slwv1kg7b

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1 Laboratório 01 - Expansão em Séries de Taylor

Aluno:

Exemplo 01 - Expansão da função $f(x) = e^x$, via Série de MacLaurin

$$f(x) = e^x \approx \sum_{k=0}^{N} \frac{x^k}{k!}$$

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
import math
```

[ ]:
```
def fatorial(n):
    result = 1
    for i in range(1, n + 1):
        result *= i
    return result

def f_exp(x,n):
    soma = 0
    for k in range(n):
        soma += x**k/fatorial(k)

    return soma

x = np.array([1,2,3])
n = 10
sn = f_exp(x,n)
print('A solução numérica com n = :',n, 'é = ',sn)
Er = np.abs(np.exp(x) - sn)/np.exp(x)
print('Com Erro relativo de', Er)
```

```
A solução numérica com n = : 10 é =  [ 2.71828153  7.38871252 20.06339286]
Com Erro relativo de [1.11425478e-07 4.64980750e-05 1.10248813e-03]
```

Exemplo 02 - Implementar a função

$$f(x) = sen(x) \approx \sum_{k=0}^{N} (-1)^k \frac{x^{2k+1}}{(2k+1)!} \tag{1}$$

[ ]:
```
def f_seno(x,n):
    soma = 0
    for k in range (n):
        num = x ** (2*k + 1)
        den = 2*k + 1
        coef = (-1)**k
        soma += coef * (num / math.factorial(den))

    return soma

x = np.pi/4
n = 10
sn = f_seno(x,n)
print(sn)
```

Exemplo 03 : Use a expansão em Série de MacLaurin para mostrar que

$$e^{ix} = cos\, x + i\, sen\, x, \quad i = \sqrt{-1}. \tag{2}$$

[76]:
```
def f_exp(x, n):
    soma = 0
    for k in range(n):
        soma += x**k/math.factorial(k)

    return soma

def f_seno(x,n):
    soma = 0
    for k in range (n):
        num = x ** (2*k + 1)
        den = 2*k + 1
        coef = (-1)**k
        soma += coef * (num / math.factorial(den))

    return soma

def f_cos(x,n):
    soma = 0
    for k in range (n):
        num = x ** (2*k)
        den = 2*k
        coef = (-1)**k
```

```
        soma += coef * (num / math.factorial(den))

    return soma

def main():
    x = float(input("Defina um x para o programa te mostrar a identidade de␣
 ↪Euler: "))
    n = 50 # aproximacao por n_termos

    i = complex(0,1) # definicao do numero complexo

    e_apr = f_exp(i*x, n)
    cos_x = f_cos(x, n)
    i_sen_x = i*f_seno(x, n)

    print(f"Valor da e^(i*{x}): {e_apr}\nComparacao com o valor da identidade␣
 ↪de Euler: {cos_x + i_sen_x}\nErro relativo: {abs(e_apr -(cos_x + i_sen_x))/
 ↪abs(e_apr)}")

main()
```

```
Valor da e^(i*10.0): (-0.839071529076596-0.5440211108892689j)
Comparacao com o valor da identidade de Euler:
(-0.8390715290765992-0.54402111088927j)
Erro relativo: 3.405689430508332e-15
```

2 Exercícios

01 - O valor de π pode ser aproximado por meio da seguinte expansão em série:

$$\pi \approx 4 \sum_{i=1}^{n} (-1)^{i-1} \frac{1}{2i-1} \tag{3}$$

escreva uma função que retorne o valor aproximado de π que receba como parâmetro de entrada os n termos da série. Calcule o erro relativo para diferentes valores de n. Por exemplo: $n = 5$, 10, 20, 30.

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
import math
```

