Cosmic ray muon sampling with CRY

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The Cosmic-ray Shower Library (CRY) is a software library to generate shower distributions of cosmic-ray particles. In this report, the principles of muon sampling with CRY are described briefly, followed by the introduction of different parametrizations for the muon energy flux spectrum at sea level. Finally, differential muon fluxes obtained with CRY are compared to these parametrizations and to data from three experiments (corresponding to different zenith angles), to evaluate their agreement.

The report was prepared following a lecture on "Neutrons and Other Backgrounds in Underground Laboratories" given by Dr. Vitaly Kudryavtsev at the Gran Sasso Science Institute in November 2013.

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1 Shower sampling with CRY

1.1 Introduction

The *Cosmic-ray Show Library* (CRY) is a software library to generate “correlated cosmic-ray particle shower distributions at one of three elevations (sea level, 2100 m, and 11300 m) for use as input to transport and detector simulation code” [1].

CRY and its documentations [1, 2] are available on `http://nuclear.llnl.gov/simulation/main.html`.

1.2 Function and parameters

CRY uses precomputed data tables to sample the particles and their properties in a shower for the given parameters: elevation, latitude and date.

Only protons are considered as primary particles. The primary proton spectrum is divided into 33 bins between 1 GeV and 10^5 GeV. The influence of the solar activity is taken into account by using the given date for the simulation to interpolate between the spectral shapes for the solar minimum and maximum, assuming an 11-year solar cycle (cf. [2]). The latitude φ determines a lower cut-off for the energy of the primary protons, given by $14.8\,\mathrm{GeV} \cdot \cos^4 \varphi$.

The numbers of each type of secondary particles are sampled from distributions dependent on the primary bin. The same is then done for the kinetic energy and direction of each particle, where the distributions from which is sampled depend on the particle type and primary bin. For this report only muons are considered.

The distribution of the kinetic energy of the secondary particles is divided into 85 bins between 10^{-9} MeV and 10^8 MeV. The $\cos\theta$ distribution of the zenith angle has 40 bins between -1 and 1. When sampling from the distributions, the distribution of the sampled values within a bin is flat for $\cos\theta$ of the secondary particles, and flat for $\log E$ in case of the (primary and secondary) energies.

The influence of solar activity and geomagnetic cutoff (determined by date and latitude) are only significant in the low-energy part of the primary proton spectrum [2], and subsequently only affect the low-energy part of the cosmic ray muon spectrum at sea level, as can be seen in Figure 1.

2 Gaisser parametrisation and modifications

2.1 Original Gaisser parametrization

The cosmic ray muon spectrum at sea level can be approximated by a parametrization from Gaisser [3]:

$$\frac{\mathrm{d}N}{\mathrm{d}E\,\mathrm{d}\Omega\,\mathrm{d}t\,\mathrm{d}A} = \frac{A \cdot (E_\mu/\mathrm{GeV})^{-\gamma}}{\mathrm{cm}^2\,\mathrm{sr}\,\mathrm{s}\,\mathrm{GeV}} \cdot \left(\frac{1}{1 + \frac{1.1 \cdot \cos\theta \cdot E_\mu}{115.0\,\mathrm{GeV}}} + \frac{0.054}{1 + \frac{1.1 \cdot \cos\theta \cdot E_\mu}{850.0\,\mathrm{GeV}}} \right), \tag{1}$$

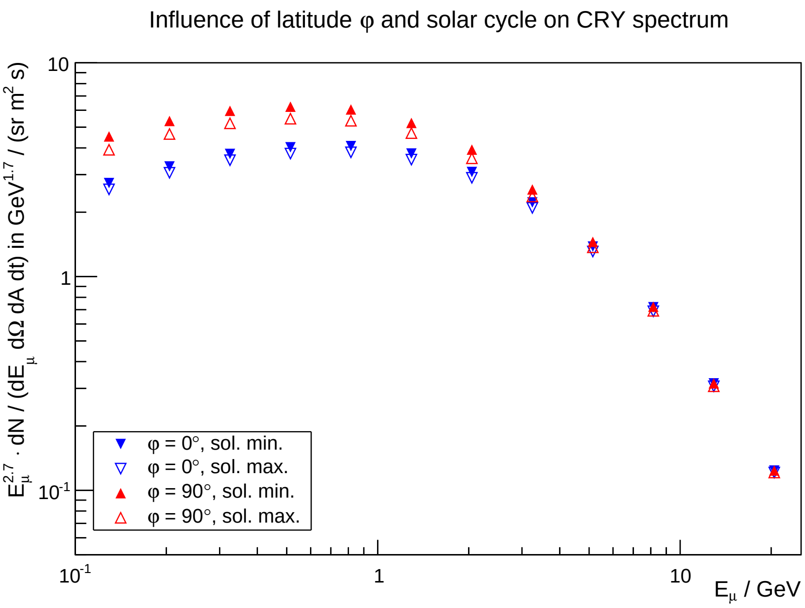


Figure 1: Influence of cut-off parameters, latitude φ and solar cycle, on the differential muon energy spectrum (averaged over the full solid angle) in CRY. As dates for the solar minimum/maximum, January 1 2008 and July 1 2013 were chosen.

with $A = 0.14$ and $\gamma = 2.7$. θ is the angle between the muon direction and the surface normal (i. e. $\cos\theta = 1$ for vertical muons), and E_μ is the muon energy on surface.

