MOBILE ROBOT ATTITUDE ESTIMATION BY FUSION OF INERTIAL DATA

J. Vaganay, M. J. Aldon, A. Fournier

LIRMM-Université Montpellier II - 161 Rue ADA
34392 Montpellier Cedex 5 - FRANCE

Abstract

This paper describes an attitude estimation system based on inertial measurements for a mobile robot. Five low- cost inertial sensors are used: two accelerometers and three gyros. The robot's attitude, represented by its roll and pitch angles, can be obtained using two different methods. The first method is based on accelerometric measurements of gravity. The second one proceeds by integration of the differential equation relating the robot's attitude and its instantaneous angular velocity which is measured by the gyrometers. The results of these two methods are fused together using an extended Kalman filter in order to take advantage of both of them. Experimental results show that this system is very sensitive and accurate.

1: Introduction

A fundamental requirement for an autonomous vehicle is its ability to localize itself with respect to its environment. Navigation on a flat and horizontal ground only requires estimations of position and heading. However, in many cases, the environment is not so well structured, and the angular orientation of the vehicle may change along its path. In this case, a real time estimation of the attitude may be necessary. All-terrain navigation, for example, requires at least rough measurements of the pitch and roll angles to ensure vehicle safety. Some specific tasks, like those executed by robots for road construction, require controlling the attitude of a tool [1]. Indoor mobile robots generally need an accurate navigation system. If they move on inclined planes, the orientation can be used to correct the position and heading estimations provided by odometry, or to interpret external sensor data (vision [2][3], proximetry,...) in order to build an accurate model of the environment.

The attitude estimation may be done by using exteroceptive sensors like cameras [4], or range-finders, which perceive the environment and allow an orientation update. But these methods are not robust and reliable enough and they are generally too time-consuming.

On the other hand, new research is being developed on the implementation of inertial sensors on robots. As a matter of fact, the inertial navigation systems developed for airplanes cannot be used for ground vehicles for two reasons. First, the motion models do not contain the same parameters and the required performances are different. Second, these systems are too expensive for robotic applications [5][6].

Our method is based on the following ideas. Pitch and roll angles may be estimated by using the gravity components deduced from the measurements of two accelerometers [7]. However, this solution is not well adapted to the dynamic localization of ground vehicles for two major reasons:
- when the robot moves, an estimation of its own acceleration must be available,
- measurements are subject to large fluctuations when the motion occurs on uneven ground.

We thus propose a multisensor fusion method which consists of correlating these angle estimations with those resulting from the integration of the angular velocities measured by three orthogonal gyrometers. This paper describes the algorithms we have developed.

The system has been implemented and successfully tested on a mobile platform equipped with low-cost sensors. Experimental results are presented and discussed in the last section.

2: Inertial measurements

Let us consider six inertial sensors rigidly attached to the robot: three accelerometers and three gyrometers. The absolute reference frame R_0 is such that $\mathbf{z_0}$ is vertical and directed upward. The frame R_r is attached to the robot. Its origin O_r is in the middle of the tread which is the distance between the front wheels. $\mathbf{x_r}$ is pointing in the direction of displacement of the robot, and $\mathbf{z_r}$ is directed upward.

An accelerometer provides an output voltage proportional to the projection on its sensitivity axis of the acceleration applied to it [8][9]. The sensitivity axes of the

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three accelerometers are set orthogonally, defining a frame R_a with origin O_a. If $\gamma(O_a/R_0)$ designates the absolute acceleration of the robot and **g** the gravity vector, the accelerometric output is:

$$\mathbf{A} = [\gamma(O_a/R_0) - \mathbf{g}]_a \tag{1}$$

The accelerometers are attached to the robot so that the transformation between R_r and R_a is reduced to the translation $\mathbf{O_rO_a}$=[dx dy dz]^T (no alignment error) which has to be calibrated. With this assumption, (1) can be rewritten as follows:

$$\mathbf{A} = [\,A_x \;\; A_y \;\; A_z\,]^T = [\gamma(O_a/R_0)]_r - [\mathbf{g}]_r \tag{2}$$