[78]:
```
valores_pi = []
erros_relativos = []

def calc_pi(n):
    soma = 0

    for i in range(1, n+1):
```

```python
        menos_1 = (-1)**(i-1)
        div = 1/(2*i-1)
        soma += menos_1 * div
        valores_pi.append(4*soma)

    return valores_pi[-1]

def erro_relativo(n):
    global erros_relativos

    for i in range(1, n):
        erros_relativos.append(abs(valores_pi[i] - valores_pi[i-1])/
 ↪abs(valores_pi[i]))

def main():

    global valores_pi
    global erros_relativos
    n = 1000

    valores_pi =[]
    print(f"Pi para {n} termos por serie: {calc_pi(n)}")

    if len(valores_pi) > 1: # erro so e possivel a partir de dois valores
        erro_relativo(len(valores_pi))

    if len(erros_relativos) > 0:
        print(f"Ultimo erro relativo: {erros_relativos[-1]}")
        print(f"Lista de erros relativos: {erros_relativos}")

    else:
        print("Nao ha como calcular o erro relativo, pois foi iterado apenas 1␣
 ↪termo")
    print(f"Lista dos valores de pi de 0 ate {n}: {valores_pi}")


    np_pi = np.array([np.pi for i in range(n)])
    plt.plot(range(len(valores_pi)), valores_pi, label="Aproximacao por series")
    plt.plot(range(len(valores_pi)), np_pi, label="Numpy Pi")
    plt.legend()
    plt.xlim(left=0)
    plt.grid()
    plt.show()


if __name__=="__main__":
    main()
```

```
Pi para 1000 termos por serie: 3.140592653839794
Ultimo erro relativo: 0.0006371410497326014
Lista de erros relativos: [0.49999999999999983, 0.2307692307692307,
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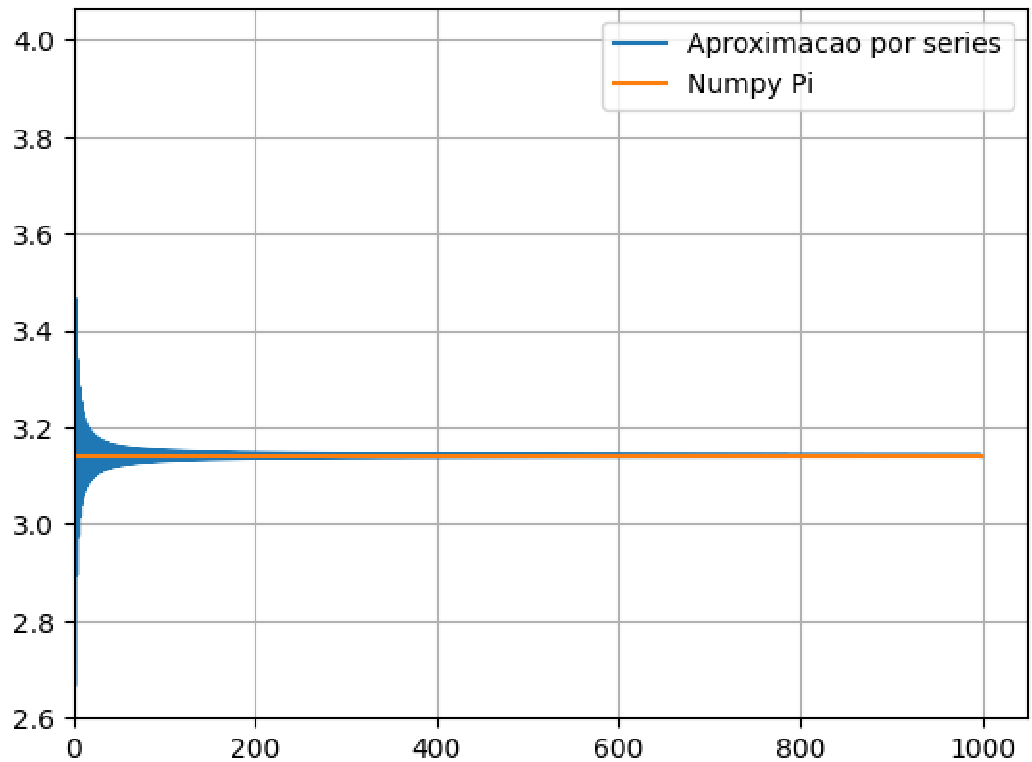


02 - Use a expansão em série de Taylor para mostrar que

$$\frac{sen\,x}{x} \approx 1 \tag{4}$$

com um valor bem pequeno em x.

