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Optimization production and distribution using production routing problem with perishable inventory (PRPPI) models

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Abstract. This paper discussed about optimization production and distribution problem of tempe using Production Routing problem with Perishable Inventory (PRPPI) models with First Produce, First Deliver (FPFD) and First Produce First Selling (FPFS) inventory management policy. Tempe is a food made from soy fermentation and distributed from the depot to retailers. There are three retailers, namely Perumnas market, Sekip market and Kebon Semai market respectively as retailer 1, 2 and 3, and Ana depot is defined as retailer 0. An exact Branch and Bound algorithm was developed and solved by Lingo 17.0 Software. The results of minimum cost production and distribution retailer 0 was 69,174 rupiah, optimal amount of tempe production 34 pieces. The shipping routes starting from retailer 0 to retailer 1, proceed to retailer 2 and then to retailer 3 finally return to retailer 0.

1. Introduction

This paper discussing about optimization of tempe production and distribution using the Production Routing Problem with Perishable Inventory (PRPPI) models with the policies of First Produce First Deliver (FPFD) and First Produce First Selling (FPFS) on Ana's Tempe Production. Tempe is a traditional Indonesian food made from soybeans. Generally, the tempe process of fermentation takes approximately three days. Tempe is a food that has a short expiration period and a selling price that is rapidly declining. Therefore, production and distribution planning is needed to minimize losses.

Several studies on perishable product among others are, Ishii [1] discussed about perishable inventory with several type costumer and difference price. Duan et al [2] discussed about perishable inventory that depends on the level of demand. Sirriuk [3] discussed about optimal of demand for perishable product. Duong et al [4] purposed used to multi-criteria performance metrics, including Order Rate Variance Ratio, Average Inventory, and Fill Rate. Perlman and Levner [5] discussed about perishable inventory for health product.

Product distribution planning is as important as production planning. Determination of shipping routes is part of the process of distributing goods. The Production Routing Problem (PRP) and Inventory Routing Problem (IRP) models can be used for production planning and shipping route determination. Some route determination studies that have been conducted are Vidovi and Ratcovi [6] by introducing Mixed integers and heuristics models for the multi product multi period IRP in fuel delivery. Chitsaz et al [7] discusses the problem of cycle Inventory Routing Problem (CIRP) to determine the cycle of delivery schedules from one type of product to several customers. Etebari and Dabiri [8] discuss a hybrid heuristic for Inventory Routing Problem under dynamic regional pricing.

Peres et al [9] discusses optimization using an IRP model with transshipment. Roldan et al [10] discuss the robustness of inventory replenishment and customer selection policies for the dynamic and stochastic IRP. Adulyasak et al [11] introduced the PRP model and provided several model completion algorithms. Qiu et al [12] introduced the mix integer programming model for the PRP problem with reverse logistics and manufacturing. Optimal production, inventory and distribution of perishable products can use the PRP model with Perishable inventory (PRPPI). Qiu et al [13] introduced the PRPPI model to optimize production, addition, shipping, routing for perishable products and completion of models using the Branch and Cut algorithm. Also introduced are nine inventory management combinations. In this study, the PRPPI model with FPFD-FPFS policy was applied to optimize tempe production and distribution. In the previous research discussed PRPPI models with the number of retailer. In this paper discussed about only three retailer and the holding cost is zero.

2. PRPPI Models

PRPPI models is the development of Production Routing Problem (PRP) models with the product studied having perishable properties. Qiu [13] explains that PRPPI can be stated as complete graph $\boldsymbol{G} = (\boldsymbol{N_0}, \boldsymbol{A})$ where the node set $\boldsymbol{N_0} = \boldsymbol{N} \cup \{\boldsymbol{0}\}$ consist of a set $\boldsymbol{N} = \{\boldsymbol{1}, \boldsymbol{2}, \boldsymbol{3} \cdots, \boldsymbol{n}\}$ of retailer and depot represented by node 0. Arc set is $\boldsymbol{A} = \{(\boldsymbol{i}, \boldsymbol{j}): \boldsymbol{i}, \boldsymbol{j} \in \boldsymbol{N_0}, \boldsymbol{i} \neq \boldsymbol{j}\}$. The product is produce at the depot and sent to retailer in planning period $\boldsymbol{T} = \{\boldsymbol{1}, \boldsymbol{2}, \cdots, |\boldsymbol{T}|\}$. [13] also gives nine combinations of inventory management, that are (1) First Produce First Deliver (FPFD)-First Produce First Sell (FPFS), (2) FPFD-LPFS (Last Produce First Sell), (3) FPFD-OS (Optimized Sell), (4) LPFD (Last Produce First Deliver)-FPFS, (5) LPFD-LPFS, (6) LPFD-OS (Optimized Selling), (7) OD (Optimized Deliver)-FPFS, (8) OD-LPFS, (9) OD-OS. The following is given the mixed integer formulation for PRPPI that was introduced by [13].