A gyrometer delivers an output voltage proportional to the projection on its sensitivity axis of the absolute angular velocity applied to it. The sensitivity axes of the three gyros are set orthogonally, defining a frame R_g, so that the gyrometric output is:

$$\mathbf{\Omega} = [\Omega(R_g/R_0)]_g \tag{3}$$

Identically, the gyrometers are attached to the robot so that the transformation between R_r and R_g is reduced to a translation (no alignment error). In this case, the gyros give a direct measure of the robot's absolute angular velocity. The gyrometric output (3) can be expressed as follows:

$$\mathbf{\Omega} = [\,p\;q\;r\,]^T = [\Omega(R_r/R_0)]_r \tag{4}$$

3: Attitude representation

Roll-pitch-yaw angles (ϕ,ψ,θ) can be used to represent the attitude and the heading of the robot. If the direction cosines matrix ${}^0[A]_r$, defining the attitude and the heading of R_r is given, the roll-pitch-yaw angles can be extracted as follows :

$${}^0[A]_r = \begin{bmatrix} s_x & n_x & a_x \\ s_y & n_y & a_y \\ s_z & n_z & a_z \end{bmatrix}$$

$$\theta = \text{Arctan}\,\frac{s_y}{s_x} \pm k.\pi \text{ (singularity when } s_x=s_y=0) \tag{5}$$

$$\psi = \text{Arctan}\,\frac{-s_z}{C\theta s_x + S\theta s_y} \tag{6}$$

$$\phi = \text{Arctan}\,\frac{S\theta a_x - C\theta a_y}{-S\theta n_x + C\theta n_y} \tag{7}$$

where $C\theta=\cos\theta$, and $S\theta=\sin\theta$

4: Accelerometric determination of the attitude

Static determination

Roll and pitch are calculated from the components of gravity in frame R_r, which are directly measured by the accelerometers. Indeed, when $[\gamma(O_a/R_0)]_r = 0$, equation (2) becomes:

$$\mathbf{A} = -[\mathbf{g}]_r = [g_x \;\; g_y \;\; g_z]^T$$

The rotation matrix ${}^r[A]_0$ from R_r to R_0, is given by :

$${}^r[A]_0 = \text{Rot}(\mathbf{x},-\phi).\text{Rot}(\mathbf{y},-\psi).\text{Rot}(\mathbf{z},-\theta)$$

$${}^r[A]_0 = \begin{bmatrix} C\psi C\theta & C\psi S\theta & -S\psi \\ S\psi S\phi C\theta - C\phi S\theta & S\psi S\phi S\theta + C\phi C\theta & C\psi S\phi \\ S\psi C\phi C\theta + S\phi S\theta & S\psi C\phi S\theta - S\phi C\theta & C\psi C\phi \end{bmatrix}$$

The components of gravity in frame R_0 are known and those in frame R_r are measured; they are related by :

$$-[\mathbf{g}]_r = -{}^r[A]_0[\mathbf{g}]_0, \quad \text{with } [\mathbf{g}]_0 = [0\;0\;-g]^T$$

Finally, the expressions of the attitude angles as functions of the components of gravity in frame R_r are:

$$\psi = -\text{Arcsin}\,\frac{g_x}{g} \tag{8}$$

$$\phi = \text{Arcsin}\,\frac{g_y}{gC\psi} \text{ or } \phi = \text{Arccos}\,\frac{g_z}{gC\psi} \tag{9}$$

Note that two accelerometers are sufficient to determine the attitude. This is why we will only consider the x and y accelerometers in the following sections.

Dynamic determination

When the robot moves, the non gravitational acceleration $[\gamma(O_a/R_0)]_r$ has to be removed from the accelerometric output (2). Since this acceleration is not directly measurable, it has to be approximated. It is given by :

$$\begin{aligned}\gamma(O_a/R_0) = \gamma(O_r/R_0) + \dot{\Omega}(R_r/R_0)\wedge \mathbf{O_rO_a} \\ + \Omega(R_r/R_0)\wedge[\Omega(R_r/R_0)\wedge \mathbf{O_rO_a}]\end{aligned} \tag{10}$$

