# A new method for robot path planning based artificial potential field

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# *A New Method for Robot Path Planning Based Artificial Potential Field*

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**Abstract—The artificial potential field method is used in mobile robot path planning extensively because of its simpleness, high efficiency and smooth path, but it also has its disadvantages. To overcome the shortcomings of the traditional artificial potential field method in mobile robot path planning, this paper analyzes the reasons that lead to the failure in path planning and puts forward an improved method, in which the attractive and repulsive potential field is optimized, also we propose a strategy of potential field filling to escape the GNRON and local minima problems. At last, we introduce regression search to optimize the path. As a result, the mobile robot can find a better and collision-free path to the goal. The simulation result proves the efficient and flexibility of our new APF.**

***Keywords—artificial potential field; path planning; GNRON; local minima***

## I. INTRODUCTION

With the development of science and technology, the application of artificial intelligence has performed in many different fields, the requirement of the capability is much higher than past. The path planning for mobile robot is a rather complicated issue. In the last few decades, scientists of robotic have investigated on service mobile robots which could execute different complex tasks and bring convenience to people, such as transport heavy objects, rescue, and guide people in shopping malls, exhibitions and museums. Path planning is aimed at searching a safe and collision-free path achieving a certain task in a complex environment [1]. Autonomous mobile robot path planning or navigation is one of the most important applications for robot control systems and the field has attracted remarkable attention from number of researchers [2]-[5]. Path planning can be divided into global path planning based on known environment and local path planning based on sensor information [6].

The artificial potential field (APF) is a local path planning method which is proposed by Khatib [7]. At first, APF is used to find a collision-free path for the arm robot, but researchers found it had a good performance in mobile robot path planning, and then it was widely used and researched. As a classical and efficient path planning algorithm, artificial potential field (APF) has a good performance, for example, it has convenient calculation, rapid search speed and better search quality (the path is collision-free and smooth), etc [8]. But during the APF's widely used in robot field, researchers have found some inherent limitations, such as local minima and GNRON (goals unreachable with obstacles nearby) problems [9]. As we can know, many improved algorithms has been proposed and some results were encouraging. These works can be classified into five approaches according to their control strategy. First, some researchers improve the distribution of the virtual force, such as reference [10]. The second kind of approach improves the repulsive potential functions, including its angle, such as reference [11] to [14]. The third kind of approach introduces the additional control force, such as reference [15]. The fourth kind of approach uses virtual obstacle or virtual goal for repelling or attracting the mobile robots from the local minima or to the target, such as reference [16]. The fifth kind of approach combines intelligent techniques, such as "follow wall" strategy in [17].

Though many improved strategies has been used to escape the disadvantages of APF in mobile robot path planning, the problems of GNRON and local minima haven't been solved perfectly. All the mentioned above APF and its improved methods still suffer from many drawbacks, and this can be vital, because they can due to failure of path planning; as a result, the robot may collide to obstacles or can't achieve the goal. It is worth mentioning that, in most of the previous studies, the goal position is set so far away from obstacles that the goal is outside of the

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repulsive potential field. Also many researchers hasn't considered that the local minima is a small area. In this paper, we take the above situations into account, analyze the reasons of the GNRON and local minima problems and propose an improved APF algorithm. The optimized attractive and repulsive potential field is used and a strategy of potential field filling is also proposed to overcome the GNRON and local minima problems. At last regression search is introduced to optimize the path, so that the path is global optimization. Some simulation studies are carried out to demonstrate the effectiveness of the new proposed algorithm.

The remainder of this paper is structured as follows. The next section discusses the fundamental of APF for mobile robot path planning. In section Ⅲ, the reasons of the defects are analyzed and the corresponding resolutions are proposed. In section Ⅳ, a mount of simulations are done to demonstrate that the improved method can perform better than the traditional APF. Finally, section Ⅴ draws conclusions and sketches future work.