$$\min \quad \sum_{t\in T}\Big(uq_t + fy_t + \sum_{i\in N_0}\sum_{\tau=0} h_{i\tau t}\; I_{i\tau t} + \sum_{(i,j)\in A}\sum_{k\in K} c_{ij}\, x_{ijkt}\Big) \tag{2.1}$$

Subject to

$$q_t - \sum_{i\in N} r_{itt} = I_{0tt} \qquad \forall t \in T, \tag{2.2}$$
$$\left(1-\alpha_{\tau,t-1}\right) I_{0t,t-1} - \sum_{i\in N} r_{i\tau t} = I_{0\tau t}\,, \qquad \forall t \in T, \tau \in T_0, \tau \le t-1 \tag{2.3}$$
$$r_{itt} - d_{itt} = I_{itt}\,, \qquad \forall i \in N, t \in T \tag{2.4}$$
$$\left(1-\alpha_{\tau,t-1}\right) I_{i\tau,t-1} - r_{i\tau t} - d_{i\tau t} = I_{i\tau t}\,, \qquad \forall i \in N, t \in T, \tau \in T_0, \tau \le t-1 \tag{2.5}$$
$$\sum_{\tau=0}^{t} d_{i\tau t} = d_{it}\,, \qquad \forall i \in N, t \in T \tag{2.6}$$
$$\sum_{\tau=0}^{t} r_{i\tau t} = \sum_{k\in K} r_{itk}\,, \qquad \forall i \in N, t \in T \tag{2.7}$$
$$q_t \le c y_t\,, \qquad \forall t \in T \tag{2.8}$$
$$r_{itk} \le Q z_{itk}\,, \qquad \forall i \in N, t \in T, k \in K \tag{2.9}$$
$$\sum_{i\in N} r_{itk} \le Q z_{0tk}\,, \qquad \forall t \in T, k \in K \tag{2.10}$$
$$\sum_{\tau=0}^{t} I_{i\tau t} \le L_i\,, \qquad \forall i \in N, t \in T \tag{2.11}$$
$$\sum_{j\in N_0} x_{ijkt} = z_{itk}\,, \qquad \forall i \in N_0, t \in T, k \in K \tag{2.12}$$
$$\sum_{j\in N_0} x_{ijkt} = \sum_{j\in N_0} x_{jikt}\,, \qquad \forall i \in N_0, t \in T, k \in K \tag{2.13}$$
$$v_{itk} - v_{jtk} + Q x_{jikt} \le Q - r_{jtk}\,, \qquad \forall (i,j) \in A, i| \in N, t \in T, k \in K \tag{2.14}$$
$$q_t \ge 0, y_t \in \{0,1\}\,, \qquad \forall t \in T \tag{2.15}$$
$$I_{i\tau t} \ge 0\,, \qquad \forall i \in N_0, \tau \in T_0, t \in T \tag{2.16}$$
$$d_{i\tau t}, r_{i\tau t} \ge 0\,, \qquad \forall i \in N_0, \tau \in T_0, t \in T \tag{2.17}$$
$$r_{itk} \ge 0\,, \qquad \forall i \in N, \tau \in T_0, t \in T \tag{2.18}$$
$$z_{itk} \in \{0,1\}\,, \qquad \forall i \in N_0, t \in T, k \in K \tag{2.19}$$
$$x_{ijkt} \in \{0,1\}\,, \qquad \forall (i,j) \in A, t \in T, k \in K \tag{2.20}$$
$$r_{itk} \le v_{itk} \le Q\,, \qquad \forall i \in N, t \in T, k \in K \tag{2.21}$$

