New analytic rotations for bifactor modeling and metric invariance in Exploratory Factor Analysis

Marcos Jiménez [1], Francisco J. Abad [1],
Eduardo Garcia-Garzon [2], Luis E. Garrido [3], Vithor R. Franco [4]

[1] Universidad Autónoma de Madrid, Spain

[2] Shakers, Spain

[3] Pontificia Universidad Católica Madre y Maestra, Dominican Republic

[4] Universidade São Francisco, Brazil

Only two kinds of factor rotation?

PSYCHOMETRIKA—VOL. 66, NO. 2, 289–306
JUNE 2001

A SIMPLE GENERAL PROCEDURE FOR ORTHOGONAL ROTATION

ROBERT I. JENNRICH

UNIVERSITY OF CALIFORNIA AT LOS ANGELES

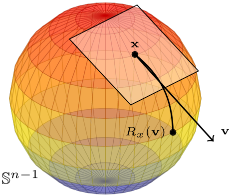
PSYCHOMETRIKA—VOL. 67, NO. 1, 7–20
MARCH 2002

A SIMPLE GENERAL METHOD FOR OBLIQUE ROTATION

ROBERT I. JENNRICH

UNIVERSITY OF CALIFORNIA AT LOS ANGELES

The gradient projection method



Rotated (Λ) = Unrotated $(\mathbf{U})$ $\times$ Rotation matrix $(\mathbf{X})$

$$\begin{pmatrix} \mathbf{0.37} & 0.07 & 0.00 \\ \mathbf{0.57} & 0.01 & 0.02 \\ \mathbf{0.53} & -0.03 & -0.02 \\ -0.02 & \mathbf{0.47} & -0.03 \\ 0.01 & \mathbf{0.44} & 0.01 \\ 0.01 & \mathbf{0.57} & 0.01 \\ -0.05 & 0.00 & \mathbf{0.52} \\ -0.02 & 0.00 & \mathbf{0.56} \\ 0.04 & 0.00 & \mathbf{0.63} \end{pmatrix} = \begin{pmatrix} 0.04 & -0.26 & -0.27 \\ 0.09 & -0.32 & -0.46 \\ 0.04 & -0.27 & -0.46 \\ -0.07 & -0.38 & 0.28 \\ -0.01 & -0.37 & 0.23 \\ -0.02 & -0.48 & 0.31 \\ 0.51 & 0.02 & 0.09 \\ 0.56 & 0.01 & 0.07 \\ 0.63 & -0.01 & 0.03 \end{pmatrix} \begin{pmatrix} -0.13 & -0.06 & 1 \\ 0.55 & -0.83 & 0.01 \\ 0.82 & 0.56 & 0.10 \end{pmatrix}$$

Orthogonal Projection

$$P(\mathbf{G}) = \mathbf{G} - 0.5\left(\mathbf{X}\mathbf{G}^\top + \mathbf{X}^\top\mathbf{G}\right)\mathbf{X}$$

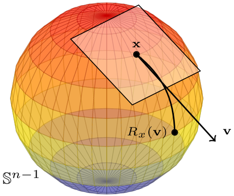
$$\mathbf{X}^\top\mathbf{X} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Oblique Projection

$$P(\mathbf{G}) = \mathbf{G} - \mathbf{X}\mathrm{diag}(\mathbf{X}^\top\mathbf{G})$$

$$\mathbf{X}^\top\mathbf{X} = \begin{pmatrix} 1 & .35 & .44 \\ .35 & 1 & .14 \\ .44 & .14 & 1 \end{pmatrix}$$