## II. THE FUNDAMENTAL OF APF

The APF is firstly introduced by Khatib. The basic idea of APF method assumes that robot is a point moves in an abstract artificial force field, and the potential function can be defined over free C-space. Potential field force is often divided into two kinds according to their functions. One is attractive force generated by the goal point, and the other is the repulsive force developed by the obstacle. The attractive force makes robots move toward the goal point, while the repulsive force keeps robots away from obstacles. Therefore, the potential function (3) is the APF of robot which is defined as the resultant of attractive field and repulsive field. Under the method of APF, robot could find a collision-free path by searching the route along the decline direction of potential function.

Different potential functions have been proposed in the literature. The most commonly used attractive potential takes the form [1]–[6]. The attractive potential field created by the goal is given as:

$$U_{att}(X)=\frac{1}{2}k\rho^2(X,X_g) \tag{1}$$

Where: $k$ is a positive coefficient for APF, $\rho(X,X_g)$ is the Euclidean distance from the location of robot to the position of goal.

The attractive force on robot is calculated as the negative gradient of attractive potential field and takes the following form:

$$F_{att}(X)=-\nabla\left[U_{att}(X)\right]=-k\rho(X,X_g)\cdot\nabla(X,X_g) \tag{2}$$

Repulsive force makes robots away from obstacles, but when robot is far from obstacles, we do not want obstacles to affect robot's motion control. Khatib uses the function FIRAS as the repulsive potential field:

$$U_{rep}(X)=\begin{cases}0.5\eta\cdot(\dfrac{1}{\rho(X,X_0)}-\dfrac{1}{\rho_0})^2 & \rho(\mathrm{X},\mathrm{X}_0)\le\rho_0\\ 0 & \rho(\mathrm{X},\mathrm{X}_0)>\rho_0\end{cases} \tag{3}$$

Where: $\eta$ is a positive scaling factor. $\rho(X,X_0)$ is the Euclidean distance from the location of robot to the obstacles. $\rho_0$ is the largest impact distance of single obstacle. There is no impact for robot when the distance between robot and obstacle is greater than $\rho_0$. Similarly to the attractive force, the repulsive force is the negative gradient of repulsive potential function, as follows:

$$\begin{aligned}F_{rep}(X)&=-\nabla\left[U_{rep}(X)\right]\\&=\begin{cases}\eta\left(\dfrac{1}{\rho(X,X_0)}-\dfrac{1}{\rho_0}\right)\dfrac{1}{\rho^2(X,X_0)}\nabla(X,X_0) & \rho(\mathrm{X},\mathrm{X}_0)<\rho_0\\ 0 & \rho(\mathrm{X},\mathrm{X}_0)<\rho_0\end{cases}\end{aligned} \tag{4}$$

While there are many obstacles in the environment, the total repulsive potential field is the sum of all obstacles' repulsive potential field. The total potential field can be expressed as function (5).

$$U_{total}(X)=U_{att}(X)+\sum_{i=1}^{n}U_{rep}(X) \tag{5}$$

Where: $n$ is the number of obstacles.

The total artificial force is:

$$F_{total}(X)=F_{att}(X)+\sum_{i=1}^{n}F_{rep}(X) \tag{6}$$

## III. ANALYZE THH DEFECTS OF APF AND IMPROVED METHODS

### *A. Analyze the factor of attractive potential field*

We set two elements of attractive and repulsive potential field in APF, for the reason that they can control the motion of robots in balance. But in the traditional APF, as shown in (1)-(2), if robots are far away from the goal, the repulsive force plays a minor role in the motion control. As a result, robots may collide into obstacles, which is unacceptable. Then we should optimize the attractive potential field to balance the motion control.

In this paper, we set the attractive potential field as (7):

$$U_{att}(X)=\begin{cases}\dfrac{1}{2}k\cdot\rho^2(X,X_g) & \rho(\mathrm{X},\mathrm{X}_g)\le d_0\\ \mathrm{k}\cdot d_0\cdot\rho(X,X_g) & \rho(\mathrm{X},\mathrm{X}_0)>d_0\end{cases} \tag{7}$$

Where: $d_0$ is a constant decided by the environment.

