

# Prob and Stat Hmwk ch5

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5.1 Pg 335: 1,4,10;

4) Sample:50 Mean:654.1 Sd:311.7

a. Find a 95% confidence interval

$$\sigma = s/\sqrt{n}$$

$$\sigma = 311.7/\sqrt{50}$$

$$311.7/\sqrt{50}$$

```
## [1] 44.08104
```

$$1-\alpha = 0.95$$

$$\alpha = 0.05$$

$$\alpha/2 = 0.025$$

$$Z = 1.96$$

$$654.1 \pm 1.96(44.08)$$

$$1.96 * 44.08104$$

```
## [1] 86.39884
```

$$654.1 + 1.96 * 44.08104$$

```
## [1] 740.4988
```

$$654.1 - 1.96 * 44.08104$$

```
## [1] 567.7012
```

(567.7012, 740.4988)

b. Find a 98% confidence interval

$$1-\alpha = 0.98$$

$$\alpha = 0.02$$

$$\alpha/2 = 0.01$$

$$Z = 2.33$$

$$654.1 \pm 2.33(44.08)$$

$$2.33 * 44.08104$$

```
## [1] 102.7088
```

$$654.1 + 2.33 * 44.08104$$

```
## [1] 756.8088
```

$$654.1 - 2.33 * 44.08104$$

```
## [1] 551.3912
```

(551.3912, 756.8088)

c. A Traffic engineer states that the mean improvement is between 581.6 and 726.6 vehicles per hour. With what level of confidence can this statement be made?

$$581.6 = 654.1 - Z 311.7/\sqrt{50}$$

$$z = (654.1 - 581.6) / 311.7 \sqrt{50}$$

$$z = 1.64469 = 1.645$$

90%

10) Sample:60 Mean:85 Sd:2

a. Find a 95% confidence interval

$$\sigma = s / \sqrt{n}$$

$$\sigma = 2 / \sqrt{60}$$

```
2/sqrt(60)
```

```
## [1] 0.2581989
```

$$1 - \alpha = 0.95$$

$$\alpha = 0.05$$

$$\alpha/2 = 0.025$$

$$Z = 1.96$$

$$85 \pm 1.96 * (0.2581)$$

$$85 \pm 1.96 * (0.2581)$$

```
## [1] 85.50588
```

$$85 - 1.96 * (0.2581)$$

```
## [1] 84.49412
```

(84.49412, 85.50588)

b. Find a 99.5% confidence interval

$$1 - \alpha = 0.995$$

$$\alpha = 0.005$$

$$\alpha/2 = 0.0025$$

$$Z = 2.81$$

$$85 \pm 2.81 * (0.2581)$$

$$85 \pm 2.81 * (0.2581)$$

```
## [1] 85.72526
```

$$85 - 2.81 * (0.2581)$$

```
## [1] 84.27474
```

(84.27474, 85.72526)

c. What is the confidence level of the interval (84.63, 85.37)?

$$85 \pm Z(0.2581)$$

$$84.63 = 85 - Z(0.2581)$$

$$(85 - 84.63) / (0.2581)$$

```
## [1] 1.433553
```

$$Z = 1.45$$

$$\alpha/2 = 0.0735$$

$$\alpha = 0.147$$

$$(1 - \alpha) = 0.853$$

85.3%

- d. How many thermostats must be sampled so that a 95% confidence interval specifies the mean to within  $\pm 0.35$ ?

$$85 \pm 0.35 = (84.65, 85.35)$$

$$0.35 = z(s/\sqrt{n})$$

$$1 - \alpha = 0.95$$

$$\alpha = 0.05$$

$$\alpha/2 = 0.025$$

$$Z = 1.96$$

$$0.35 = 1.96(2/\sqrt{n})$$

$$\sqrt{n} = 1.96 \cdot 2 / 0.35$$

$$(1.96 \cdot 2 / 0.35)^2$$

```
## [1] 125.44
```

```
n = 126
```

- e. How many thermostats must be sampled so that a 99.5% confidence interval specifies the mean to within  $\pm 0.35$ ?

$$85 \pm 0.35 = (84.65, 85.35)$$

$$0.35 = z(s/\sqrt{n})$$

$$1 - \alpha = 0.995$$

$$\alpha = 0.005$$

$$\alpha/2 = 0.0025$$

$$Z = 2.81$$

$$0.35 = 2.81(2/\sqrt{n})$$

$$\sqrt{n} = 2.81 \cdot 2 / 0.35$$

$$(2.81 \cdot 2 / 0.35)^2$$

```
## [1] 257.8318
```

```
n = 258
```

5.3 Pg 353: 1,2,14,16;

2) Find the value of  $t_{[n-1, \alpha]}$  needed to construct an upper or lower confidence bound in each of the situation in Exercise 1.

a. Level 90%, sample size 12.

$$t \cdot s / \sqrt{n}$$

$$t_{[11, 0.05]} = 1.796$$

b. Level 90%, sample size 7.

$$t_{[6, 0.1]} = 1.440$$

c. Level 99%, sample size 2.

$$t_{[1, 0.01]} = 318.309$$

d. Level 95%, sample size 29.

