Dynamic ETM integration

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Previous works analysis

The system under analysis both in [1] and [2] is characterized by a *Feed Forward Neural Network* controller characterized by l layers that takes the system state x as input.

The controller π_{ETM} of l-layers is defined by:

$$\begin{aligned} \hat{\omega}^0(k) &= x(k) \\ \nu_i(k) &= W^i \hat{\omega}^{i-1}(k) + b^i, \quad i \in \{1, \dots, l\} \\ \omega^i(k) &= \text{sat}(\nu^i(k)) \\ u(k) &= W^{l+1} \hat{\omega}^l(k) + b^{l+1} \end{aligned} \tag{1}$$

Where ν^i is the input to the i^{th} activation function, ω^i and $\hat{\omega^i}$ are the current output and the last updated output from the i^{th} layer.

To isolate the non linearity everything is collected with the vectors:

$$\begin{aligned} \nu_\phi &= [\nu^{1\top} \ \dots \ \nu^{l\top}]^\top \\ \omega_\phi &= [\omega^{1\top} \ \dots \ \omega^{l\top}]^\top \\ \hat{\omega}_\phi &= [\hat{\omega}^{1\top} \ \dots \ \hat{\omega}^{l\top}]^\top \\ \text{sat}(\nu_\phi) &= [\text{sat}(\nu^1)^\top \ \dots \ \text{sat}(\nu^l)^\top]^\top \\ \omega_\phi(k) &= \text{sat}(\nu_\phi) \end{aligned}$$

Denoting with $n_\phi = \sum_{i=1}^{l} n_i$ and n_i the number of neurons of the i^{th} layer. This leads to

$$\begin{bmatrix} u(k) \\ \nu_\phi(k) \end{bmatrix} = N \begin{bmatrix} x(k) \\ \hat{\omega}_\phi(k) \\ 1 \end{bmatrix} \tag{2}$$

Ideas so far

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \tag{3}$$

References

[1] C. de Souza, S. Tarbouriech, and A. Girard. "Event-Triggered Neural Network Control for LTI Systems". In: *IEEE Control Systems Letters* 7 (2023), pp. 1381–1386. DOI: 10.1109/LCSYS.2023.3242835.

[2] Sophie Tarbouriech, Carla De Souza, and Antoine Girard. "Layers Update of Neural Network Control via Event-Triggering Mechanism". In: *Hybrid and Networked Dynamical Systems: Modeling, Analysis and Control*. Ed. by Romain Postoyan et al. Cham: Springer Nature Switzerland, 2024, pp. 253–272. ISBN: 978-3-031-49555-7. DOI: 10.1007/978-3-031-49555-7_11. URL: https://doi.org/10.1007/978-3-031-49555-7_11.