Computational Modelling of Surface Tension Driven Flow and Experimental Measurements of Metallic Wicks

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Outline

- Introduction and Motivation
- Governing Equations
- Interface Reconstruction and Advection
- Surface Tension and Curvature
- Validation Cases
- Current and Future Work
- Conclusion

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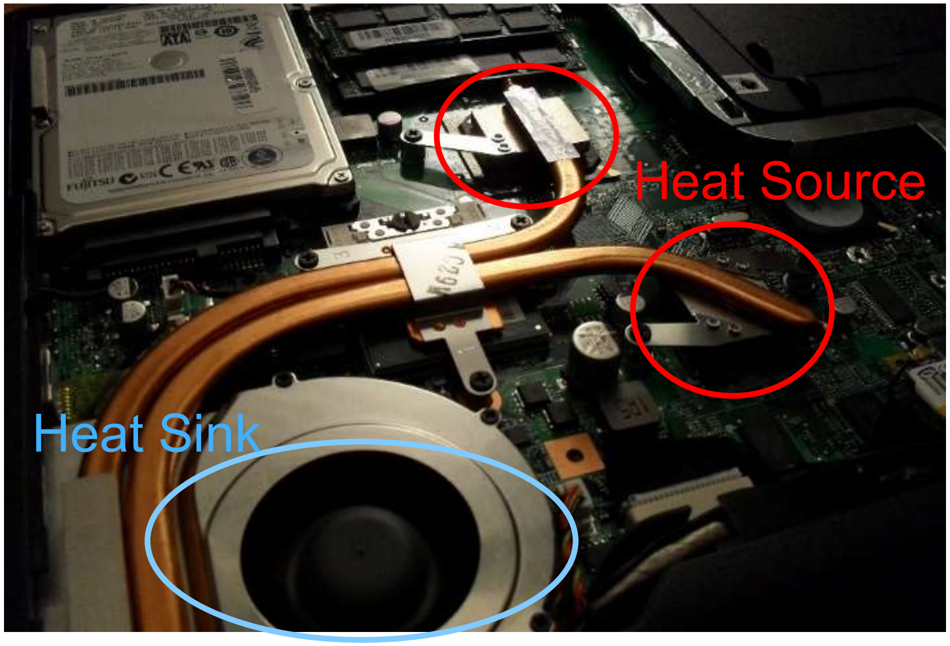
Introduction

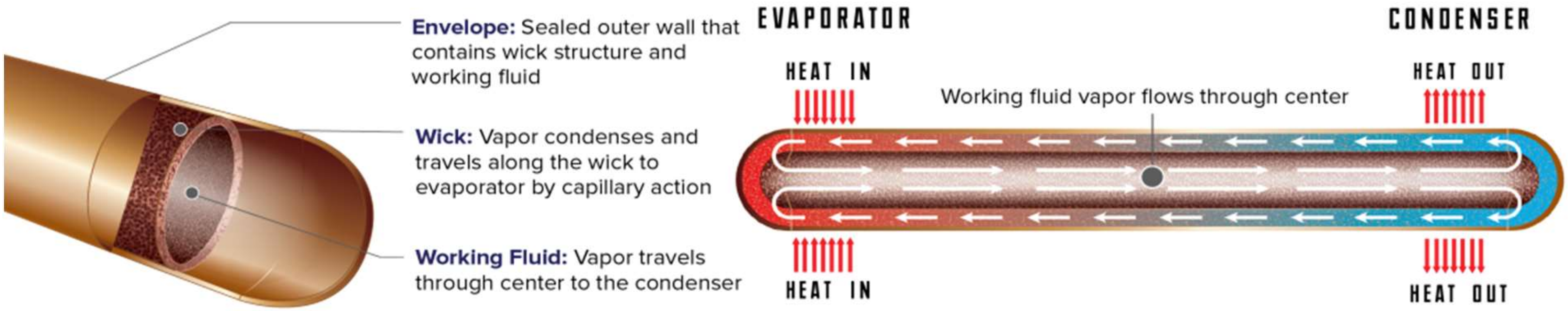
- Poor thermal management leads to total system failure
- Two-phase thermal management, is crucial to improve efficiency in miniaturized electronic, energy, and spacecraft systems
- Latent heat ➔Higher heat flux
- Passive cooling ➔ Reliability, No Power!
- Long distances ➔ Flexibility



https://commons.wikimedia.org/w/index.php?curid=13144188

Why wicks?

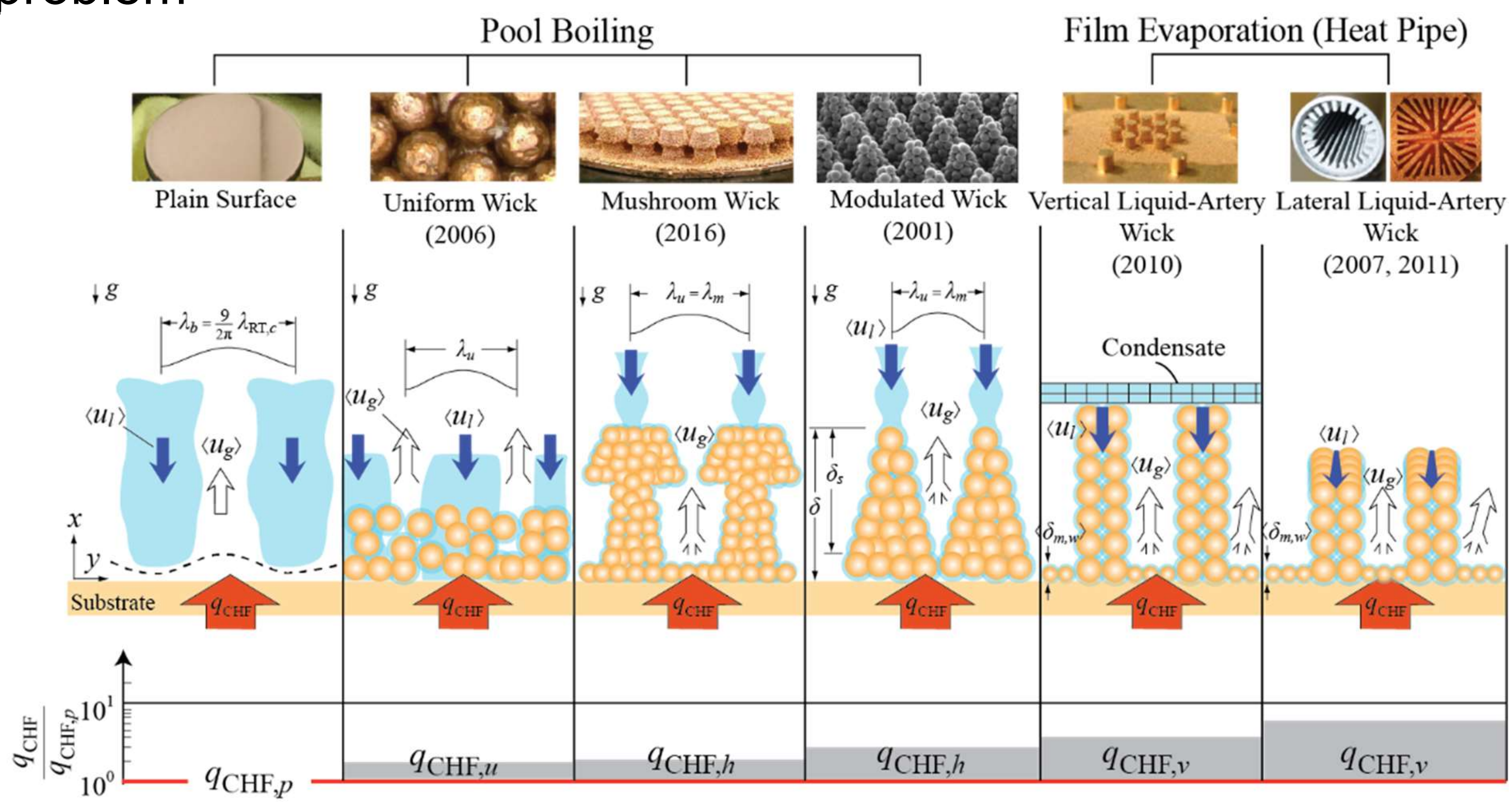
- Evaporator: Coolant is vaporized by absorbing heat
- Condenser: Phase change to liquid
- A wick structure is needed so that condensed liquid returns back to the heating region



https://www.1-act.com/resources/heat-pipe-resources/faq/

Why wicks?

