

# DFA Minimization Algorithm, In-class Assignment 5

Benjamin Sanders, MS October 27, 2020

## 1 Instructions

When directed, complete one problem at a time. You may work in groups of up to two or three students. Write all team names at the top of the assignment. Turn in your work to Blackboard before the deadline to receive credit.

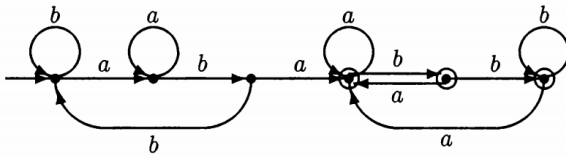
## 2 The DFA Minimization Algorithm

Here is an algorithm for computing the collapsing relation  $\approx$  for a given DFA  $M$  with no inaccessible states. Our algorithm will mark (unordered) pairs of states  $\{p, q\}$ . A pair  $\{p, q\}$  will be marked as soon as a reason is discovered why  $p$  and  $q$  are not equivalent.

1. Write down a table of all pairs  $\{p, q\}$ , initially unmarked.
2. Mark  $\{p, q\}$  if  $p \in F$  and  $q \notin F$  or vice versa.
3. Repeat the following until no more changes occur:  
if there exists an unmarked pair  $\{p, q\}$  such that  $\{\delta(p, a), \delta(q, a)\}$  is marked for some  $a \in \Sigma$ , then mark  $\{p, q\}$ .
4. When done,  $p \approx q \iff \{p, q\}$  is not marked.

## 3 Example DFA to Collapse

Collapse the following DFA, using the DFA Minimization Algorithm.



Write the DFA in table form. This will be helpful in step 3.

	$a$	$b$
$\rightarrow 0$	1	2
1	1	2
2	3F	0
3F	3F	4F
4F	3F	5F
5F	3F	5F

1. Write down a table of all pairs  $\{p, q\}$ , initially unmarked.

	0	1	2	3F	4F	5F
0	unmarked	repeat	repeat	repeat	repeat	repeat
1	$\{0, 1\}$	unmarked	repeat	repeat	repeat	repeat
2	$\{0, 2\}$	$\{1, 2\}$	unmarked	repeat	repeat	repeat
3F	$\{0, 3F\}$	$\{1, 3F\}$	$\{2, 3F\}$	unmarked	repeat	repeat
4F	$\{0, 4F\}$	$\{1, 4F\}$	$\{2, 4F\}$	$\{3F, 4F\}$	unmarked	repeat
5F	$\{0, 5F\}$	$\{1, 5F\}$	$\{2, 5F\}$	$\{3F, 5F\}$	$\{4F, 5F\}$	unmarked

2. Mark  $\{p, q\}$  if  $p \in F$  and  $q \notin F$  or vice versa.

	0	1	2	$3F$	$4F$	$5F$
0	unmarked	repeat	repeat	repeat	repeat	repeat
1	$\{0, 1\}$	unmarked	repeat	repeat	repeat	repeat
2	$\{0, 2\}$	$\{1, 2\}$	unmarked	repeat	repeat	repeat
$3F$	$\{0, 3F\}$	$\{1, 3F\}$	$\{2, 3F\}$	unmarked	repeat	repeat
$4F$	$\{0, 4F\}$	$\{1, 4F\}$	$\{2, 4F\}$	$\{3F, 4F\}$	unmarked	repeat
$5F$	$\{0, 5F\}$	$\{1, 5F\}$	$\{2, 5F\}$	$\{3F, 5F\}$	$\{4F, 5F\}$	unmarked

3. Repeat the following until no more changes occur:

if there exists an unmarked pair  $\{p, q\}$  such that  $\{\delta(p, a), \delta(q, a)\}$  is marked for some  $a \in \Sigma$ , then mark  $\{p, q\}$ .