The Gaisser parametrization is applicable when muon decay and energy loss is negligible ($E_\mu > 100\,\mathrm{GeV}/\cos\theta$) and the curvature of the Earth can be neglected ($\theta < 70°$) [4].

2.2 Modified parametrizations

Outside the applicable parameter range, i. e. at low muon energies and large zenith angles, the fluxes given by the Gaisser parameterization are too large.

Different modifications to (1) have been proposed. They include (partially empiric) modifications to the expressions $\cos\theta$ (to account for the curvature of the Earth), A and E_μ (to account for muon energy loss and decay) or adding another term r_c (to account for prompt muons). The following expression will be used to describe the modified parametrizations:

$$\frac{\mathrm{d}N}{\mathrm{d}E\,\mathrm{d}\Omega\,\mathrm{d}t\,\mathrm{d}A} = \frac{A\cdot(E_{\mu,1}/\mathrm{GeV})^{-\gamma}}{\mathrm{cm}^2\,\mathrm{sr}\,\mathrm{s}\,\mathrm{GeV}}\cdot\left(\frac{1}{1+\frac{1.1\cdot\cos\theta^*\cdot E_{\mu,2}}{115.0\,\mathrm{GeV}}}+\frac{0.054}{1+\frac{1.1\cdot\cos\theta^*\cdot E_{\mu,2}}{850.0\,\mathrm{GeV}}}+r_c\right), \qquad (2)$$

For $A = 0.14$, $\gamma = 2.7$, $E_{\mu,1} = E_{\mu,2} = E_\mu$ and $r_c = 0$, this yields the standard Gaisser parametrization. Three parametrizations which modify some or all of the parameters will be considered for this report and are briefly described in the following sections in order of the cited publications.

A: Tang et. al. [5] present a parametrization similar to [6]. The modified zenith angle is calculated as given by [7]:

$$\cos\theta^* = \sqrt{\frac{(\cos\theta)^2 + P_1^2 + P_2 \cdot (\cos\theta)^{P_3} + P_4 \cdot (\cos\theta)^{P_5}}{1 + P_1^2 + P_2 + P_4}}, \tag{3}$$

where the parameters are $P_1 = 0.102573$, $P_2 = -0.068287$, $P_3 = 0.958633$, $P_4 = 0.0407253$ and $P_5 = 0.817285$.

For $E_\mu > 100\,\mathrm{GeV} \cdot \sec\theta$, the standard Gaisser parametrization (1) is used, only replacing $\cos\theta$ with $\cos\theta^*$. Otherwise, i. e. for $E_\mu < 100\,\mathrm{GeV} \cdot \sec\theta$, the following further modifications apply, using the notation of equation (2):

$$r_c = 10^{-4} \tag{4}$$

$$\Delta = 2.06 \cdot 10^{-3}\,\mathrm{GeV} \cdot (950 \cdot \sec\theta^* - 90) \tag{5}$$

$$E_{\mu,1} = \begin{cases} \frac{3 \cdot E_\mu + 7\,\mathrm{GeV} \cdot \sec\theta^*}{10} & (E_\mu/\mathrm{GeV}) < \sec\theta \\ E_\mu & (E_\mu/\mathrm{GeV}) \geq \sec\theta \end{cases} \tag{6}$$

$$E_{\mu,2} = E_{\mu,1} + \Delta \tag{7}$$

$$A = 0.14 \cdot 1.1 \cdot \left(\frac{9 \cdot \sqrt{\cos\theta + 0.001}}{103}\right)^{4.5\,\mathrm{GeV} \cdot (E_{\mu,2})^{-1} \cdot \sec\theta^*} \tag{8}$$

This parametrization is therefore a piecewise function with three pieces.

