Notes on Dynamic Triggering Mechanisms for Event-Triggered Control paper

Marco Sterlini

From **Dynamic Triggering Mechanisms for Event-Triggered Control** paper it is useful to extract the dynamic ETM applied to the linear system case scenario:

$$\dot{x} = Ax + Bu, x \in \mathbb{R}^n, u \in \mathbb{R}^m$$

Assuming linear feedback controller $u = Kx$ we obtain the ideal closed-loop system, we add the error considered in the paper e

$$\dot{x} = Ax + BK(x + e)$$

That being asymptotically stable presents a Lyapunov function $V(x) = x^\top Px$ with P p.d. and symmetric such That

$$(A + BK)^\top P + P(A + BK) = -Q$$

We have then

$$\begin{aligned}
&\frac{d}{dt}V(x(t)) = \dot{x}^\top Px + xP\dot{x} = \\
&= (Ax + BKx + BKe)^\top Px + x^\top P(Ax + BKx + BKe) = \\
&= x^\top \left[(A + BK)^\top P + P(A + BK)\right] x + x^\top PBKe + e^\top K^\top B^\top Px =
\end{aligned}$$

since the result is a scalar I can sum up the last two terms

$$= x^\top \left[(A + BK)^\top P + P(A + BK)\right] x + 2x^\top PBKe = -x^\top Qx + 2x^\top PBKe$$

As a first approach the static ETM that follows can be used:

$$\begin{aligned}
&t_0 = 0, \\
&t_{i+1} = \inf\left\{t \in \mathbb{R} | t > t_i \wedge \sigma x(t)^\top Qx(t) - 2x(t)^\top PBKe(t^-) \leq 0\right\}
\end{aligned} \tag{1}$$

This ensures that the next event will occur whenever the derivative of the Lyapunov function will cease to be negative. Ensuring a forced definite negativeness.

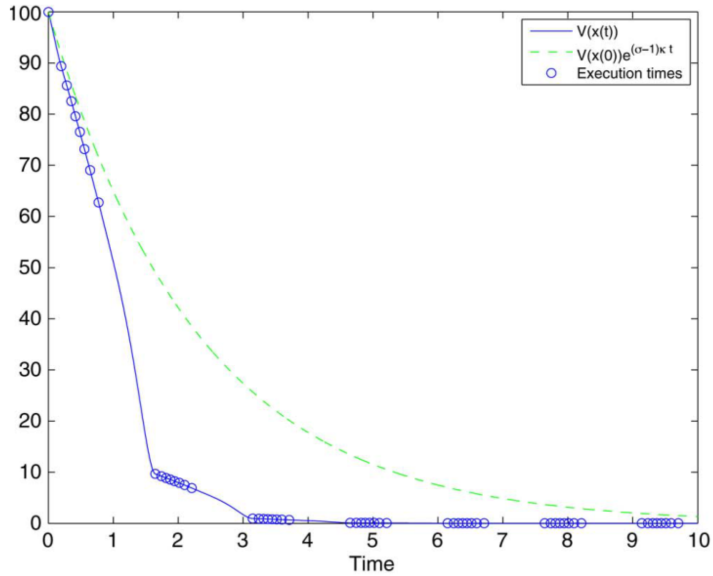

Figure 1: It is clear how the Lyapunov function $V(x)$ is strictly decreasing

Now a dynamic ETM is proposed based on the dynamic variable η governed by the differential equation

$$\dot{\eta} = -\lambda\eta + \sigma x^{\top}Qx - 2x^{\top}PBKe, \eta(0) = \eta_0 \tag{2}$$

With $\sigma \in (0,1), \lambda > 0, \eta_0 \in \mathbb{R}_0^+$ the design parameters

It is important noticing how this addition of dynamic changes the overall behavior of the ETM mechanism. In the static case (eq. 1) the value is updated every time the condition is violated. By adding a dynamic term the update will not be immediate, $\eta(t)$ integrates the previous triggering signal over time rather than responding to the current value of the signal. The term $-\lambda\eta(t)$ makes $\eta(t)$ decay over time to that it doesn't accumulate the signal history indefinitely, it smoothly adjusts based on the previous value of the signal. Overall it's a filter of the previous signal since it gives a "smoothed" version of the signal used for the static ETM.

It is then defined the new dynamic ETM:

$$\begin{aligned} t_0 &= 0, \\ t_{i+1} &= \inf\left\{t \in \mathbb{R} | t > t_i \wedge \eta(t) + \theta\left(\sigma x(t)^{\top}Qx(t) - 2x(t)^{\top}PBKe(t^-)\right) \leq 0\right\} \end{aligned} \tag{3}$$

It has been added the parameter $\theta \in \mathbb{R}_0^+$. The static ETM (eq. 1) can now be seen as the limit case scenario of the dynamic one (eq. 3) where θ tends to $+\infty$ and hence the $\eta(t)$ term can be ignored.

