What are we weighting for?

A mechanistic model for probability weighting

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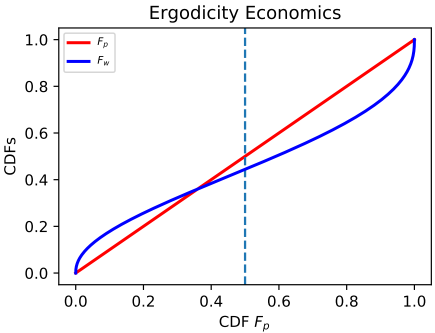
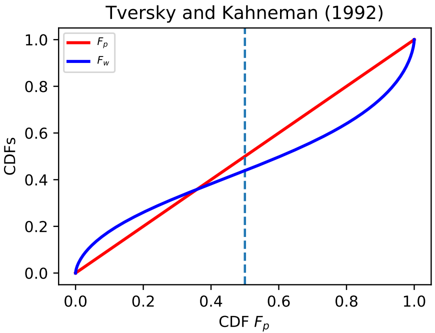
Main results

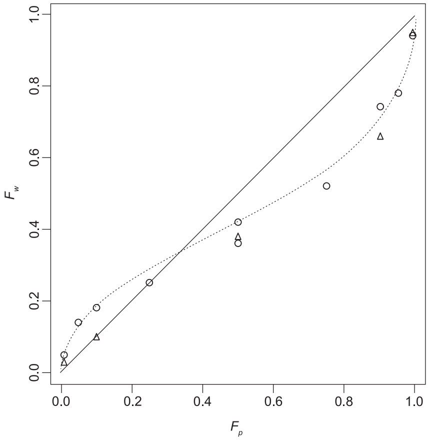




1. inverse-S shape can be explained by difference in uncertainty
2. cautious estimation of probabilities generates such differences

▸ PW K&T 1979

Definition of Probability Weighting (PW)



(Tversky and Kahneman 1992, p. 310, Fig. 1, relabelled axes)

- low probabilities treated as higher; high probabilities treated as lower
- stable empirical pattern: inverse-S shape

Received wisdom:

- PW = maladaptive irrational cognitive bias

In search of a mechanism

$\hookrightarrow$ How does this pattern emerge?

$\hookrightarrow$ Can we derive a functional form (rather than fit a function)?

Setup

Task: model payout, x, of a gamble as a random variable.

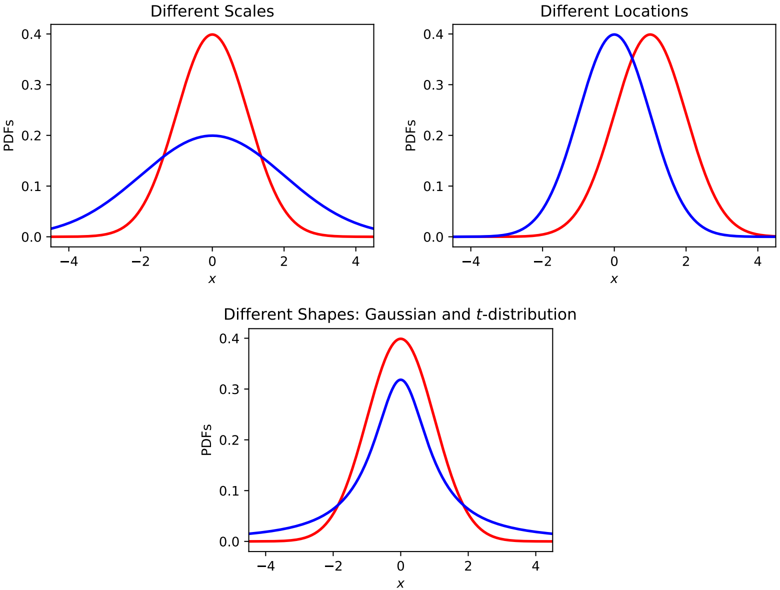
Disinterested Observer (DO)