B: Mengyun et. al. [8] also use equation (3) to determine $\cos\theta^*$. Furthermore for this parametrization $A = 0.14$, $\gamma = 2.7$, $r_c = 0$, and:

$$E_{\mu,1} = E_\mu \cdot \left(1 + \frac{3.64\,\mathrm{GeV}}{E_\mu \cdot (\cos\theta^*)^{1.29}}\right) \tag{9}$$

$$E_{\mu,2} = E_\mu \tag{10}$$

C: Kudryavtsev [9] sets $\gamma = 2.7$, $r_c = 10^{-4}$ and:

$$\cos\theta^* = \sqrt{1 - \frac{1 - \cos^2\theta}{\left(1 + \frac{32}{6370}\right)^2}}$$

$$\Delta E = 2.06 \cdot 10^{-3}\,\mathrm{GeV} \cdot \left(\frac{1030}{\cos\theta^*} - 120\right) \tag{11}$$

$$A = 0.14 \cdot \left(\frac{120 \cdot \cos\theta^*}{1030} \right)^{1.04\,\mathrm{GeV} \cdot \left(E_\mu + \frac{\Delta E}{2}\right)^{-1} \cdot \sec\theta^*} \tag{12}$$

$$E_{\mu,1} = E_{\mu,2} = E_\mu + \Delta E \tag{13}$$

2.3 Comparison of parametrizations

In Figures 2-4, the parametrizations are shown for different zenith angles θ. From the three cases, the original Gaisser parametrization is only applicable for the first with $\theta = 0°$, shown in Fig. 2. The effect of the modifications to the parametrization can be seen clearly for muon energies below 100 GeV (i. e. outside the applicable range for the original parametrization).

Differences between the individual modified parametrizations occur in particular for large zenith angles and low muon energies. For small angles and energies above 1 GeV they agree within about 20%. Outside this range they can differ much more.

3 Comparison between CRY results, parametrizations and experimental data

3.1 Methods

Starting point for the comparisons are the experimental datasets. CRY results and parametrizations have to be adjusted in order to be compared to this data.

The parameters of CRY (date and latitude) were set according to the experiment to which its results were to be compared to. If no date was given for the measurements in the cited publications, it was estimated from the publication date. Only setups at sea level were simulated and investigated.

An experimental setup is sensitive to muons in a certain range of $\cos\theta$. The flux parametrization is averaged over these values. For CRY, the data is binned in $\cos\theta$, so that the average over the bins covered by the experimental range of $\cos\theta$ was used. The contribution of partially covered bins was determined with the help of one spectrum parametrization (C), using the fraction of the flux in the covered interval and the whole bin.

CRY returns the particles for a given horizontal area. Hence the effective area for a given zenith angle scales with $\cos\theta$, which has to be corrected for the comparison. The parametrization C was used to calculate the average of $\cos\theta$ weighted by the flux according to the parametrization.

The CRY pdfs are used directly instead of using distributions of sampled muons, to avoid additional statistical fluctuations.