```
[ ]: valores_sn = []

     def f_seno(x,n): # aproximacao do sen x em serie de MacLaurin
         soma = 0

         for k in range (n):
```

```
        num = x ** (2*k + 1)
        den = 2*k + 1
        coef = (-1)**k
        soma += coef * (num / math.factorial(den))
        valores_sn.append(soma)

    return soma

def main():
    global valores_sn
    x = float(input("Coloque um valor bem pequeno para X:"))
    n = 40

    sn = float(f_seno(x,n))
    fn = sn/x

    print(f"Para x = {x}, n = {n} => fn = sen(x)/x = {fn} ou com 4 casas␣
 ↪decimais: {fn:.4f}")

main()
```

```
Para x = 0.001, n = 40 => fn = sen(x)/x = 0.9999998333333416 ou com 4 casas
decimais: 1.0000
```

03 - Escreva a expansão em série de Taylor para $f(x) = e^{x^2}$ em torno da origem. Defina uma função $exp\_dupla(x, n)$, para calcular uma aproximação para $f(x)$ utilizando os n primeiros termos da expansão. Adapte a função para receber como parâmetro de entrada um vetor.

[79]:
```
def exp_dupla(x, n):
    soma = 0

    for i in range(n):
        soma+= ( x ** (2*i) )/math.factorial(i)

    return soma

def main():
    vetor = np.array(list(map(float, input("Digite um vetor de numeros,␣
 ↪separado por vírgulas, para fazer a expansao em serie de Taylor: \n").
 ↪split(', '))))
    n_termos = 50
    resultados = np.array([])

    for item in vetor:
        item_resultado = exp_dupla(item, n_termos)
        resultados = np.append(resultados, item_resultado)
```

```
        print(f"Iterador {resultados.size-1} com valor {item} apresenta o␣
↪seguinte resultado para {n_termos} termos: {item_resultado}")

    print(f"Vetor com todos os resultados: {resultados}")

main()
```

```
Iterador 0 com valor 10.0 apresenta o seguinte resultado para 50 termos:
3.1678118395809394e+35
Iterador 1 com valor 15.0 apresenta o seguinte resultado para 50 termos:
3.7916796653005296e+52
Iterador 2 com valor 30.0 apresenta o seguinte resultado para 50 termos:
9.955469193109487e+81
Vetor com todos os resultados: [3.16781184e+35 3.79167967e+52 9.95546919e+81]
```

04 - Sabendo que o seno hiperbólico pode ser obtido por meio da seguinte expansão,

$$senh\, x = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!} \tag{5}$$

o cosseno hiperbólico por,

$$cosh\, x = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!} \tag{6}$$

e a tangente hiperbólica é dada por,

$$tanh\, x = \frac{senh\, x}{cosh\, x} \tag{7}$$

calcule a tangente hiperbólica de $x_0 = 0,5$ a partir das funções $ senh,x$ e $ cosh,x$, com um erro relativo de aproximadamente 10^{-5}. Gerar os gráficos e comparar com a função $np.tanh(x)$.

[80]:
```
def sen_hip(x, n):
    soma = 0

    for i in range(n+1):
        soma += float((x**(2*i+1))/math.factorial(2*i+1))

    return soma

def cos_hip(x, n):
    soma = 0

    for i in range(n+1):
        soma += float((x**(2*i))/math.factorial(2*i))

    return soma

def main():
    x = 0.5
    erro_relativo = 1e-5
```

```
    tang_h = np.array([])

    n = 0
    erro = 2 # numero arbitrario maior que qualquer valor de tang_h, para␣
↪iniciar o while

    while erro > erro_relativo:
        ultima_tang = sen_hip(x, n)/cos_hip(x, n)

        tang_h = np.append(tang_h, ultima_tang)

        if n>0:
            erro = abs(tang_h[-1] - tang_h[-2])/abs(tang_h[-1])
            print(f"Calculando os valores de tang_h de x={x}, para {n} termos,␣
↪temos: {ultima_tang}, com erro de {erro}")
        else:
            print(f"Calculando os valores de tang_h de x={x}, para {n} termos,␣
↪temos: {ultima_tang}, com erro ainda nao calculado por ser a primeira␣
↪iteracao")