- Fundamentally, wicks are used to supply cooland to the heated surface and can be applied to film evaporation and pool boiling problem



G. Hwang et al., "High-Heat-Flux, Distributed, Capillary-Artery Evaporators", *Handbook of Porous Media*, Taylor and Francis & CRC Press, 2015

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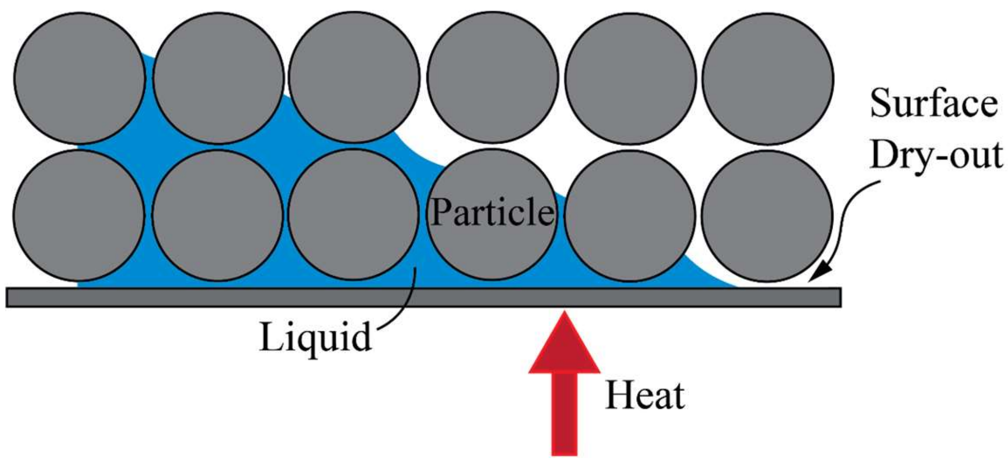
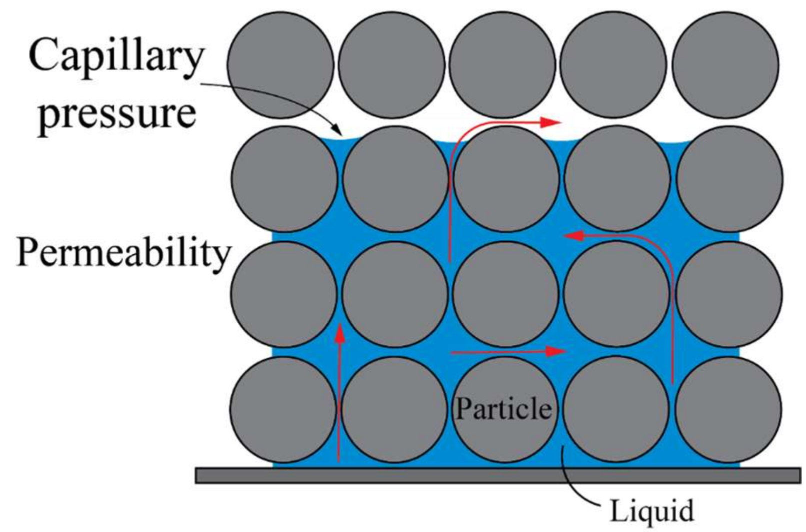
Preventing Surface Dry-Out

- It is important to keep the surface wet and prevent surface dry-out
- Limits the maximum cooling capability (critical heat flux)
- Continuous heat transfer can lead to system failure!

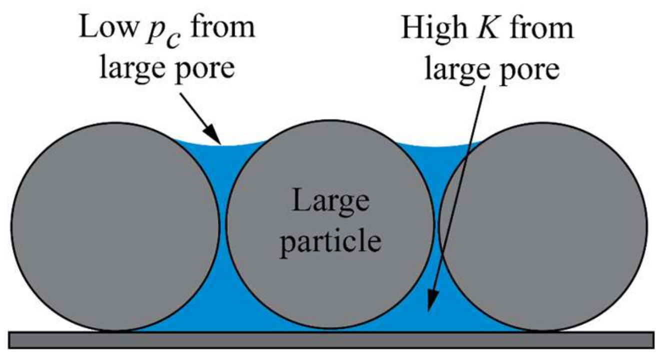


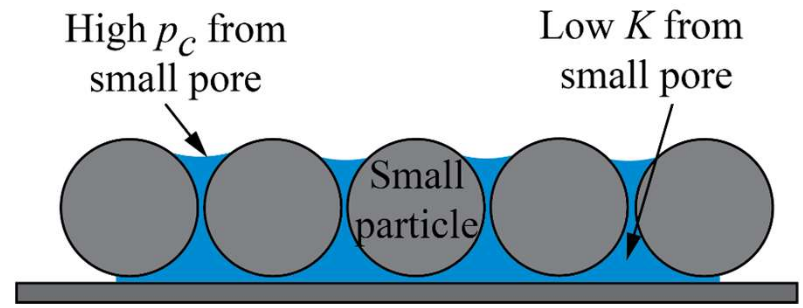


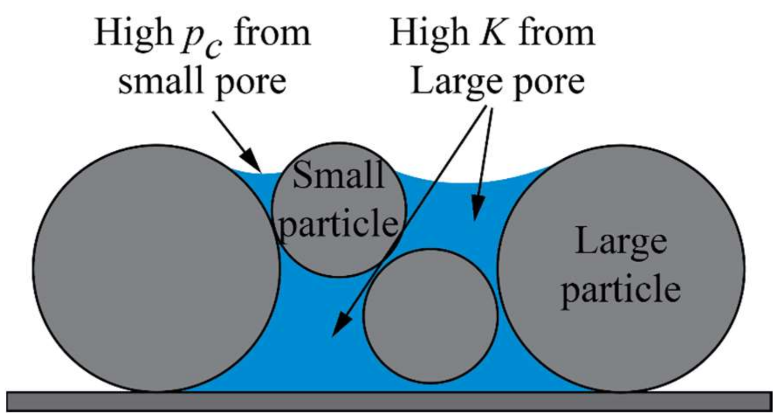
M. Borumand, T. Lee, and G. Hwang, “Enhanced wickability of single-columnar, non-uniform pore-size wick using lattice Boltzmann method,” *Computers and Fluids*, vol. 238, p. 105376, 2022. doi:10.1016/j.compfluid.2022.105376

Wickability

- High permeability $K = a\dfrac{\varepsilon^3 r_p{}^2}{(1-\varepsilon)^2}$
- High capillary pressure $p_c = \dfrac{2\sigma}{r_c}$

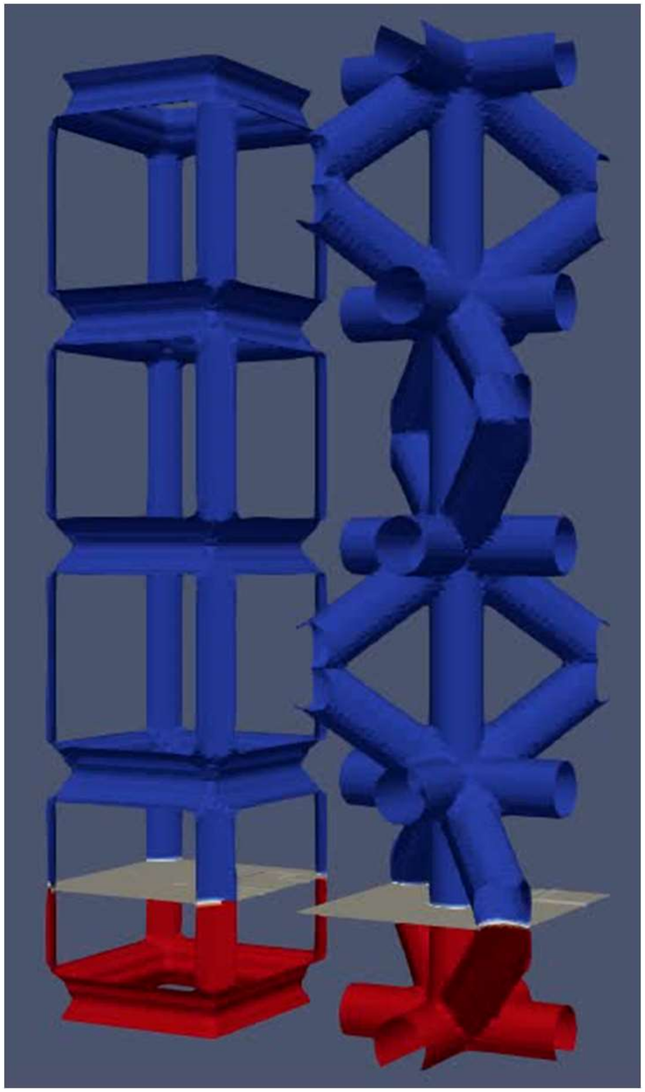






Modelling using Finite Volume Method

- Permeability requires single phase, steady state, pressure driven simulation.
  ➔ straight forward
  ➔ cheap
- Capillary pressure requires two phase, transient, buoyancy driven simulation!
  ➔ modelling of the vapor-liquid interface
  ➔ accurate contact angle modelling
  ➔ computationally expensive



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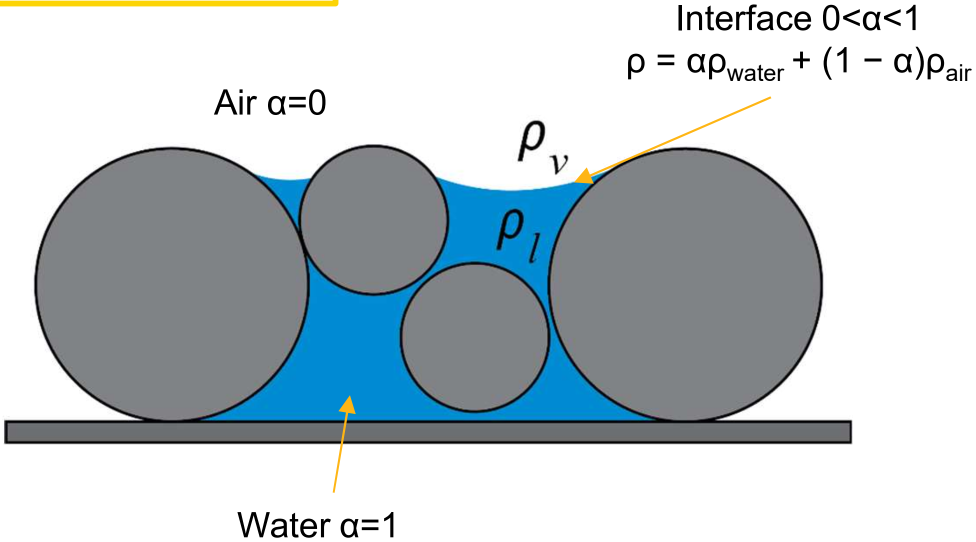
Governing Equations