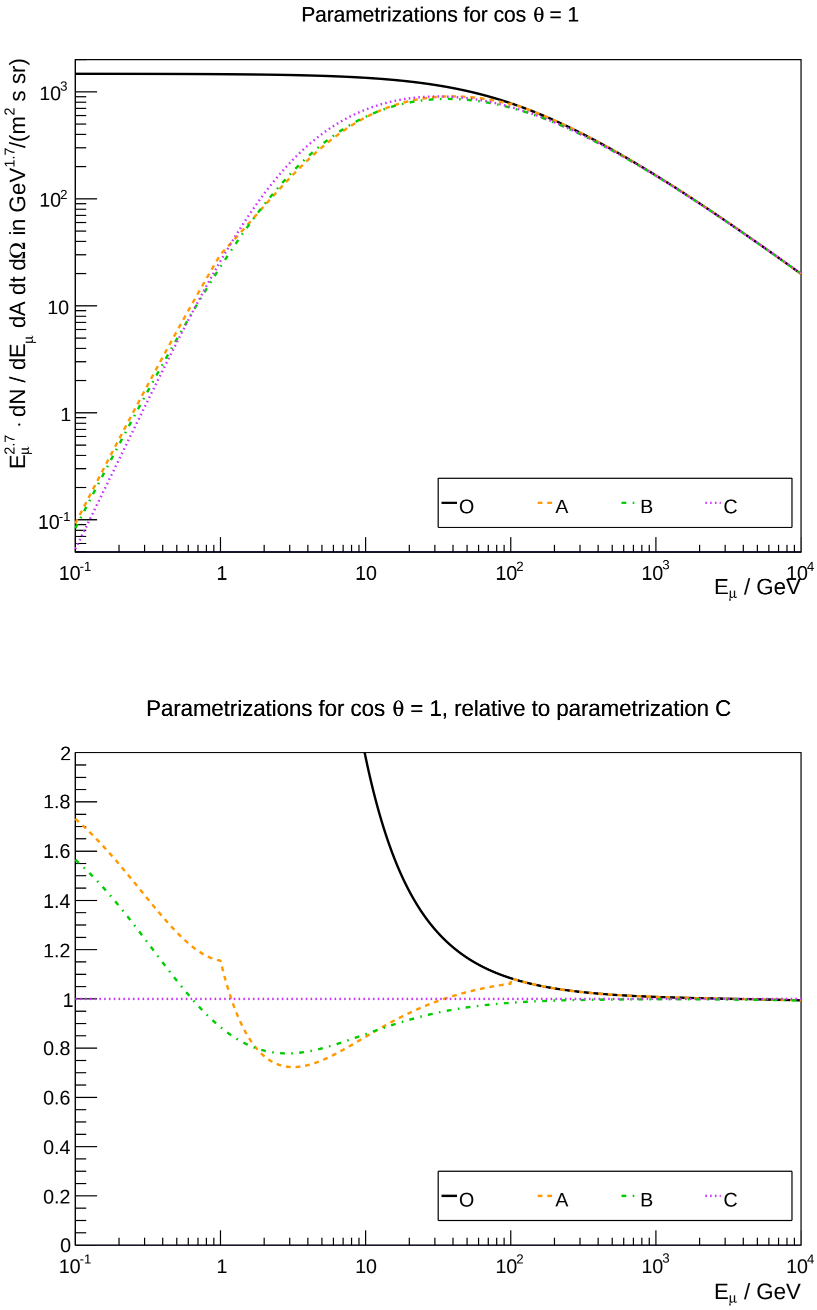


Figure 2: Comparison of parametrizations for the differential muon energy spectrum for vertical muons. Top: values of the parametrizations. Bottom: values relative to one parametrization.

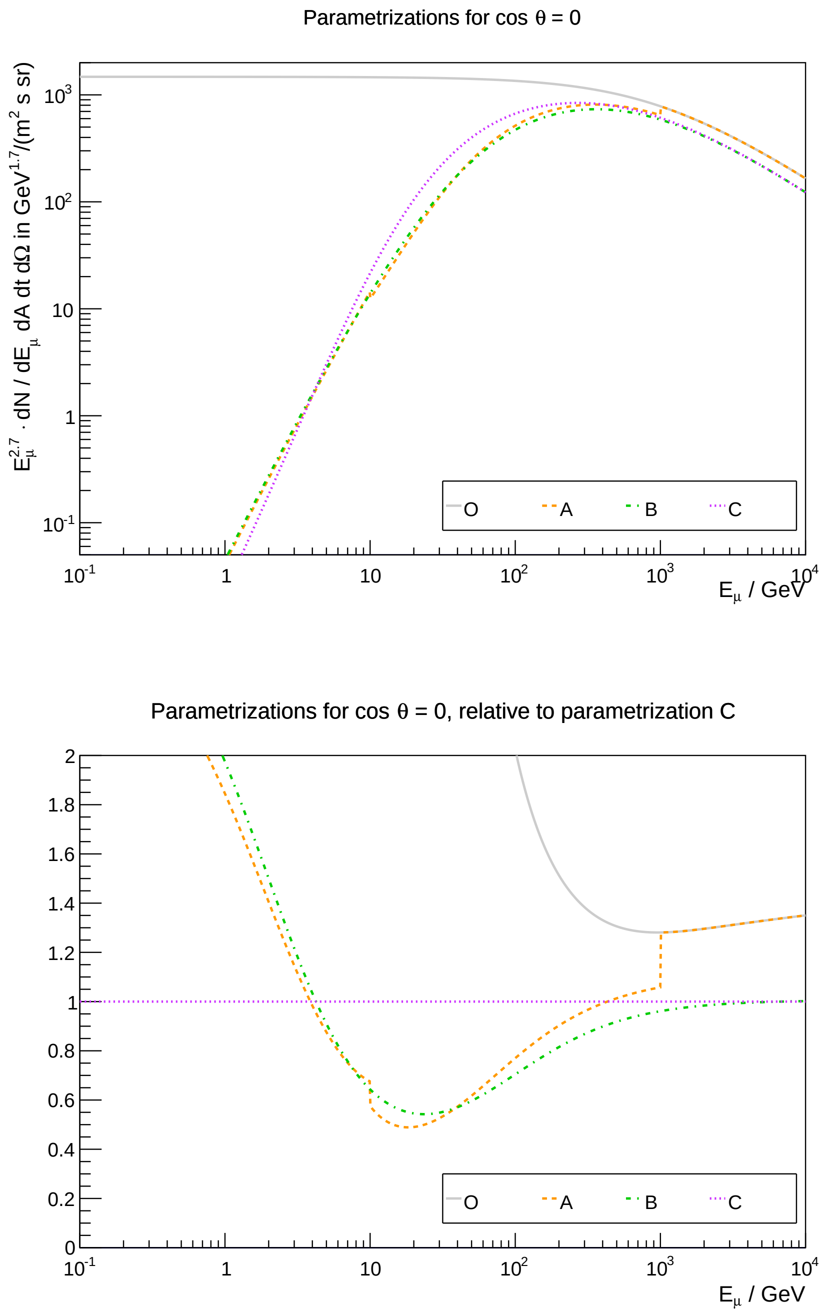
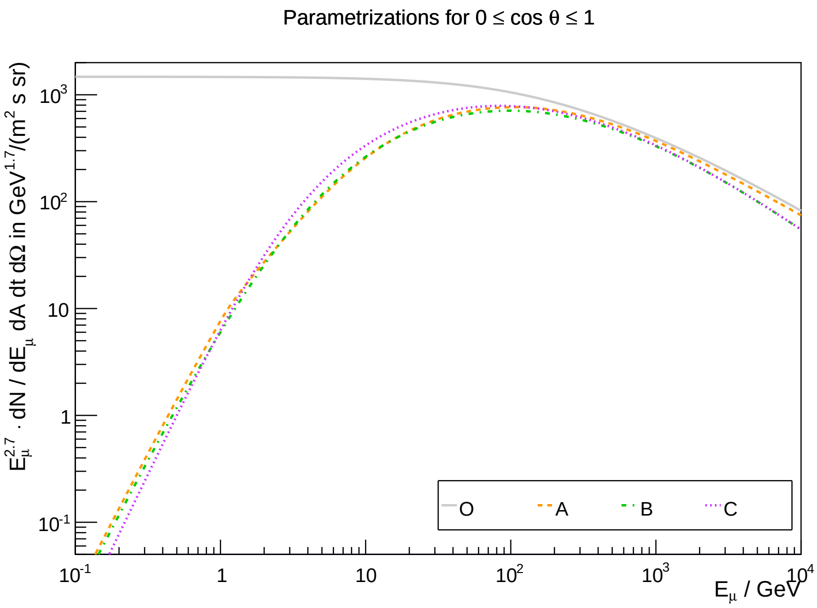


