

Contents

1	Template	1
2	Algebra	2
2.1	All divisors	2
2.2	Primality test	2
2.3	Binary exponentiation	2
2.4	Greatest common divisor	3
2.4.1	Least common multiple	3
3	Graphs	3
3.1	DFS	3
3.2	BFS	3
3.2.1	Shortest path on unweighted graph	4
3.3	Flood Fill	5
3.4	Topological Sort (Directed Acyclic Graph)	5
3.4.1	DFS Variation	5
3.4.2	Kahn's Algorithm	6
3.5	Bipartite Graph Check (Undirected Graph)	6
3.6	Cycle Check (Directed Graph)	7
4	Math Formulas	8
4.1	Sum of an arithmetic progression	8
4.2	Permutation with repeated elements	8
4.3	Check if is geometric progression	8
5	Facts	8
5.1	XOR	8
5.1.1	Self-inverse property	8
5.1.2	Identity element	8
5.1.3	Commutative	8
5.1.4	Associative	8

1 Template

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 #define ll          long long
5 #define vll         vector<ll>
6 #define pll         pair<ll, ll>
7 #define vpll        vector<pll>
8 #define endl        '\n'
9 #define all(xs)      xs.begin(), xs.end()
10 #define found(x, xs) (xs.find(x) != xs.end())
```

2 Algebra

2.1 All divisors

$O(\sqrt{n})$

```
1  vll divisors(ll n) {
2      vll divs;
3      for (ll i = 1; 1LL * i * i <= n; i++) {
4          if (n % i == 0) {
5              divs.push_back(i);
6              if (i != n / i) {
7                  divs.push_back(n / i);
8              }
9          }
10     }
11
12     return divs;
13 }
```

2.2 Primality test

$O(\sqrt{n})$

```
1  bool isPrime(ll n)
2  {
3      if(n!=2 && n % 2==0)
4          return false;
5
6      for(ll d=3; d*d <= n; d+=2)
7      {
8          if(n % d==0)
9              return false;
10     }
11
12     return n >= 2;
13 }
```

2.3 Binary exponentiation

$O(\log n)$

```
1  ll binpow(ll a, ll b) {
2      ll res = 1;
3      while (b > 0) {
4          if (b & 1)
5              res = res * a;
6          a = a * a;
7          b >>= 1;
8      }
```

```

8     }
9     return res;
10 }

```

2.4 Greatest common divisor

$O(\log \min(a, b))$

```

1 ll gcd (ll a, ll b) {
2     while (b) {
3         a %= b;
4         swap(a, b);
5     }
6     return a;
7 }

```

2.4.1 Least common multiple

```

1 ll lcm(ll a, ll b) {
2     return a / gcd(a, b) * b;
3 }

```

3 Graphs

3.1 DFS

$O(n + m)$

```

1 void dfs(ll at, ll n, vll adj[], bool visited[]) {
2     if(visited[at])
3         return;
4
5     visited[at] = true;
6
7     vll neighbours = adj[at];
8     for(auto nex: neighbours)
9         dfs(nex.first, n, adj, visited);
10 }

```

3.2 BFS

$O(n + m)$

```

1 void bfs(ll s, ll n, vll adj[]) {
2     bool visited[n] = {0};
3     visited[s] = true;

```

```

4
5     queue<ll> q;
6     q.push(s);
7     while (!q.empty())
8     {
9         vll neighbours = adj[q.front()];
10        for(auto nex: neighbours) {
11            if(!visited[nex]) {
12                visited[nex]=true;
13                q.push(nex);
14            }
15        }
16        cout << q.front() << '\n';
17        q.pop();
18    }
19 }

```

3.2.1 Shortest path on unweighted graph

$O(n + m)$

```

1 vll solve(ll s, ll n, vll adj[]) {
2     bool visited[n] = {0};
3     visited[s] = true;
4
5     queue<ll> q;
6     q.push(s);
7     vll prev(n, -1);
8     while (!q.empty())
9     {
10        vll neighbours = adj[q.front()];
11        for(auto nex: neighbours) {
12            if(!visited[nex]) {
13                visited[nex]=true;
14                q.push(nex);
15                prev[nex] = q.front();
16            }
17        }
18        q.pop();
19    }
20
21    return prev;
22 }
23
24 vll reconstructPath(ll s, ll e, vll prev) {
25     vll path;
26     for(ll i=e; i!=-1; i=prev[i])
27         path.push_back(i);
28 }

```