DO assigns
probabilities $p(x)$
CDF $F_p(x)$

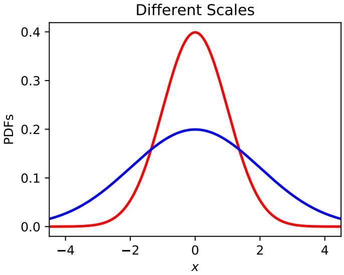
Decision Maker (DM)

DM assigns different
probabilities $w(x)$ (decision weights)
CDF $F_w(x)$

Possible model differences



Thought experiment: DM assumes greater scale



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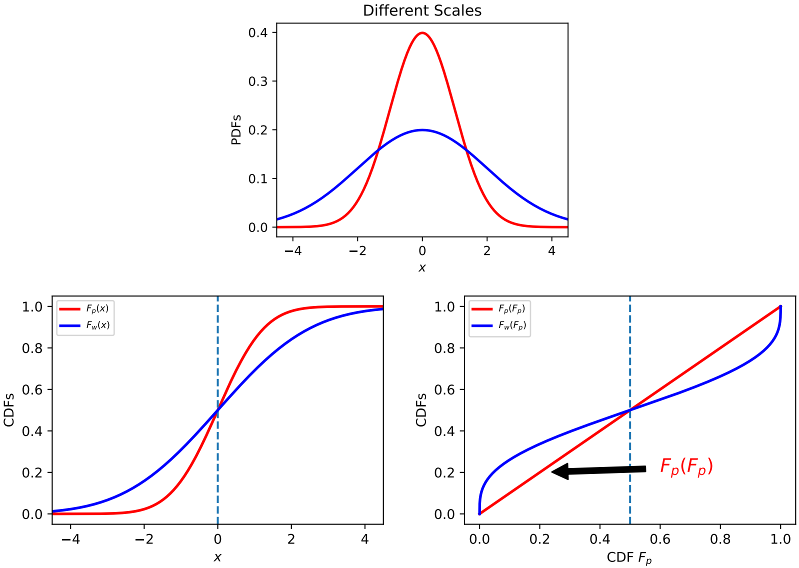

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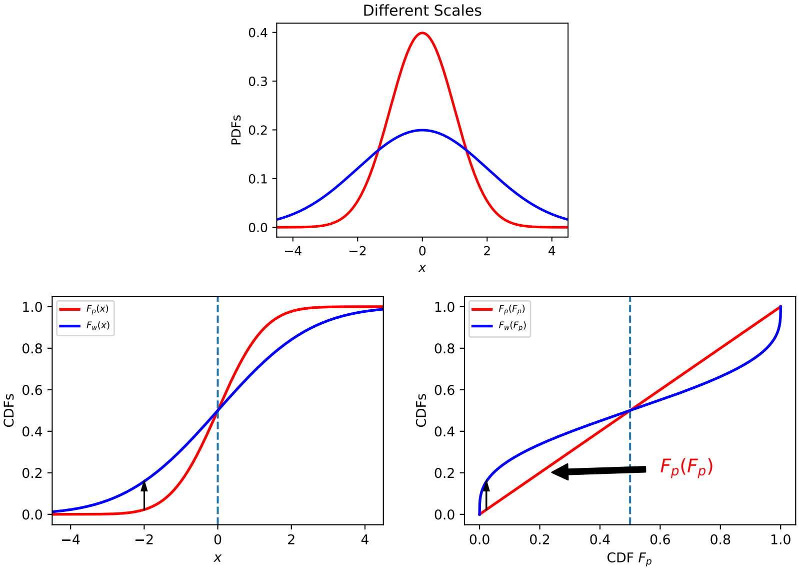

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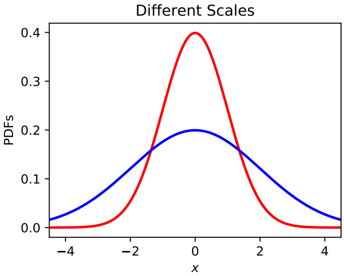