Where

T_0 the set of time, indexed by $\tau \in \{0,1,\dots,|T|\}$;

K the set of identical vehicles, indexed by $k \in \{1,\dots,|K|\}$;

f fixed set up cost;

u unit production cost;

α_{0t} deterioration rate in period t for products from initial inventory;

$\alpha_{\tau t}$ deterioration rate in period t for products manufactured in period τ;

h_{i0t} unit inventory holding cost at node i at the end of period t for products from initial inventory;

$h_{i\tau t}$ unit inventory holding cost at node i at the end of period t for product manufactured in period τ, $1 \le \tau \le |T|$;

c_{ij} transportation cost over arc (i,j) ;

d_{it} customer demand at retailer i in period t ;

Q vehicle capacity;

L_i maximum or target inventory level at node i ;

C production capacity;

d_{i0t} customer demand at retailer i $(i \in N)$ in period t $(t \in T)$ statisfied by initial inventory;

$d_{i\tau t}$ customer demand at retailer i $(i \in N)$ in period t $(t \in T)$ statisfied by product manufactured in period τ $(\tau \in T)$;

I_{i0t} inventory amount at node i $(i \in N_0)$ by the end of period t $(t \in T)$ remaining from initial inventory and let I_{i00} be the initial inventory of node i ;

$I_{i\tau t}$ inventory amount at node i $(i \in N_0)$ by the end period t $(t \in T)$ remaining from products manufactured in period τ $(\tau \in T)$;

$$P_{i0t} = \begin{cases} 1, & \text{If delivery amount of products from initial inventory to retailer } i\ (i \in N) \text{ in period } t\ (t \in T) \text{ is positive;} \\ 0, & \text{otherwise;} \end{cases}$$

$$P_{i\tau t} = \begin{cases} 1, & \text{If delivery amount of products in manufactured in } \tau\ (\tau \in T) \text{ to retailer } i\ (i \in N) \text{ in period } t\ (t \in T) \text{ is positive} \\ 0, & \text{otherwise;} \end{cases}$$

q_t amount of production in period$t$ $(t \in T)$;

r_{i0t} delivery amount of product from initial inventory to retailer i $(i \in N)$ in period t $(t \in T)$;

$r_{i\tau t}$ delivery amount of products manufactured in period τ $(\tau \in T)$ to retailer i $(i \in N)$ in period t $(t \in T)$;

r_{itk} delivery amount to retailer i in period t by vehicle k;

v_{itk} product amount carried by vehicle k before visiting retailer i in period t if retailer i is visited by vehicle k in period t, and 0 otherwise;

$$w_{i0t} = \begin{cases} 1, & \text{If customer demand at retailer } i \text{ in period } t \text{ is patially satisfied by initial al inventory;} \\ 0 & \text{otherwise;} \end{cases}$$

$$w_{i\tau t} = \begin{cases} 1 & \text{If customer demand at retailer } i \text{ in period } t \text{ partially satisfied by product manufactured in period } t; \\ 0, & \text{otherwise;} \end{cases}$$

$$x_{ijkt} = \begin{cases} 1, & \text{If arc } (i,j) \text{ traversed in period } t \text{ by vehicle } k; \\ 0, & \text{otherwise;} \end{cases}$$

$$y_t = \begin{cases} 1, & \text{If product is set up for production in period } t; \\ 0, & \text{otherwise;} \end{cases}$$

$$z_{itk} = \begin{cases} 1, & \text{If arc } i \text{ visiting in period } t \text{ by vehicle } k; \\ 0, & \text{otherwise;} \end{cases}$$

Some assumptions for the PRPPI models formulation.
1. The production begins in each period.
2. Inventory at each retailer in period t are defined as the total number of products until the end of period *t*, not including the amount used to satisfied demand at each retailer in period *t*. Inventory of the in period *t* defined as the total number of products until the end of period *t*, excluding shipments to all retailers in period *t*. Inventory in the previous period are assumed to deteriorate in the current period with a known level of decline.
3. $\alpha_{\tau_1 t} \geq \alpha_{\tau_2 t}$, $\forall t \in T, \tau_1 \in T_0, \tau_2 \in T_0, 0 \leq \tau_1 \leq \tau_2 \leq t$ (2.22)
4. $h_{i\tau_1 t} \geq h_{i\tau_2 t}, \forall i \in N_0, t \in T, \tau_1 \in T_0, \tau_2 \in T_0, 0 \leq \tau_1 \leq \tau_2 \leq t$ (2.23)
5. Backorder is not allowed.

