CS 5012

Homework 1: Logic, Sets, Functions and Relations

Due date: Friday, September 18, 2015

Learning Objectives:

* Analyze the meanings of predicates
* Convert logical formulas to and from English sentences
* Analyze logical formulas
* Describe and explain sets and set-elements
* Identify and analyze functions and relations

Questions:

**(Q1)** [20 pts.]

Given the following predicates and their meanings

1. P(x,y) : x > y

2. Q(x,y) : x ≤ y

3. R(x) : x–7 = 2

4. S(x) : x > 9

If the universe of discourse is the real numbers, give the truth value (**true** or **false**) of each of the following propositions:

(i) (∃x) R(x)

**True**

(ii) (∀y)[¬S(y)]

**False** (x = 7)

(iii) (∀x)(∃y) P(x,y)

**True**

(iv) (∃y)(∀x) Q(x,y)

**False** (y = 1 and x = 2)

(v) (∀x)(∀y)[P(x,y) ˅ Q(x,y)]

**True**

(vi) (∃x) S(x) ∧ ¬(∀x) R(x)

**True**

(vii) (∃y)(∀x)[S(y) ∧ Q(x,y)]

**False** (y=10 and x = 11)

(viii) (∀x)(∀y)[{R(x) ∧ S(y)} → Q(x,y)]

**True**

**(Q2)** [10 pts.]

Which of the following sentences has the logical form **(p ^ q)** → **r**?

1. If you don’t attend the wedding, then Sam will be angry with you

**1 is not right**

1. Matt is happy and so are Sam and Fae

**2 is not right**

1. If it rains and it snows then flooding will result

**3 is right**

1. Students will play football or students will play soccer; but they will not attend classes

**4 is not right**

1. Gene is smart and strong, additionally he is a good swimmer

**5 is not right**

**(Q3)** [10 pts.]

Which of the following formulas represents the sentence, “*If there are no fruit in the market then the farmers didn’t plant fruit trees or the farmers didn’t water the trees*”

p means *There are no fruit in the market*

q means *Farmers didn’t plant fruit trees*

r means *Farmers didn’t water the trees*

1. http://www.cs.bham.ac.uk/research/lics/tutor/chap1/pics/not.gifp → q

**Not this one**

1. p → q ˅ r

**This one!**

1. (p → q) ˅ http://www.cs.bham.ac.uk/research/lics/tutor/chap1/pics/not.gifr

**Not this one**

1. p → q ˅ http://www.cs.bham.ac.uk/research/lics/tutor/chap1/pics/not.gifr

**Not this one**

1. p ˅ q → http://www.cs.bham.ac.uk/research/lics/tutor/chap1/pics/not.gifr

**Not this one**

**(Q4)** [15 pts.]

Show **[p ^ (p → q)] → q** is a tautology.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **P** | **Q** | **P → Q** | **P ^ (P → Q)** | **(P ^ P → Q) -> Q** |
| **T** | **T** | **T** | **T** | **T** |
| **T** | **F** | **F** | **F** | **T** |
| **F** | **T** | **T** | **F** | **T** |
| **F** | **F** | **T** | **F** | **T** |

**(Q5)** [15 pts.]

Argue that set A and set A' (the complement of A) are disjoint.

**By contradiction assume there exists and x in A such that x is also in A'. Recall the definition of a complement set: U – A = A'. Therefore if x is in A then A cannot be in A'. Hence a contradiction, there A and A' are disjoint.**

**(Q6)** [10 pts.]

Which of the following is a one-to-one function?

1. { (1,2), (2,3), (3,4), (4,5), (3,7), (2,2) }

**1→ 2 and 2 → 2 therefore it is not 1-1**

1. x = 5

**This isn't even a function**

1. x=5, 10 < y < 25

**Also not a function**

1. { (1,2), (2,3), (3,4), (2,5), (3,7) }

**This is not a function since 2→3 and 2→5**

1. { (1,2), (2,4), (3,6), (4,8) }

**This is a function**

**(Q7)** [20 pts.]

4 9 16 25

Let U = {x : x is an integer and 2 ≤ x ≤ 10}.

In each of the following cases, determine whether A ⊆ B, B ⊆ A, both or neither:

**odd: 3 5 7 9 // Even: 2 4 6 8 10 // mul3: 3 6 9 // even(x^2): 2 4 6 8 10 // pow2: 2 4 8**

**ivA: 4-10 // ivB: 5 – 10 // Sqrt x<=2: 4 9 // perfSq: 4 9 // sqrt<=2: 2-4 // +7perfSq: 2**

1. A = {x : x is odd } B = {x : x is a multiple of 3}

**Neither**

1. A = {x : x is even} B = {x : x2 is even}

**Both**

1. A = {x : x is even } B = {x : x is a power of 2}

**B ⊆ A**

1. A = {x : 2x + 1 > 7} B = {x : x2 > 20}

**B ⊆ A**

1. A = {x : √x ∈ ℤ} B = {x : x is a power of 2 or 3} (see **note** below)

**A ⊆ B**

1. A = {x : √x ≤ 2} B = {x : x is a perfect square}

**Neither**

1. A = {x : x2 − 3x + 2 = 0} B = {x : x + 7 is a perfect square}

**A ⊆ B**

**Note**: ℤ denotes the set of all integers

Grading:

* A maximum of **100 points** can be obtained on this homework assignment.

Submitting:

* Submit on Collab
* Submit 1 **PDF** document as your homework
* You must work individually on this homework
* Your submitted homework must be typed
* At the top of your document be sure to include your name and computing ID
* The submission deadline is **5:00pm** on the date the assignment is due, mentioned above