Figure 3: Comparison of parametrizations for the differential muon energy spectrum for horizontal muons (cf. Fig. 2).



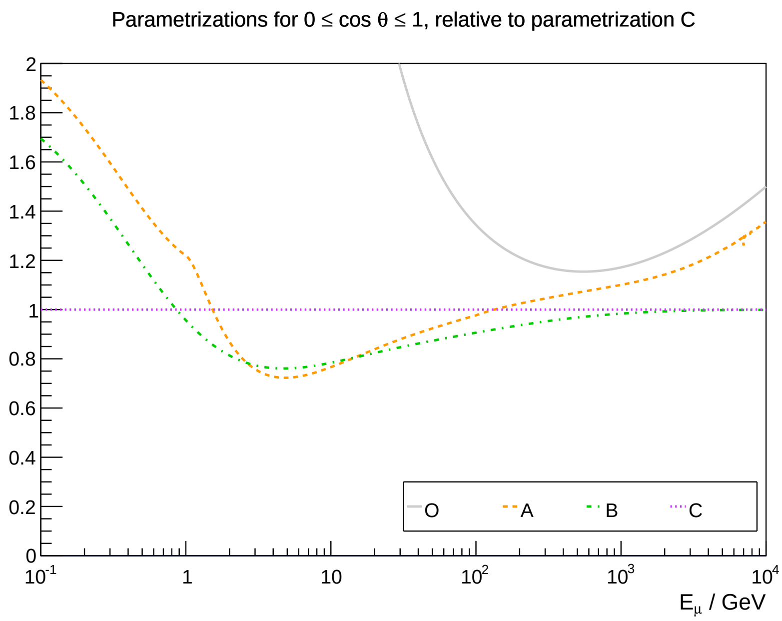


Figure 4: Comparison of parametrizations for the differential muon energy spectrum averaged over the full solid angle (cf. Fig. 2).

3.2 Zenith angle distribution

The distribution of $\cos\theta$ obtained with CRY and from the parametrizations is shown in Figure 5. For the marginalization of the energy in the given parametrizations a lower limit on the muon energy has to be given.

For a lower limit of 0.1 GeV, the curves of all parametrizations lie above the CRY data, i. e. their total flux is larger. The shape is resembled well, however. For a cut-off of 1.0 GeV, the CRY results resemble the parametrization well, with A and B matching CRY's data at 0° latitude, and C being more close to the CRY results at 90° latitude for $\cos\theta$ close to 1 (and slightly above the CRY data for smaller values of $\cos\theta$).

3.3 Differential energy spectra

The plots in Figures 6 to 8 show sampled CRY data (resulting from 10^9 sampled primary protons) and the underlying CRY pdf, together with experimental data and the different parametrizations.