We are only interested in the x and y components of this equation. Odometry gives an estimate of the translational velocity $V_x(O_r/R_0)$ which can be used to approximate $\gamma_x(O_r/R_0)$ by numerical differentiation :

$$\gamma_x(O_r/R_0) = \frac{d}{dt} V_x(O_r/R_0)$$

Odometry also gives an estimate of the heading velocity r of the robot. The centrifugal acceleration can then be approximated by :

$$\gamma_y(O_r/R_0) = V_x(O_r/R_0).r$$

The angular acceleration is obtained by numerical differentiation :

$$\dot{\mathbf{\Omega}}(R_r/R_0) = [\ \dot{p}\ \dot{q}\ \dot{r}\ ]^T = \frac{d}{dt}\mathbf{\Omega}(R_r/R_0)$$

Using (10), the x and y components of the non-gravitational acceleration can be expressed by :

$$\gamma_x(O_a/R_0)=\gamma_x(O_r/R_0)+\dot{q}.dz-\dot{r}.dy+q(p.dy-q.dx)-r(r.dx-p.dz)$$

$$\gamma_y(O_a/R_0)=\gamma_y(O_r/R_0)+\dot{r}.dx-\dot{p}.dz+r(q.dz-r.dy)-p(p.d_y-q.dx)$$

The components of gravity become :

$$g_x = A_x - \gamma_x(O_a/R_0)$$
$$g_y = A_y - \gamma_y(O_a/R_0)$$

Roll and pitch angles can then be determined using (8) and (9).

It can be noted that the accelerometers by themselves are not sufficient to determine the attitude. Gyrometric and odometric measurements are necessary to estimate $\boldsymbol{\gamma}(O_a/R_0)$.

5: Gyrometric determination of the attitude

The attitude can be determined using gyrometric measurements. This method also allows to estimate the heading (yaw), which is not possible with the accelerometers. In this case, a differential equation relating the attitude and the instantaneous angular velocity has to be integrated. The form of this equation depends on the choice made in representing the attitude [8][10]. Roll and pitch angles are used as output of the system to define the robot's attitude because they have direct physical interpretation, but this representation is not used in the differential equation. We chose to use quaternions mainly because they never lead to singularities. Once the quaternion is obtained, the direction cosines matrix is computed. Roll, pitch, and yaw are then extracted (5,6,7) so that the singularity problem is eliminated from the integration process. Using quaternions, the differential equation to be solved takes the form :

$$\dot{Q} = \frac{1}{2} Q\,\Omega$$

$$\begin{bmatrix} \dot{Q}_0 \\ \dot{Q}_1 \\ \dot{Q}_2 \\ \dot{Q}_3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix}\begin{bmatrix} Q_0 \\ Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}$$

where $Q=Q_0+Q_1.i+Q_2.j+Q_3.k$ is the quaternion associated with the attitude of the robot, and $\mathbf{\Omega}=[p\ q\ r]^T$ is its instantaneous angular velocity.

This differential equation has an analytical solution only when the robot rotates around a fixed axis in R_r and R_0 [8], which is generally not the case. A numerical integration method must then be used. C. Grubin [10] compared different integration algorithms (rectangular, trapezoidal, third-order Runge-Kutta) and found that the best results were given by a third-order algorithm derived by Edwards in [11] after minor changes. We decided to use this algorithm to solve our problem.

Quaternions

A quaternion is defined by :

$$q = \chi+\xi.i+\eta.j+\zeta.k$$

where i, j, k are such that :

$$ii = jj = kk = -1$$
$$ij = -ji = k,\ jk = -kj = i,\ ki = -ik = j$$

Euler's theorem states that an arbitrary sequence of rotations with one point fixed applied to a rigid body leads to a final attitude that can be obtained using a single rotation about an axis which passes through the fixed point. If $[u_x\ u_y\ u_z]^T$ are the direction cosines of the axis and β the angle of rotation which allow to rotate from the absolute reference frame to the robot's frame, then we have :

$$Q_0=\cos\frac{\beta}{2},\ Q_1=u_x.\sin\frac{\beta}{2},\ Q_2=u_y.\sin\frac{\beta}{2},\ Q_3=u_z.\sin\frac{\beta}{2}$$

