Notes on LMI implementation with dynamic ETM

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Introduction

I am controlling a DT system of the type:

$$x^{+} = Ax + Bu \tag{1}$$

I have a pre-trained FNN controller trained to bring the system to its equilibrium. I am implementing a dynamic ETM between each layer of the FNN to reduce computational load.

I have to build up the LMI conditions in order to estimate the parameters for the dynamic ETM as follows:

With the notation χ it is intended the last state (of the neural network) used to compute an input, with s the corresponding time instant.

$$\begin{pmatrix}\chi \\ s\end{pmatrix} = \begin{cases} \begin{pmatrix}\omega \\ k\end{pmatrix}, & \text{if the memory is updated} \\ \begin{pmatrix}\chi_{k-1} \\ s_{k-1}\end{pmatrix} & \text{otherwise} \end{cases} \tag{2}$$

The rule to determine if we have to update the values can be summarized as follows:

$$s^{+} = \min_{m \in \mathbb{N}} \left\{ m \geq s+1 | \psi\left(\omega_m, \chi\right) \geq \rho \eta_m \right\} \tag{3}$$

With the triggering condition parametrized as a quadratic function

$$\psi\left(\omega, \chi\right) = \begin{bmatrix}\omega \\ \chi\end{bmatrix}^{\top} \begin{bmatrix}\Psi_1 & \Psi_2 \\ \Psi_2^{\top} & \Psi_3\end{bmatrix} \begin{bmatrix}\omega \\ \chi\end{bmatrix}, \forall \left(\omega, \chi\right) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_x} \tag{4}$$

With $\Psi = \begin{bmatrix}\Psi_1 & \Psi_2 \\ \Psi_2^{\top} & \Psi 3\end{bmatrix}$ being symmetric and satisfying:

$$\begin{bmatrix}I \\ I\end{bmatrix}^{\top} \begin{bmatrix}\Psi_1 & \Psi_2 \\ \Psi_2^{\top} & \Psi_3\end{bmatrix} \begin{bmatrix}I \\ I\end{bmatrix} = \Psi_1 + \Psi_2 + \Psi_2^{\top} + \Psi_3 \leq 0 \tag{5}$$

So that right after an update we don't have a new triggering whenever $\omega = \chi$

Also we give a dynamic to the threshold ρ as follows:

$$\eta^+ = (\lambda + \rho)\eta - \psi\left(\omega, \chi\right), \forall \eta_0 \geq 0 \tag{6}$$

That for $\rho \geq 0$ and $(\lambda+\rho) \in [0, 1)$ makes that the threshold stays initially high right after an update when ψ is still non negative, decays with time with rate $(\lambda+\rho)$ and takes into account the incresing values of ψ

The design parameters that need to be found are (Ψ, ρ, λ).

As a first step it will be attempted to adapt the work done in the CSS paper with a dynamic ETM.

Using the quadratic abstraction that folllows we are able to deal with the non linearities of the activation functions.

Assumption 1 *There exist $S \in \mathbb{S}^s_+$,$T \in \mathbb{R}^{s\times p}$ and $R \in \mathbb{S}^p_-$ such that, for all $v_1, v_2 \in \mathbb{R}^s$,*

$$\begin{bmatrix} v_1 - v_2 \\ \phi(v_1) - \phi(v_2) \end{bmatrix}^\top \begin{bmatrix} S & T \\ \star & R \end{bmatrix} \begin{bmatrix} v_1 - v_2 \\ \phi(v_1) - \phi(v_2) \end{bmatrix} \geq 0 \tag{7}$$

Now using the Lyapunov function $V(x, \eta) = x^\top P x + \eta$ and considering the sign of $\Delta V = V(x^+, \eta^+) - V(x, \eta)$

$$
\begin{aligned}
&\Delta V = (x^+)^\top P x - x^\top P x + \eta^+ - \eta \\
&\Delta V \leq \Delta V + \tau \begin{bmatrix} v - v^* \\ \phi(v) - \phi(v^*) \end{bmatrix}^\top \begin{bmatrix} S & T \\ \star & R \end{bmatrix} \begin{bmatrix} v - v^* \\ \phi(v) - \phi(v^*) \end{bmatrix} \text{ with } \tau \geq 0 \\
&\Delta V \leq (Ax + Bu)^\top P (Ax + Bu) - x^\top P x + (\lambda + \rho)\eta - \psi(\omega, \chi) - \eta + \\
&+ \tau \begin{bmatrix} v - v^* \\ \phi(v) - \phi(v^*) \end{bmatrix}^\top \begin{bmatrix} S & T \\ \star & R \end{bmatrix} \begin{bmatrix} v - v^* \\ \phi(v) - \phi(v^*) \end{bmatrix}
\end{aligned}
$$

