1. a) $t(n) = \begin{cases} 1 & , n < 2 \\ 2t(n-2) & , n > 1 \end{cases}$

$$t(n) = 2t(n-2)$$
$$= 2(2t(n-2-2)) = 4t(n-4)$$
$$= 4(2t(n-4-2)) = 8t(n-6)$$
$$= 2^k t(n-2k)$$

Para $t(n) = 1$ $\quad \rightarrow 2^{\frac{n-1}{2}} \cdot t(1) = 2^{(n-1)/2}$

$$n - 2k = 1$$
$$k = \frac{n-1}{2}$$

Classes: $\lim\limits_{n \to \infty} \frac{2^{(n-1)/2}}{n} = \infty$, Somente Ω

b) $t(n) = \begin{cases} 1 & , n <= 1 \\ 2t(n/4) + n \log(n) & , n > 1 \end{cases}$

É possível aplicar o teorema mestre:

$a = 2$, $b = 4$; $f(n) = n \log(n)$

$$n^{\log_4 2} = n^{1/2}$$

Para $f(n) = \Omega(n^{\log_4 2 + \varepsilon})$, com $\varepsilon = \frac{1}{2}$ aplica-se o 3º caso, logo:

$t(n) = \Theta(n \log_2 n)$, então as classes são Y, Z e T

c) $t(n) = \begin{cases} 1 & , n <= 1 \\ 2t(n|2) & , n > 1 \end{cases}$

$t(n) = 2t(n|2)$

$= 2.(2t(n|2|2)) = 4t(n|4)$

$= 2.(2.(2t(n|2|2|2))) = 8t(n|8)$

$= 2^k t(n|2^k)$

Para $t(n) = 1$ $\rightarrow 2^{\log_2 n} . t(1) = 2^{\log_2 n} = n$

$\frac{n}{2^k} = 1$

$k = \log_2 n$

Classes: $\lim_{n \to \infty} \frac{n}{n^2} = 0 \therefore$ T

$\lim_{n \to \infty} \frac{n}{n} = 1 \therefore$ Z

d) $t(n) = \begin{cases} 1 & , n <= 1 \\ 4t(n/8) + \log(n) & , n > 1 \end{cases}$

$a = 4$

$b = 8$

$f(n) = \log(n)$

$n^{\log_8 4} = n^{0,666}$

$\varepsilon = 0,333\ldots$

$$\lim_{n \to \infty} \frac{\log(n)}{n^{0,666 - \varepsilon}} = \lim_{n \to \infty} \frac{\frac{1}{n \ln(10)}}{n^{0,333}} = 0$$

1º Caso. Logo $t(n) = \Theta(n^{0,666})$, então podemos apenas afirmar que $t(n)$ é T

2 $\Theta(2^n) \sim 2^{40}$ — 1 M

2^{80} — X

$2^{40} X = 2^{80}$

$X = 2^{40}$ minutos

Como o tempo é constante, cresce a mesma medida que 2^n, em relação à quantidade de entradas, podemos fazer a regra de três.

a) $t(n) = \begin{cases} 1 & , n <= 1 \\ 4t(n/2) + n^2\sqrt{n} & , n > 1 \end{cases}$

$a = 4$

$b = 2$

$f(n) = n^2 . n^{1/2} = n^{2,5}$

$n^{\log_2 4} = n^2$

$\varepsilon = 0,5$

$$\lim_{n \to \infty} \frac{n^{2,5}}{n^{2+0,5}} = \frac{n^{2,5}}{n^{2,5}} = 1$$

3º Caso: Logo $T(n) = \Theta(n^2\sqrt{n})$

b) $t(n) = \begin{cases} 1 & , n <= 1 \\ 2t(n/4) + n^2, & n > 1 \end{cases}$

$a = 2$

$b = 4$

$f(n) = n^2$

$n^{\log_4 2}$ $n^{1/2}$

$\varepsilon = 0{,}5$

$$\lim_{n \to \infty} \frac{n^2}{n^{\frac{1}{2} + \varepsilon}} = \infty$$

3º Caso: Logo $t(n) = \Theta(n^2)$

4.
```
int faz_algo(){
    return 1;
}

int F(int n){
    if(n<=1){
        faz_algo();
    } else {
        F(n/8);
        F(n/8);
        F(n/8);
        F(n/8);
        for( ; n>0 ; n/=2){
            faz_algo();
        }
    }
    return;
```