### *B. Analyze the factor of repulsive potential field*

In the previous researches, people usually put the goal outside repulsive potential fields' scope. From the analysis before, if the goal is within the effective scope of the obstacles, robots may can't reach the goal, which is the problem of GNRON. This situation is seldom considered in the previous literature. But the problem does exist in reality and is worth investigating.

The essential cause of the GNRON problem is that the goal position is not a minimum of the total potential function. For example, consider a one-dimensional (1-D) case as shown in Fig. 1, where the robot $X=(x,0)$ is moving along x-axis toward the $X_g=(0,0)$, while the obstacle $X_0=(r,0)$ is on the right-hand side of the goal. The goal is within the distance of influence of the obstacle.

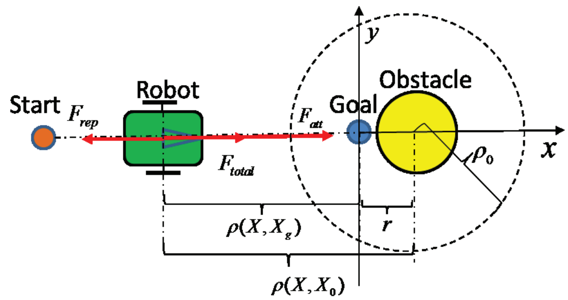


Fig.1. Locations of the robot, goal, and obstacle in a 1-D case.

The attractive potential and the repulsive potential are given by (1)-(4). Assuming that $k=\eta=1$, and $X_g=(0,0)$, $X_0=(0.5,0), d_0=1.5$, Fig 2 shows the total potential $U_{total}(X)=U_{att}(X)+U_{rep}(X)$ with respect to X. It is clear that $X_g=(0,0)$ is not the minimum of the total potential function. In fact ,the robot will be trapped at the minimum at nearly X = (-0.5,0), where the total force becomes zero and the forces at positions on both sides of the minimum are pointing to the minimum position. The APF is much like the principle of flow , moving from the higher potential filed to lower potential field. So the robot cannot reach the goal, though there is no obstacle in its way.

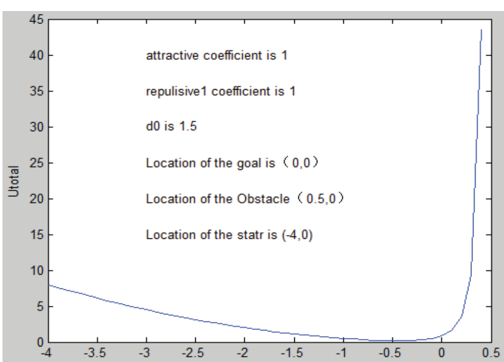


Fig.2. Total potential function in a 1-D case.

From previous analysis, we can conclude that the GNRON problem arises while the goal is within the influence distance of the obstacle, which makes the global minimum of the total potential field is not at the goal position. This situation is due to the fact that as the robot approaches the goal, on one hand the attractive potential decreases, on the other hand the repulsive potential increases quickly. So we should do some optimizations of the repulsive potential field to balance the changes of two kind of force, especially the rapid increase of the repulsive force. It is found that if the repulsive potential approaches zero as the robot approaches the goal, the total potential will take the global minimum at the goal. This motivates us to construct a new repulsive potential function which takes the relative distance between the robot and the goal into consideration as (8):

$$U_{rep}(X) = \begin{cases} \frac{1}{2}\eta\left(\frac{1}{\rho(X,X_0)}-\frac{1}{\rho_0}\right)^2 \rho^n(X,X_g) & \rho(X,X_0)\le\rho_0 \\ 0 & \rho(X,X_0)>\rho_0 \end{cases} \quad (8)$$

where: $n$ is a index coefficient, and we make $n=2$ based our analysis of mathematical modeling, then $U_{rep}$ will do a better performance. Then:

$$F_{rep}(X) = -\nabla\left[U_{rep}(X)\right] = \begin{cases} F_{rep1}\cdot N_1 + F_{rep2}\cdot N_2 & \rho(X,X_0)\le\rho_0 \\ 0 & \rho(X,X_0)>\rho_0 \end{cases} \quad (9)$$