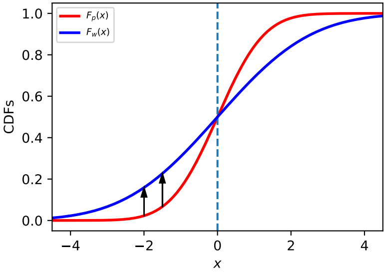

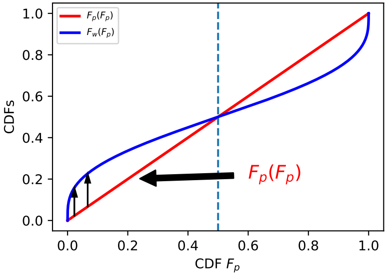

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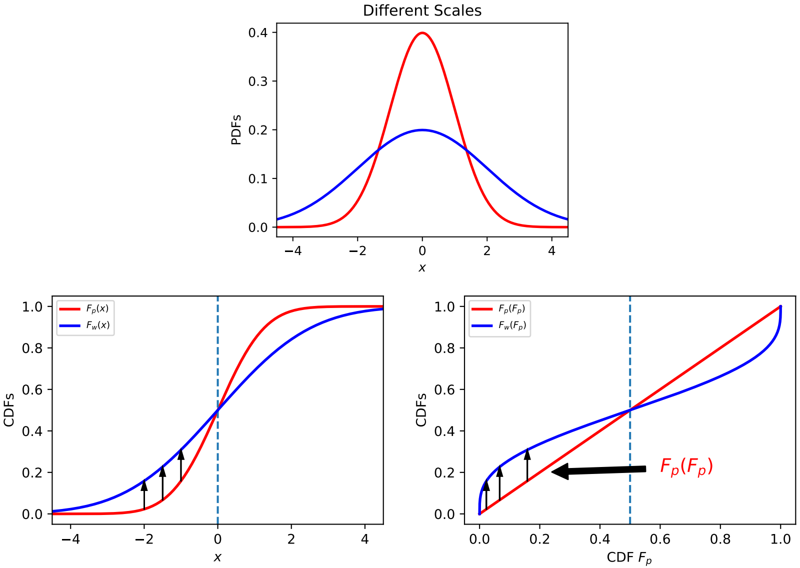

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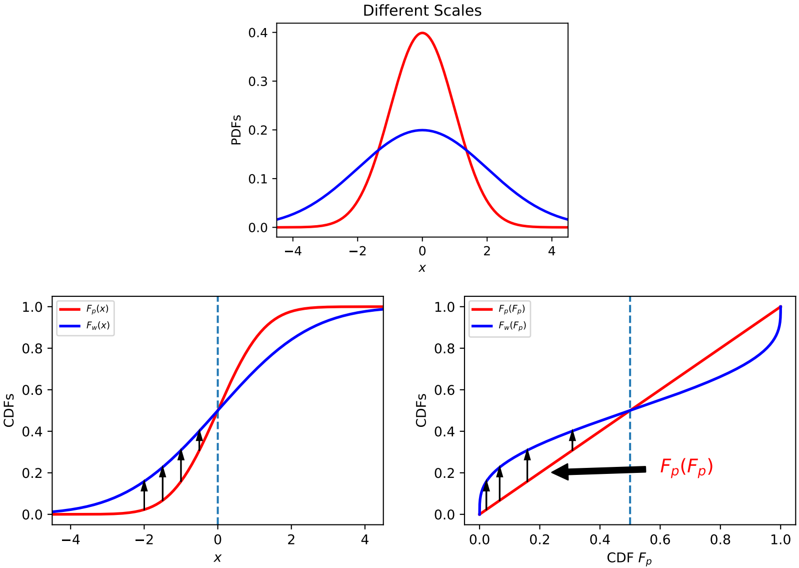