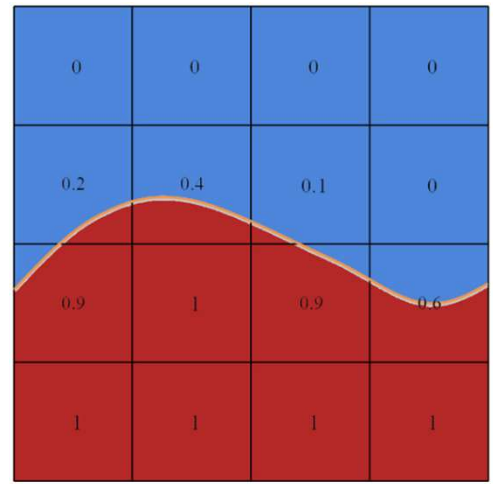
$$\nabla \cdot \mathbf{u} = 0$$

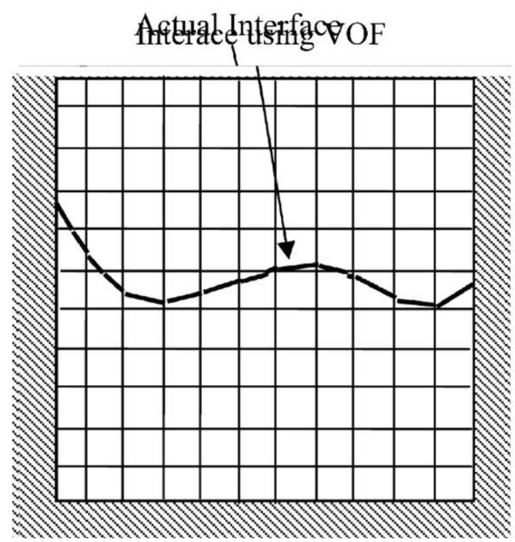
$$p_{rgh} = p - \rho(\mathbf{g} \cdot \mathbf{x})$$

$$\frac{\partial(\rho \mathbf{u})}{\partial t} + \mathbf{u} \cdot \nabla(\rho \mathbf{u}) - \nabla \cdot (\mu \nabla \mathbf{u}) = -\nabla p_{rgh} - \mathbf{g} \cdot \mathbf{x} \nabla \rho + \mathbf{f}$$

$$\boxed{\frac{\partial \alpha}{\partial t} + \nabla \cdot (\mathbf{u}\alpha) = 0}$$







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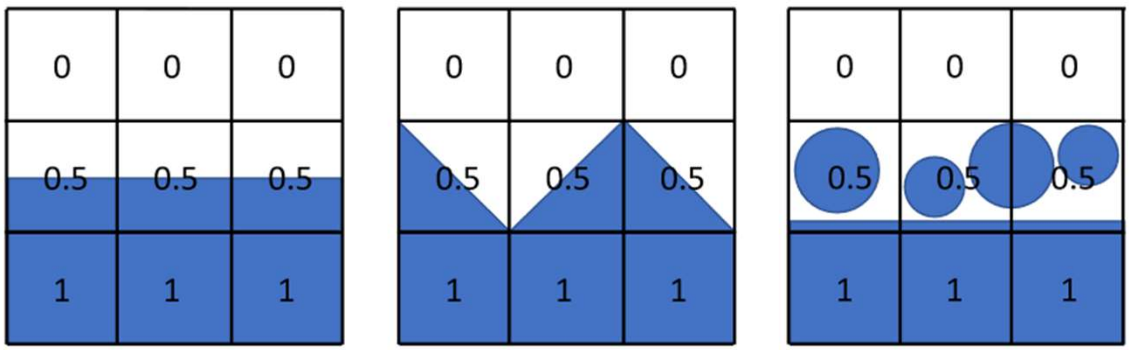
Interface Reconstruction

- Because we are not tracking the interface, when we solve the transport equation for the volume fraction, we only have knowledge of the scalar values.
- We have no idea on the distribution of the two fluids at the cell faces.

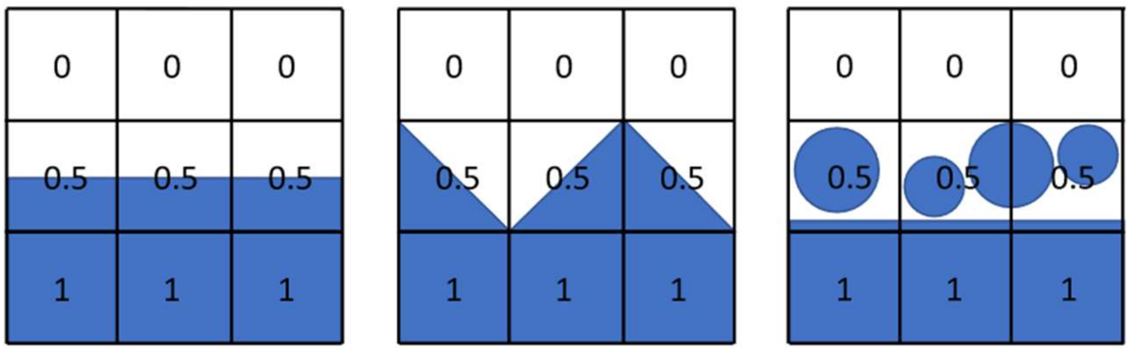


- A subgrid scale model is needed for cells with 0<α<1

J. Roenby, H. Bredmose, and H. Jasak, "A computational method for sharp interface advection," *R. Soc. open sci.*, vol. 3, no. 11, p. 160405, Nov. 2016, doi: 10.1098/rsos.160405.

Interface Reconstruction

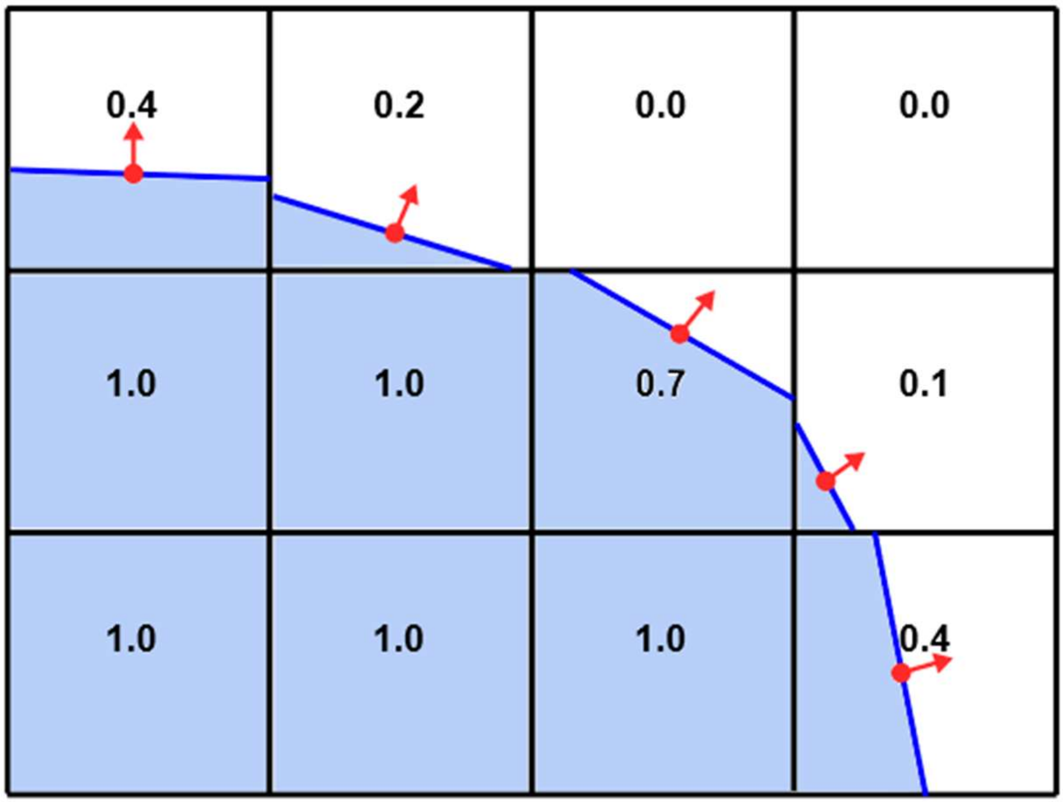
- The key assumption is that the interface is smooth (relative to grid size).
- This means that if the mesh size is much smaller than the local radius of curvature, $\Delta x << R$, then every cell can be approximated as a linear plane.



