$$q = [q_0 \quad q_1 \quad q_2 \quad q_3]$$

$$p = [p_0 \quad p_1 \quad p_2 \quad p_3]$$

$$q = q_0 + q_1 i + q_2 j + q_3 k$$

$ij = k \qquad ji = -k$

$jk = i \qquad kj = -i$

$ki = j \qquad ik = -j$

$i^2 = j^2 = k^2 = -1$

$$q \cdot p = (q_0 + q_1 i + q_2 j + q_3 k)(p_0 + p_1 i + p_2 j + p_3 k)$$

$$q_0 p_0 + q_0 p_1 i + q_0 p_2 j + q_0 p_3 k$$

$$+ \; q_1 i \cdot p_0 + \underbrace{q_1 i \cdot p_1 i}_{-1} + \underbrace{q_1 i \cdot p_2 j}_{k} + \underbrace{q_1 i \cdot p_3 k}_{-j}$$

$$+ \; q_2 j \cdot p_0 + \underbrace{q_2 j \cdot p_1 i}_{-k} + \underbrace{q_2 j \cdot p_2 j}_{-1} + \underbrace{q_2 j \cdot p_3 k}_{i}$$

$$+ \; q_3 k \cdot p_0 + \underbrace{q_3 k \, p_1 i}_{j} + \underbrace{q_3 k \cdot p_2 j}_{-i} + \underbrace{q_3 k \cdot p_3 k}_{-1}$$

$$q \cdot p = (q_0 p_0 - q_1 p_1 - q_2 p_2 - q_3 p_3)$$

$$+ i (q_0 p_1 + q_1 p_0 + q_2 p_3 - q_3 p_2)$$

$$+ j (q_0 p_2 - q_1 p_3 + q_2 p_0 + q_3 p_1)$$

$$+ k (q_0 p_3 + q_1 p_2 - q_2 p_1 + q_3 p_0)$$

$$\boxed{\dot{q} = \frac{1}{2} q \otimes \omega} \qquad \omega_q = \begin{bmatrix} 0 \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

$$\dot{q} = \frac{1}{2} \begin{bmatrix} q_0 & -q_1 & -q_2 & -q_3 \\ q_1 & q_0 & -q_3 & q_2 \\ q_2 & q_3 & q_0 & -q_1 \\ q_3 & -q_2 & q_1 & q_0 \end{bmatrix} \begin{bmatrix} 0 \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

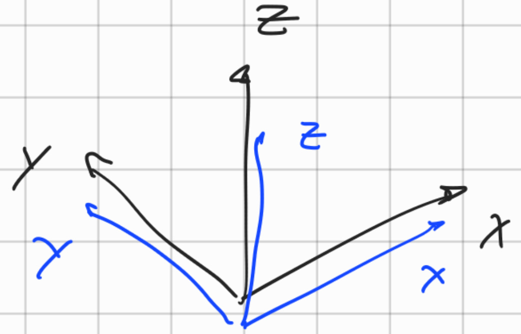
$$\dot{q} \simeq \frac{q_n - q_{n-1}}{\Delta T}$$

$$\boxed{q_{n+1} = q_n + \frac{1}{2} q_n \otimes \omega \cdot \Delta T}$$

$$\omega = \begin{bmatrix} 0 \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \rightarrow \omega_c$$

$9{,}8\ m/s^2$

$$\vec{g}_I = [0 \quad 0 \quad g]$$



$$\vec{g}_S = [Ax \quad Ay \quad Az]$$

$$\boxed{q_I^S}$$

$$\vec{g}_S = R(\varphi, \theta, \psi) \cdot \vec{g}_I$$

$$\vec{g}_S = q_I^{S*} \otimes g_I \otimes q_I^S$$

$$\begin{bmatrix} 0 \\ Ax \\ Ay \\ Az \end{bmatrix} \quad \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ g \end{bmatrix}$$

Euler

$$\begin{bmatrix} Ax \\ Ay \\ Az \end{bmatrix} = R_\theta \cdot \vec{g}$$

R

Predict

$$\hat{x}^- = f(x, u)$$

$$P^- = J^T P J + Q_N$$

4x4

4x4

$$J = \frac{\partial f(x,u)}{\partial x}$$

$$\hat{y} = h(x, u)$$

$$e = y - \hat{y}$$

$$K = P^- H^T (H \cdot P \cdot H^T + R_N)^{-1}$$

$$\hat{x}^+ = \hat{x}^- + K \cdot e$$

$$P^+ = (I - K H) P^-$$

$$H = \frac{\partial h(x,u)}{\partial x}$$

$$q_{n+1} = q_n + \frac{1}{2} q_n \otimes \omega \cdot \Delta T \qquad J = \frac{\partial f}{\partial x}$$

$$f(x, u) = q_n + \frac{1}{2} q_n \otimes \omega \cdot \Delta T$$

$x = q$

$$= \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} + \frac{\Delta T}{2} \begin{bmatrix} q_0 & -q_1 & -q_2 & -q_3 \\ q_1 & q_0 & -q_3 & q_2 \\ q_2 & q_3 & q_0 & -q_1 \\ q_3 & -q_2 & q_1 & q_0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \omega x \\ \omega y \\ \omega z \end{bmatrix}$$

$$f(x, u) = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} -q_1 \omega x - q_2 \omega y - q_3 \omega z \\ q_0 \omega x - q_3 \omega y + q_2 \omega z \\ q_3 \omega x + q_0 \omega y - q_1 \omega z \\ -q_2 \omega x + q_1 \omega y + q_0 \omega z \end{bmatrix} \cdot \frac{\Delta T}{2}$$

$u = [0 \ \omega x \ \omega y \ \omega z]$

4×1 $\qquad x = [q_0 \ q_1 \ q_2 \ q_3]$

$$= f(x, u) = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \end{bmatrix} \qquad \frac{\partial f(x, u)}{\partial x} = \begin{bmatrix} \frac{\partial f_0}{\partial q_0} & \cdots & \frac{\partial f_0}{\partial q_3} \\ \vdots & & \\ \frac{\partial f_3}{\partial q_0} & \cdots & \frac{\partial f_3}{\partial q_3} \end{bmatrix}_{4 \times 4}$$

$$= \begin{bmatrix} 2/\Delta T & -\omega x & -\omega y & -\omega z \\ \omega x & 2/\Delta T & \omega z & -\omega y \\ \omega y & -\omega z & 2/\Delta T & \omega x \\ \omega z & \omega y & -\omega x & 2/\Delta T \end{bmatrix} \Delta T/2$$