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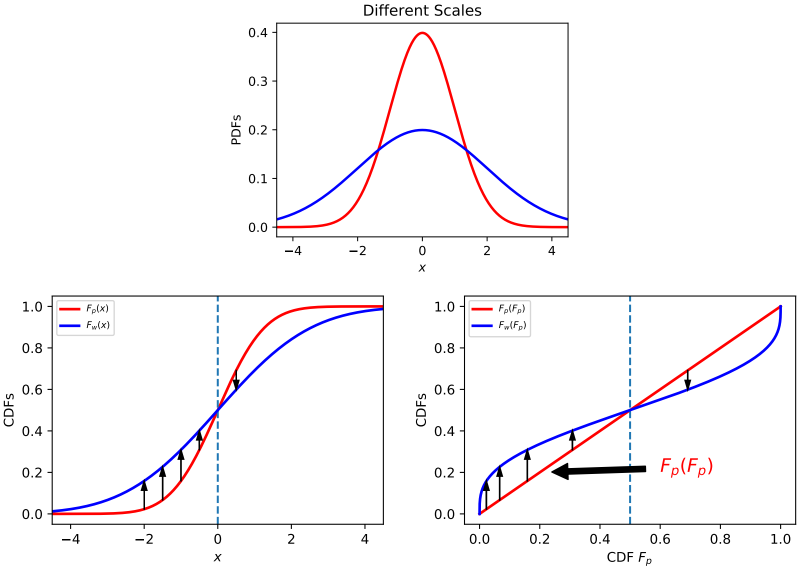

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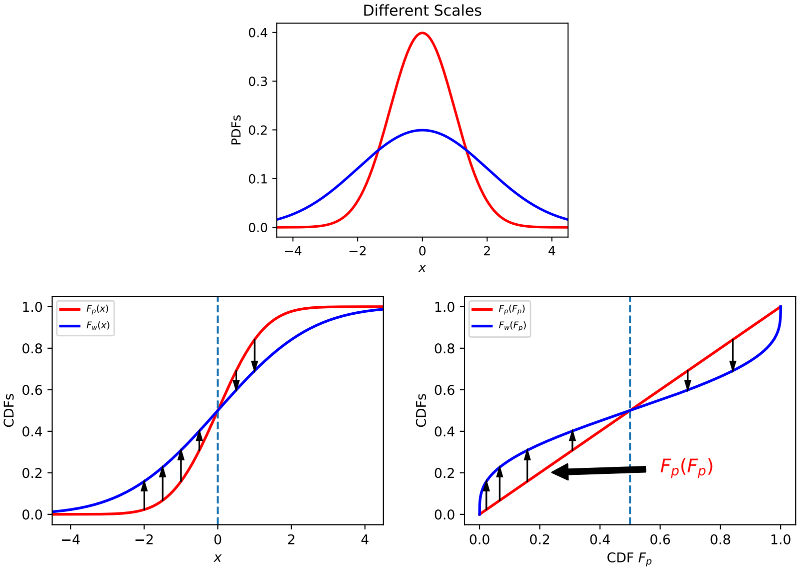

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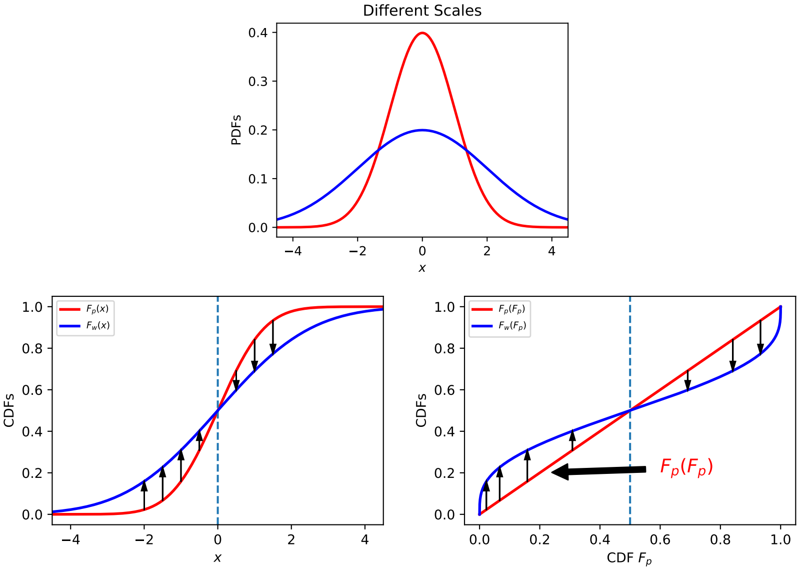