In the top plot, the CRY and experimental results are shown as points, the different parametrizations as lines. The bottom plot shows all the data relative to the CRY pdf. To interpolate between the CRY pdf data points, a spline was used. The spline was applied to the data points scaled with $E_\mu^{2.7}$, rather than the actual values.

Close-to-vertical

A close-to-vertical muon spectrum was adapted from measurements made by the BESS-TeV spectrometer [10]. The data was taken at KEK (36.2°N, 140.1°W) between October 1 and 6, 2002. The date in CRY was set to October 3, 2002. For the measurement, a cut was set on $\cos\theta > 0.98$ below 20 GeV and $\cos\theta > 0.9$ above.

The μ^+ and μ^- flux data given in Table 3 of [10] was used for the comparison, with the respective statistical and systematic errors added linearly.

Experimental and CRY data are shown in Figure 6. The agreement between CRY and data is very good in the center of the energy range, roughly between 10 and 50 GeV. For lower energies, the experimental flux is significantly larger than the CRY result, and most closely resembled by parametrization B. In the high-energy range, a similar behavior is observed, even though the uncertainty of the data points is relatively large in this region.

Inclined

As an example of an inclined spectrum, a measurement made at DESY [11] is considered. Events with zenith angles between 68° and 82° were accepted by the experiment, parametrization and CRY data were averaged accordingly. CRY was set to a latitude of 53.5° N and an estimated date of September 15, 1978.

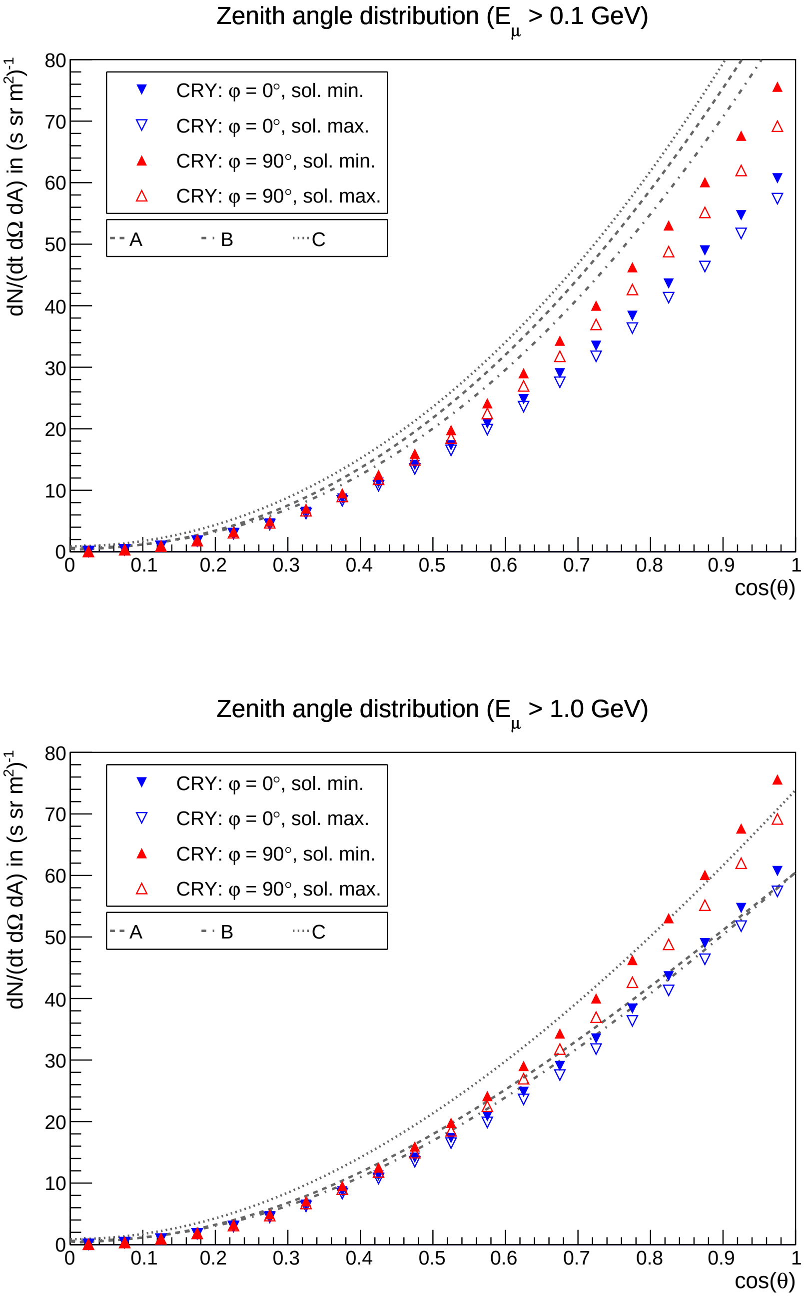


Figure 5: Zenith angle distribution for the different parametrizations and from CRY (for different latitude and solar activity), for muon energies above 0.1 GeV (top) and above 1.0 GeV (bottom).