With condition from eq. 3 we have $\eta \leq \psi(\omega, \chi)/\rho$

$$
\begin{aligned}
&\leq x^\top (A^\top P A - P) x + x^\top A^\top P B u + u^\top B^\top P A x + u^\top B^\top P B u + \\
&\frac{(\lambda + \rho - 1)}{\rho} \psi(\omega, \chi) - \psi(\omega, \chi) + \tau \begin{bmatrix} v - v^* \\ \phi(v) - \phi(v^*) \end{bmatrix}^\top \begin{bmatrix} S & T \\ \star & R \end{bmatrix} \begin{bmatrix} v - v^* \\ \phi(v) - \phi(v^*) \end{bmatrix} =
\end{aligned}
$$

$$
\begin{aligned}
&= x^\top (A^\top P A - P) x + x^\top A^\top P B u + u^\top B^\top P A x + u^\top B^\top P B u + \\
&\frac{(\lambda - 1)}{\rho} \psi(\omega, \chi) + \tau \begin{bmatrix} v - v^* \\ \phi(v) - \phi(v^*) \end{bmatrix}^\top \begin{bmatrix} S & T \\ \star & R \end{bmatrix} \begin{bmatrix} v - v^* \\ \phi(v) - \phi(v^*) \end{bmatrix} =
\end{aligned}
$$

$$
\begin{aligned}
&= \begin{bmatrix} x \\ u \end{bmatrix}^\top \begin{bmatrix} A^\top P A - P & A^T P B \\ \star & B^\top P B \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} + \frac{\lambda - 1}{\rho} \begin{bmatrix} \omega \\ \chi \end{bmatrix}^\top \begin{bmatrix} \Psi_1 & \Psi_2 \\ \Psi_2^\top & \Psi_3 \end{bmatrix} \begin{bmatrix} \omega \\ \chi \end{bmatrix} + \\
&+ \begin{bmatrix} v - v^* \\ \phi(v) - \phi(v^*) \end{bmatrix}^\top \begin{bmatrix} S & T \\ \star & R \end{bmatrix} \begin{bmatrix} v - v^* \\ \phi(v) - \phi(v^*) \end{bmatrix}
\end{aligned}
$$

Now the quantities u, v, ω must be expressed as a function of x and $\phi(\cdot)$ to simplify the expression. In particular we have

$$
\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} N_{ux} & N_{u\omega} & N_{ub} \\ N_{vx} & N_{v\omega} & N_{vb} \end{bmatrix} \begin{bmatrix} x \\ \omega \\ 1 \end{bmatrix} \tag{8}
$$

We have that $N_{ux} = 0$ so

$$
u = N_{u\omega}\omega + N_{ub} \tag{9}
$$
$$
v = V_{vx} x + N_{v\omega}\omega + N_{vb} \tag{10}
$$

Hence by posing

- $\omega \to \phi(v)$
- $u \to N_{u\omega}\phi(v) + N_{ub}$
- $v \to N_{vx} x + N_{v\omega}\phi(v) + N_{vb}$

$$
\begin{bmatrix} x \\ N_{u\omega}\phi(v)+N_{ub} \end{bmatrix}^\top \begin{bmatrix} A^\top PA-P & A^T PB \\ \star & B^\top PB \end{bmatrix} \begin{bmatrix} x \\ N_{u\omega}\phi(v)+N_{ub} \end{bmatrix} +
\frac{\lambda-1}{\rho} \begin{bmatrix} \phi(v) \\ \chi \end{bmatrix}^\top \begin{bmatrix} \Psi_1 & \Psi_2 \\ \Psi_2^\top & \Psi_3 \end{bmatrix} \begin{bmatrix} \phi(v) \\ \chi \end{bmatrix} +
+ \begin{bmatrix} N_{vx}x+N_{v\omega}\phi(v)+N_{vb} \\ \phi(v) \end{bmatrix}^\top \begin{bmatrix} S & T \\ \star & R \end{bmatrix} \begin{bmatrix} N_{vx}x+N_{v\omega}\phi(v)+N_{vb} \\ \phi(v) \end{bmatrix}
$$