The Partially Oblique Manifold



Rotated (Λ) $=$ Unrotated $(\mathbf{U})$ $\times$ Rotation matrix $(\mathbf{X})$

$$\begin{pmatrix} \mathbf{0.37} & 0.07 & 0.00 \\ \mathbf{0.57} & 0.01 & 0.02 \\ \mathbf{0.53} & -0.03 & -0.02 \\ -0.02 & \mathbf{0.47} & -0.03 \\ 0.01 & \mathbf{0.44} & 0.01 \\ 0.01 & \mathbf{0.57} & 0.01 \\ -0.05 & 0.00 & \mathbf{0.52} \\ -0.02 & 0.00 & \mathbf{0.56} \\ 0.04 & 0.00 & \mathbf{0.63} \end{pmatrix} = \begin{pmatrix} 0.04 & -0.26 & -0.27 \\ 0.09 & -0.32 & -0.46 \\ 0.04 & -0.27 & -0.46 \\ -0.07 & -0.38 & 0.28 \\ -0.01 & -0.37 & 0.23 \\ -0.02 & -0.48 & 0.31 \\ 0.51 & 0.02 & 0.09 \\ 0.56 & 0.01 & 0.07 \\ 0.63 & -0.01 & 0.03 \end{pmatrix} \begin{pmatrix} -0.13 & -0.06 & 1 \\ 0.55 & -0.83 & 0.01 \\ 0.82 & 0.56 & 0.10 \end{pmatrix}$$

bifactor

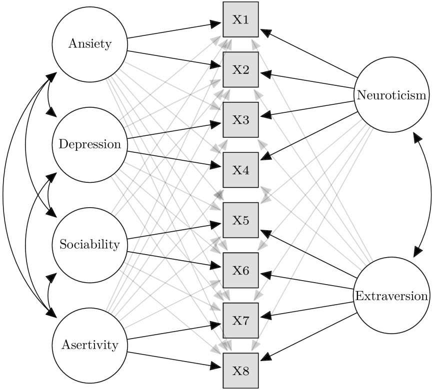
Partially Oblique Projection

$$P(\mathbf{G}) = \mathbf{G} - \mathbf{X}\left(I \circ \mathbf{V}\left(\frac{\mathbf{V}^\top \mathbf{Q} \mathbf{V}}{\mathbf{1}\mathbf{d}^\top + \mathbf{d}\mathbf{1}^\top}\right)\mathbf{V}^\top\right)$$

$$\mathbf{X}^\top \mathbf{X} = \begin{pmatrix} 1 & \mathbf{0} & .44 \\ \mathbf{0} & 1 & .14 \\ .44 & .14 & 1 \end{pmatrix}$$

Exploratory Bi-factor Model with Multiple General factors

[✓] Disentangle the variance due to the general and specific factors.
[✓] Use any rotation criteria that you want (i.e., target, oblimin, etc.).



Factor Correlation Matrix

$$\begin{pmatrix} 1 & \mathbf{.30} & 0 & 0 & 0 & 0 \\ \mathbf{.30} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \mathbf{.30} & \mathbf{.30} & \mathbf{.30} \\ 0 & 0 & \mathbf{.30} & 1 & \mathbf{.30} & \mathbf{.30} \\ 0 & 0 & \mathbf{.30} & \mathbf{.30} & 1 & \mathbf{.30} \\ 0 & 0 & \mathbf{.30} & \mathbf{.30} & \mathbf{.30} & 1 \end{pmatrix}$$

Estimation with the `bifactor` package

Loading Matrix

Factor Correlation Matrix

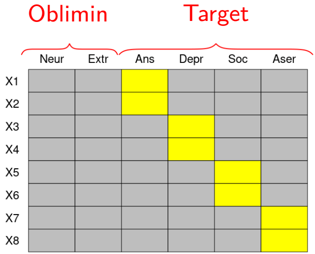
```
efast(Sigma, nfactors = 6, estimator = "ULS",
      rotation = "target", Target = Target,
      projection = "poblq", oblq_factors = c(2, 4))

rotate(Lambda, rotation = "target", Target = Target,
       projection = "poblq", oblq_factors = c(2, 4))
```

bifactor

Mixed rotation criteria

[✓] Use Oblimin for the general factors and Target for the specific factors.




```r
efast(Sigma, nfactors = 6, estimator = "ULS", Target = Target,
      projection = "poblq", oblq_factors = c(2, 4),
      rotation = c("oblimin", "target"), blocks = list(1:2, 3:6))
```