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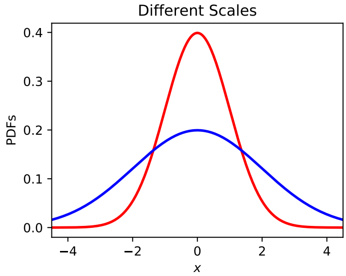

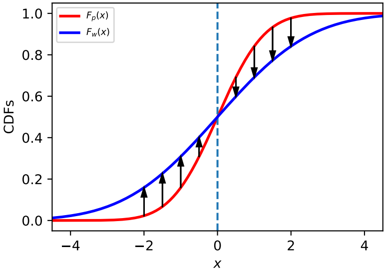

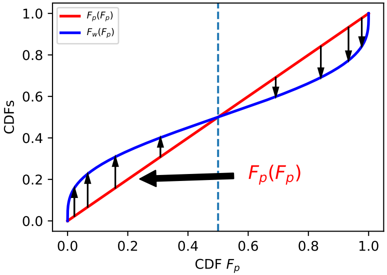

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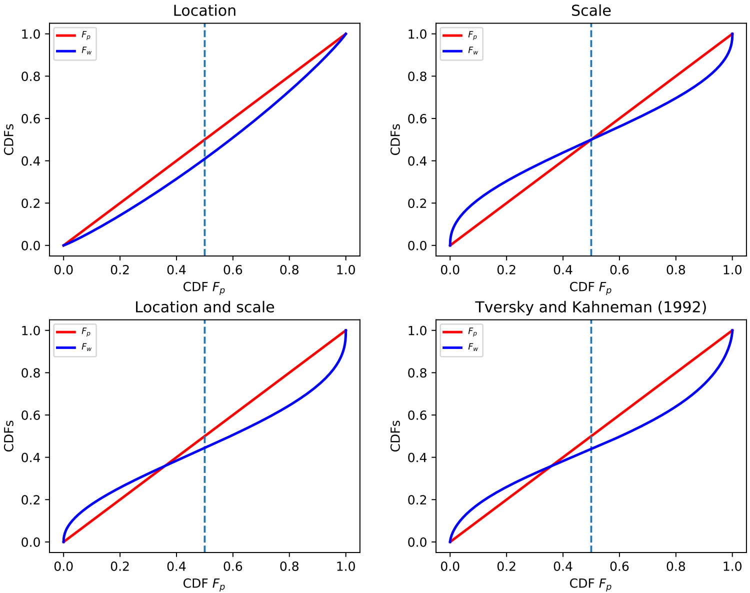
Numerically easy for any pair of distributions (models):

1. list values of DO's CDF, $F_p(x)$, at set x_i
2. list values of DM's CDF, $F_w(x)$, at same x_i
3. plot $F_w(x)$ *vs.* $F_p(x)$

Asymmetry from different locations

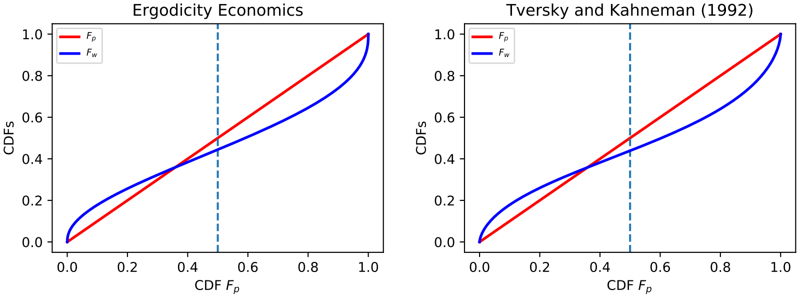


Interim conclusion

- DM's greater scale gives inverse-S shape (unimodal distributions)
- difference in locations gives asymmetry
- reproduces observations of probability weighting

Job done. Thank you for your attention ;)

▸ Functional Forms



The Ergodicity Question

Typical DO concern

What happens on average to the ensemble of subjects?