It is possible to rewrite everything with respect to the augmented state

$$
\zeta = (x^\top \chi^\top \phi(v)^\top I)
$$

I start by the first matrix multiplication indicated by $\mathbb{B}_1$ using the temporary variables:

- $N_{u\omega} \to A$
- $N_{ub} \to B$
- $A^\top PA - P \to W$
- $A^\top PB \to Y$
- $B^\top PB \to Z$

$$
\begin{bmatrix} x \\ A\phi(v)+B \end{bmatrix}^\top \begin{bmatrix} W & Y \\ Y^\top & Z \end{bmatrix} \begin{bmatrix} x \\ A\phi(v)+B \end{bmatrix} =
$$
$$
= \begin{bmatrix} x^\top W + \phi(v)^\top A^\top Y^\top + B^\top Y^\top \\ x^\top Y + \phi(v)^\top A^\top Z + B^\top Z \end{bmatrix}^\top \begin{bmatrix} x \\ A\phi(v)+B \end{bmatrix} =
$$

$$
= x^\top W x + \phi(v)^\top A^\top Y^\top x + B^\top Y^\top x + x^\top Y A\phi(v) +
+ x^T YB + \phi(v)^\top A^\top ZA\phi(v) + \phi(v)^\top A^\top ZB + B^T ZA\phi(v) + B^\top ZB
$$

Regrouping everything accordingly

$$
\begin{bmatrix} x \\ \chi \\ \phi(v) \\ I \end{bmatrix}^\top \begin{bmatrix} W & 0 & YA & YB \\ \star & 0 & 0 & 0 \\ \star & \star & A^\top ZA & A^\top ZB \\ \star & \star & \star & B^\top ZB \end{bmatrix} \begin{bmatrix} x \\ \chi \\ \phi(v) \\ I \end{bmatrix} =
$$
$$
\mathbb{B}_1 = \zeta^\top \begin{bmatrix} A^T PA - P & 0 & A^\top PBN_{u\omega} & A^\top PBN_{ub} \\ \star & 0 & 0 & 0 \\ \star & \star & N_{u\omega}^\top B^\top PBN_{u\omega} & N_{u\omega}^\top B^\top PBN_{ub} \\ \star & \star & \star & N_{ub}^\top B^\top PBN_{ub} \end{bmatrix} \zeta \tag{11}
$$

I proceed with the second block $\mathbb{B}_2$:

$$\mathbb{B}_2 = \zeta^\top \begin{bmatrix} 0 & 0 & 0 & 0 \\ \star & \frac{\lambda-1}{\rho}\Psi_3 & \frac{\lambda-1}{\rho}\Psi_2^\top & 0 \\ \star & \star & \frac{\lambda-1}{\rho}\Psi_1 & 0 \\ \star & \star & \star & 0 \end{bmatrix} \zeta \tag{12}$$

Lastly I treat $\mathbb{B}_3$

This time I use the temporary variables:

- $N_{vx} \to A$
- $N_{v\omega} \to B$
- $N_{vb} \to C$

$$\begin{aligned}
&\begin{bmatrix} Ax + B\phi(v) + C \\ \phi(v) \end{bmatrix}^\top \begin{bmatrix} S & T \\ \star & R \end{bmatrix} \begin{bmatrix} Ax + B\phi(v) + C \\ \phi(v) \end{bmatrix} = \\
&\begin{bmatrix} x^\top A^\top S + \phi(v)^\top B^\top S + C^T S + \phi(v)^\top T^\top \\ x^\top A^\top T + \phi(v)^\top B^\top T + C^T T + \phi(v)^\top R \end{bmatrix}^\top \begin{bmatrix} Ax + B\phi(v) + C \\ \phi(v) \end{bmatrix} = \\
&= x^\top A^\top SAx + \phi(v)^\top B^\top SAx + C^T SAx + \phi(v)^\top T^\top Ax + x^\top A^\top SB\phi(v) + \\
&+ \phi(v)^\top B^\top SB\phi(v) + C^T SB\phi(v) + \phi(v)^\top T^\top B\phi(v) + x^\top A^\top SC + \\
&+ \phi(v)^\top B^\top SC + C^T SC + \phi(v)^\top T^\top C + x^\top A^\top T\phi(v) + \\
&+ \phi(v)^\top B^\top T\phi(v) + C^T T\phi(v) + \phi(v)^\top R\phi(v)
\end{aligned}$$