The direction cosines matrix can be expressed in terms of the quaternion components by :

$$A(q)=\begin{bmatrix} 2(\chi^2+\xi^2)^2-1 & 2(\xi\eta-\chi\zeta) & 2(\xi\zeta+\chi\eta) \\ 2(\xi\eta+\chi\zeta) & 2(\chi^2+\eta^2)^2-1 & 2(\eta\zeta-\chi\xi) \\ 2(\xi\zeta-\chi\eta) & 2(\eta\zeta+\chi\xi) & 2(\chi^2+\zeta^2)^2-1 \end{bmatrix} \quad (11)$$

Integration algorithm

Integrated gyro outputs on time intervals $[t-2\Delta t;t-\Delta t]$ and $[t-\Delta t;t]$ are needed to compute $Q(t)$ knowing $Q(t-\Delta t)$. Let's define :

$$\alpha_1^* = \int_{t-2\Delta t}^{t-\Delta t} p.d\tau \quad \alpha_2^* = \int_{t-2\Delta t}^{t-\Delta t} q.d\tau \quad \alpha_3^* = \int_{t-2\Delta t}^{t-\Delta t} r.d\tau$$

$$\alpha_1 = \int_{t-\Delta t}^{t} p.d\tau \quad \alpha_2 = \int_{t-\Delta t}^{t} q.d\tau \quad \alpha_3 = \int_{t-\Delta t}^{t} r.d\tau$$

and the quaternions :

$$\begin{cases} \alpha^* = \alpha_1^*.i+\alpha_2^*.j+\alpha_3^*.k \\ \alpha \ = \alpha_1.i+\alpha_2.j+\alpha_3.k \end{cases}$$

$$Q(t) = Q(t-\Delta t)\left[1 + \frac{\alpha}{2} + \frac{\alpha^2}{8} + \frac{1}{24}\left(\alpha^*.\alpha - \alpha.\alpha^*\right)\right]$$

The analytical solution of $\dot{Q} = \frac{1}{2} Q\,\Omega$ (with the restriction concerning the rotation axis) is

$$Q(t) = Q_0.\exp\left(\int_0^t \frac{\Omega}{2}\,d\tau\right) = Q_0.\exp\left(\frac{\alpha}{2}\right)$$
$$= Q_0.\left(1+\frac{\alpha}{2}+\frac{1}{2!}\left(\frac{\alpha}{2}\right)^2+\frac{1}{3!}\left(\frac{\alpha}{2}\right)^3+\ldots\right)$$

It appears that the solution given by this algorithm corresponds to a third-order approximation of the analytical solution. The term $\frac{1}{24}\left(\alpha^*.\alpha - \alpha.\alpha^*\right)$ corresponds to a correction applied to account for the fact that the rotation axis is not fixed.

At each step, a normalization of the updated quaternion is performed :

$$|Q| = \sqrt{Q_0^2 + Q_1^2 + Q_2^2 + Q_3^2}$$
$$Q \rightarrow Q / |Q|$$

Once the quaternion is determined, the corresponding direction cosines matrix is computed (11) and the roll-pitch-yaw angles are extracted (5,6,7).

$Q(t_0)$ is initialized by accelerometric measurement of gravity when the robot is static at the initial position.

6: Fusion of accelerometric and gyrometric attitude estimations

The fusion only concerns roll and pitch estimations since yaw cannot be estimated with the accelerometers. Accelerometers allow a long-term control of the attitude because this attitude estimation has a good mean value. On the other hand, the instantaneous estimate can be completely wrong because robot vibrations are converted into angles and because numerical differentiations are used. Gyrometers give a good short time evolution of the attitude but this estimation is subject to drift with time (gyrometer drift, integration process). A fusion by extended Kalman filtering is performed to take advantage of both methods.

Kalman filter

We wish to estimate the mean drifts on pitch and roll, due to gyrometric drifts and to the integration process, to correct the gyrometric estimation of the attitude. Measurements on the experimental site tend to show that the following model can be used :

$$\psi_g(t) = \psi(t) + d_\psi(t).t \qquad (12)$$
$$\phi_g(t) = \phi(t) + d_\phi(t).t \qquad (13)$$

where ψ_g and ϕ_g are determined by the gyrometric method, ψ and ϕ define the real attitude, and d_ψ and d_ϕ are the mean drifts between the initial time and time instant t.