$\neq$

Typical DM concern

What happens to me on average over time?

Why DM's greater scale?

- DM has no control over experiment
- experiment may be unclear to DM
- DM may not trust DO
- ...

Experiencing probabilities

- probabilities are not observable
- probabilities encountered as
  - known frequency in ensemble of experiments (DO)
  - frequencies estimated over time (DM)

$\hookrightarrow$ **estimates have uncertainties – cautious DM accounts for these**

Estimating probabilities

Rare Event

- $p(x) = 0.001$
- 100 observations
- $\sim 99.5\%$ of such time series will contain 0 or 1 events
- Naïve estimation: $\hat{p}(x) = 0$ or $\hat{p}(x) = 0.01$

$\hookrightarrow$ either impossible or ten times (over)estimation

Common Event

- $p(x) = 0.5$
- 100 observations
- $\sim 99.5\%$ of time series would contain between 35 and 65 events,
- Naïve estimation: $0.35 < \hat{p}(x) < 0.65$

$\hookrightarrow$ $< 50\%$ error in $\hat{p}(x)$

$\hookrightarrow$ small $p(x)$, small count
$\hookrightarrow$ small count, big uncertainty

Relative estimation error is large for rare events

Asymptotic probability	Most likely count	Standard error in count	Standard error in probability	Relative error in probability
0.1	1000	32	0.003	3%
0.01	100	10	0.001	10%
0.001	10	3	0.0003	30%
0.0001	1	1	0.0001	100%

Table: $T = 10\,000$, assuming Poisson statistics, relative estimation errors $\sim {}^{1}/\sqrt{\text{count}}$

$\hookrightarrow$ small $p(x)$, small count
$\hookrightarrow$ small count, big uncertainty

DMs don't like surprises

To avoid surprises, let's say DMs add estimation uncertainty $\varepsilon\left[p(x)\right]$ to every estimated probability, then normalize, s.t.

$$w(x) = \frac{p(x) + \varepsilon\left[p(x)\right]}{\int \left(p(s) + \varepsilon\left[p(s)\right]\right) ds}$$

This allows us to derive a functional form, *e.g.* for the Gaussian case

$$w(p) = p^{\frac{1}{\alpha^2}} \frac{\left(2\pi\sigma^2\right)^{\frac{1-\alpha^2}{2\alpha^2}}}{\alpha},$$

where DM's greater scale is $(\alpha\sigma)^2$

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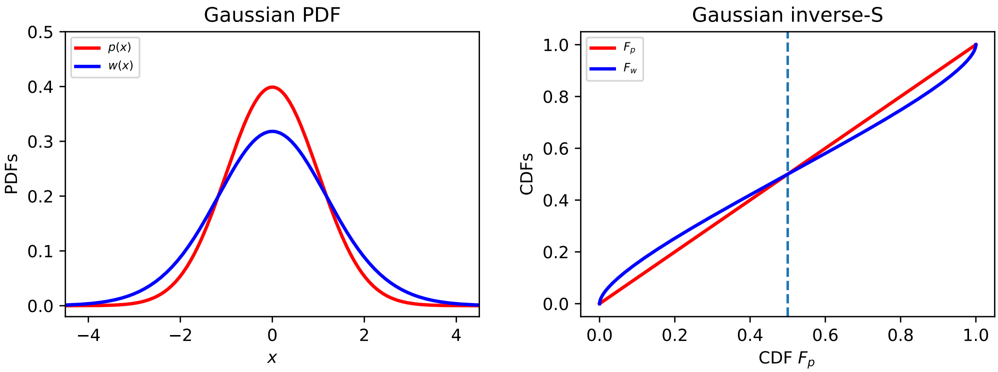
Estimating probabilities for two Gaussians



Conclusion

Ergodicity Economics and probability weighting

- inverse-S shape appears as neutral indicator of a difference in opinion
- reported observations consistent with DM's extra uncertainty
- may arise from DM estimating probabilities over time

$\hookrightarrow$ Probability weighting is rational cautious behaviour under uncertainty over time

- Manuscript at https://www.researchers.one/article/2020-04-14
- Interactive code at https://bit.ly/lml-pw-count-b

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Thank you for your attention!