It follows that:

$$\zeta^\top \begin{bmatrix} A^\top SA & 0 & A^\top SB + A^\top T & A^\top SC \\ \star & 0 & 0 & 0 \\ \star & \star & B^\top SB + T^\top B + B^\top T + R & B^\top SC + T^\top C \\ \star & \star & \star & C^\top SC \end{bmatrix} \zeta$$

$$\mathbb{B}_3 = \tau\zeta^\top \begin{bmatrix} N_{vx}^\top SN_{vx} & 0 & N_{vx}^\top SN_{v\omega} + N_{vx}^\top T & N_{vx}^\top SN_{vb} \\ \star & 0 & 0 & 0 \\ \star & \star & N_{v\omega}^\top SN_{v\omega} + T^\top N_{v\omega} + N_{v\omega}^\top T + R & N_{v\omega}^\top SN_{vb} + T^\top N_{vb} \\ \star & \star & \star & N_{vb}^\top SN_{vb} \end{bmatrix} \zeta \tag{13}$$

Finally by summing the blocks $M = \mathbb{B}_1 + \mathbb{B}_2 + \mathbb{B}_3$

$$M = \begin{bmatrix} A^\top PA - P + \tau N_{vx}^\top SN_{vx} & 0 & A^\top PBN_{u\omega} + \tau\left(N_{vx}^\top SN_{v\omega} + N_{vx}^\top T\right) & A^\top PBN_{ub} + \tau N_{vx}^\top SN_{vb} \\ \star & \frac{\lambda-1}{\rho}\Psi_3 & \frac{\lambda-1}{\rho}\Psi_2^\top & 0 \\ \star & \star & N_{u\omega}^\top B^\top PBN_{u\omega} + \frac{\lambda-1}{\rho}\Psi_1 + \tau\left(N_{v\omega}^\top SN_{v\omega} + T^\top N_{v\omega} + N_{v\omega}^\top T + R\right) & N_{u\omega}^\top B^\top PBN_{ub} + \tau\left(N_{v\omega}^\top SN_{vb} + T^\top N_{vb}\right) \\ \star & \star & \star & N_{ub}^\top B^\top PBN_{ub} + \tau N_{vb}^\top SN_{vb} \end{bmatrix}$$

$$(14)$$

I consider the equilibrium points:

- $x_\star = [I - A - BR_\omega]^{-1} BR_b$
- $u_\star = R_\omega x_\star + R_b$
- $\nu_\star = RN_{vx}x_\star + RN_{vb}$
- $\omega_\star = \nu_\star$

with

- $R_\omega = N_{ux} + N_{u\omega}RN_{vx}$
- $R = (I - N_{v\omega}^{-1})$
- $R_b = N_{u\omega}RN_{vb} + N_{ub}$

Solved doubts

- What is the correct point where I should plug the equilibrium conditions? $(x_\star, u_\star, \nu_\star, \omega_\star)$ taken from CSS paper.
  - They should be injected at the step when it is first considered $\Delta V = (x^+)^\top Px^+ + \eta^+ - x^\top Px - \eta$ as a function of the incremental variables:

$$\Delta V = (x^+ - x_\star^+)^\top P(x^+ - x_\star^+) + \eta^+ - (x - x_\star)^\top P(x - x_\star) - \eta \quad (15)$$

- As regards χ equilibrium, could I use $\chi_\star = \phi(v)_\star = \omega_\star = v_\star = RN_{vx}x_\star + RN_{vb}$?
  - Yes
- I considered $N_{ux} = 0$ since $u(k)$ depends only on the last layer of the NN controller. In the paper it is considered non null since they potentially consider cases where $u(k)$ is the sum of the output of the NN and a direct state feedback. Should I consider it non null too?
  - For ease of computation consider it null, analogous computations can be portrayed for $N_{ux} \neq \mathbf{0}$
- In the notes we have both the presence of P and P^{-1} due to Schur complement. Since matrix A is known the term $A^\top PA - P$ is affine in P and can be directly fed into a LMI, why it the Schur complement applied there? In the paper of reference I think the change of variables made sense to handle the uncertainty on $[\mathcal{A}, \mathcal{B}]$ but here don't we have the matrices A and B known?
  - Yes, this was justified by the unknown $[\mathcal{A}, \mathcal{B}]$ in the paper, with change of variables and the application of **Finsler's lemma** the presence of P and P^{-1} was handled, it could be left like this too here but, at least for me, it doesn't make sense to make things harder since the term $A^\top PA - P$ can be directly handled by LMI solvers.