Multitrait-Multimethod analysis

Trait	Trait	Trait	Method	Method
0.37	0.07	0.00	**0.56**	0.00
0.57	0.01	0.02	**0.56**	0.00
0.53	0.03	0.02	0.00	**0.51**
0.49	0.02	0.03	0.00	**0.43**
0.02	**0.47**	0.03	**0.61**	0.00
0.01	**0.44**	0.01	**0.45**	0.00
0.01	**0.57**	0.01	0.00	**0.39**
0.08	**0.34**	0.08	0.00	**0.55**
0.05	0.00	**0.52**	**0.47**	0.00
0.02	0.00	**0.56**	**0.67**	0.00
0.04	0.00	**0.63**	0.00	**0.44**
0.06	0.07	**0.58**	0.00	**0.50**

Loading Matrix

	Trait	Trait	Trait	Method	Method
Trait	1	**.33**	**.45**	0	0
Trait	**.33**	1	**.25**	0	0
Trait	**.45**	**.25**	1	0	0
Method	0	0	0	1	**.44**
Method	0	0	0	**.44**	1

Factor Correlation Matrix

```
efast(Sigma, nfactors = 5, estimator = "ULS",
      rotation = "target", Target = Target,
      projection = "poblq", oblq_factors = c(3, 2))
```

Model identification in CFA

[✗] `warning: covariance matrix of latent variables is not positive definite`

[✓] Solution: Parameterize the covariance matrix of latent variables as $\mathbf{X}^\top\mathbf{X}$ and optimize it on the partially oblique manifold.

$$\begin{pmatrix} 1 & \mathbf{.33} & \mathbf{.45} & 0 & \mathbf{.68} \\ \mathbf{.33} & 1 & \mathbf{.25} & 0 & \mathbf{.77} \\ \mathbf{.45} & \mathbf{.25} & 1 & \mathbf{.56} & 0 \\ 0 & 0 & \mathbf{.56} & 1 & \mathbf{.44} \\ \mathbf{.68} & \mathbf{.77} & 0 & \mathbf{.44} & 1 \end{pmatrix}$$

Factor correlation matrix optimized
on the Partially Oblique manifold

Summary

[✗] Not possible to estimate complex exploratory models with

[✓] Uncorrelate the general and specific factors in Exploratory Bi-factor analysis with either one or multiple general factors.

[✓] Uncorrelate trait and method factors in multitrait-multimethod analysis.

[✓] Identify CFA models where the correlations between the latent factors are, at least, positive semi-definite.

[✓] Similar rotations can be developed to obtain a method for metric invariance in EFA.

Thank you

Contact: marcosjnezhquez@gmail.com

https://github.com/Marcosjnez/bifactor

\@skeptpsych

Referencias

Jennrich, R. I. (2002). A simple general method for oblique rotation. *Psychometrika*, *67*(1), 7–19. https://doi.org/10.1007/BF02294706

Jennrich, R. I. (2001). A simple general procedure for orthogonal rotation. *Psychometrika*, *66*(2), 289–306. https://doi.org/10.1007/BF02294840

Jiménez, M., Abad, F. J., Garcia-Garzon, E., & Garrido, L. E. (2023). Exploratory bi-factor analysis with multiple general factors. *Multivariate Behavioral Research*, *0*(0), 1–18. https://doi.org/10.1080/00273171.2023.2189571

Jiménez, M., Abad, F. J., Garcia-Garzon, E., Garrido, L. E., & Franco, V. R. (2022). *Bifactor: Exploratory factor and bi-factor modeling with multiple general fators* [Manual]. https://github.com/Marcosjnez/bifactor

Jiménez, M., Abad, F. J., Garcia-Garzon, E., Golino, H., Christensen, A. P., & Garrido, L. E. (2023). Dimensionality assessment in bifactor structures with multiple general factors: A network psychometrics approach. *Psychological Methods*, No Pagination Specified–No Pagination Specified. https://doi.org/10.1037/met0000590