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# 1 Template

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 using ll =          long long;
5 #define vll         vector<ll>
6 #define vvll        vector<vll>
7 #define pll         pair<ll, ll>
8 #define vpll        vector<pll>
9 #define vvpll       vector<vpll>
10 #define endl '\n'
11 #define all(xs)      xs.begin(), xs.end()
12 #define found(x, xs) (xs.find(x) != xs.end())
```

## 2 Search

### 2.1 Ternary Search

$O(\log n)$

Function  $f(x)$  is unimodal on an interval  $[l, r]$ . Unimodal means: the function strictly increases first, reaches a maximum, and then strictly decreases OR the function strictly decreases first, reaches a minimum and then strictly increases

```
1 double ternary_search(double l, double r) {
2     double eps = 1e-9; // error limit
3     while(r - l > eps) {
4         double m1 = l + (r-l) / 3;
5         double m2 = r - (r-l) / 3;
6
7         double f1 = f(m1);
8         double f2 = f(m2);
9
10        if(f1 < f2)
11            l = m1;
12        else
13            r = m2;
14    }
15
16    return f(l);
17 }
```

## 3 Algebra

### 3.1 All divisors

$O(\sqrt{n})$

```

1  vll divisors(ll n) {
2      vll divs;
3      for (ll i = 1; 1LL * i * i <= n; i++) {
4          if (n % i == 0) {
5              divs.push_back(i);
6              if (i != n / i) {
7                  divs.push_back(n / i);
8              }
9          }
10     }
11
12     return divs;
13 }

```

### 3.2 Primality test

$O(\sqrt{n})$

```

1  bool isPrime(ll n)
2  {
3      if(n!=2 && n % 2==0)
4          return false;
5
6      for(ll d=3; d*d <= n; d+=2)
7      {
8          if(n % d==0)
9              return false;
10     }
11
12     return n >= 2;
13 }

```

### 3.3 Binary exponentiation

$O(\log n)$

```

1  ll binpow(ll a, ll b) {
2      ll res = 1;
3      while (b > 0) {
4          if (b & 1)
5              res = res * a;
6          a = a * a;
7          b >>= 1;
8      }
9      return res;
10 }

```

### 3.4 Greatest common divisor

$O(\log \min(a, b))$

```
1 ll gcd (ll a, ll b) {  
2     while (b) {  
3         a %= b;  
4         swap(a, b);  
5     }  
6     return a;  
7 }
```

#### 3.4.1 Least common multiple

```
1 ll lcm(ll a, ll b) {  
2     return a / gcd(a, b) * b;  
3 }
```

#### 3.4.2 Extended Euclides Algorithm

```
1 ll gcd(ll a, ll b, ll& x, ll& y) {  
2     if (b == 0) {  
3         x = 1;  
4         y = 0;  
5         return a;  
6     }  
7     ll x1, y1;  
8     ll d = gcd(b, a % b, x1, y1);  
9     x = y1;  
10    y = x1 - y1 * (a / b);  
11    return d;  
12 }
```

### 3.5 Linear Diophantine Equations

$O(\log \min(a, b))$

#### 3.5.1 Any solution

```
1 bool find_any_solution(ll a, ll b, ll c, ll &x0, ll &y0, ll  
2     &g) {  
3     g = gcd(abs(a), abs(b), x0, y0);  
4     if (c % g) {  
5         return false;  
6     }  
7 }
```

```

7     x0 *= c / g;
8     y0 *= c / g;
9     if (a < 0) x0 = -x0;
10    if (b < 0) y0 = -y0;
11    return true;
12 }

```

## 3.6 Integer Factorization

### 3.6.1 Pollard's Rho

$O(\sqrt[3]{n} \log n)$

```

1  /**
2   * @param a first multiplier
3   * @param b second multiplier
4   * @param mod
5   * @return a * b mod n (without overflow)
6   * @brief Multiplies two numbers >= 10^18
7   * Time Complexity: O(log b)
8  */
9  ll mult(ll a, ll b, ll mod) {
10     ll result = 0;
11     while (b) {
12         if (b & 1)
13             result = (result + a) % mod;
14         a = (a + a) % mod;
15         b >>= 1;
16     }
17     return result;
18 }
19
20 /**
21 * @param x first multiplier
22 * @param c second multiplier
23 * @param mod
24 * @return f(x) = x^2 + c mod (mod)
25 * @brief Polynomial function chosen for pollard's rho
26 * Time Complexity: O(1)
27 */
28 ll f(ll x, ll c, ll mod) {
29     return (mult(x, x, mod) + c) % mod;
30 }
31
32 /**
33 * @param n number that we want to find a factor p
34 * @param x0 number where we will start
35 * @param c constant in polynomial function
36 * @return fac

```