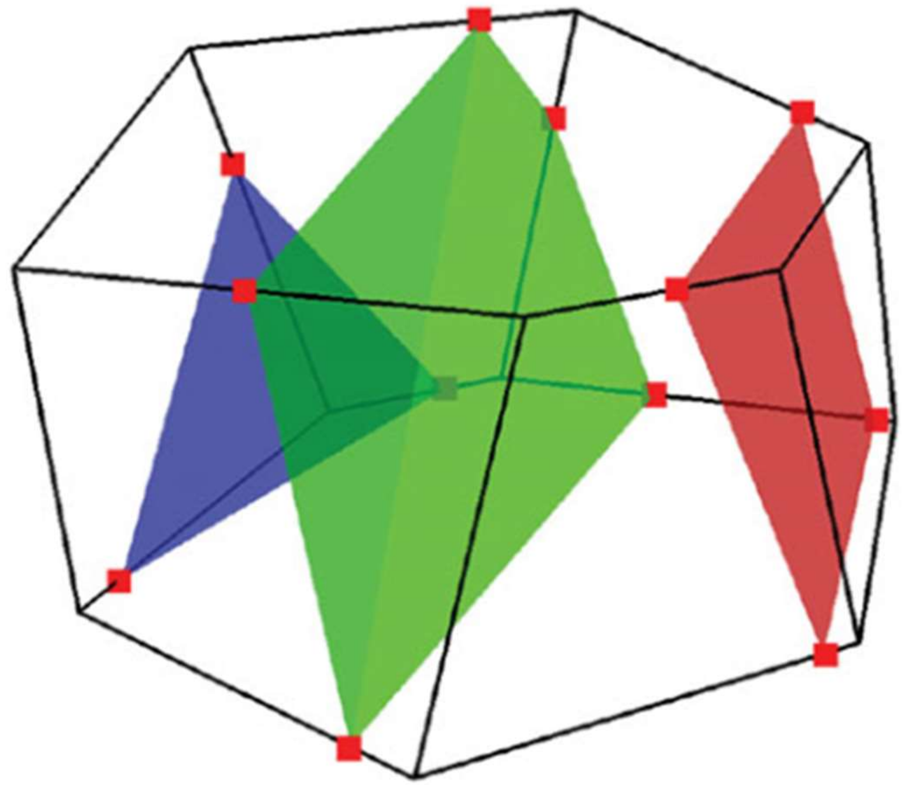
- Inherent restriction on mesh size
- Typical grid convergence not possible because refining the grids changes the subgrid modelling!

Interface Reconstruction

- We now know $\alpha(\mathbf{x},t)$ and the linear interface, however what is $\alpha(\mathbf{x},t+\Delta t)$?



Interface Advection



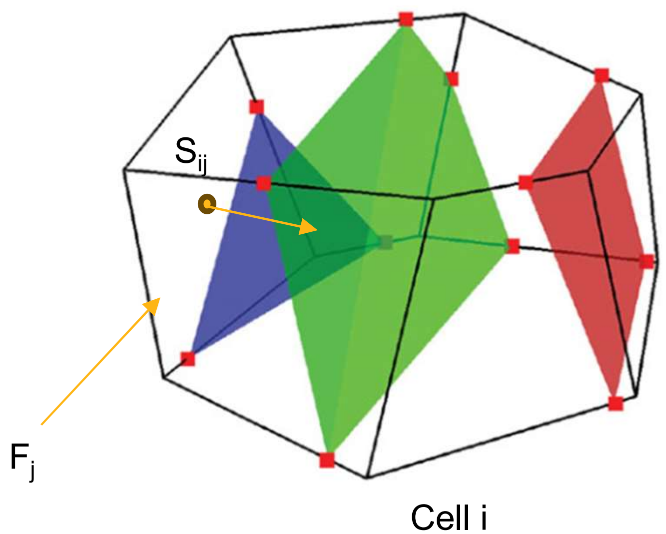
Interface Advection

This sums the net volume transported for all face of cell i

This is an exact solution and is valid for all two-phase solver

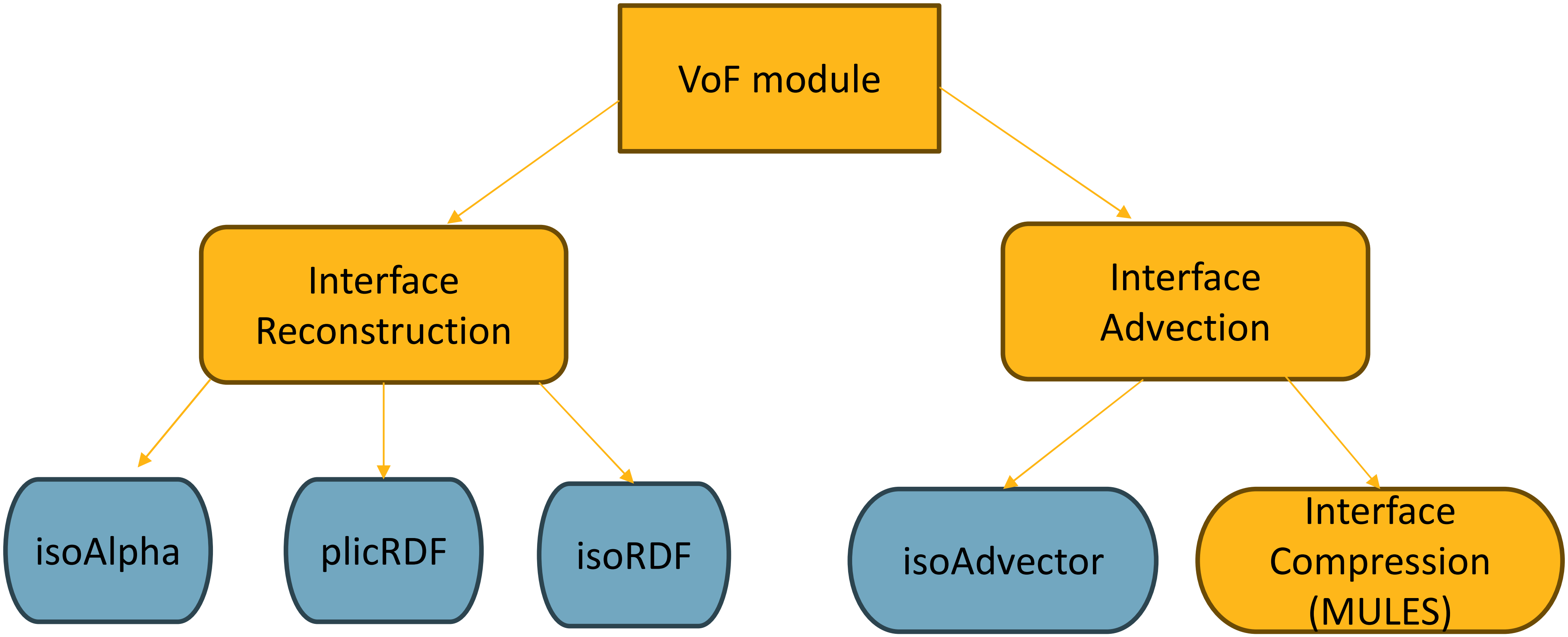
We have no idea what's going on at the future time step!

$$\alpha_i(t+\Delta t) = \alpha_i(t) - \frac{1}{V_i}\sum_{j \epsilon B_i} s_{ij} \int_t^{t+\Delta t} \int_{F_j} H(\mathbf{x},t)\mathbf{u}(\mathbf{x},t)d\mathbf{S}dt$$



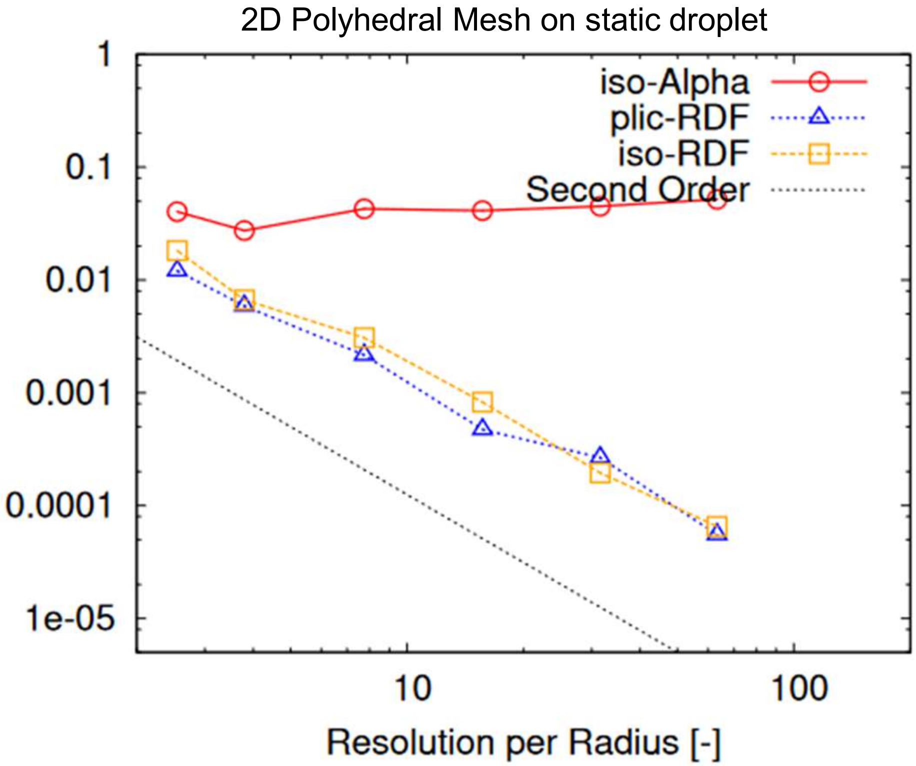
This is just the total volume of water that is transported across a single face j between t and t+Δt

