

New analytic rotations for bifactor modeling and metric invariance in Exploratory Factor Analysis

Marcos Jiménez ¹, Francisco J. Abad ¹,
Eduardo Garcia-Garzon ², Luis E. Garrido ³, Vithor R. Franco ⁴

¹ Universidad Autónoma de Madrid, Spain

² Shakers, Spain

³ Pontificia Universidad Católica Madre y Maestra, Dominican Republic

⁴ Universidade São Francisco, Brazil

UAM Universidad Autónoma
de Madrid

 Psychometric Models
& Applications Lab.



Only two kinds of factor rotation?

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A SIMPLE GENERAL PROCEDURE FOR ORTHOGONAL ROTATION

ROBERT I. JENNRICH

UNIVERSITY OF CALIFORNIA AT LOS ANGELES

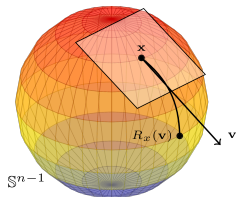
PSYCHOMETRIKA—VOL. 67, NO. 1, 7–20
MARCH 2002

A SIMPLE GENERAL METHOD FOR OBLIQUE ROTATION

ROBERT I. JENNRICH

UNIVERSITY OF CALIFORNIA AT LOS ANGELES

The gradient projection method



$$\text{Rotated } (\Lambda) = \text{Unrotated } (U) \times \text{Rotation matrix } (X)$$

$$\begin{pmatrix} \mathbf{0.37} & 0.07 & 0.00 \\ \mathbf{0.57} & 0.01 & 0.02 \\ \mathbf{0.53} & -0.03 & -0.02 \\ -0.02 & \mathbf{0.47} & -0.03 \\ 0.01 & \mathbf{0.44} & 0.01 \\ 0.01 & \mathbf{0.57} & 0.01 \\ -0.05 & 0.00 & \mathbf{0.52} \\ -0.02 & 0.00 & \mathbf{0.56} \\ 0.04 & 0.00 & \mathbf{0.63} \end{pmatrix} = \begin{pmatrix} 0.04 & -0.26 & -0.27 \\ 0.09 & -0.32 & -0.46 \\ 0.04 & -0.27 & -0.46 \\ -0.07 & -0.38 & 0.28 \\ -0.01 & -0.37 & 0.23 \\ -0.02 & -0.48 & 0.31 \\ 0.51 & 0.02 & 0.09 \\ 0.56 & 0.01 & 0.07 \\ 0.63 & -0.01 & 0.03 \end{pmatrix} \begin{pmatrix} -0.13 & -0.06 & 1 \\ 0.55 & -0.83 & 0.01 \\ 0.82 & 0.56 & 0.10 \end{pmatrix}$$

Orthogonal Projection

$$P(G) = G - 0.5(XG^T + X^T G)X$$

$$X^T X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

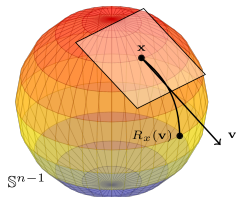
Oblique Projection

$$P(G) = G - X \text{diag}(X^T G)$$

$$X^T X = \begin{pmatrix} 1 & .35 & .44 \\ .35 & 1 & .14 \\ .44 & .14 & 1 \end{pmatrix}$$

The Partially Oblique Manifold

$$\text{Rotated } (\Lambda) = \text{Unrotated } (\mathbf{U}) \times \text{Rotation matrix } (\mathbf{X})$$



