CS 5012
Homework 1: Logic, Sets, Functions and Relations
Due date: Friday, September 18, 2015

LEARNING OBJECTIVES:

- Analyze the meanings of predicates
- Convert logical formulas to and from English sentences
- Analyze logical formulas
- Describe and explain sets and set-elements
- Identify and analyze functions and relations

QUESTIONS:

(Q1) [20 pts.]

Given the following predicates and their meanings

1. P(x,y) : $x > y$
2. Q(x,y) : $x \leq y$
3. R(x) : $x-7 = 2$
4. S(x) : $x > 9$

If the universe of discourse is the real numbers, give the truth value (**true** or **false**) of each of the following propositions:

(i) $(\exists x)\ R(x)$
True

(ii) $(\forall y)[\neg S(y)]$
False ($x = 7$)

(iii) $(\forall x)(\exists y)\ P(x,y)$
True

(iv) $(\exists y)(\forall x)\ Q(x,y)$
False ($y = 1$ and $x = 2$)

(v) $(\forall x)(\forall y)[P(x,y) \vee Q(x,y)]$
True

(vi) $(\exists x)\ S(x) \wedge \neg(\forall x)\ R(x)$
True

(vii) $(\exists y)(\forall x)[S(y) \wedge Q(x,y)]$
False ($y=10$ and $x = 11$)

(viii) $(\forall x)(\forall y)[\{R(x) \wedge S(y)\} \rightarrow Q(x,y)]$
True

(Q2) [10 pts.]

Which of the following sentences has the logical form **(p ^ q) → r**?

1. If you don't attend the wedding, then Sam will be angry with you
   1 is not right
2. Matt is happy and so are Sam and Fae
   2 is not right
3. If it rains and it snows then flooding will result
   3 is right
4. Students will play football or students will play soccer; but they will not attend classes
   4 is not right
5. Gene is smart and strong, additionally he is a good swimmer
   5 is not right

(Q3) [10 pts.]

Which of the following formulas represents the sentence, "*If there are no fruit in the market then the farmers didn't plant fruit trees or the farmers didn't water the trees*"

p means *There are no fruit in the market*
q means *Farmers didn't plant fruit trees*
r means *Farmers didn't water the trees*

1. $\neg p \rightarrow q$
Not this one
2. $p \rightarrow q \vee r$
This one!
3. $(p \rightarrow q) \vee \neg r$
Not this one
4. $p \rightarrow q \vee \neg r$
Not this one
5. $p \vee q \rightarrow \neg r$
Not this one

(Q4) [15 pts.]

Show **[p ^ (p → q)] → q** is a tautology.

P	Q	P → Q	P ^ (P → Q)	(P ^ P → Q) -> Q
T	**T**	**T**	**T**	**T**
T	**F**	**F**	**F**	**T**
F	**T**	**T**	**F**	**T**
F	**F**	**T**	**F**	**T**

(Q5) [15 pts.]

Argue that set A and set A' (the complement of A) are disjoint.

By contradiction assume there exists and x in A such that x is also in A'. Recall the definition of a complement set: U – A = A'. Therefore if x is in A then A cannot be in A'. Hence a contradiction, there A and A' are disjoint.

(Q6) [10 pts.]

Which of the following is a one-to-one function?

1. { (1,2), (2,3), (3,4), (4,5), (3,7), (2,2) }

1→ 2 and 2 → 2 therefore it is not 1-1

2. x = 5

This isn't even a function

3. x=5, 10 < y < 25

Also not a function

4. { (1,2), (2,3), (3,4), (2,5), (3,7) }

This is not a function since 2→3 and 2→5

5. { (1,2), (2,4), (3,6), (4,8) }

This is a function

(Q7) [20 pts.]

4 9 16 25

Let U = {x : x is an integer and $2 \leq x \leq 10$}.

In each of the following cases, determine whether $A \subseteq B$, $B \subseteq A$, both or neither:

odd: 3 5 7 9 // Even: 2 4 6 8 10 // mul3: 3 6 9 // even(x^2): 2 4 6 8 10 // pow2: 2 4 8
ivA: 4-10 // ivB: 5 – 10 // Sqrt x<=2: 4 9 // perfSq: 4 9 // sqrt<=2: 2-4 // +7perfSq: 2

(i) A = {x : x is odd } B = {x : x is a multiple of 3}

Neither

(ii) A = {x : x is even} B = {x : x^2 is even}

Both

(iii) $A = \{x : x \text{ is even } \}$ $B = \{x : x \text{ is a power of } 2\}$

$B \subseteq A$

(iv) $A = \{x : 2x + 1 > 7\}$ $B = \{x : x^2 > 20\}$

$B \subseteq A$

(v) $A = \{x : \sqrt{x} \in \mathbb{Z}\}$ $B = \{x : x \text{ is a power of 2 or 3}\}$ (see **note** below)

$A \subseteq B$

(vi) $A = \{x : \sqrt{x} \leq 2\}$ $B = \{x : x \text{ is a perfect square}\}$

Neither

(vii) $A = \{x : x^2 - 3x + 2 = 0\}$ $B = \{x : x + 7 \text{ is a perfect square}\}$

$A \subseteq B$

Note: $\mathbb{Z}$ denotes the set of all integers

GRADING:

- A maximum of **100 points** can be obtained on this homework assignment.

SUBMITTING:

- Submit on Collab
- Submit 1 **PDF** document as your homework
- You must work individually on this homework
- Your submitted homework must be typed
- At the top of your document be sure to include your name and computing ID
- The submission deadline is **5:00pm** on the date the assignment is due, mentioned above