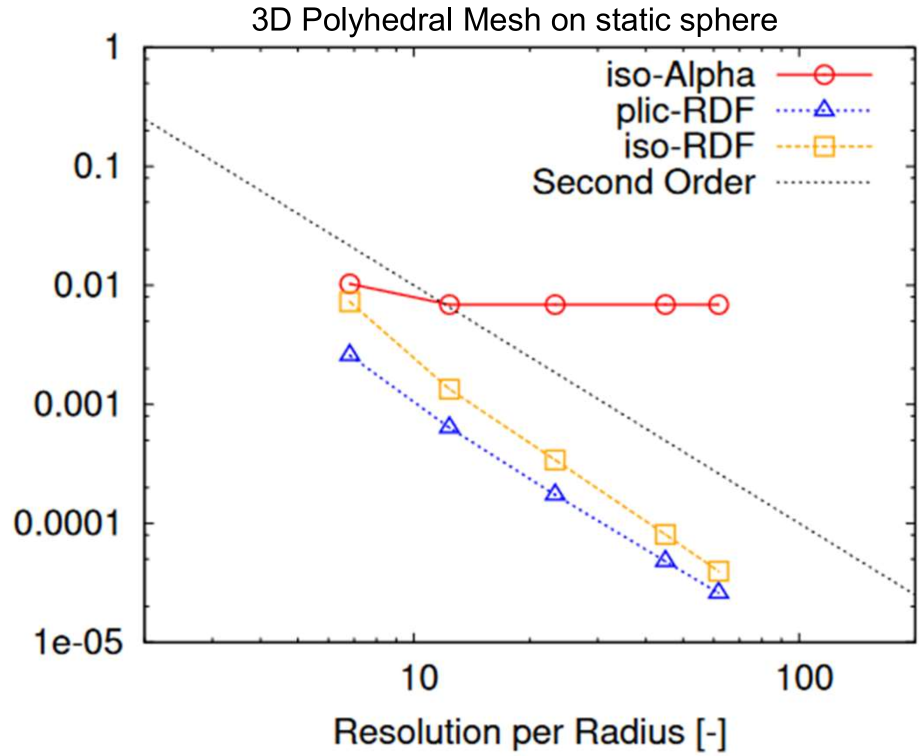
Overview of twoPhaseFlow library



H. Scheufler and J. Roenby, “TwoPhaseFlow: An OpenFOAM based framework for development of two phase flow solvers.” arXiv, Mar. 01, 2021. Accessed: Aug. 09, 2023. [Online]. Available: http://arxiv.org/abs/2103.00870

Notes on Accuracy





- plicRDF and isoRDF yields second-order convergence for 2D and 3D unstructured meshes in static equilibrium conditions
- Much better accuracy than iso-Alpha (standard OpenFOAM)

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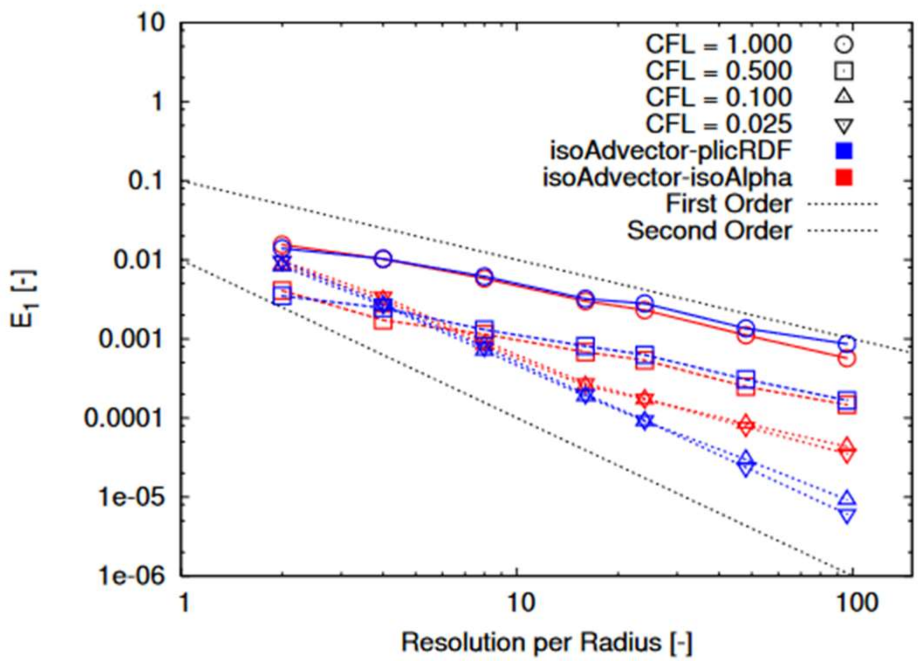
H. Scheufler and J. Roenby, “Accurate and efficient surface reconstruction from volume fraction data on general meshes,” *Journal of Computational Physics*, vol. 383, pp. 1–23, Apr. 2019, doi: 10.1016/j.jcp.2019.01.009.

Notes on Accuracy

3D Polyhedral Mesh on advected sphere



- plicRDF generally yields second-order convergence for 3D unstructured meshes
- plicRDF with isoAdvector scheme should be used for most applications

H. Scheufler and J. Roenby, "Accurate and efficient surface reconstruction from volume fraction data on general meshes," *Journal of Computational Physics*, vol. 383, pp. 1–23, Apr. 2019, doi: [10.1016/j.jcp.2019.01.009](10.1016/j.jcp.2019.01.009).

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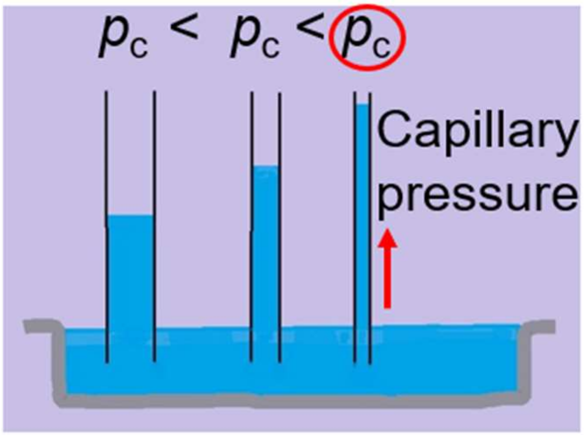
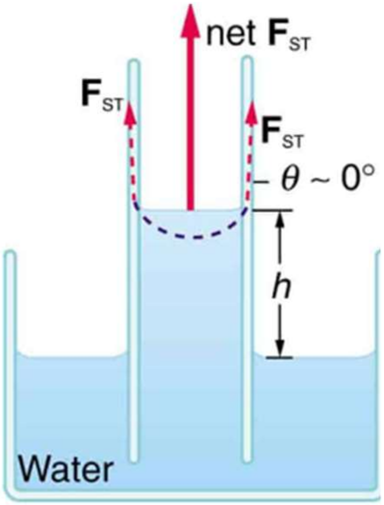
Surface Tension and Curvature

$$\frac{\partial(\rho\mathbf{u})}{\partial t} + \mathbf{u}\cdot\nabla(\rho\mathbf{u}) - \nabla\cdot(\mu\nabla\mathbf{u}) = -\nabla p_{rgh} - \mathbf{g}\cdot\mathbf{x}\nabla\rho + \mathbf{f}$$

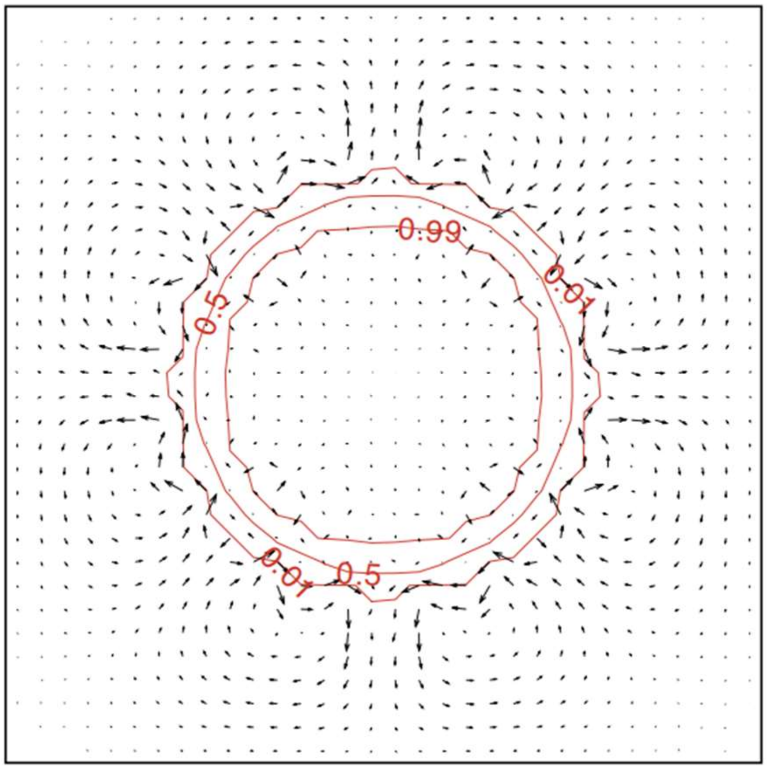
- Capillary Pressure ➔Surface Tension
- Errors:
  - Interface reconstruction and advection
  - Temporal and spatial discretization
  - External force discretization
- In flows that is driven by surface tension, these errors have a noticeable impact in predicted equilibrium height

