Tarea 4

Se entrega el Miércoles 24 de Septiembre

1. If L is a Lagrangian for a system of n degrees of freedom satisfying Euler-Lagrange's equations, show that

$$L' = L + \frac{dF(q_1, \ldots, q_n, t)}{dt} \tag{1}$$

also satisfies Euler-Lagrange's equations where F is any arbitrary, but differentiable, function of its arguments.

2. Write down the Lagrangian for a projectile (subject to no air resistance), in terms of its Cartesian coordinates (x, y, z), with z measure vertically upward. Find the three Eurler-Lagrange equations and show that they are exactly what you would expect for the equations of motion.

3. The Lagrangian for a relativistic point particle, of mass m, is

$$L = -mc^2\sqrt{1 - (\dot{\mathbf{r}} \cdot \dot{\mathbf{r}})/c^2} - U(\mathbf{r}) \tag{2}$$

where c is the speed of light. Derive the equation of motion, and show that it reduces to Newton's equation of motion in the limit $|\dot{\mathbf{r}}| \lll c$.

4. A bead slides along a smooth wire that has the shape of a parabola $z = cr^2$. At equilibrium, the bead rotates in a circle of radius R when the wire is rotating about its vertical symmetry axis with angular velocity ω . Find the value of c .