To resctrict choice of products with different manufacturing periods for the delivery or sale, add additional variable and constraints to PRPPI models (2.1) - (2.21).

$$d_{i\tau t} \leq d_{it} w_{i\tau t} \qquad \forall i \in N, \tau \in T_0, t \in T \tag{2.24}$$
$$r_{i\tau t} \leq Q p_{i\tau t} \qquad \forall i \in N, \tau \in T_0, t \in T \tag{2.25}$$
$$w_{i\tau t} \in \{0,1\} \qquad \forall i \in N, \tau \in T_0, t \in T \tag{2.26}$$
$$p_{i\tau t} \in \{0,1\} \qquad \forall i \in N, \tau \in T_0, t \in T \tag{2.27}$$

PRPPI formulation with management policy FPFD can be modelled by adding constraints

$$\sum_{\tau=0}^{\eta}\left(1-\alpha_{\tau,t-1}\right) I_{0t,t-1} - \sum_{i \in N}\sum_{\tau=0}^{\eta} r_{i\tau t} \leq L_0(1-p_{i,n+1,t}), \ \forall i \in N, t \in T, 0 \leq \eta \leq t-1 \tag{2.28}$$

Into formula (2.1) - (2.21), (2.25) and (2.27).

PRPPI models with management policy FPFS can be modelled

$$\sum_{\tau=0}^{\eta}\left(1-\alpha_{\tau,t-1}\right) I_{i\tau,t-1} + \sum_{\tau=0}^{\eta} r_{i\tau t} - d_{it} + 1 \leq L_i(1-w_{i,n+1,t}) \qquad \forall i \in N, t \in T, 0 \leq \eta \leq t-1 \tag{2.29}$$

Into formula (2.1) - (2.21), (2.24) dan (2.26).

3. Implementation PRPPI Models in Tempe Production

Mrs. Ana tempe production is an industry that is lacaly in tempe industry on Tanjung Sari I Street Bukit Sangkal Palembang South Sumatera. She produces tempe every day and start at 1-5 pm. The ready tempe sell distributed by bike which can hold to 100 pieces of tempe. The tempe is sent to retailer in the market such as Perumnas, Sekip Ujung and Kebon Semai every morning at 6 am. There are three kinds of tempe packaging. They are leafed tempe, pieces plastic tempe, and bar plastic tempe. In this paper discuss the problem of the production and the distribution by using PRPPI model for one kind of tempe, that is leafed tempe.

3.1. Data Description

The following is given data amount of tempe production, cost production, maximal production capacity, etc.

Table 1. The Amount of Tempe Production

No.	Day	Leafed Tempe	Piece Plastic Tempe	Bar Plastic Tempe
1.	Monday	52	75	52
2.	Tuesday	39	82	52
3.	Wednesday	52	67	52

Source : Ana's Industry, Juli 2018. The amount of Tempe Production at Ana's Home Industry for three days.

In producing the tempe, Mrs Ana spend additional cost such as salary, water, fuel and the wood. In this paper modelled PRPPI use production data on Monday. Additional cost data production on Monday given in Table 2.

Table 2. The Additional Cost Data Production Leafed Tempe

No	**Category**	**Additional Cost**
1	Salary	Rp 10.000,-
2	Water	Rp 9.000,-
3	Wood	Rp 8.500
4	Fuel	Rp 5.500,-
Total		Rp 33.000,-
The Additional Cost of Leafed Tempe for piece		Rp 634,-

The additional cost production is Rp 33.000 used for producing 52 piece of leafed tempe. Additional cost production leafed tempe for piece get from $\frac{33.000}{52} = 634$. The calculation of production cost for leafed tempe is given in Table 3.