Where:

$$F_{rep1} = \eta\cdot\left(\frac{1}{\rho(X,X_0)}-\frac{1}{\rho_0}\right)\cdot\frac{\rho^n(X,X_g)}{\rho^2(X,X_0)}$$
$$F_{rep2} = -\frac{n}{2}\eta\left(\frac{1}{\rho(X,X_0)}-\frac{1}{\rho_0}\right)^2\cdot\rho^{n-1}(X,X_g) \quad (10)$$

and $N_1=\nabla\rho(X,X_0), N_2=-\nabla\rho(X,X_g)$ are two unit vectors pointing from the obstacle to the robot and from the robot to the goal, respectively.

The relationship between the repulsive force and its two components is shown in Fig 3. It is clear that while the component $F_{rep1}\cdot N_1$ repulses the robot away from the obstacle, the other component $F_{rep2}\cdot N_2$ attracts the robot toward the goal, which is what we want.

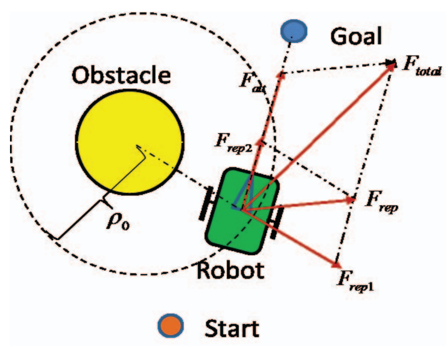


Fig.3. Repulsive force derived by the new potential function

In comparison with (3), the introduction of $\rho^n(X,X_g)$ ensures that the total potential $U_{total}$ arrives its global minimum 0, if and only if $\rho^n(X, X_g) = 0$. Fig.4 shows the total potential distruction in 2-D environment, where $k = \eta = 1, n = 2, \rho_0 = 8, X_0 = [29,45;40,45;65,45;74,74]$, $X_g = (70,70)$, so the goal is within the effective scope of the obstacle (74,74). It is obvious that at the goal, the total potential reaches its global minimum zero. For other choices of n, we have similar potential distribution as shown in Fig 3 by properly tuning the scaling parameters $k$ and $\eta$.

If $k$ and $\eta$ are not chosen properly, local minima does exist though the goal is the global minimum of the total potential. So we should do furtur research about the two coefficients. These two scaling parameters determine the relative intensity of the attractive force and the repulsive force.If we want the totall force pushes the robot away from the obstacles and pulls toward the goal,the following (11) should be satisfied:

$$F_{total} = F_{att} - F_{rep1} + F_{rep2} > 0 \tag{11}$$

From (11) we get:

$$\frac{k}{\eta} > \left(\frac{1}{\rho(X, X_0)} - \frac{1}{\rho_0}\right) \cdot \frac{\rho^{n-1}(X, X_g)}{\rho^2(X, X_0)} - \frac{n}{2}\left(\frac{1}{\rho(X, X_0)} - \frac{1}{\rho_0}\right)^2 \cdot \rho^{n-2}(X, X_g) \tag{12}$$

Then from mathematics simplify computing (n=2), we get:

$$\frac{k}{\eta} > \left(\frac{2}{9\rho_0^{\,2}} + \frac{2r}{27\rho_0^{\,3}}\right) \cdot \sqrt{1 + \frac{3\rho_0}{r}} - \frac{2}{3\rho_0^{\,2}} + \frac{2r}{27\rho_0^{\,3}} \tag{13}$$

Where $r$ is the smallest distance between goal and obstacles, as shown in Fig.1. Here $r$ and $\rho_0$ is known previously.