```

37  * @brief Pollard's Rho algorithm (works only for composite
    numbers)
38  * if(g==n) try other starting values
39  * Time Complexity:  $O(n^{1/4} \log n)$ 
40  */
41  ll rho(ll n, ll x0=2, ll c=1) {
42      ll x = x0;
43      ll y = x0;
44      ll g = 1;
45      while (g == 1) {
46          x = f(x, c, n);
47          y = f(y, c, n);
48          y = f(y, c, n);
49          g = gcd(abs(x - y), n);
50      }
51      return g;
52  }

```

## 4 Graphs

### 4.1 DFS

$O(n + m)$

```

1  void dfs(ll at, ll n, vll adj[], bool visited[]) {
2      if(visited[at])
3          return;
4
5      visited[at] = true;
6
7      vll neighbours = adj[at];
8      for(auto nex: neighbours)
9          dfs(nex.first, n, adj, visited);
10 }

```

### 4.2 BFS

$O(n + m)$

```

1  void bfs(ll s, ll n, vll adj[]) {
2      bool visited[n] = {0};
3      visited[s] = true;
4
5      queue<ll> q;
6      q.push(s);
7      while (!q.empty())
8      {
9          vll neighbours = adj[q.front()];

```

```

10         for(auto nex: neighbours) {
11             if(!visited[nex]) {
12                 visited[nex]=true;
13                 q.push(nex);
14             }
15         }
16         cout << q.front() << '\n';
17         q.pop();
18     }
19 }

```

#### 4.2.1 Shortest path on unweighted graph

$O(n + m)$

```

1  vll solve(ll s, ll n, vll adj[]) {
2      bool visited[n] = {0};
3      visited[s] = true;
4
5      queue<ll> q;
6      q.push(s);
7      vll prev(n, -1);
8      while (!q.empty())
9      {
10         vll neighbours = adj[q.front()];
11         for(auto nex: neighbours) {
12             if(!visited[nex]) {
13                 visited[nex]=true;
14                 q.push(nex);
15                 prev[nex] = q.front();
16             }
17         }
18         q.pop();
19     }
20
21     return prev;
22 }
23
24 vll reconstructPath(ll s, ll e, vll prev) {
25     vll path;
26     for(ll i=e; i!=-1; i=prev[i])
27         path.push_back(i);
28
29     reverse(path.begin(), path.end());
30
31     if(path[0]==s)
32         return path;
33     else {
34         vll place;

```

```

35         return place;
36     }
37 }
38
39 vll bfs(ll s, ll e, ll n, vll adj[]) {
40     vll prev = solve(s, n, adj);
41
42     return reconstructPath(s, e, prev);
43 }

```

### 4.3 Flood Fill

$O(n + m)$

```

1  int dir_y[] = {};
2  int dir_x[] = {};
3
4  int ff(int i, int j, char c1, char c2) {
5      if ((i < 0) || (i >= n)) return 0;
6      if ((j < 0) || (j >= m)) return 0;
7      if (grid[i][j] != c1) return 0;
8
9      int ans = 1;
10     grid[i][j] = c2;
11
12     for (int d = 0; d < 8; ++d)
13         ans += floodfill(i+dir_y[d], j+dir_x[d], c1, c2);
14
15     return ans;
16 }

```

;

### 4.4 Topological Sort (Directed Acyclic Graph)

#### 4.4.1 DFS Variation

$O(n + m)$

```

1  void dfs(ll at, ll n, vll adj[], bool visited[], vll &ts) {
2      if(visited[at])
3          return;
4
5      visited[at] = true;
6
7      vll neighbours = adj[at];
8      for(auto nex: neighbours)
9          dfs(nex.first, n, adj, visited);
10     ts.push_back(at);           // Only change
11 }

```



#### 4.4.2 Kahn's Algorithm

```
1 priority_queue<ll, vll, greater<ll>> pq;
2 for(ll at=0; at<n; at++) // Push all sources of
3   connected components in graph
4   if(in_degree[at] == 0)
5     pq.push(at);
6 while(!pq.empty()) {
7   ll at = pq.top(); pq.pop();
8   vll neighbors = adj[at];
9   for(auto nex: neighbors) {
10     in_degree[nex]--;
11     if(in_degree[nex]>0) continue;
12     pq.push(nex);
13   }
14 }
```

#### 4.5 Bipartite Graph Check (Undirected Graph)

$O(n + m)$