Table 3. The Cost Production of Leafed Tempe

No.	Ingredients	Cost (Rupiah)	The amount	Production cost (Rupiah)
1.	Soy Been	7600/kg	14,5 kg	Rp 110.200,-
2.	Fermentation Material	5000/100gr	25 gram	Rp 1.250,-
3.	Leaf	5000/pack	5 pack	Rp 25.000,-
The Cost Production of Leafed Tempe				Rp 136.450,-
The Cost Production of Leafed Tempe/piece				Rp 2.624,-
The additional cost of Leafed Tempe/piece				Rp 634,-

The calculation of cost production obtained from the sum of price ingredients that have been used in producing leafed tempe. The cost production for piece obtained from divide the cost production in Table 1. with amount of production leafed tempe, $(7600 \times 14{,}5) + \left(\frac{5000}{100} \times 25\right) + (5000 \times 5) = 136.450$ for 52 piece, so cost production for piece $\frac{136.450}{52} = 2.624$. The capacity data of leafed tempe production that produced can be seen on Table 4.

Table 4. The Capacity Data of Leafed Tempe

No.	Day	The Amount Of Tempe
1.	Monday	60
2.	Tuesday	45
3.	Wednesday	60

Source : Ana's Tempe Industri, Juli 2018. The amount of Leafed tempe produced for three days.

Data on the number of leaf tempe products sent to each retailer can be seen in table 5.

Table 5. Number of Leafed Tempe Products Sent to Retailer

		The Amount of Tempe to Retailer -		
No.	Day	Perumnas	Sekip Ujung	Kebon Semai
1.	Monday	17	17	18
2.	Tuesday	15	12	12
3.	Wednesday	20	17	15

Source : Ana's Tempe Industry, Juli 2018. The amount of Tempe sent to three retailers, namely Perumnas, Sekip Ujung and Kebon Semai.

Data on leafed tempe left at retailers on Monday, Tuesday and Wednesday are outlined in Table 6

Table 6. Data on leafed tempe left at retailers

No.	**Day**	**The Amount of Leafed Tempe-**		
		Perumnas	Sekip Ujung	Kebon Semai
1.	Monday	4	3	1
2.	Tuesday	2	4	2
3.	Wednesday	3	2	0

Source : Ana's Tempe Industry, Juli 2018. The amount of Leafed Tempe sent to each market.

Maximum inventory of leafed tempe at retailers in Perumnas market, Sekip Ujung market and Kebon Semai market can be seen in Table 7 below.

Table 7. Maximum Inventory of Leaf Tempe at Retailers

No.	**Retailer**	**Persediaan tempe Maks**
1.	Perumnas Market	35
2.	Sekip Ujung Market	30
3.	Kebon Semai Market	35

Source : Ana's Tempe Industry, Juli 2018. Maximum Capacity at each retailer.

The route used by Ana in distributing leafed tempe to retailers in Perumnas market, Sekip Ujung market and Kebon Semai market can be seen in the Figure 1.

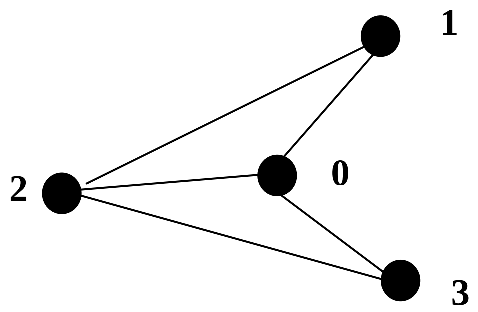

Figure 1. Distribution Routes

Figure 1 describes the route for each retailer in Perumnas market, Sekip Ujung market and Kebon Semai market in the distribution of leafed tempe. Ana's distribution route can be described in the form of a graph which can be seen in Figure 1. In the picture there is node 0 is the depot, node 1 is Perumnas market, node 2 is Sekip Ujung market and node 3 is Kebon Semai market. Tour in the distribution is Depot - Perumnas Market – Sekip Ujung Market - Kebon Semai Market - return Depot. The PRPPI model will make a tour built from several subtour. Subtour used is $\{(0,1),\ (1,2),\ (2,3),\ (3,0),\ (0,1),\ (1,3),\ (3,2),\ (2,0),\ (0,2),\ (2,1),\ (1,3),(3,0),\ (0,2),\ (2,3),\ (3,1),\ (1,0),\ (0,3),\ (3,1),\ (1,2),\ (2,0),\ (0,3),(3,2),(2,1),(1,0)\}$.