(a) Iteration

	0	1	2	$3F$	$4F$	$5F$
0	unmarked	repeat	repeat	repeat	repeat	repeat
1	$\{0, 1\}$ $a \rightarrow \{1, 1\}$ $b \rightarrow \{0, 2\}$	unmarked	repeat	repeat	repeat	repeat
2	$\{0, 2\}$ $a \rightarrow \{1, 3F\}$ $b \rightarrow \{0, 0\}$	$\{1, 2\}$ $a \rightarrow \{1, 3F\}$ $b \rightarrow \{2, 0\}$	unmarked	repeat	repeat	repeat
$3F$	$\{0, 3F\}$	$\{1, 3F\}$	$\{2, 3F\}$	unmarked	repeat	repeat
$4F$	$\{0, 4F\}$	$\{1, 4F\}$	$\{2, 4F\}$	$\{3F, 4F\}$ $a \rightarrow \{3F, 3F\}$ $b \rightarrow \{4F, 5F\}$	unmarked	repeat
$5F$	$\{0, 5F\}$	$\{1, 5F\}$	$\{2, 5F\}$	$\{3F, 5F\}$ $a \rightarrow \{3F, 3F\}$ $b \rightarrow \{4F, 5F\}$	$\{4F, 5F\}$ $a \rightarrow \{3F, 3F\}$ $b \rightarrow \{5F, 5F\}$	unmarked

(b) Iteration

	0	1	2	$3F$	$4F$	$5F$
0	unmarked	repeat	repeat	repeat	repeat	repeat
1	$\{0, 1\}$ $a \rightarrow \{1, 1\}$ $b \rightarrow \{0, 2\}$	unmarked	repeat	repeat	repeat	repeat
2	$\{0, 2\}$ $a \rightarrow \{1, 3F\}$ $b \rightarrow \{0, 0\}$	$\{1, 2\}$ $a \rightarrow \{1, 3F\}$ $b \rightarrow \{2, 0\}$	unmarked	repeat	repeat	repeat
$3F$	$\{0, 3F\}$	$\{1, 3F\}$	$\{2, 3F\}$	unmarked	repeat	repeat
$4F$	$\{0, 4F\}$	$\{1, 4F\}$	$\{2, 4F\}$	$\{3F, 4F\}$ $a \rightarrow \{3F, 3F\}$ $b \rightarrow \{4F, 5F\}$	unmarked	repeat
$5F$	$\{0, 5F\}$	$\{1, 5F\}$	$\{2, 5F\}$	$\{3F, 5F\}$ $a \rightarrow \{3F, 3F\}$ $b \rightarrow \{4F, 5F\}$	$\{4F, 5F\}$ $a \rightarrow \{3F, 3F\}$ $b \rightarrow \{5F, 5F\}$	unmarked

(c) Iteration

	0	1	2	$3F$	$4F$	$5F$
0	unmarked	repeat	repeat	repeat	repeat	repeat
1	$\{0, 1\}$ $a \rightarrow \{1, 1\}$ $b \rightarrow \{0, 2\}$	unmarked	repeat	repeat	repeat	repeat
2	$\{0, 2\}$ $a \rightarrow \{1, 3F\}$ $b \rightarrow \{0, 0\}$	$\{1, 2\}$ $a \rightarrow \{1, 3F\}$ $b \rightarrow \{2, 0\}$	unmarked	repeat	repeat	repeat
$3F$	$\{0, 3F\}$	$\{1, 3F\}$	$\{2, 3F\}$	unmarked	repeat	repeat
$4F$	$\{0, 4F\}$	$\{1, 4F\}$	$\{2, 4F\}$	$\{3F, 4F\}$ $a \rightarrow \{3F, 3F\}$ $b \rightarrow \{4F, 5F\}$	unmarked	repeat
$5F$	$\{0, 5F\}$	$\{1, 5F\}$	$\{2, 5F\}$	$\{3F, 5F\}$ $a \rightarrow \{3F, 3F\}$ $b \rightarrow \{4F, 5F\}$	$\{4F, 5F\}$ $a \rightarrow \{3F, 3F\}$ $b \rightarrow \{5F, 5F\}$	unmarked

4. When done,  $p \approx q \iff \{p, q\}$  is not marked.

Therefore, from the table, the only pairs left unmarked are  $\{3F, 4F\}$ ,  $\{3F, 5F\}$ , and  $\{4F, 5F\}$ . Consequently,  $3F \approx 4F$ ,  $3F \approx 5F$ , and  $4F \approx 5F$ .