$$t_{[28, 0.05]} = 1.701$$

14) One sample T: X N:10 Mean:6.59635 Sd:0.11213 SeMean:0.03546 95% CI:(6.51613, 6.67656)

a. How many degrees of freedom does the Student's t distribution have?

$$n - 1 = \text{dof}$$

$$10 - 1 = 9$$

b. Use the information in the output, along with the t table, to compute a 99% confidence interval.

$$t_{[9, .01]} = 2.821$$

$$6.59635 \pm 2.821 \cdot 0.03546$$

```
6.59635+2.821*0.03546
```

```
## [1] 6.696383
```

```
6.59635-2.821*0.03546
```

```
## [1] 6.496317
```

```
(6.496317,6.696383)
```

- 16) The concentration of carbon monoxide (CO) in a gas sample is measured by a spectrophotometer and found to be 85 ppm. Through long experience with this instrument, it is believed that its measurements are unbiased and normally distributed, with an uncertainty (Sd): 8ppm. Find a 95% confidence interval for the concentration of CO in this sample.

$$X \pm Z[a/2]sd$$

$$z[0.025] = 1.96$$

$$85 \pm 1.968$$

```
85+1.96*8
```

```
## [1] 100.68
```

```
85-1.96*8
```

```
## [1] 69.32
```

```
(69.32,100.68)
```

5.2 Pg 341: 1,2,11,12,16;

- 2) During a recent drought, a water utility in a certain town sampled 100 residential water bills and found that 73 of the residences has reduced their water consumption over that of the previous year.

- a. Find a 95% confidence interval for the proportion of residences that reduced their water consumption.

$$P \pm Z[\alpha/2] \sqrt{P(1-P)/m}$$

$$m = 100 + 4 = 104$$

$$P = (X+2)/m$$

$$P = (73+2)/104 = 75/104 = 0.721$$

$$0.721 \pm 1.96 \sqrt{0.721(1-0.721)/104}$$

```
0.721+1.96*sqrt(0.721*(1-.721)/104)
```

```
## [1] 0.8072004
```

```
0.721-1.96*sqrt(0.721*(1-.721)/104)
```

```
## [1] 0.6347996
```

```
(0.6347996,0.8072004)
```

- b. Find a 99% confidence interval for the proportion of residences that reduced their water consumption.

$$0.721 \pm 2.58 \sqrt{0.721(1-0.721)/104}$$

```
0.721+2.58*sqrt(0.721*(1-.721)/104)
```

```
## [1] 0.8344678
```

```
0.721-2.58*sqrt(0.721*(1-.721)/104)
```

```
## [1] 0.6075322
```

```
(.6075322,0.8344678)
```

- c. Find the sample size needed for a 95% confidence interval to specify the proportion to within  $\pm 0.05$

$$0.721 - 0.05 = 0.671$$

$$0.671 = 0.721 - Z[0.025] \sqrt{0.721(1-0.721)/m}$$

$$(.721(1-0.721))/((0.721-0.671)/1.96)^2=m$$

$$m=309.109$$

n=310

- d. Find the sample size needed for a 99% confidence interval to specify the proportion to within  $\pm 0.05$   
 $0.721 - .05 = 0.671$

$$0.671 = 0.721 - Z[0.025] \sqrt{0.721(1-0.721)/m}$$

$$(.721(1-0.721))/((0.721-0.671)/2.58)^2$$

$$m=535.5979$$

n=536

- e. Someone claims that more than 70% of residences reduced their water consumption with what level of confidence can this statement be made?

$$.7 = .721 - Z \sqrt{0.721(1-.721)/104}$$

$$z = (.721 - .7) / (\sqrt{0.721 * (1 - .721) / 104})$$

z

```
## [1] 0.4774921
```

```
z*2
```

```
## [1] 0.9549843
```

```
1-z*2
```

```
## [1] 0.04501571
```

95% confidence

- f. If 95% confidence intervals are computed for 200 towns, what is the probability that more than 192 of the confidence intervals cover the true proportions?

$$1 - \text{pbinom}(192, 200, 0.95)$$

```
## [1] 0.2133047
```

- 16) A stock market analyst notices that in a certain year, the price of IBM stock increased on 131 out of 252 trading days. Can these data be used to find a 95% confidence interval for the proportion of days that IBM stock increases? Explain.

$$P \pm Z[\alpha/2] \sqrt{P(1-P)/m}$$

$$m = 252 + 4 = 256$$

$$P = (X + 2) / m$$

$$P = (131 + 2) / 256 = 133 / 256 = 0.519$$

$$0.519 \pm 1.96 \sqrt{0.519(1-0.519)/256}$$

$$0.519 \pm 1.96 * \sqrt{0.519 * (1 - 0.519) / 256}$$

```
## [1] 0.5802058
```

$$0.519 - 1.96 * \sqrt{0.519 * (1 - 0.519) / 256}$$

```
## [1] 0.4577942
```

(0.4577, 0.5819)

I would probably shy away from his conclusion. The probably is already low and the interval is a little too large for me to be comfortable. Although its not a terrible range and some conclusions could be made but I wouldn't act to quickly on this day.