Data on transportation costs issued by Ana in distributing leafed tempe to retailers in Perumnas market, Sekip Ujung market and Kebon Semai market can be seen in Table 8 below.

Table 8. Transportation Costs

No.	Route	Cost
1.	Depot – Perumnas Market	Rp 2000
2.	Perumnas Market - Sekip Ujung Market	Rp 3000
3.	Sekip Ujung Market - Kebon Semai Market	Rp 2000
4.	Depot - Kebon Semai Market	Rp 3000
5.	Kebon Semai Market – Perumnas Market	Rp 5000
6.	Depot - Sekip Ujung Market	Rp 3000

Source : Ana’s Tempe Industry, Juli 2018. Distribution cost of tempe for each route

3.2. Parameter and variabel

The Parameter and variable used on PRPPI models given on table 9.

Tabel 9. Parameter and Variable

No.	Parameter	Index
1.	T_0	with $\tau = 0, 1$
2.	K	with $k = 1$
3.	f	
4.	u	
5.	α_{0t}	with $t = 1$
6.	$\alpha_{\tau t}$	with $\tau = 0, 1$, $t = 1$
7.	h_{i0t}	with $i = 1,2,3$ $t = 1$
8.	$h_{i\tau t}$	with $i = 1,2,3$ $\tau = 0, 1$ $t = 1$
9.	c_{ij}	with $i = 0,1,2,3$ $j = 0,1,2,3$
10.	d_{it}	with $i = 1,2,3$ $t = 1$

11.	Q	
12.	L_i	with $i = 1,2,3$
13.	C	
	Variable	Index
1.	$d_{i\tau t}$	with $i = 1,2,3$ $\tau = 0,1$ $t = 1$
2.	$I_{i\tau t}$	with $i = 1,2,3$ $\tau = 0,1$ $t = 1$
3.	$P_{i\tau t}$	with $i = 1,2,3$ $\tau = 0,1$ $t = 1$
4.	q_t	with $t = 1$
5.	$r_{i\tau t}$	with $i = 1,2,3$ $\tau = 0,1$ $t = 1$
6.	v_{itk}	with $i = 0,1,2,3$ $k = 1$ $t = 1$
7.	$w_{i\tau t}$	with $i = 1,2,3$ $\tau = 0,1$ $t = 1$
8.	x_{ijkt}	with $i = 0,1,2,3$ $j = 0,1,2,3$ $k = 1$ $t = 1$
9.	y_t	with $t = 1$
10.	z_{itk}	with $i = 0,1,2,3$ $k = 1$ $t = 1$

3.3. Formulatioan PRPPI Models For Optimal Production, Distributian and Inventory at Ana's Tempe Production

Min $A = 2624q_1 + 634y_1 + (2000x_{0111} + 3000x_{1211} + 2000x_{2311} + 3000x_{3011} + 2000x_{0111} + 5000x_{1311} + 2000x_{3211} + 3000x_{2011} + 3000x_{0211} + 3000x_{2111} + 5000x_{1311} + 3000x_{3011} + 3000x_{0211} + 2000x_{2311} + 5000x_{3111} + 2000x_{1011} + 3000x_{0311} + 5000x_{3111} + 3000x_{1211} + 3000x_{2011} + 3000x_{0311} + 2000x_{3211} + 3000x_{2111} + 2000x_{1011}));$

Subject to (3.1)

$q_1 - (r_{111} + r_{211} + r_{311}) = I_{011}$

$(1-1)\quad I_{1010} - (r_{101} + r_{201} + r_{301}) = I_{001}$

$r_{111} - d_{111} = I_{111},\ \ r_{211} - d_{211} = I_{211},\ \ r_{311} - d_{311} = I_{311}$