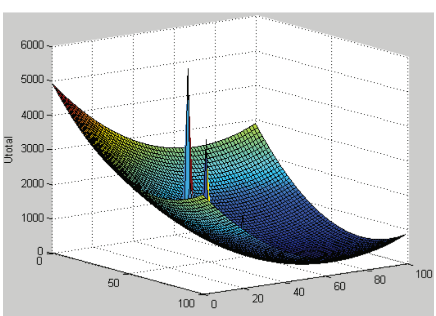


Fig.4. Total potential in a 2-D environment without local minima

*C Analyze the problem of local minima*

In the previous section, the GNRON problem that the goal is within the effective scope of obstacles is discussed and solved, but when the attractive force and repulsive force is equal or almost equal and collinear on the opposite direction in the process of moving to target, the potential force of robot is zero, then it will cause robot to be trapped in local minima, as shown in Fig.5.

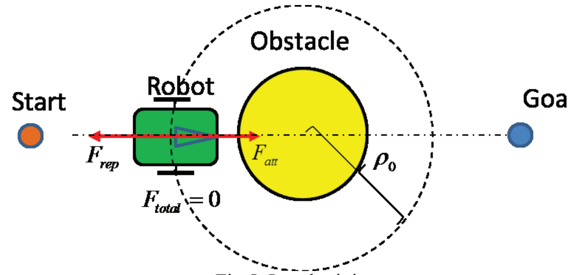


Fig.5. Local minima

In previous research, people usually set additional goals to make robots escape from local minima, but how to set the locations of additional goals has been a difficult subject. Also, some researchers try the method of random step to solve the problem, but this isn't a good idea for the local minima is a small area. While the local minima is a vital problem and occurs frequently in complex environment, so we propose a strategy of potential field filling to escape the local minima in this paper. Our algorithm flowchart is shown as Fig. 6.

In the flowchart, n and N respectively represents the search time that it has executed and the maximum search time. D represents a small positive value. If the distance between the robot and goal is smaller than the value, we regard the current location as the destination. In the improved APF flowchart, there includes three links mainly. We will describe them in the following paper.

*1) Detect the search time n and go ahead*

There may be no path from start to goal, so we must set a maximum search time N to make sure our algorithm will not go into a dead loop. If robot has not achieve the goal and search time doesn't reach to N, the force will make robot approach the goal step by step. So robot will change its position as (14):

$$\begin{cases} x_{k+1} = x_k + \gamma \cdot \cos(\theta) \\ y_{k+1} = y_k + \gamma \cdot \sin(\theta) \end{cases} \tag{14}$$

Where: X = (x , y) is the location of robot, $\gamma$ represents the length of robot moving step. $\theta$ is decided by the vector of $F_{total}(X)$.

*2) Detect local minima and potential field filling*

Robot recorded his previous position and orientation information in all the process of path planning. Assuming that the current location of robot is $X_k(x_k, y_k)$, then we should campare $X_k(x_k, y_k)$ and $X_{k-2}(x_{k-2}, y_{k-2})$. If they are equal, we make a decision that this is a local minima area. Then the robot has to reintilize the potential field and the virtual 2-D environment. In this step, we use the potential filling to overcome the local minima problem, in other words we set an additional potential field in the area to pull the robot away from the local minima. The additional potential field is like (15):

$$U_{add}(X)=\begin{cases} s\cdot\dfrac{1}{\rho^{2}(X,X_{local})} & \rho(\mathrm{X},\mathrm{X}_{\mathrm{local}})\le\rho_{1} \\ 0 & \rho(\mathrm{X},\mathrm{X}_{\mathrm{local}})>\rho_{1} \end{cases} \tag{15}$$

Where: $s$ is a positive scaling factor. $X_{local}$ represents the position of the local minima, and $\rho(X,X_{local})$ is the distance between robt and local minima. $\rho_1$ is the the largest impact distance of the additional potential field.