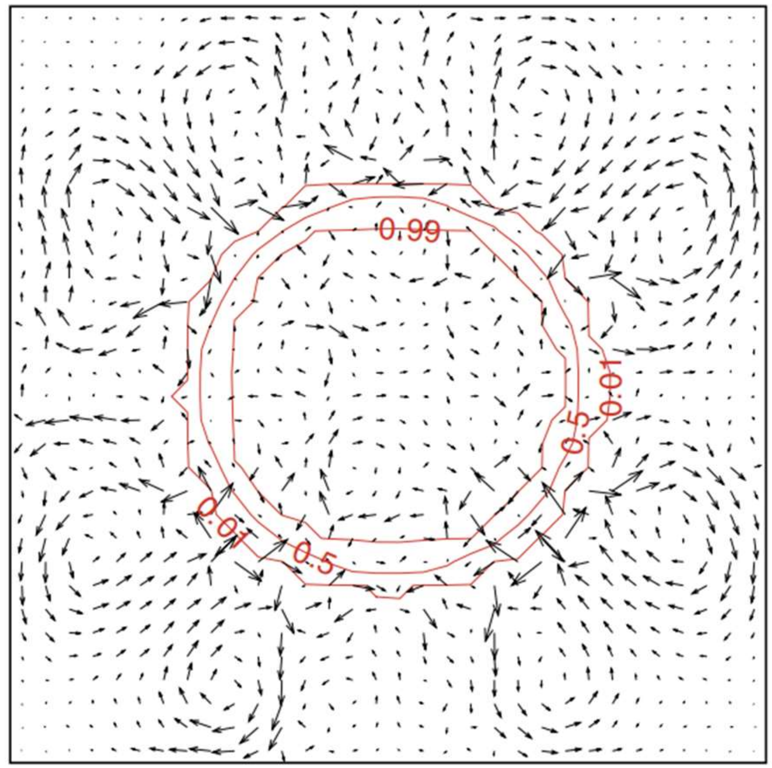




Surface Tension and Curvature

- Imperfect discretization of the pressure discontinuity leads to artificially induced velocities around the interface causing parasitic currents