$$\begin{pmatrix} \mathbf{0.37} & 0.07 & 0.00 \\ \mathbf{0.57} & 0.01 & 0.02 \\ \mathbf{0.53} & -0.03 & -0.02 \\ -0.02 & \mathbf{0.47} & -0.03 \\ 0.01 & \mathbf{0.44} & 0.01 \\ 0.01 & \mathbf{0.57} & 0.01 \\ -0.05 & 0.00 & \mathbf{0.52} \\ -0.02 & 0.00 & \mathbf{0.56} \\ 0.04 & 0.00 & \mathbf{0.63} \end{pmatrix} = \begin{pmatrix} 0.04 & -0.26 & -0.27 \\ 0.09 & -0.32 & -0.46 \\ 0.04 & -0.27 & -0.46 \\ -0.07 & -0.38 & 0.28 \\ -0.01 & -0.37 & 0.23 \\ -0.02 & -0.48 & 0.31 \\ 0.51 & 0.02 & 0.09 \\ 0.56 & 0.01 & 0.07 \\ 0.63 & -0.01 & 0.03 \end{pmatrix} \begin{pmatrix} -0.13 & -0.06 & 1 \\ 0.55 & -0.83 & 0.01 \\ 0.82 & 0.56 & 0.10 \end{pmatrix}$$



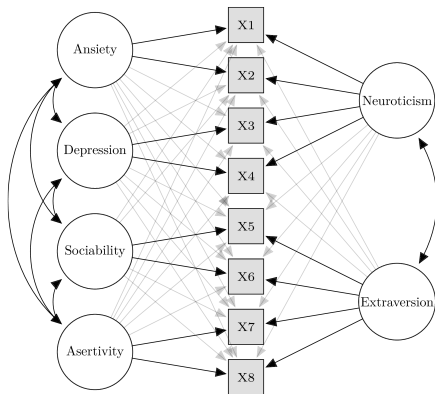
Partially Oblique Projection

$$P(\mathbf{G}) = \mathbf{G} - \mathbf{X} \left(I \circ \mathbf{V} \left(\frac{\mathbf{V}^\top \mathbf{Q} \mathbf{V}}{\mathbf{1d}^\top + \mathbf{d1}^\top} \right) \mathbf{V}^\top \right)$$

$$\mathbf{X}^\top \mathbf{X} = \begin{pmatrix} 1 & \mathbf{0} & .44 \\ \mathbf{0} & 1 & .14 \\ .44 & .14 & 1 \end{pmatrix}$$

Exploratory Bi-factor Model with Multiple General factors

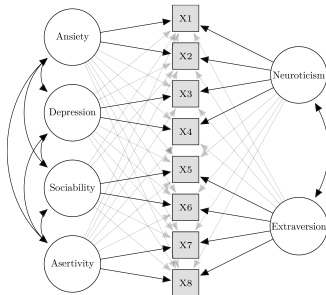
- [✓] Disentangle the variance due to the general and specific factors.
- [✓] Use any rotation criteria that you want (i.e., target, oblimin, etc.).



Factor Correlation Matrix

$$\begin{pmatrix} 1 & .30 & 0 & 0 & 0 & 0 \\ .30 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & .30 & .30 & .30 \\ 0 & 0 & .30 & 1 & .30 & .30 \\ 0 & 0 & .30 & .30 & 1 & .30 \\ 0 & 0 & .30 & .30 & .30 & 1 \end{pmatrix}$$

Estimation with the bifactor package



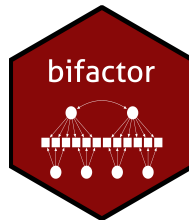
```
efast(Sigma, nfactors = 6, estimator = "ULS",  
      rotation = "target", Target = Target,  
      projection = "poblq", oblq_factors = c(2, 4))  
  
rotate(Lambda, rotation = "target", Target = Target,  
       projection = "poblq", oblq_factors = c(2, 4))
```



Mixed rotation criteria

[✓] Use Oblimin for the general factors and Target for the specific factors.