*3) Detect the goal*

During the mobile robot navigation, errors are unavoidable, which comes from hardware and control. We set a small constant D as the acceptable error of the robot destination. If robot gets to the destination area, then it can accept the next task, or the algorithm will keep on searching.

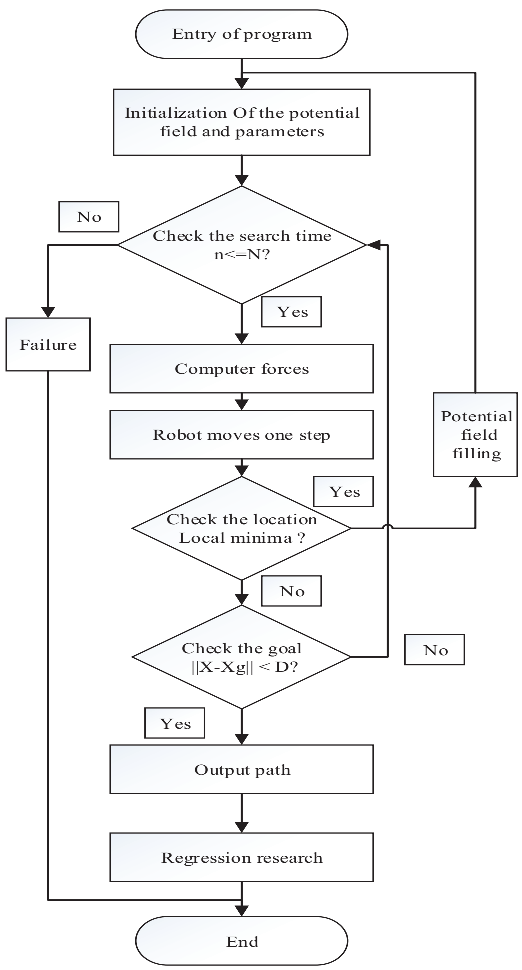


Fig.6. The improved APF flowchart

*D Optimize the path*

Although our improved APF which could resolve the local minima and GNRON problems successfully, but when it comes to global optimization, especially for the time/energy constrain robots, there are defects also. This paper introduce the regession search to optimize the planned path points. The optimization path is produced by connecting the path points, which are produced by our optimized APF.

In order to illustrate the regression search method based on our optimized APF, we take Fig. 7 as an example. As we can see in Fig. 7, Pi {P1, P2, P3, …, Pi, Pi+1, …, Pn} are the path points produced by our optimized APF. Robot can reach the goal moving along the path points without colliding obstacles without taking the path distance or time into account. Of course, we want robot to get the goal in the shortest possible time. Then, let me introduce the regression search step by step. Firstly, we get line $L_{(1,2)}$ by connecting the point P1 and P2. Then we should judge two conditions: if $L_{(1,2)}$ crosses any obstacles or not, and if the shortest distance H between $L_{(1,2)}$ and obstacles is bigger than H0 or not, where H0 is a constant we set previously based on the environment. If $L_{(1,2)}$ satisfies both two conditions simultaneously, then we can connect P1 with P3 as $L_{(1,3)}$, which is better than $L_{(1,2,3)}$ in distance and safety; otherwise we should take $L_{(1,2,3)}$ as the best. Then we should do the similar steps mentioned above circularly. But when we connect P1 and Pi, we find that the corresponding H is smaller than H0, so the optimized local optimal path is $L_{(1,i)}$. Then we should begin from point Pi, connect Pi and Pi+1 getting $L_{(i,i+1)}$, and do the similar work. Finally, we get the optimal path of this example, which is the line $L_{(1,i)}$ ,$L_{(i,i+1)}$ and $L_{(i+1,n)}$. In this example, robot will consume the least energy and time achieving the goal moving along the path $L_{(1,i)}$ , $L_{(i,i+1)}$ and $L_{(i+1,n)}$. We can say that the new path is global optimization.