Doubts

- How should I inject $\omega_\star, \chi_\star$? Directly at the step when I first introduce $\psi(\omega, \chi)$? So instead of $\psi(\omega, \chi)$ I introduce $\psi(\omega - \omega_\star, \chi - \chi_\star)$?

Now I reconsider everything with respect to the incremental variables

$$\Delta V = (x^+ - x_\star^+)^\top P(x^+ - x_\star^+) + \eta^+ - (x - x_\star)^\top P(x - x_\star) - \eta$$

$$\Delta V \leq \Delta V + \tau \begin{bmatrix} v - v_\star \\ \phi(v) - \phi(v_\star) \end{bmatrix}^\top \begin{bmatrix} S & T \\ \star & R \end{bmatrix} \begin{bmatrix} v - v_\star \\ \phi(v) - \phi(v_\star) \end{bmatrix}$$

Expanding now the expressions and introducing ψ as $\psi(\omega - \omega_\star, \chi - \chi_\star)$

$$\Delta V \leq (Ax + Bu - Ax_\star - Bu_\star)^\top P(Ax + Bu - Ax_\star - Bu_\star) - (x - x_\star)^\top P(x - x_\star) +$$

$$+ (\lambda + \rho - 1)\eta - \psi(\omega - \omega_\star, \chi - \chi_\star) + \tau \begin{bmatrix} v - v_\star \\ \phi(v) - \phi(v_\star) \end{bmatrix}^\top \begin{bmatrix} S & T \\ \star & R \end{bmatrix} \begin{bmatrix} v - v_\star \\ \phi(v) - \phi(v_\star) \end{bmatrix}$$

Now I regroup everything with respect to the incremental variables and with the previous assumptions on η and, due to a merging of notation, considering $\omega = \phi(v)$

$$\Delta V \leq \begin{bmatrix} x - x_\star \\ u - u_\star \end{bmatrix}^\top \begin{bmatrix} A^\top PA - P & A^\top PB \\ \star & B^\top PB \end{bmatrix} \begin{bmatrix} x - x_\star \\ u - u_\star \end{bmatrix} +$$

$$+ \frac{\lambda - 1}{\rho} \begin{bmatrix} \phi(v) - \phi(v_\star) \\ \chi - \chi_\star \end{bmatrix}^\top \begin{bmatrix} \Psi_1 & \Psi_2 \\ \star & \Psi_3 \end{bmatrix} \begin{bmatrix} \phi(v) - \phi(v_\star) \\ \chi - \chi_\star \end{bmatrix} +$$

$$+ \tau \begin{bmatrix} v - v_\star \\ \phi(v) - \phi(v_\star) \end{bmatrix}^\top \begin{bmatrix} S & T \\ \star & R \end{bmatrix} \begin{bmatrix} v - v_\star \\ \phi(v) - \phi(v_\star) \end{bmatrix}$$

Now I should regroup with respect to the augmented vector $\zeta = \left(x^\top \chi^\top \phi(v)^\top\right)$, to do so I should express the current variables as a function of the latter.

Recalling that

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} N_{ux} & N_{u\omega} & N_{ub} \\ N_{vx} & N_{v\omega} & N_{vb} \end{bmatrix} \begin{bmatrix} x \\ \omega \\ 1 \end{bmatrix} \tag{16}$$

And I want to get rid of u, v, ω, in this case due to the merge of notation of two different papers I have $\omega = \phi(v)$. Then I have

- $u = N_{ux}x + N_{u\omega}\phi(v) + N_{ub}$
- $v = N_{vx}x + N_{v\omega}\phi(v) + N_{vb}$

Hence I rewrite the incremental variables as

- $(u - u_\star) = N_{ux}x + N_{u\omega}\phi(v) + N_{ub} - N_{ux}x_\star - N_{u\omega}\phi(v_\star) - N_{ub}$ That is equal to

$$(u - u_\star) = N_{ux}(x - x_\star) + N_{u\omega}(\phi(v) - \phi(v_\star)) \tag{17}$$

- $(v - v_\star) = N_{vx}x + N_{v\omega}\phi(v) + N_{vb} - N_{vx}x_\star - N_{v\omega}\phi(v_\star) - N_{vb}$ That is equal to

$$(v - v_\star) = N_{vx}(x - x_\star) + N_{v\omega}(\phi(v) - \phi(v_\star)) \tag{18}$$