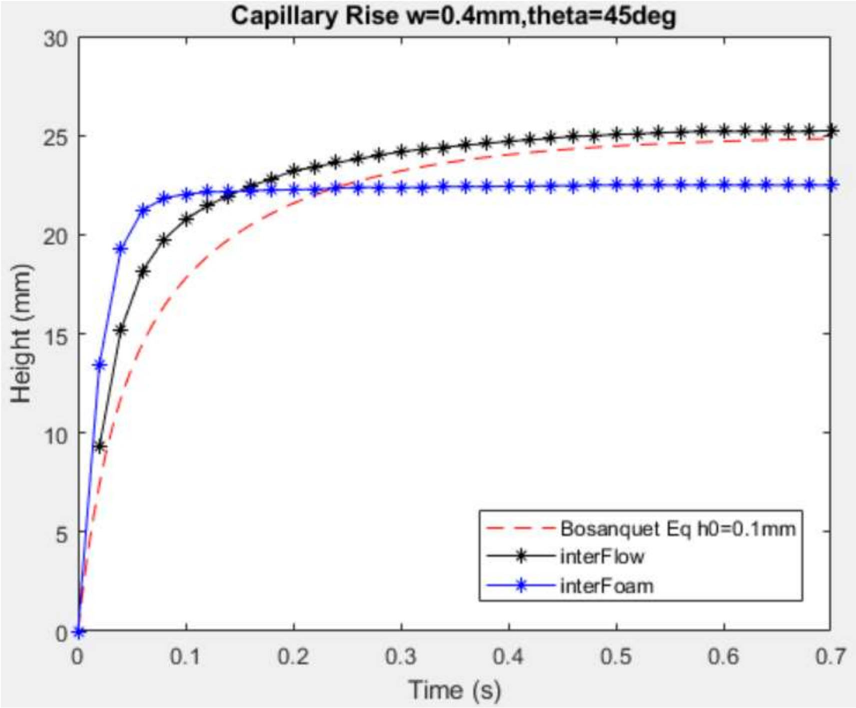


Surface Tension of Curvature

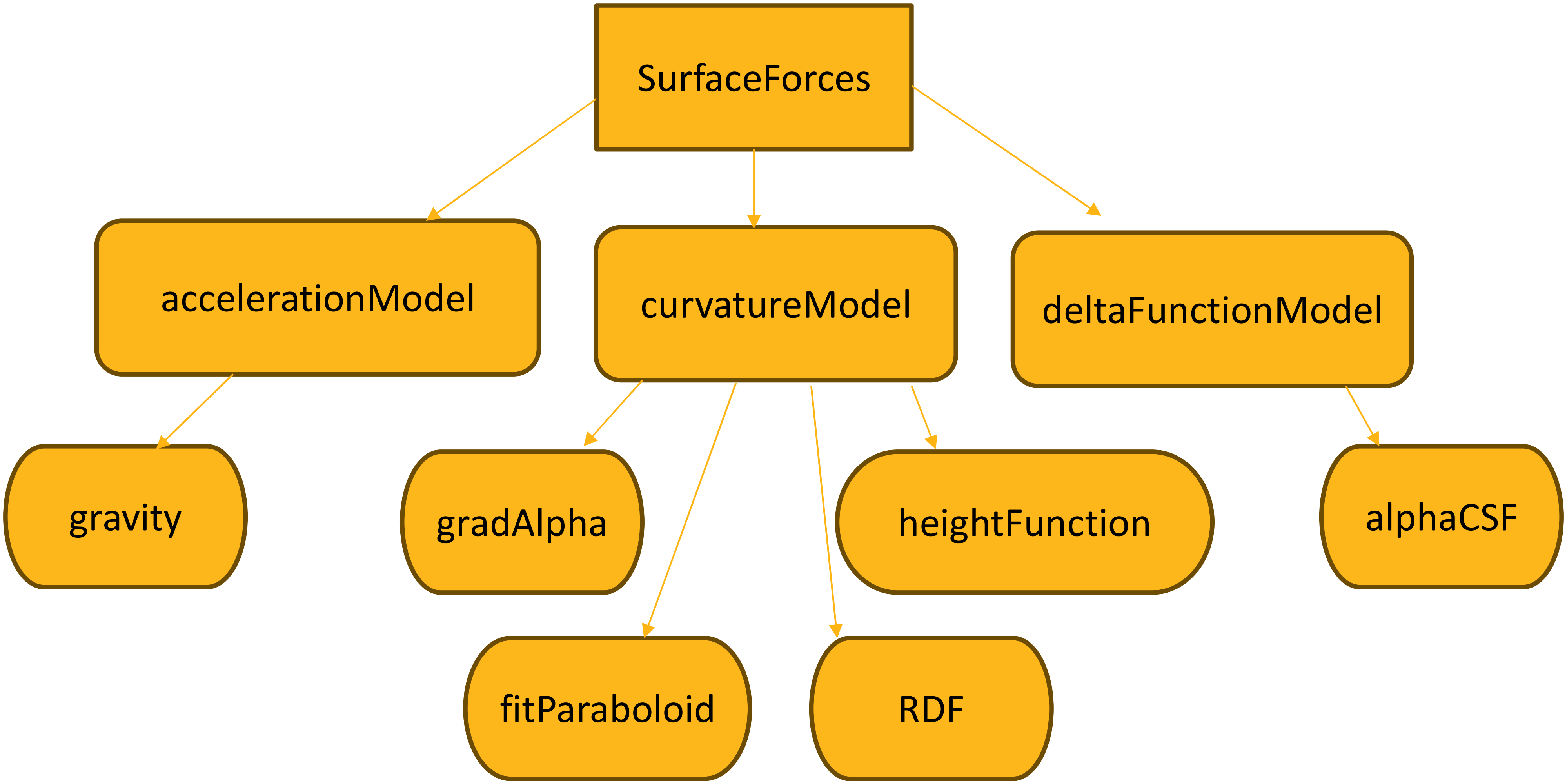
Surface Tension Force

$$\mathbf{F_{st}} = \sigma\kappa\hat{\mathbf{n}}_\mathbf{s}\delta_s$$

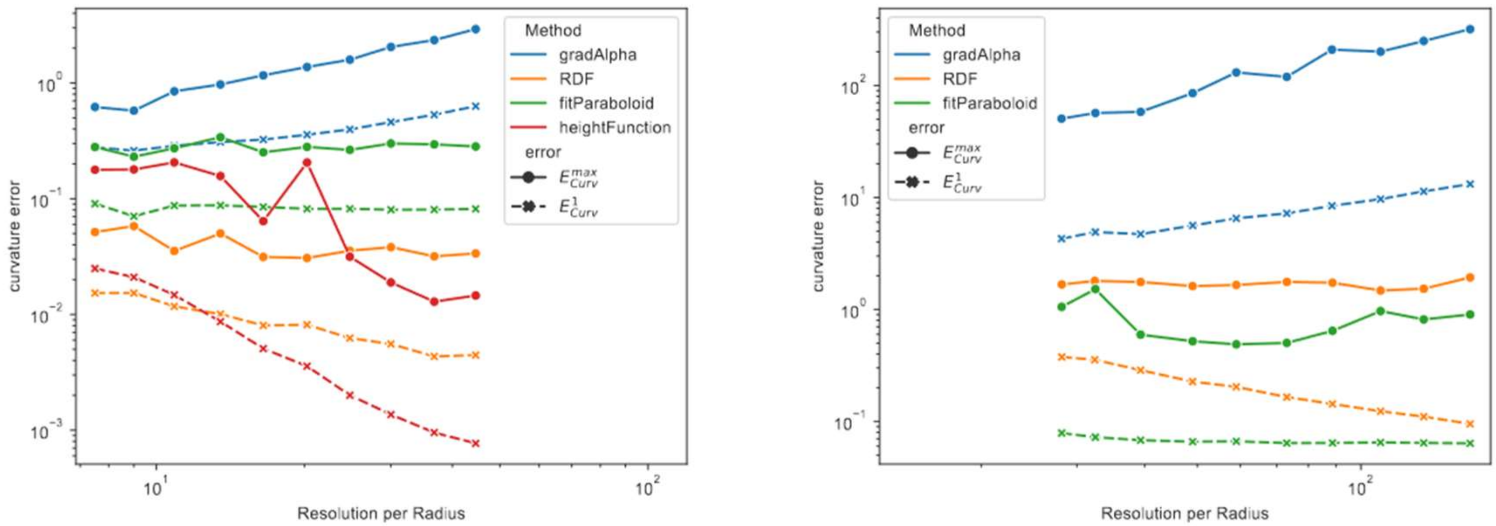
where
σ is surface tension
κ is the curvature
$\hat{\mathbf{n}}_\mathbf{s}$ is the unit interface normal (pointing into the heavy liquid)
δ_s is the Dirac delta function and is only nonzero at the fluid interface



Overview of Surface Tension Module



Notes on Accuracy



(a) Hexahedral grid

(b) Tetrahedral grid

- For unstructured 3D grids, RDF is first order accurate for all grids and accuracy can be improved by using least square method for the gradient discretization
- Much better than gradAlpha (standard OpenFOAM)

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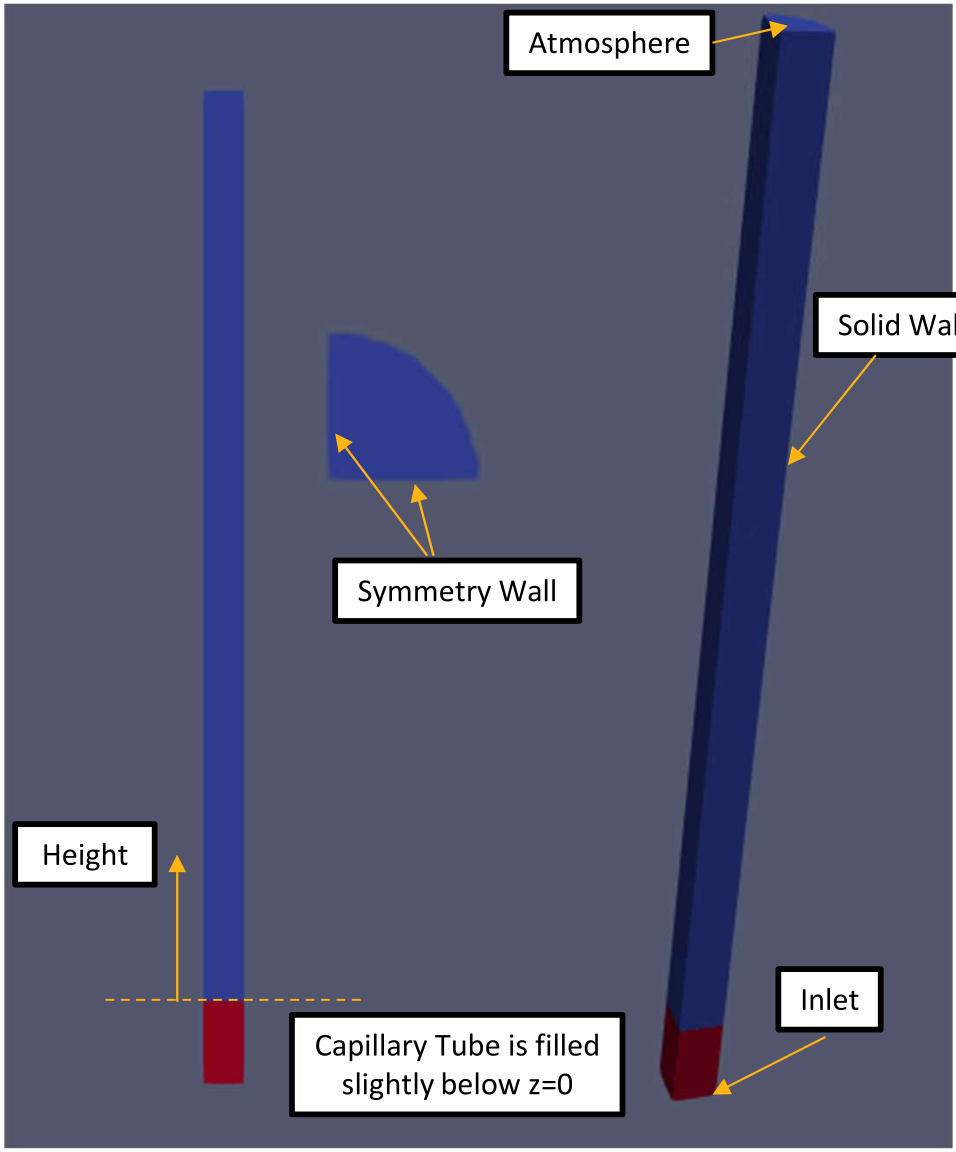
Validation 1

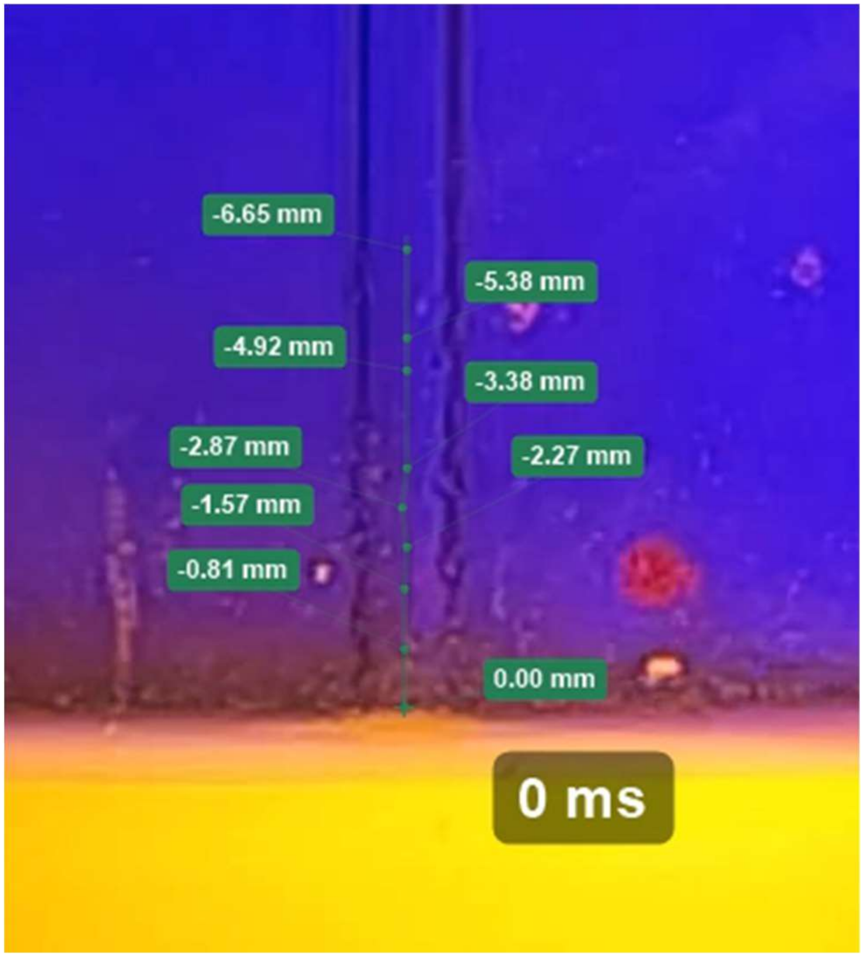
- Capillary rise of Alcohol in 1.1mm capillary tube
- Analytical solution is obtained by solving using ODE45

$$\frac{4\sigma \cos\theta}{D} = \frac{32\mu h}{D^2}\dot{h} + \rho\frac{d(h\dot{h})}{dt} + \rho g h$$

