# Tarea 1

**Entrega**: 22 de agosto de 2022

## Problema 1

Resolver los siguientes ejercicios de *Introducción al formalismo de la Mecánica Cuántica no relativista* (Spinel):

(a) Con base en la propiedad (1.1.3), demostrar que el dual de $c|\beta\rangle = \langle\beta|c^*$.

Sean $|\gamma\rangle$ y $|\eta\rangle$ los ket definidos por: $|\gamma\rangle = (3+i)|a_1\rangle + 4|a_2\rangle - 6i|a_3\rangle$ y $|\eta\rangle = 2i|a_1\rangle + 3|a_3\rangle$, donde los kets $|a_i\rangle$ son ortonormales

(b) Calcule la norma de los kets $|\gamma\rangle$ y $|\eta\rangle$ y determine sus kets normalizados $|\gamma'\rangle$ y $|\eta'\rangle$.

(c) Encuentre los bras correspondientes a los kets $|\gamma'\rangle$ y $|\eta'\rangle$.

(d) Calcule el producto interior $\langle\gamma', \eta'\rangle$ y demuestre por cálculo directo que es igual a $\{\langle\eta', \gamma'\rangle\}^*$.

(e) Calcules los productos interiores $\langle a_1, \eta'\rangle$, $\langle a_2, \eta'\rangle$ y $\langle a_3, \eta'\rangle$. De acuerdo con sus resultados ¿qué interpretación geométrica puede dar al producto interior?

## Problema 2

(a) Find the condition under which two vectors

$$|v_1\rangle = \begin{pmatrix} x \\ y \\ 3 \end{pmatrix}, \quad |v_2\rangle = \begin{pmatrix} 2 \\ x-y \\ 1 \end{pmatrix} \in \mathbb{R}^3$$

are linearly independent.

(b) Show that a set of vectors

$$|v_1\rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad |v_2\rangle = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad |v_3\rangle = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

is a basis of $\mathbb{C}^3$.

(c) Let

$$|x\rangle = \begin{pmatrix} 1 \\ i \\ 2+i \end{pmatrix}, \quad |y\rangle = \begin{pmatrix} 2-i \\ 1 \\ 2+i \end{pmatrix} \Gamma$$

Find $\||x\rangle\|$, $\langle x, y\rangle$ and $\langle y, x\rangle$.

(d) 1. Use the Gram-Schmidt orthonormalization to find an orthonormal basis $\{|e_k\rangle\}$ from a linearly independent set of vectors

$$|v_1\rangle = \begin{pmatrix} -1, & 2, & 2 \end{pmatrix}^t, \quad |v_2\rangle = \begin{pmatrix} 2, & -1, & 2 \end{pmatrix}^t, \quad |v_3\rangle = \begin{pmatrix} 3, & 0, & -3 \end{pmatrix}^t$$

2. Let

$$|u\rangle = \begin{pmatrix} 1, & -2, & 7 \end{pmatrix}^t = \sum_k c_k |e_k\rangle$$

Find the coefficients $c_k$.

3. Let

$$|v_1\rangle = \begin{pmatrix} 1, & i, & 1 \end{pmatrix}^t, \quad |v_2\rangle = \begin{pmatrix} 3, & 1, & i \end{pmatrix}^t \Gamma$$

Find the orthonormal basis for a two-dimensional subspace spanned by $\{|v_1\rangle, |v_2\rangle\}$.

# Problema 3

(a) Let $x \neq 0$ and $y \neq 0$. (a) If $x \perp y$, show that $\{x, y\}$ is a linearly independent set. (b) Extend the result to mutually orthogonal nonzero vectors $x_1, \ldots, x_m$.

(b) Let $z_1$ and $z_2$ denote complex numbers. Show that $\langle z_1, z_2\rangle = z_1 \bar{z}_2$ defines an inner product, which yields the usual metric on the complex plane. Under what condition do we have orthogonality?

(c) Show that the norm on $C[a, b]$ is invariant under a linear transformation $t = \alpha\tau + \beta$. Use this to prove that the statement in 3.1-8 by mapping $[a, b]$ onto $[0, 1]$ and then considering the functions defined by $\bar{x}(\tau) = 1,\ \bar{y}(\tau) = \tau$, where $\tau \in [0, 1]$.

(d) If $X$ is a finite dimensional vector space and $(e_j)$ is a basis for $X$, show that an inner product on $X$ is completely determined by its values $\gamma_{jk} = \langle e_j, e_k\rangle$. Can we choose such scalar $\gamma_{jk}$ in a completely arbitrary fashion?

(e) If $(e_k)$ is an orthonormal sequence in an inner product space $X$, and $x \in X$, show that $x - y$ with $y$ given by

$$y = \sum_{k=1}^{n} \alpha_k e_k, \quad \alpha_k = \langle x, e_k\rangle$$

is orthogonal to the subspace $Y_n = \operatorname{span}\{e_1, \ldots, \mathrm{e}_n\}$.

(f) Orthonormalize the first three terms of the sequence $(x_0, x_1, x_2, \cdots)$, where $x_j(t) = t^j$, on the interval $[-1, 1]$, where

$$\langle x, y\rangle = \int_{-1}^{1} x(t)y(t)\,\mathrm{d}t\mathit{\Gamma}$$