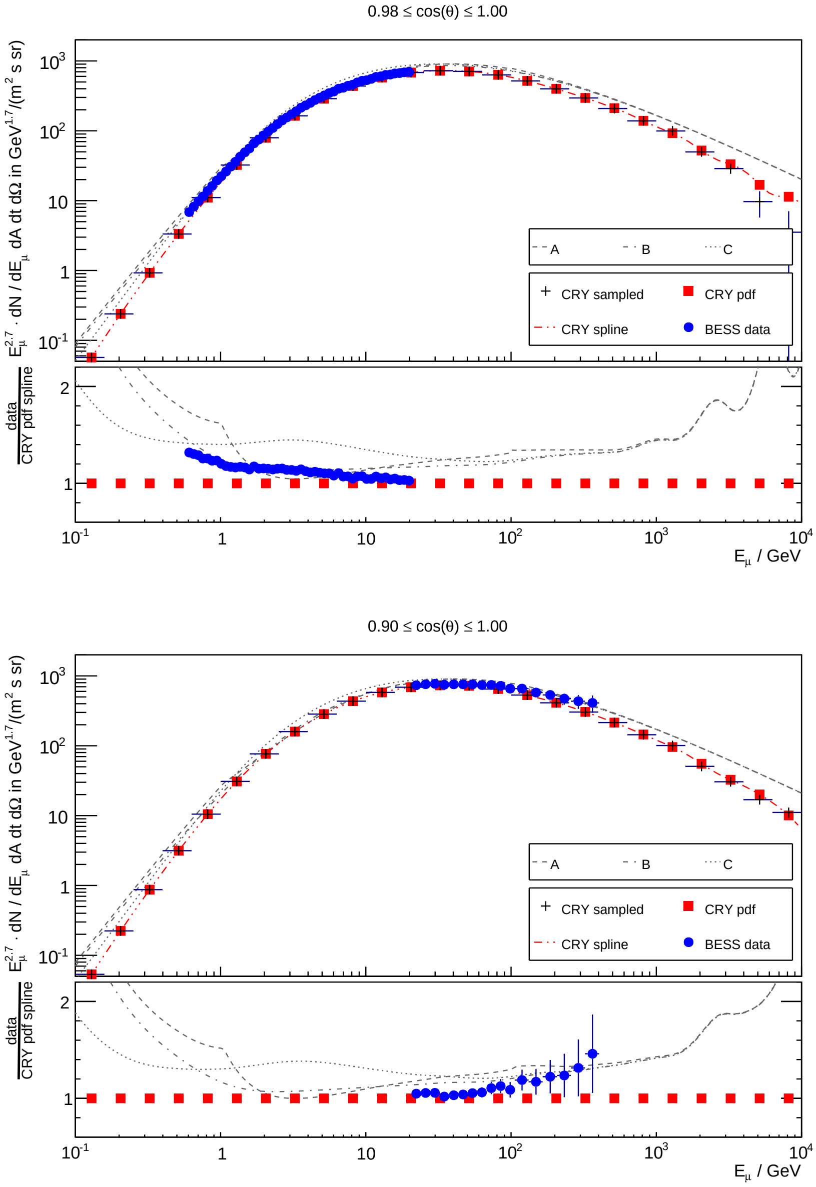


Figure 6: Comparison of parametrizations, CRY and BESS [10] data for the differential flux of almost vertical muons. The two plots result from the two slightly different cuts on $\cos\theta$ in the two energy ranges.

Data from Table II of [11] was used for the comparison, using the relative errors given in the same Table.

The comparison is shown in Figure 7. The CRY pdf starts at about 20% above the experimental data. Agreement is reached at about 50 GeV, after which the data surpasses the CRY results by up to about 40% at 400 GeV and even larger values for the following three data points with the highest energies. The parametrizations follow the data more closely than the CRY results in the energy range above 10 GeV.

