Title

Marco Sterlini

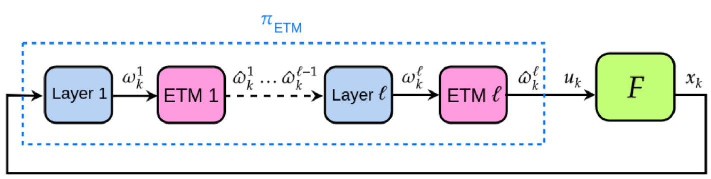
Theory recap

First works

My contribution started by exploring the works [2] and [3]. The problem addressed is the design of an event-triggered control (ETC) for discrete-time linear-time-invariant (LTI) systems. The aim was to decrease the computational load that follows the application of non-linear activation functions inside the neural network (NN) controller. Also thanks to previous works [4], [5], [1] several tools to treat these non-linearities have been studied and applied allowing the use of classical stability analysis methods to highly non-linear NN controllers. Lyapunov theory will be used to guarantee the stability of the NN controlled plant.

System under analysis

The feedback system under examination is the following:



The NN controller is called π_{ETM}, the plant to stabilize is F, and it is described by the dynamics:

$$x(k+1) = A_F x(k) + B_F u(k)$$

$$\begin{aligned}
&\hat{\omega}^0(k) = x(k),\\
&\nu^i(k) = W^i \hat{\omega}^{i-1}(k) + b^i, \quad i \in \{1, \dots, l\},\\
&\omega^i(k) = \text{sat}(\nu^i(k)),\\
&u(k) = W^{l+1} \hat{\omega}^l(k) + b^{l+1}
\end{aligned}$$

Where the first equation describes the plant F as a discrete-time linear time-invariant system, the other equations describe the behavior of the NN controller: $\hat{\omega}^i$ and ω^i are respectively the last and current output of the i^{th} layer, ν^i is the current input to the i^{th} layer. The control input $u(k)$ is hence the output of the last layer of the NN, also the weights and the biases of the i^{th} layer are indicated by W^i and b^i. As per the activation function the saturation function was used and is defined by:

$$\text{sat}(\nu_j^i(k)) = \text{sign}(\nu_j^i(k)) min(|\nu_j^i(k)|, \bar{\nu}_j^i)$$

For the j^{th} neuron of the i^{th} layer where $\bar{\nu}_j^i$ is the relative maximum limit value for the input signal. The activation function is applied element wise but a notation to isolate the non linearities is used:

$$\nu_\phi = \left[\nu^{1^\top}, \dots \nu^{l^\top}\right]^\top, \omega_\phi = \left[\omega^{1^\top}, \dots \omega^{l^\top}\right]^\top, \hat{\omega}_\phi = \left[\hat{\omega}^{1^\top}, \dots \hat{\omega}^{l^\top}\right]^\top \in \mathbb{R}^{n_\phi}$$

With n_ϕ the total number of neurons involved in the controller. It is then possible to express:

$$\mathrm{sat}(v_\phi) = \left[\mathrm{sat}(\nu^1)^\top, \dots, \mathrm{sat}(\nu^l)^\top\right]^\top \in \mathbb{R}^{n_\phi}$$

The control input $u(k)$ and the input layers' vector ν_ϕ can be expressed as a function of the state $x(k)$ and the last layers' output vector $\hat{\omega}_\phi(k)$

$$\begin{bmatrix} u(k) \\ \nu_\phi(k) \end{bmatrix} = \left[\begin{array}{c|cccc|c} \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & W^{l+1} & b^{l+1} \\ \hline W^1 & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & b^1 \\ \mathbf{0} & W^2 & \dots & \mathbf{0} & \mathbf{0} & b^2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & W^l & \mathbf{0} & b^l \end{array}\right] \begin{bmatrix} x(k) \\ \hat{\omega}_\phi(k) \\ 1 \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} u(k) \\ \nu_\phi(k) \end{bmatrix} = N \begin{bmatrix} x(k) \\ \hat{\omega}_\phi(k) \\ 1 \end{bmatrix} \text{ and } N = \begin{bmatrix} N_{ux} & N_{u\omega} & N_{ub} \\ N_{vx} & N_{v\omega} & N_{vb} \end{bmatrix}$$

Then the equilibrium point $(x_*, u_*, \nu_*, \omega_*)$ is computed,

References

[1] Carsten W Scherer. “Dissipativity and integral quadratic constraints: Tailored computational robustness tests for complex interconnections”. In: *IEEE Control Systems Magazine* 42.3 (2022), pp. 115–139.

[2] C. de Souza, S. Tarbouriech, and A. Girard. “Event-Triggered Neural Network Control for LTI Systems”. In: *IEEE Control Systems Letters* 7 (2023), pp. 1381–1386. DOI: 10.1109/LCSYS.2023.3242835.

[3] Sophie Tarbouriech, Carla De Souza, and Antoine Girard. “Layers Update of Neural Network Control via Event-Triggering Mechanism”. In: *Hybrid and Networked Dynamical Systems: Modeling, Analysis and Control*. Ed. by Romain Postoyan et al. Cham: Springer Nature Switzerland, 2024, pp. 253–272. ISBN: 978-3-031-49555-7. DOI: 10.1007/978-3-031-49555-7_11. URL: https://doi.org/10.1007/978-3-031-49555-7_11.

[4] Sophie Tarbouriech et al. *Stability and stabilization of linear systems with saturating actuators*. Springer Science & Business Media, 2011.

[5] He Yin, Peter Seiler, and Murat Arcak. “Stability analysis using quadratic constraints for systems with neural network controllers”. In: *IEEE Transactions on Automatic Control* 67.4 (2021), pp. 1980–1987.