```

29     reverse(path.begin(), path.end());
30
31     if(path[0]==s)
32         return path;
33     else {
34         vll place;
35         return place;
36     }
37 }
38
39 vll bfs(ll s, ll e, ll n, vll adj[]) {
40     vll prev = solve(s, n, adj);
41
42     return reconstructPath(s, e, prev);
43 }

```

3.3 Flood Fill

$O(n + m)$

```

1  int dir_y[] = {};
2  int dir_x[] = {};
3
4  int ff(int i, int j, char c1, char c2) {
5      if ((i < 0) || (i >= n)) return 0;
6      if ((j < 0) || (j >= m)) return 0;
7      if (grid[i][j] != c1) return 0;
8
9      int ans = 1;
10     grid[i][j] = c2;
11
12     for (int d = 0; d < 8; ++d)
13         ans += floodfill(i+dir_y[d], j+dir_x[d], c1, c2);
14
15     return ans;
16 }

```

;

3.4 Topological Sort (Directed Acyclic Graph)

3.4.1 DFS Variation

$O(n + m)$

```

1  void dfs(ll at, ll n, vpll adj[], bool visited[], vll &ts) {
2      if(visited[at])
3          return;
4
5      visited[at] = true;

```

```

6
7     vpll neighbours = adj[at];
8     for(auto nex: neighbours)
9         dfs(nex.first, n, adj, visited);
10    ts.push_back(at);           // Only change
11 }

```

3.4.2 Kahn's Algorithm

```

1 priority_queue<ll, vll, greater<ll>> pq;
2 for(ll at=0; at<n; at++)           // Push all sources of
3     if(in_degree[at] == 0)         connected components in graph
4         pq.push(at);
5
6 while(!pq.empty()) {
7     ll at = pq.top(); pq.pop();
8     vll neighbors = adj[at];
9     for(auto nex: neighbors) {
10         in_degree[nex]--;
11         if(in_degree[nex]>0) continue;
12         pq.push(nex);
13     }
14 }

```

3.5 Bipartite Graph Check (Undirected Graph)

$O(n + m)$

```

1 bool isBipartite(ll s, ll n, vll adj[]) {
2     queue<ll> q;
3     q.push(s);
4     vll color(n, -1); color[s]=0;
5     bool flag = true;
6     while (!q.empty())
7     {
8         vll neighbours = adj[q.front()];
9         for(auto nex: neighbours) {
10             if(color[nex] == -1) {
11                 color[nex] = 1-(color[q.front()]);
12                 q.push(nex);
13             }
14             else if(color[nex] == color[q.front()]) {
15                 flag = false;
16                 break;
17             }
18         }
19         q.pop();
20     }
21 }

```

```

20     }
21
22     return flag;
23 }

```

3.6 Cycle Check (Directed Graph)

$O(n + m)$

```

1  enum { UNVISITED = -1, VISITED = -2,  EXPLORED=-3};
2
3  void cycleCheck(ll at, ll n ,vll adj[], int visited[], ll
4      dfs_parent[]) {
5      visited[at] = EXPLORED;
6
7      vll neighbours = adj[at];
8      for(auto nex: neighbours) {
9          if(visited[nex] == UNVISITED) {
10             // Tree edges (part of the DFS spanning tree)
11             dfs_parent[nex] = at;
12             cycleCheck(nex, n, adj, visited);
13         }
14         else if(visited[nex] == EXPLORED) {
15             if(nex == dfs_parent[at]) {
16                 // Trivial cycle
17                 // Do something
18             }
19             else {
20                 // Non trivial cycle - Back Edge ((u, v)
21                 // such that v is the ancestor of node u but
22                 // is not part of the DFS tree)
23                 // Do something
24             }
25         }
26         else if(visited[nex] == VISITED) {
27             // Forward/Cross edge ((u, v) such that v is a
28             // descendant but not part of the DFS tree)
29             // Do something
30         }
31     }
32     visited[at] = VISITED;
33 }

```

4 Math Formulas

4.1 Sum of an arithmetic progression

$$S_n = \frac{n}{2}(a_1 + a_n)$$

4.2 Permutation with repeated elements

$$P_n = \frac{n!}{n_1!n_2!\dots n_k!}$$

4.3 Check if is geometric progression

$$a_i^2 = a_{i-1}a_{i+1}$$

5 Facts

5.1 XOR

5.1.1 Self-inverse property

To cancel a XOR, you can XOR again the same value because $a \oplus a = 0$, so $(value \oplus a) \oplus a = value$

5.1.2 Identity element

$$a \oplus 0 = a$$

5.1.3 Commutative

$$a \oplus b = b \oplus a$$

5.1.4 Associative

$$(a \oplus b) \oplus c = a \oplus (b \oplus c)$$