It is then proved how the inter-execution times with the dynamic ETM are larger with respect to the static ETM use. A new candidate Lyapunov function $W : \mathbb{R}^n \times \mathbb{R}_0^+ \to \mathbb{R}_0^+$ designed for the augmented system that sees the introduction of e and η variables:

$$W(x,\eta) = V(x) + \eta \tag{4}$$

$$\frac{d}{dt}W(x(t),\eta(t)) = (\sigma - 1)\, x(t)^\top Q x(t) - \lambda\eta(t) \tag{5}$$

That with the previous assumptions (not declared here but in the paper) that $\sigma \in (0,1)$ and $\eta(t) \geq 0$ with $\lambda > 0$ Assures that $\dot{W}$ stays definite negative and both $x(t)$ and $\eta(t)$ converge asymptotically to the origin.

It's then treated how each of the design parameters influence the overall system behavior. 3 different cases were portrayed where an explicit expression of the inter-execution time τ is drawn out.

It follows the treatment of the Lyapunov function decay rate, it is strongly influenced by σ, not influenced by θ and to understand the influence of this last term it has been used the *Quadratic Integral Performance Index*.

To choose the parameters it was first tuned the Lyapunov function decay rate by imposing $\eta_0 = 0$ and $\lambda = (1-\sigma)\kappa$ (what this κ means it is explained in the paper), here σ represents the degradation of Lyapunov function decay with respect to the ideal close-loop system. It was then chosen θ taking into account the considerations on τ formulation.

Simulation results

In comparison to the results of figure (fig. 1) the dynamic ETM has been applied in the following:

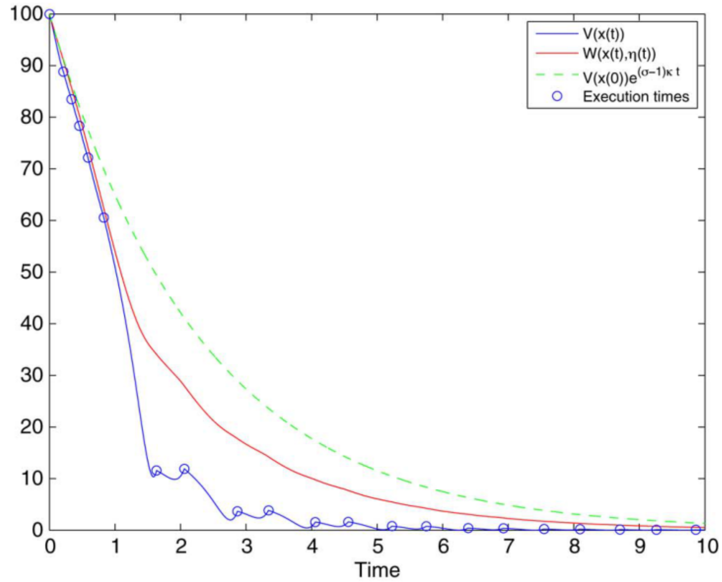

Figure 2:

It was also done the comparison with a dynamic ETM taken from one of the references that worked as follow:

$$t_0 = 0$$
$$t_{i+1} = \inf\left\{t \in \mathbb{R} | t > t_i \wedge V(x(t)) \geq e^{(\sigma-1)\kappa(t-t_i)} V(x(t_i))\right\} \tag{6}$$

In this case the update will occur every time the value of the Lyapunov function will exceed the one of the exponential as shown in the following:

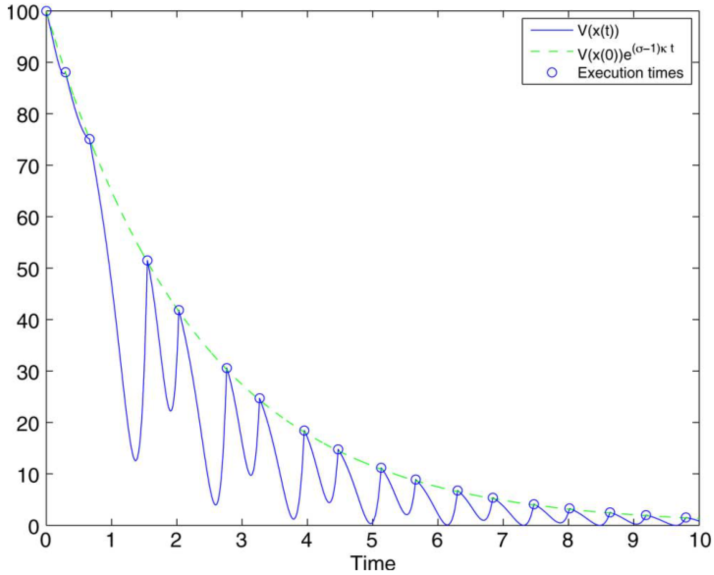

Figure 3:

In the final comment it's shown how the approach of this paper puts itself as a good compromise between the good monotonic behavior of the static ETM and the large inter-execution times of the reference approach (fig. 3) that, however, exhibit large variations in V value.

Also it is shown that using a dynamic ETM the coefficient of variability of the inter-execution times is reduced by a lot, meaning that the controller acts in a more predictable way.