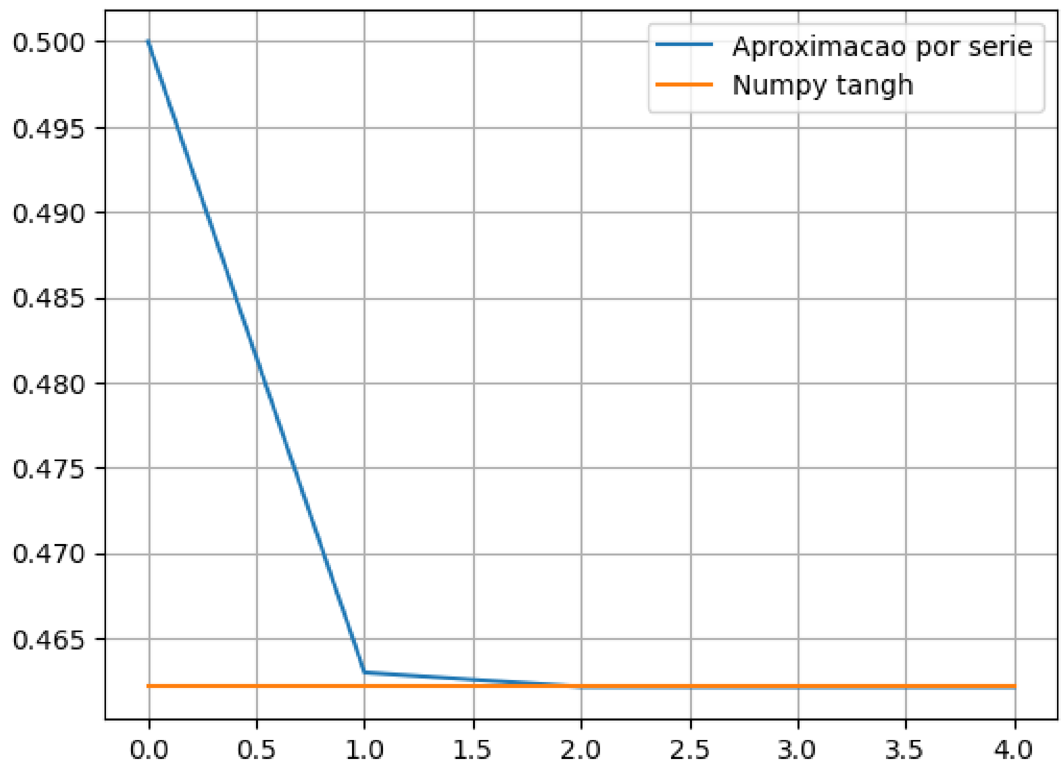
        n+=1


    np_tang_h = np.full(n, np.tanh(x))

    plt.plot(range(n), tang_h, label="Aproximacao por serie")
    plt.plot(range(n), np_tang_h, label="Numpy tangh")
    plt.legend()
    plt.grid()
    plt.show()

main()
```

```
Calculando os valores de tang_h de x=0.5, para 0 termos, temos: 0.5, com erro
ainda nao calculado por ser a primeira iteracao
Calculando os valores de tang_h de x=0.5, para 1 termos, temos:
0.46296296296296296, com erro de 0.07999999999999986
Calculando os valores de tang_h de x=0.5, para 2 termos, temos:
0.4621247113163972, com erro de 0.001813907860884543
Calculando os valores de tang_h de x=0.5, para 3 termos, temos:
0.4621171922898218, com erro de 1.6270821992496396e-05
Calculando os valores de tang_h de x=0.5, para 4 termos, temos:
0.4621171573596405, com erro de 7.558728505675665e-08
```


Aproximacao por serie
Numpy tangh
0.500
0.495
0.490
0.485
0.480
0.475
0.470
0.465
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0