The state vector **X** is made up of the pitch and roll angles as well as their drifts. Since the angular movement of the robot is unknown, the state transition equation is :

$$\begin{bmatrix} \psi \\ \phi \\ d_\psi \\ d_\phi \end{bmatrix}_{k+1} = \begin{bmatrix} \psi \\ \phi \\ d_\psi \\ d_\phi \end{bmatrix}_k + v_k \iff \mathbf{X}_{k+1} = \mathbf{X}_k + v_k$$

where v_k is the state noise vector presumed uncorrelated, zero-mean, and Gaussian with a constant variance Q_k.

The observation vector **Z**, given below, contains the x and y components of gravity and the gyrometric attitude estimate. Using equations (8,9,12,13), the observation equation can be written :

$$\begin{bmatrix} g_x \\ g_y \\ \psi_g \\ \phi_g \end{bmatrix}_k = \begin{bmatrix} -g\sin\psi \\ g\cos\psi\sin\phi \\ \psi+d_\psi.t \\ \phi+d_\phi.t \end{bmatrix}_k + w_k \iff \mathbf{Z}_k = h(\mathbf{X}_k,k) + w_k$$

where w_k is the observation noise vector presumed uncorrelated, zero-mean, and Gaussian with a constant variance R_k.

Given the inital state estimate $\hat{X}_{0/0}$ with its associated variance $P_{0/0}$, the attitude is calculated at step k+1 using the following formulas :

$$\hat{X}_{k+1/k} = \hat{X}_{k/k}$$

$$P_{k+1/k} = P_{k/k}+Q_k$$

$$K_{k+1} = P_{k+1/k}H^T_{k+1}(H_{k+1}P_{k+1/k}H^T_{k+1}+R_{k+1})^{-1}$$

$$P_{k+1/k+1} = (I - K_{k+1}H_{k+1})P_{k+1/k}$$

$$\hat{X}_{k+1/k+1}= \hat{X}_{k+1/k} + K_{k+1} (Z_{k+1}-h(\hat{X}_{k+1/k},k+1))$$

$$\text{with } H_{k+1} = \left.\frac{\partial h(X_{k+1},k+1)}{\partial X_{k+1}}\right|_{X_{k+1}=\hat{X}_{k+1/k}}$$

7: Experimental results

The system described above has been implemented and tested on the two-differential-wheels mobile platform CHARLY available in our laboratory. All software (robot's control and inertial measurements processing) is programmed in C and executed on a T805 transputer network. The real-time implementation allows the attitude estimation to be updated every 10 ms. Three gyrometers and a three-axis accelerometer are fixed on a precision-made support to ensure a good alignment of the sensitivity axes. This support is attached to the robot.

In the experiment, the robot moves in a straight line for 5 meters, turns 180° with zero radius, and moves again in a straight line for 5 meters. This motion is executed on flat ground, except during a part of the linear phases. During the first linear part, at time t≈7s, the right wheel climbs onto a plank which is 2 cm thick and 150 cm long, using a small inclined plane. At time t≈12s, the wheel gets back onto flat ground. After the half turn, in the second linear part, the robot's left wheel climbs onto the plank (t≈32s and t≈37s). All inertial measurements are filtered by a second order Butterworth with a 10Hz cut frequency.

Figure 1 shows the estimations of the roll angle throughout the motion. The graphic sampling interval is 0.15s. With a 64.8 cm tread, the theoretical roll is -1.76° during the first linear phase and +1.76° during the second. It can be seen that the accelerometric estimates have a good mean value (0° when the robot moves on flat ground, and around ±1.5° when one wheel of the robot is on the plank), but that instantaneous values can be completely wrong because of shocks or vibrations (example: ≈-4.5° at t≈8s). It can also be verified that the gyrometric estimates give a precise short time evolution of roll. For example, between t≈7s and t≈12s, the variation of roll is very well represented. On the other hand, the values of this angle get worse with time, because of the drift due to the gyros and the integration process. The roll estimation obtained by extended Kalman filtering gives a precise evolution and value of the angle at the same time. The variation of roll observed between t≈22s and t≈26s, corresponds to the rotating phase. Before the rotation, the gyrometric roll estimation is about -2°. Since this angle is calculated by integration, after a +180° rotation, it becomes +2°. Everything happens as if the robot had turned on a 2° inclined plane. This is why a +2° variation can be observed on the pitch estimation (figure 2).