```
1 bool isBipartite(ll s, ll n, vll adj[]) {
2   queue<ll> q;
3   q.push(s);
4   vll color(n, -1); color[s]=0;
5   bool flag = true;
6   while (!q.empty())
7   {
8     vll neighbours = adj[q.front()];
9     for(auto nex: neighbours) {
10       if(color[nex] == -1) {
11         color[nex] = 1-(color[q.front()]);
12         q.push(nex);
13       }
14       else if(color[nex] == color[q.front()]) {
15         flag= false;
16         break;
17       }
18     }
19     q.pop();
20   }
21   return flag;
22 }
23 }
```

## 4.6 Cycle Check (Directed Graph)

$O(n + m)$

```
1  enum { UNVISITED = -1, VISITED = -2,  EXPLORED=-3};
2
3  void cycleCheck(ll at, ll n ,vll adj[], int visited[], ll
4      dfs_parent[]) {
5      visited[at] = EXPLORED;
6
7      vll neighbours = adj[at];
8      for(auto nex: neighbours) {
9          if(visited[nex] == UNVISITED) {
10             // Tree edges (part of the DFS spanning tree)
11             dfs_parent[nex] = at;
12             cycleCheck(nex, n, adj, visited);
13         }
14         else if(visited[nex] == EXPLORED) {
15             if(nex == dfs_parent[at]) {
16                 // Trivial cycle
17                 // Do something
18             }
19             else {
20                 // Non trivial cycle - Back Edge ((u, v)
21                 // such that v is the ancestor of node u but
22                 // is not part of the DFS tree)
23                 // Do something
24             }
25         }
26         else if(visited[nex] == VISITED) {
27             // Forward/Cross edge ((u, v) such that v is a
28             // descendant but not part of the DFS tree)
29             // Do something
30         }
31     }
32     visited[at] = VISITED;
33 }
```

## 4.7 Dijkstra

$O(n \log n + m \log n)$

```
1  void dijkstra(ll s, vll & d, vll & p) {
2      d.assign(n, LLONG_MAX);
3      p.assign(n, -1);
4
5      d[s] = 0;
```

```

6     priority_queue<pll, pll, greater<pll>> q;
7     q.push({0, s});
8     while (!q.empty()) {
9         ll v = q.top().second;
10        ll d_v = q.top().first;
11        q.pop();
12        if (d_v != d[v])
13            continue;
14
15        for (auto edge : adj[v]) {
16            ll to = edge.first;
17            ll len = edge.second;
18
19            if (d[v] + len < d[to]) {
20                d[to] = d[v] + len;
21                p[to] = v;
22                q.push({d[to], to});
23            }
24        }
25    }
26 }

```

## 5 Math Formulas

### 5.1 Sum of an arithmetic progression

$$S_n = \frac{n}{2}(a_1 + a_n)$$

### 5.2 Permutation with repeated elements

$$P_n = \frac{n!}{n_1!n_2!\dots n_k!}$$

### 5.3 Check if is geometric progression

$$a_i^2 = a_{i-1}a_{i+1}$$

### 5.4 Bitwise equations

$$a|b = a \oplus b + a \& b$$

$$a \oplus (a \& b) = (a|b) \oplus b$$

$$(a \& b) \oplus (a|b) = a \oplus b$$

$$a + b = a|b + a \& b$$

$$a + b = a \oplus b + 2(a \& b)$$

$$a - b = (a \oplus (a \& b)) - ((a|b) \oplus a)$$

$$a - b = ((a|b) \oplus b) - ((a|b) \oplus a)$$

$$a - b = (a \oplus (a \& b)) - (b \oplus (a \& b))$$

$$a - b = ((a|b) \oplus b) - (b \oplus (a \& b))$$

## 5.5 Cube of Binomial

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

### 5.5.1 Sum of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

### 5.5.2 Difference of Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

## 5.6 Binomial expansion

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}$$

## 6 Facts

### 6.1 XOR

#### 6.1.1 Self-inverse property

To cancel a XOR, you can XOR again the same value because  $a \oplus a = 0$ , so  $(value \oplus a) \oplus a = value$

#### 6.1.2 Identity element

$$a \oplus 0 = a$$

#### 6.1.3 Commutative

$$a \oplus b = b \oplus a$$

#### 6.1.4 Associative

$$(a \oplus b) \oplus c = a \oplus (b \oplus c)$$