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5. a) $f(n) = 2^{1,5}$ $\therefore$ $f(n)$ é $O(2^n)$ $\rightsquigarrow$ Falso

$$\lim_{n \to \infty} \frac{2^{1,5n}}{2^n} = 2^{0,5n} = \infty$$

b) $f(n) = 2,2n - \log(n)$ $\therefore$ $f(n)$ é $\Omega(3n)$ $\rightsquigarrow$ Verdadeiro

$$\lim_{n \to \infty} \frac{2,2n - \log(n)}{3n} = \lim_{n \to \infty} \frac{2,2\cancel{n}}{3\cancel{n}} - \lim_{n \to \infty} \frac{\log(n)}{3n}$$

$$= \frac{2,2}{3} - 0 = \frac{2,2}{3}$$

c) $f(n) = n \log(n)$ $\therefore$ $f(n) = O(n^2)$ e $f(n) = \Omega(n)$ e $f(n) = \Theta(n \log(n))$ $\rightsquigarrow$ Verdadeiro

$$\lim_{n \to \infty} \frac{\cancel{n} \log(n)}{n^{\cancel{2}}} = \lim_{n \to \infty} \frac{\log(n)}{n} = 0$$

$$\lim_{n \to \infty} \frac{\cancel{n} \log(n)}{\cancel{n}} = \lim_{n \to \infty} \log(n) = \infty$$

$$\lim_{n \to \infty} \frac{n \cancel{\log}(n)}{n \cancel{\log}(n)} = \lim_{n \to \infty} 1 = 1$$

6\. a) $t(n) = \begin{cases} 0 & , n \leq 3 \\ 2t(n-4)+1 & , n \geq 4 \end{cases}$

$t(n) = 2t(n-4)+1$

$= 2.(2t(n-4-4)+1)+1$

$= 2^2 t(n-8)+2+1$

$= 2^2(2t(n-8-4)+1)+2+1$

$= 2^3 t(n-12)+4+2+1$

$= 2^K t(n-4K) + \sum_{i=0}^{K-1} 2^i$

$= 2^K t(n-4K) + 2^{(K-1)+1} - 1$

$\hookrightarrow 0 + 2^{(n-3)/4} - 1 \therefore t(n) = 2^{(n-3)/4} - 1$

Para $t(n)=0$

$n - 4K = 3$

$K = (n-3)/4$

Logo $t(n) = O(2^n)$

b) $t(n) = \begin{cases} 0 & , n = 0 \\ 1 & , n = 1 \\ 3t(n-1)+4t(n-2) & , n \geq 2 \end{cases}$

$t(n) = 3t(n-1)+4t(n-2)$

$t(n) - 3t(n-1) - 4t(n-2) = 0$

$E^2 - 3E - 4 = 0$

$a = 1$
$b = -3$
$c = -4$

$$\frac{3 \pm \sqrt{9 - 4.1 \cdot -4}}{2} \quad \frac{3+5}{2} = 4 \quad \frac{3-5}{2} = -1$$

$\hookrightarrow t(n) = C_1 4^n + C_2 . -1^n$

$t(0) = C_1 + C_2 = 0$
$t(1) = 4C_1 - C_2 = 1$

$5C_1 = 1$
$C_1 = 1/5$
$C_2 = -1/5$

$\hookrightarrow t(n) = \frac{4^n}{5} - \frac{(-1)^n}{5}$

Logo $t(n) = O(4^n)$

c) $t(n) = \begin{cases} 1 & , n = 1 \\ 4t(n/2) + n^2 \log n & , n > 1 \end{cases}$

$t(n) = 4t(n/2) + n^2 \log n$

$= 4(4t(n/2/2) + \left(\frac{n}{2}\right)^2 \log \frac{n}{2}) + n^2 \log n$

$= 4^2 t(n/2^2) + 4\left(\frac{n}{2}\right)^2 \log \frac{n}{2} + n^2 \log n$

$= 4^2(4t(n/2^2/2) + \left(\frac{n}{2^2}\right)^2 \log \frac{n}{2^2}) + 4\left(\frac{n}{2}\right)^2 \log \frac{n}{2} + n^2 \log n$

$= 4^3 t(n/2^3) + 4^2\left(\frac{n}{2^2}\right)^2 \log \frac{n}{2^2} + 4\left(\frac{n}{2}\right)^2 \log \frac{n}{2} + n^2 \log n$

$= 4^k t(n/2^k) + \sum_{i=0}^{k-1} 4^i \left(\frac{n}{2^i}\right)^2 \log \frac{n}{2^i}$

