat{p}(x) = 0$ or $\hat{p}(x) = 0.01$

$\hookrightarrow$ 1000% uncertainty in $\hat{p}(x)$

Common Event

- $p(x) = 0.5$
- 100 observations
- $\sim 99.5\%$ get between 35 and 65 events,
- $0.35 < \hat{p}(x) < 0.65$

$\hookrightarrow$ $\pm 15\%$ uncertainty in $\hat{p}(x)$

$\hookrightarrow$ small $p(x)$, small count
$\hookrightarrow$ small count, big uncertainty

DMs don't like surprises

To avoid surprises, DMs add estimation uncertainty $\varepsilon\left[p(x)\right]$ to every estimated probability, then normalize, s.t.

$$w(x)=\frac{p(x)+\varepsilon\left[p(x)\right]}{\int\left(p(s)+\varepsilon\left[p(s)\right]\right)ds}$$

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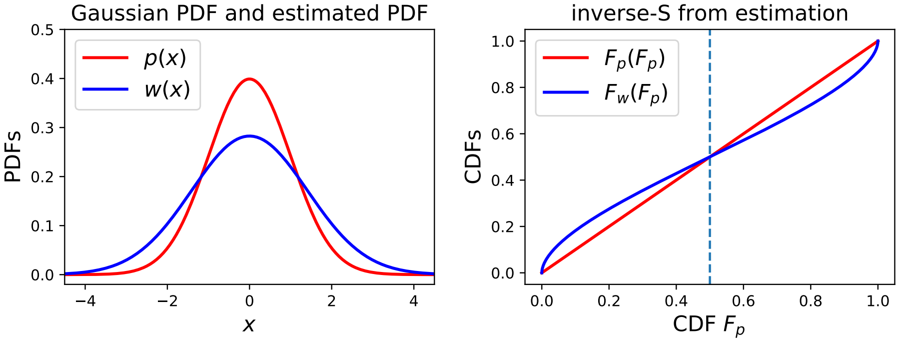

Conclusion

Classical interpretation of PW:

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Ergodicity Economics and probability weighting

- inverse-S shape: neutral indicator of different models of the world
- reported observations consistent with DM's extra uncertainty
- may arise from DM estimating probabilities over time

$\hookrightarrow$ Probability weighting is rational cautious behaviour under uncertainty over time

- Manuscript at https://www.researchers.one/article/2020-04-14
- Interactive code at https://bit.ly/lml-pw-count-b

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Thank you for your attention!

References I

Tversky, Amos and Daniel Kahneman (1992). "Advances in Prospect Theory: Cumulative Representation of Uncertainty". *Journal of Risk and Uncertainty* 5 (4), pp. 297–323. DOI:10.1007/BF00122574 (cit. on p. 4).