	Oblimin		Target			
	Neur	Extr	Ans	Depr	Soc	Aser
X1						
X2						
X3						
X4						
X5						
X6						
X7						
X8						



```
efast(Sigma, nfactors = 6, estimator = "ULS", Target = Target,  
      projection = "poblq", oblq_factors = c(2, 4),  
      rotation = c("oblimin", "target"), blocks = list(1:2, 3:6))
```

Multitrait-Multimethod analysis

Trait	Trait	Trait	Method	Method
0.37	0.07	0.00	0.56	0.00
0.57	0.01	0.02	0.56	0.00
0.53	0.03	0.02	0.00	0.51
0.49	0.02	0.03	0.00	0.43
0.02	0.47	0.03	0.61	0.00
0.01	0.44	0.01	0.45	0.00
0.01	0.57	0.01	0.00	0.39
0.08	0.34	0.08	0.00	0.55
0.05	0.00	0.52	0.47	0.00
0.02	0.00	0.56	0.67	0.00
0.04	0.00	0.63	0.00	0.44
0.06	0.07	0.58	0.00	0.50

Loading Matrix

	Trait	Trait	Trait	Method	Method
Trait	1	.33	.45	0	0
Trait	.33	1	.25	0	0
Trait	.45	.25	1	0	0
Method	0	0	0	1	.44
Method	0	0	0	.44	1

Factor Correlation Matrix

```
efast(Sigma, nfactors = 5, estimator = "ULS",
      rotation = "target", Target = Target,
      projection = "poblq", oblq_factors = c(3, 2))
```


Model identification in CFA

[X] warning: covariance matrix of latent variables is not positive definite

[✓] Solution: Parameterize the covariance matrix of latent variables as $\mathbf{X}^\top \mathbf{X}$ and optimize it on the partially oblique manifold.

$$\begin{pmatrix} 1 & .33 & .45 & 0 & .68 \\ .33 & 1 & .25 & 0 & .77 \\ .45 & .25 & 1 & .56 & 0 \\ 0 & 0 & .56 & 1 & .44 \\ .68 & .77 & 0 & .44 & 1 \end{pmatrix}$$

Factor correlation matrix optimized
on the Partially Oblique manifold

Summary

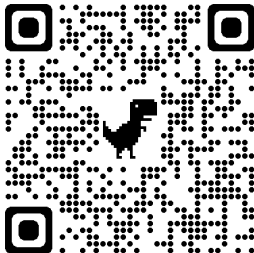
- [X] Not possible to estimate complex exploratory models with
- [✓] Uncorrelate the general and specific factors in Exploratory Bi-factor analysis with either one or multiple general factors.
- [✓] Uncorrelate trait and method factors in multitrait-multimethod analysis.
- [✓] Identify CFA models where the correlations between the latent factors are, at least, positive semi-definite.
- [✓] Similar rotations can be developed to obtain a method for metric invariance in EFA.

Thank you

Contact: marcosjnezhquez@gmail.com

 <https://github.com/Marcosjnezh/bifactor>

 [@skeptpsych](https://twitter.com/skeptpsych)



Referencias

- Jennrich, R. I. (2002). A simple general method for oblique rotation. *Psychometrika*, 67(1), 7–19.
<https://doi.org/10.1007/BF02294706>
- Jennrich, R. I. (2001). A simple general procedure for orthogonal rotation. *Psychometrika*, 66(2), 289–306. <https://doi.org/10.1007/BF02294840>
- Jiménez, M., Abad, F. J., Garcia-Garzon, E., & Garrido, L. E. (2023). Exploratory bi-factor analysis with multiple general factors. *Multivariate Behavioral Research*, 0(0), 1–18.
<https://doi.org/10.1080/00273171.2023.2189571>
- Jiménez, M., Abad, F. J., Garcia-Garzon, E., Garrido, L. E., & Franco, V. R. (2022). *Bifactor: Exploratory factor and bi-factor modeling with multiple general factors* [Manual].
<https://github.com/Marcosjnez/bifactor>
- Jiménez, M., Abad, F. J., Garcia-Garzon, E., Golino, H., Christensen, A. P., & Garrido, L. E. (2023). Dimensionality assessment in bifactor structures with multiple general factors: A network psychometrics approach. *Psychological Methods*, No Pagination Specified–No Pagination Specified. <https://doi.org/10.1037/met0000590>