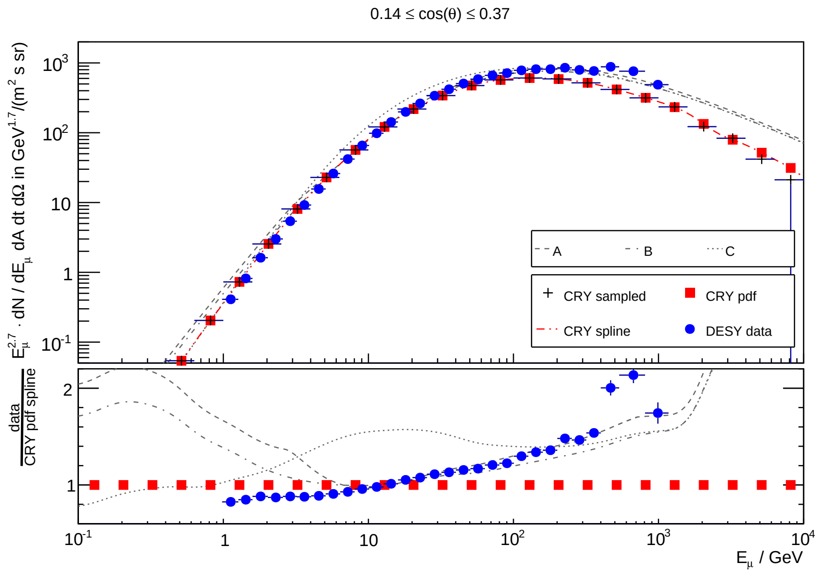


Figure 7: Comparison of parametrizations, CRY and data taken at DESY [11] for the differential flux of muons at an azimuth of $(75 \pm 7)°$.

Almost horizontal

To consider almost horizontal muons, data taken by the MUTRON spectrometer [12] was analyzed. The setup at a latitude of 35.75° N accepted muons with zenith angles between 86° and 90°. The date in CRY was set to an estimated date of April 15, 1983.

The data is taken from Table IV in [12], only the given statistical errors are considered.

In Figure 8, the data is compared to the parametrizations and CRY. The CRY pdf has a dip between 1 and 4 TeV, which might be a hint at statistical fluctuations of the simulations which were used to create the CRY data tables. Reasonable agreement between CRY and the experimental data is seen up to about 2 TeV. In the dip region of the CRY pdf, experimental data exceeds the CRY pdf by about 50%. In this region, the data lies between CRY and the parametrizations B/C. However, the parametrizations describe the data much worse outside the dip region. Also in regions without experimental data, large differences are observed between CRY and the individual parametrizations, underlining the difficulty of a correct description of the spectrum at large zenith angles.

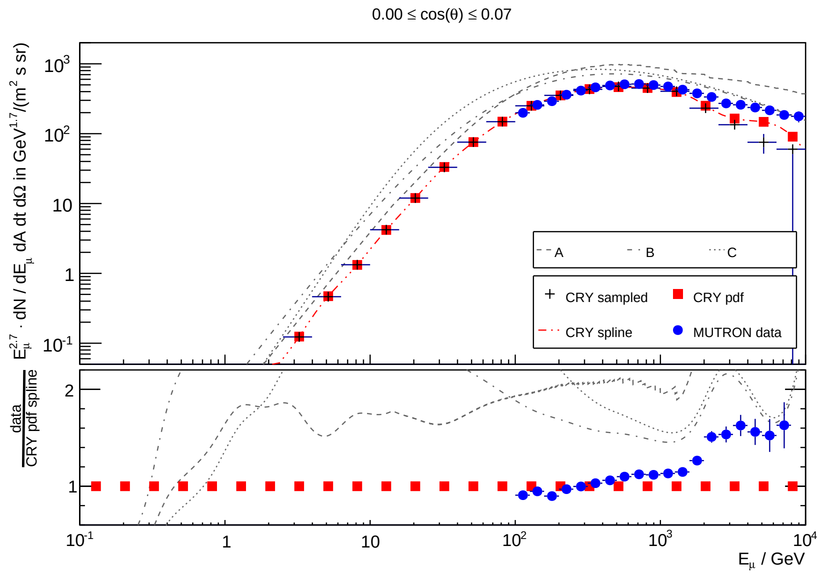


Figure 8: Comparison of MUTRON data [12] for almost horizontal muons.

3.4 Conclusions

The comparison of the muon properties sampled with CRY to parametrized spectra and experimental results confirmed the general validity of the results, even though significant differences were observed in some ranges of the datasets for both CRY and the modified parametrizations. From the three examples it is hardly possible to draw any general conclusion. More datasets (for example comparing experiments at similar zenith angles) would need to be considered to assess the observed deviations, and identify possible trends.

To possibly explain the observed deviations, systematic uncertainties would need to be considered in much more detail. The procedures of the generation of the data tables for CRY are only outlined in [2], the following points could be investigated in more detail for CRY:

1. The influence of *other primary particles*, as only protons were considered.
2. *Applied cuts* in the Monte Carlo simulations to generate the tables. These could affect the low-energy region of the simulated spectrum.
3. The *simulated geometry*, in particular the influence of the curvature of the Earth and the atmosphere. These could influence the results for large zenith angles.

CRY provides a comfortable interface to sample shower particles for further use, e. g. in Monte Carlo simulations. When using muons sampled with CRY as an input, the deviations from the data or parametrizations should be taken into account for the uncertainty of the obtained results.

Acknowledgements

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