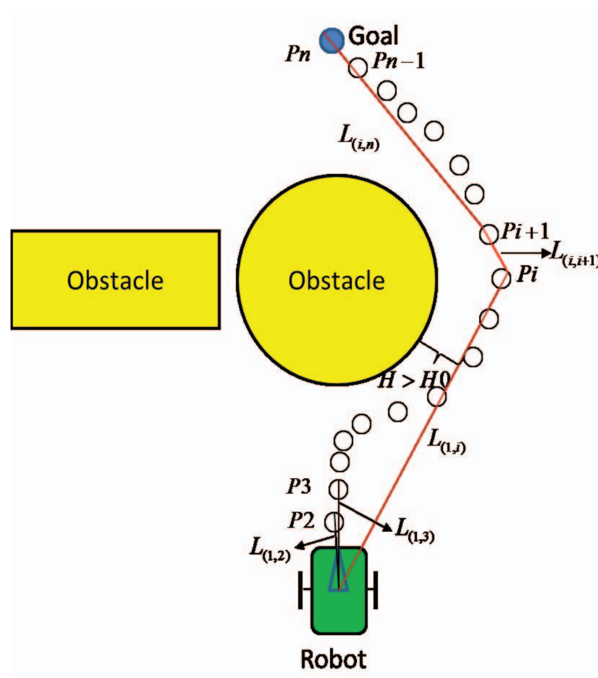


Fig.7. Regression search

## IV. SIMULATION

Some simulation experiments are carried out to prove the efficient and flexibility of our optimized APF using MATLAB. In the following simulations, the paths are found by assuming that the robot moves at constant speed, and the resultant virtual force applied to it only determines the direction of its motion. The workspace is within a 100 × 100 room and there are some obstacles at $X_0 = [10,90;40,45;65,45;74,74;...]$ . We also set some parameters, such as coefficients and distances are as follows: $k = \eta = 1, n = 2, \rho_0 = \rho_1 = 8, s = 0.5, H0 = 1.5$ , $\gamma = 0.1$, $X_g = (70,70)$ . The goal is in the effective scope of the up-right obstacle and there are some local minima areas, for example the area near (15,15). Fig. 8 shows that traditional APF is failed to plan path in the complex environment beacause of a local minima, and the optimized path planned by our new APF. The green path planned by our new APF is collision-free and global optimization, which is fit for mobile robots' control.

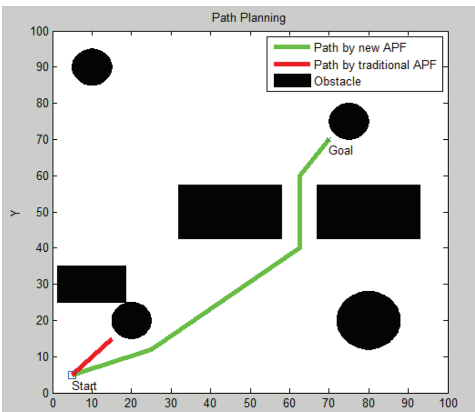


Fig.8. Path planned by our new APF and traditional APF

## V. CONCLUSIONS

Path planning problem is a very important branch of the navigation for autonomous mobile robots. APF has been usually used for collision avoidance for mobile robots. Many researchers have been studying APF and a few resolutions are presented to improve its inherent disadvantages. This paper improves the attractive and repulsive potential field, proposes a strategy of potential field filling to escape the GNRON and local minima problems and introduces the regression search to get a global optimized path. The result proves our new APF method is very feasibility and efficiency to solve path planning problems.

In the future works, we attend to solve the oscillation and smoothness defects of APF. And we also will try to make a combination of local and global path planning for mobile robots.


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