Write the resulting DFA in table form, using the original DFA table, but now including all  $\approx$  relations.

		<i>a</i>	<i>b</i>
→	0	1	0
	1	1	2
	2	$3F \approx 4F \approx 5F$	0
	$3F \approx 4F \approx 5F$	$3F \approx 4F \approx 5F$	$3F \approx 4F \approx 5F$
	$3F \approx 4F \approx 5F$	$3F \approx 4F \approx 5F$	$3F \approx 4F \approx 5F$
	$3F \approx 4F \approx 5F$	$3F \approx 4F \approx 5F$	$3F \approx 4F \approx 5F$

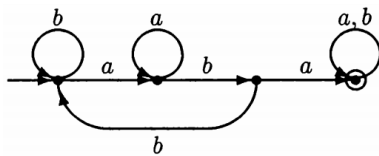
Eliminate any redundant rows from the resulting DFA table.

		<i>a</i>	<i>b</i>
→	0	1	0
	1	1	2
	2	$3F \approx 4F \approx 5F$	0
	$3F \approx 4F \approx 5F$	$3F \approx 4F \approx 5F$	$3F \approx 4F \approx 5F$

Collapse all  $\approx$  relations in the resulting table.

		<i>a</i>	<i>b</i>
→	0	1	0
	1	1	2
	2	$3F$	0
	$3F$	$3F$	$3F$

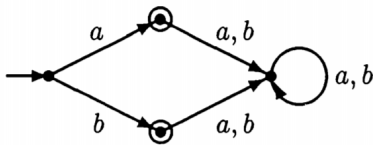
Finally, draw the resulting DFA.



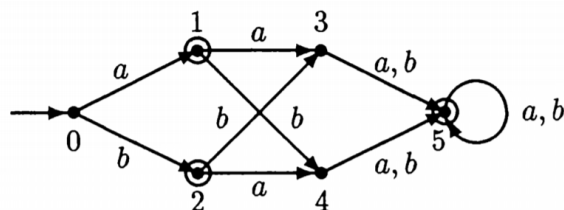
## 4 Assignment

Collapse the following DFAs using the DFA Minimization Algorithm. Show your work (all steps of the algorithm, as described above) to receive credit for correct answers.

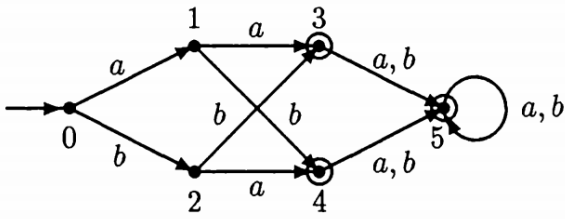
### 4.1 DFA to Collapse



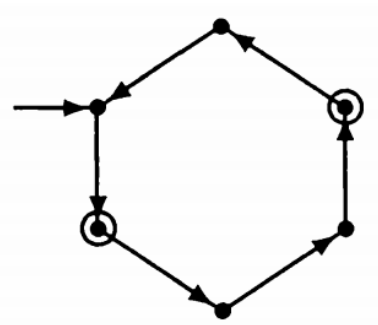
### 4.2 DFA to Collapse



### 4.3 DFA to Collapse



### 4.4 DFA to Collapse



## 5 What to Turn In

Turn in one PDF or Word document on Blackboard, containing the following items.

1. All pages scanned or photographed of the work you used to complete the In Class Assignment 5.
2. All team member names written at the top of your turnin.