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BACK UP

Probability Weighting as an Estimation Issue

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Back Up

References

"It is important to distinguish overweighting, which refers to a property of decision weights, from the overestimation that is commonly found in the assessment of the probability of rare events. [...] In many real-life situations, overestimation and overweighting may both operate to increase the impact of rare events." (Kahneman and Tversky 1979, p. 281)

$\hookrightarrow$ distinguish between

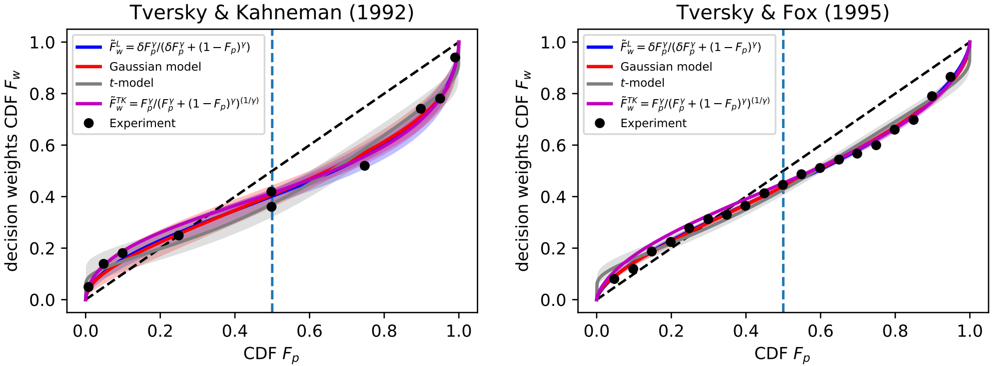
- uncertainty estimation and
- "weighting"

we analyse the former and find very good agreement with the empirical inverse-S pattern

$\hookrightarrow$ How big is the residual "probability weighting" after accounting for uncertainty estimation?

Main Results

Estimation Error Explains 99% of Probability Weighting



- similar fits of Gaussian & t-distributed model

↪ How big is the residual "probability weighting" after accounting for estimation errors?

◂ Main Results

Functional Forms

◂ Gaussian

Tversky and Kahneman (1992, $\gamma = 0.68$)

$$\tilde{F}_w^{TK}\left(F_p;\gamma\right) = \left(F_p\right)^\gamma \frac{1}{\left[\left(F_p\right)^\gamma + \left(1-F_p\right)^\gamma\right]^{1/\gamma}} \tag{1}$$

Lattimore, Baker, and Witte (1992)

$$\tilde{F}_w^{L}\left(F_p;\delta,\gamma\right) = \frac{\delta F_p^\gamma}{\delta F_p^\gamma + \left(1-F_p\right)^\gamma} \tag{2}$$

We derive decision weight as a function of probability with $(\alpha\sigma)^2$ as the DM's scale

$$w(p) = p^{\frac{1}{\alpha^2}} \frac{\left(2\pi\sigma^2\right)^{\frac{1-\alpha^2}{2\alpha^2}}}{\alpha}\,, \tag{3}$$

which is a power law in p with a pre-factor to ensure normalisation

References I

Kahneman, Daniel and Amos Tversky (1979). "Prospect Theory: An Analysis of Decision under Risk". *Econometrica* 47 (2), pp. 263–291. DOI:10.2307/1914185 (cit. on p. 29).

Lattimore, Pamela K., Joanna R. Baker, and A. Dryden Witte (1992). "Influence of Probability on Risky Choice: A Parametric Examination". *Journal of Economic Behavior and Organization* 17 (3), pp. 377–400. DOI:10.1016/S0167-2681(95)90015-2 (cit. on p. 31).

Tversky, Amos and Daniel Kahneman (1992). "Advances in Prospect Theory: Cumulative Representation of Uncertainty". *Journal of Risk and Uncertainty* 5 (4), pp. 297–323. DOI:10.1007/BF00122574 (cit. on pp. 3, 31).