$(1-1)I_{100} + r_{101} - d_{101} = I_{101}$

$(1-1)I_{200} + r_{201} - d_{201} = I_{201}$

$(1-1)I_{300} + r_{301} - d_{301} = I_{301}$

$d_{101} + d_{111} = 11,\ \ d_{201} + d_{211} = 12,\ \ d_{301} + d_{311} = 14$

$r_{101} + r_{111} = r_{111},\ \ r_{101} = 0,\ \ r_{201} + r_{211} = r_{211},\ \ r_{201} = 0,\ \ r_{301} + r_{311} = r_{311},\ \ r_{301} = 0$

$q_1 \leq 60y_1$

$r_{111} \leq 100z_{111},\ \ r_{211} \leq 100z_{211},\ \ r_{311} \leq 100z_{311}$

$r_{111} + r_{211} + r_{311} \leq 100z_{011}$

$I_{101} + I_{111} \leq 35,\ \ I_{201} + I_{211} \leq 30,\ \ I_{301} + I_{311} \leq 35$

$x_{1011} + x_{1211} + x_{1311} = z_{111},\ \ x_{2011} + x_{2111} + x_{2311} = z_{211},\ \ x_{3011} + x_{3111} + x_{3211} = z_{311}$

$x_{1011} + x_{1211} + x_{1311} = x_{1011} + x_{2111} + x_{3111}$

$x_{2011} + x_{2111} + x_{2311} = x_{0211} + x_{1211} + x_{3211}$

$x_{3011} + x_{3111} + x_{3211} = x_{0311} + x_{1311} + x_{2311}$

$v_{011} - v_{111} + 100x_{0111} \leq 100 - r_{111},$

$v_{111} - v_{211} + 100x_{1211} \leq 100 - r_{211}$

$v_{211} - v_{311} + 100x_{2311} \leq 100 - r_{311}$

$v_{011} - v_{111} + 100x_{0111} \leq 100 - r_{111}$

$v_{111} - v_{311} + 100x_{1311} \leq 100 - r_{311}$

$v_{311} - v_{211} + 100x_{3211} \leq 100 - r_{211}$
$v_{011} - v_{211} + 100x_{0211} \leq 100 - r_{211}$
$v_{211} - v_{111} + 100x_{2111} \leq 100 - r_{111}$
$v_{111} - v_{311} + 100x_{1311} \leq 100 - r_{311}$
$v_{011} - v_{211} + 100x_{0211} \leq 100 - r_{211}$
$v_{211} - v_{311} + 100x_{2311} \leq 100 - r_{311}$
$v_{311} - v_{111} + 100x_{3111} \leq 100 - r_{111}$
$v_{011} - v_{311} + 100x_{0311} \leq 100 - r_{311}$
$v_{311} - v_{111} + 100x_{3111} \leq 100 - r_{111}$
$v_{111} - v_{211} + 100x_{1211} \leq 100 - r_{211}$
$v_{011} - v_{311} + 100x_{0311} \leq 100 - r_{311}$
$v_{311} - v_{211} + 100x_{3211} \leq 100 - r_{211}$
$v_{211} - v_{111} + 100x_{2111} \leq 100 - r_{111}$
$q_1 \geq 0 \;\;,\;\; y_1 \in \{0,1\}$
$I_{001}, I_{011}, I_{101}, I_{111}, I_{201}, I_{211}, I_{301}, I_{311} \geq 0$
$d_{101}, r_{101} \geq 0,\;\; d_{111}, r_{111} \geq 0,\;\; d_{201}, r_{201} \geq 0$
$d_{211}, r_{211} \geq 0,\;\; d_{301}, r_{301} \geq 0,\;\; d_{311}, r_{311} \geq 0$
$r_{111}, r_{211}, r_{311} \geq 0,\;\; z_{011}, z_{111}, z_{211}, z_{311} \in \{0,1\}$
$x_{0111}, x_{1211}, x_{2311}, x_{3011}, x_{0111}, x_{1311}, x_{3211}, x_{2011}, x_{0211}, x_{2111}, x_{1311}, x_{3011}, x_{0211},$
$x_{2311}, x_{3111}, x_{1011}, x_{0311}, x_{3111}, x_{1211}, x_{2011}, x_{0311}, x_{3211}, x_{2111}, x_{1011} \in \{0,1\}$
$r_{111} \leq v_{111} \leq 100,\;\; r_{211} \leq v_{211} \leq 100,\;\; r_{311} \leq v_{311} \leq 100$
$d_{101} \leq 2w_{101},\;\; d_{201} \leq 2w_{201},\;\; d_{301} \leq 2w_{301}$
$d_{111} \leq 11w_{111},\;\; d_{211} \leq 12w_{211},\;\; d_{311} \leq 14w_{311}$
$w_{101}, w_{201}, w_{301}, w_{111}, w_{211}, w_{311} \in \{0,1\}$
$r_{101} \leq 100P_{101},\;\; r_{201} \leq 100P_{201},\;\; r_{301} \leq 100P_{301}$
$r_{111} \leq 100P_{111},\;\; r_{211} \leq 100P_{211},\;\; r_{311} \leq 100P_{311}$
$P_{101}, P_{201}, P_{301}, P_{111}, P_{211}, P_{311} \in \{0,1\}$
$(1-1)I_{000} - (r_{101} + r_{201} + r_{301}) \leq 10(1 - P_{101})$
$(1-1)I_{000} - (r_{101} + r_{201} + r_{301}) \leq 10(1 - P_{201})$
$(1-1)I_{000} - (r_{101} + r_{201} + r_{301}) \leq 10(1 - P_{301})$
$(1-1)I_{100} - (r_{101} + r_{111}) - 11 + 1 \leq 35(1 - w_{101})$
$(1-1)I_{100} - (r_{201} + r_{211}) - 12 + 1 \leq 30(1 - w_{201})$
$(1-1)I_{100} - (r_{301} + r_{311}) - 14 + 1 \leq 35(1 - w_{301})$