figure 1: ROLL ESTIMATION

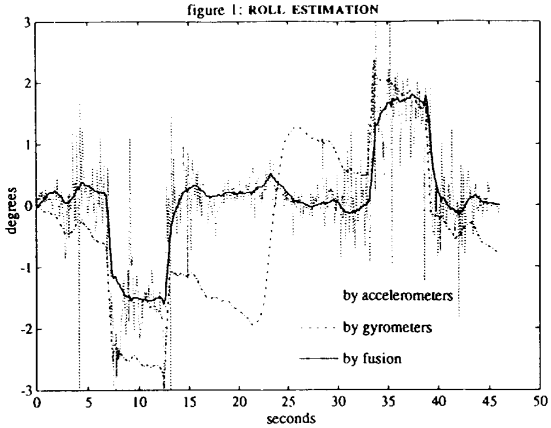


figure 2: PITCH ESTIMATION

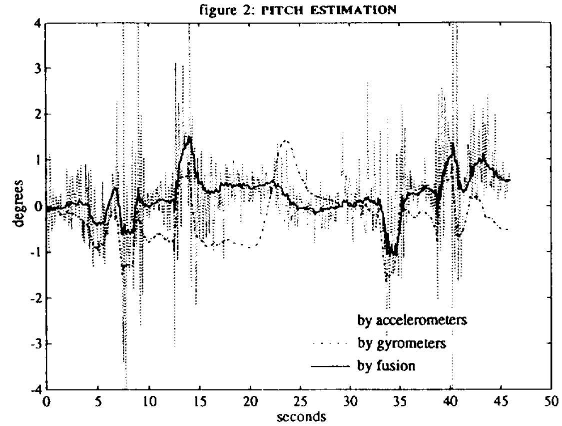
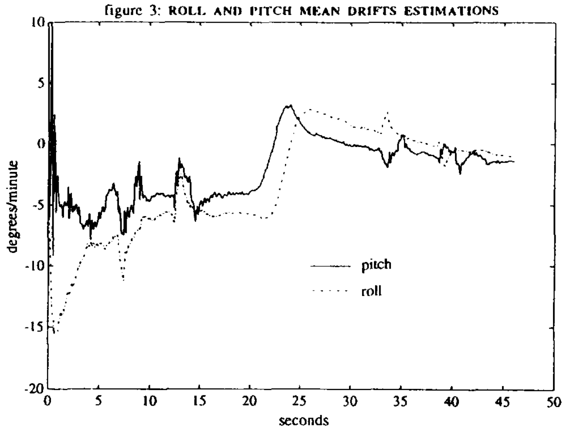


Figure 2 shows the estimations of the pitch angle throughout the motion. The same comments are valid concerning the accelerometric and gyrometric estimations. Variations in the pitch can be observed when the robot climbs on the plank (negative pitch around t≈8s and t≈34s) or when it climbs down from it (positive pitch around t≈14s and t≈39s).

Figure 3 shows the estimations of the pitch and roll mean drifts by the Kalman filter. Let's consider the roll mean drift estimation. The drift mean value between t=0 and t≈22s is quite constant and close to -5°/min, which can be verified on figure 1. At time t≈24s, the mean drift is close to 0°/min since gyrometric and accelerometric estimations are very close. Because of the rotation, the drift becomes positive and then decreases. If the movement had continued it would probably have converged to a value of -5°/min.

figure 3: ROLL AND PITCH MEAN DRIFTS ESTIMATIONS



8: Conclusion

This paper described an attitude estimation system based on inertial measurements. Even with low-cost inertial sensors, this system appears to be very sensitive and accurate when used on an indoor mobile robot. The method takes advantage of both types of sensors by fusing the attitude estimations they provide via a Kalman filter. Real-time implementation of the system on our mobile platform CHARLY allowed to show that it works well, but the variation domain of the attitude being very restricted, it would be interesting to test it on other types of vehicles. To employ this system on other types of vehicles (all-terrain, under-water), an estimation of the absolute acceleration of the vehicle should be available to extract the gravity components from the accelerometric measurements. The sensors' full scales and resolutions should also be adapted to the vehicle's dynamics.

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