I rewrite everything with respect to the incremental variables indicated with $(y - y_\star) = \delta y$ for brevity

$$\begin{aligned}
\Delta V \leq & \begin{bmatrix} \delta x \\ N_{ux}\delta x + N_{u\omega}\delta\phi(v) \end{bmatrix}^\top \begin{bmatrix} A^\top PA - P & A^\top PB \\ \star & B^\top PB \end{bmatrix} \begin{bmatrix} \delta x \\ N_{ux}\delta x + N_{u\omega}\delta\phi(v) \end{bmatrix} + \\
& + \frac{\lambda - 1}{\rho} \begin{bmatrix} \delta\phi(v) \\ \delta\chi \end{bmatrix}^\top \begin{bmatrix} \Psi_1 & \Psi_2 \\ \star & \Psi_3 \end{bmatrix} \begin{bmatrix} \delta\phi(v) \\ \delta\chi \end{bmatrix} + \\
& + \tau \begin{bmatrix} N_{vx}\delta x + N_{v\omega}\delta\phi(v) \\ \delta\phi(v) \end{bmatrix}^\top \begin{bmatrix} S & T \\ \star & R \end{bmatrix} \begin{bmatrix} N_{vx}\delta x + N_{v\omega}\delta\phi(v) \\ \delta\phi(v) \end{bmatrix}
\end{aligned}$$

Regrouping with respect to $\zeta_\star = (\delta x, \delta\chi, \delta\phi(v))$

$$\Delta V \leq \zeta_\star^\top \left[\mathbb{B}_1 + \mathbb{B}_2 + \mathbb{B}_3\right] \zeta_\star = \zeta_\star^\top M \zeta_\star$$

$$\mathbb{B}_1 = \begin{bmatrix} A^\top PA - P + He\left\{A^\top PBN_{ux}\right\} + N_{ux}^\top B^\top PBN_{ux} & 0 & A^\top PBN_{u\omega} + N_{ux}^\top B^\top PBN_{u\omega} \\ \star & 0 & 0 \\ \star & \star & N_{u\omega}^\top B^\top PBN_{u\omega} \end{bmatrix} \tag{19}$$

$$\mathbb{B}_2 = \begin{bmatrix} 0 & 0 & 0 \\ \star & \frac{\lambda-1}{\rho}\Psi_3 & \frac{\lambda-1}{\rho}\Psi_2^\top \\ \star & \star & \frac{\lambda-1}{\rho}\Psi_1 \end{bmatrix} \tag{20}$$

$$\mathbb{B}_3 = \tau \begin{bmatrix} N_{vx}^\top SN_{vx} & 0 & N_{vx}^\top SN_{v\omega} + N_{vx}^\top T \\ \star & 0 & 0 \\ \star & \star & R + N_{v\omega}^\top SN_{v\omega} + He\left\{N_{v\omega}^\top T\right\} \end{bmatrix} \tag{21}$$

$$M = \begin{bmatrix} A^\top PA - P + He\left\{A^\top PBN_{ux}\right\} + N_{ux}^\top B^\top PBN_{ux} + \tau N_{vx}^\top SN_{vx} & 0 & A^\top PBN_{u\omega} + N_{ux}^\top B^\top PBN_{u\omega} + \tau\left(N_{vx}^\top SN_{v\omega} + N_{vx}^\top T\right) \\ \star & \frac{\lambda-1}{\rho}\Psi_3 & \frac{\lambda-1}{\rho}\Psi_2^\top \\ \star & \star & N_{u\omega}^\top B^\top PBN_{u\omega} + \frac{\lambda-1}{\rho}\Psi_1 + \tau\left(R + N_{v\omega}^\top SN_{v\omega} + He\left\{N_{v\omega}^\top T\right\}\right) \end{bmatrix} \tag{22}$$

LMI conditions

The conditions that need to be taken into account are:

- $P > 0$
- $\rho \geq 0$
- $\tau \geq 0$
- $\lambda + \rho \geq 0$
- $\lambda + \rho < 1$
- $\Psi_1 + \Psi_2 + \Psi_2^\top + \Psi_3 \leq 0$
- $M < 0$

1 FINAL CONSIDERATIONS

After implementing everything I forgot the conditions on CSS paper are local and the sector conditions here are global hence nothing makes sense. I will start over trying to implement the static etm solution with the trained neural network that is given by `https://github.com/heyinUCB/\Stability-Analysis-using-Quadratic-Constraints-for-Sys`
I will have also an introduction to the argument in Sophie's book "Stability and Stabilization of Linear Systems with saturating actuators"