3.4. Solution of The Problem (3.1) using Branch and Bound method. Completion Branch and Bound method used Lingo 17 sofware, obtaining

A = 117.722 , $q_1 = 37$, y_1 = 1, x_{0111} = 0 , x_{1211} = 0, x_{2311} = 0 , x_{1311} = 0, x_{3211} = 1, x_{0211} = 0, x_{2111} = 1, x_{3111} = 0, x_{0311} = 1, x_{1011} = 1, x_{2011} = 0, x_{3011} = 0, v_{011} = 0, v_{111} = 37, v_{211} = 26, v_{311} = 14, r_{111} = 11, r_{211} = 12, r_{311} = 14, r_{101} = 0, r_{201} = 0, r_{301} = 0, I_{011} = 0, I_{010} = 0, I_{001} = 0, I_{111} = 0, I_{211} = 0, I_{311} = 0, I_{100} = 0, I_{101} = 0, I_{200} = 0, I_{201} = 0, I_{300} = 0, I_{301} = 0, I_{000} = 0, d_{111} = 11, d_{211} = 12, d_{311} = 14, d_{101} = 0, d_{201} = 0, d_{301} = 0, z_{011} = 1, z_{111} = 1, z_{211} = 1, z_{311} = 1, w_{101} = 0, w_{201} = 0, w_{301} = 0, w_{111} = 1, w_{211} = 1, w_{311} = 1, P_{101} = 0, P_{201} = 0, P_{301} = 0, P_{111} = 1, P_{211} = 1, P_{311} = 1.

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Based on the solution of the PRPPI model using the Lingo 17, it can be seen that the cost of producing leafed tempe for the PRPPI model is Rp. 117,722. The production cost that Ana spent without using PRPPI was Rp. 169,418. The optimal route formed for the distribution of industrial leafed tempe products for Ana is Depot → Kebon Semai market, Sekip Ujung market → Perumnas market → Depot, the number of productions is 37.

4. Conclusion

Based on the results and discussion, conclusions are obtained as follows.

1. By using the PRPPI model in the production of leafed tempe, Mrs Ana can spend cheaper production costs. The difference in production costs of leafed tempe using the PRPPI model with the production cost that Mrs. Ana spent was Rp. 57,696.
2. The optimal route using the PRPPI model is the route that distributes tempe to retailers that has the highest number of tempe shipments first.

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