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Para $t(n) = 1$

$$\frac{n}{2^k} = 1$$

$$2^k$$

$$K = \log_2 n$$

$$4^k t(n/2^k) + \sum_{i=0}^{k-1} 4^i \left(\frac{n}{2^i}\right)^2 \log \frac{n}{2^i}$$

$$4^{\log_2 n} t(1) + \sum_{i=0}^{\log_2 n - 1} 4^i \left(\frac{n}{2^i}\right)^2 \log \frac{n}{2^i}$$

$$n^2 + \sum_{i=0}^{\log_2 n - 1} 4^i \left(\frac{n}{2^i}\right)^2 \log \frac{n}{2^i}$$

$$= n^2 + \sum_{i=0}^{\log_2 n - 1} 4^i \left(\frac{n}{2^i}\right)^2 [\log n - \log 2^i]$$

$$= n^2 + \sum_{i=0}^{\log_2 n - 1} 4^i \left(\frac{n}{2^i}\right)^2 [\log n - i] = n^2 + \sum_{i=0}^{\log_2 n - 1} \cancel{2^i} \cdot \frac{n^2}{\cancel{2^i}} \cdot [\log n - i]$$

$$= n^2 + \sum_{i=0}^{\log_2 n - 1} n^2 . [\log n - i] = n^2 + n^2 \sum_{i=0}^{\log_2 n - 1} \log n - i$$

$$= n^2 + n^2 \sum_{i=0}^{\log_2 n - 1} \log n - \sum_{i=0}^{\log_2 n - 1} i$$

$$= n^2 + n^2 \left[\log n (\log n) - \sum_{i=0}^{\log_2 n - 1} i\right] = n^2 + n^2 \left[2 \log n - \sum_{i=1}^{\log n - 1} (i-1)\right]$$

$$= n^2 + n^2 \log n - n^2 \left[\sum_{i=1}^{\log - 1} i - \sum_{i=1}^{\log - 1} 1\right]$$

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$$= n^2 + 2n^2 \log n - n^2 \left[\frac{(\log n - 1) \cdot \log n}{2} - \log n - 1 \right]$$

$$= n^2 + 2n^2 \log n - n^2 \left[\frac{\log n^2 - \log n}{2} - \log n - 1 \right]$$

$$= n^2 + 2n^2 \log n - n^2 \left[\frac{\log n^2}{2} - \log n - 1 \right]$$

$$= n^2 + 2n^2 \log n - \frac{n^2}{2} \left[\log n - 2 \log n - 2 \right]$$

$$= n^2 + 2n^2 \log n - \frac{n^2}{2} \left[- \log n - 2 \right]$$

$$= n^2 + 2n^2 \log n + \frac{n^2 \log n + 2}{2}$$

$$= \frac{2n^2 + 4n^2 \log n + n^2 \log n + 2}{2}$$

$$= \frac{2n^2 + 5n^2 \log n + 2}{2}$$

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7. $$n = \sum_{x=0}^{n^2} \sum_{y=0}^{x-1} \sum_{z=y}^{x-1} 1$$

$$= \sum_{x=0}^{n^2} \sum_{y=0}^{x-1} (x-y) = \sum_{x=0}^{n^2} \sum_{y=0}^{x-1} x - \sum_{x=0}^{n^2-1} \sum_{y=0}^{x-1} y$$

$$= \sum_{x=0}^{n^2} (x^2) - \sum_{x=0}^{n^2-1} \frac{(x-1+0)x}{2}$$

$$= \sum_{x=0}^{n^2} (x^2) - \sum_{x=0}^{n^2-1} \frac{x^2-x}{2} = \sum_{x=0}^{n^2} \frac{x^2+x}{2}$$

$$= \frac{1}{2}\left(\sum_{x=0}^{n^2} x^2 + \sum_{x=0}^{n^2} x\right)$$

$$= \frac{1}{2}\left(\frac{n^2(n^2+1)(2n^2+1)}{6} + \frac{n^2(n^2+1)}{2}\right)$$

$$= \frac{1}{2}\left(\frac{n^2(n^2+1)(2n^2+1) + 3n^2(n^2+1)}{6}\right)$$

$$= \frac{1}{2}\left(\frac{n^2(n^2+1)(2n^2+1+3)}{6}\right)$$

$$= \frac{1}{2}\left(\frac{n^2(n^2+1)(2n^2+4)}{6}\right) = \frac{2n^2(n^2+1)(n^2+2)}{12}$$

$$n = \frac{n^2(n^2+1).(n^2+2)}{6}$$