Tarea 5

Se entrega el Miércoles 1 de Octubre

1. A particle moves in one dimension, in a potential $V(x)$, where x is the spatial coordinate. The dynamics is governed by the Lagrangian

$$L = \frac{1}{12}m^2\dot{x}^4 + m\dot{x}^2V(x) - V(x)^2, \tag{1}$$

   Show that the resulting equation of motion is identical to that which arises from the more traditional Lagrangian, $L = \frac{1}{2}m\dot{x}^2 - V(x)$.

2. A bead is constrained to move without friction on a helix whose equation in cylindrical polar coordinates is $\rho = b$, $z = a\phi$, with the potential $V = \frac{1}{2}k(\rho^2 + z^2)$. Use the Lagrange multiplier method to find the constraint forces.

3. Use the Lagrange multiplier method to solve the following problem: A particle in a uniform gravitational field is free to move without friction on a paraboloid of revolution whose symmetry axis is vertical (opening upward). Obtain the force of constraint. Prove that for a given energy and angular momentum about the symmetry axis there are a minimum and maximum height to which the particle will go.