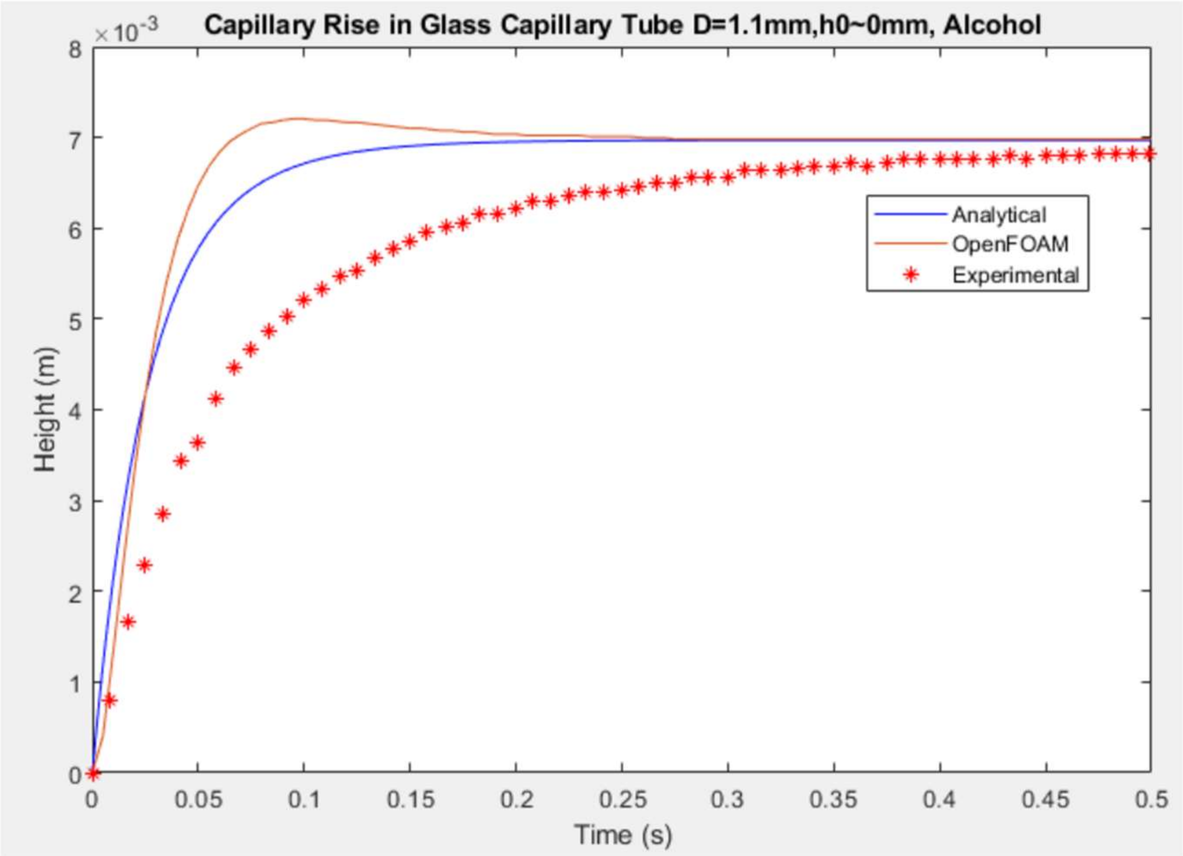
- A constant contact angle is assumed which will lead to error in the transient behaviour.



Results



-6.65 mm
-5.38 mm
-4.92 mm
-3.38 mm
-2.87 mm
-2.27 mm
-1.57 mm
-0.81 mm
0.00 mm
0 ms



Capillary Rise in Glass Capillary Tube D=1.1mm,h0~0mm, Alcohol
×10^-3
Height (m)
Time (s)
Analytical
OpenFOAM
Experimental
0
1
2
3
4
5
6
7
8
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5

Validation 2

- Capillary rise in Body Centered Cubic lattice column
- Lattice properties

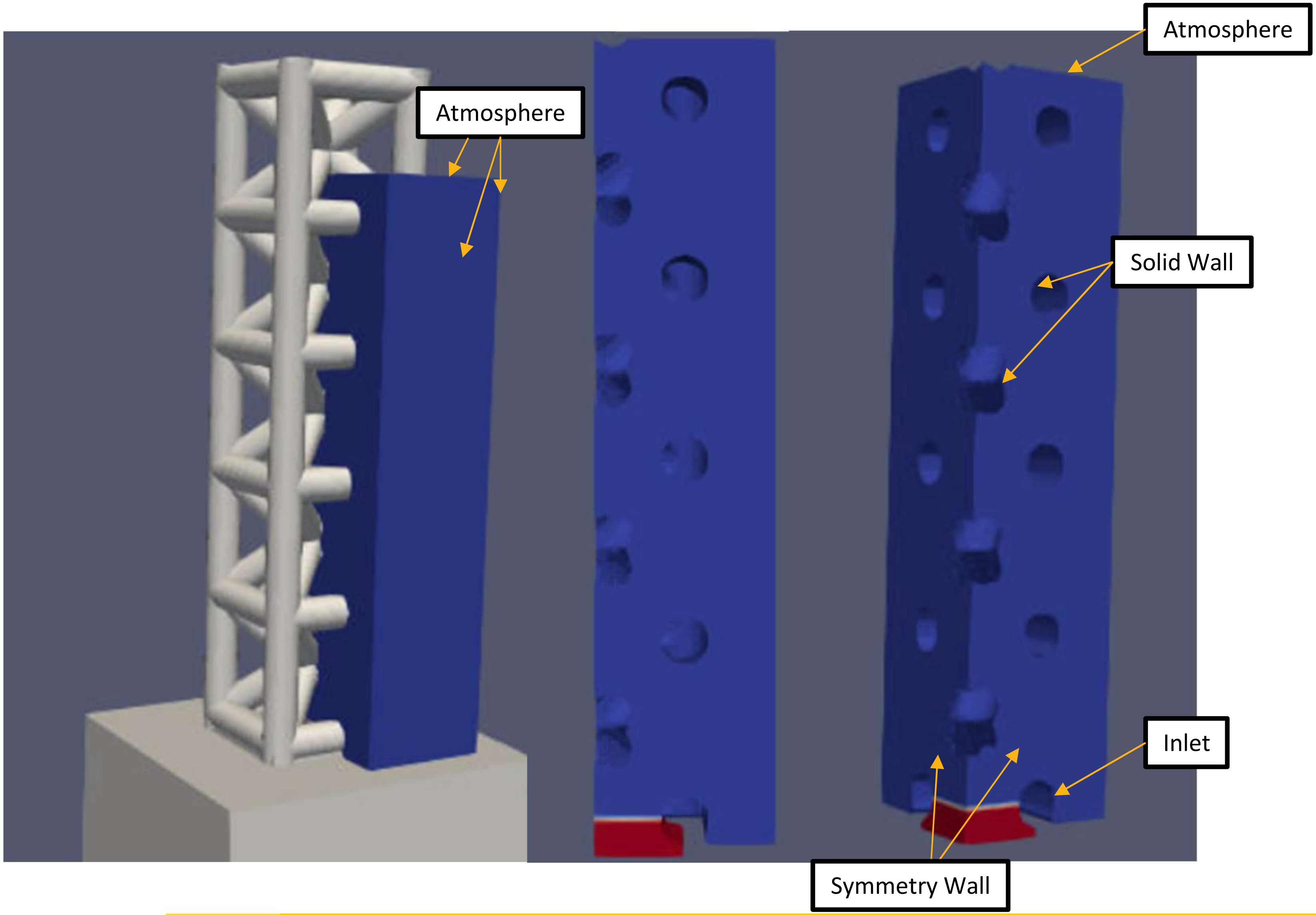
Unit length, L=1.5mm

Strut diameter, D=0.4mm

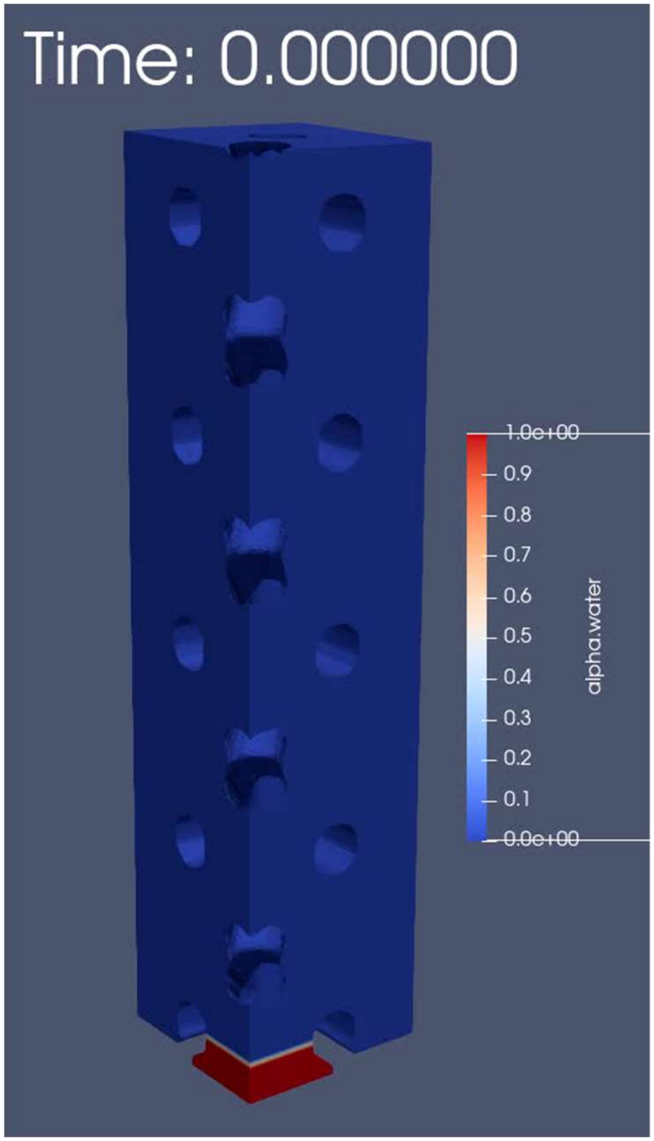
- Dimensionless height, h*=h/L
- Dimensionless time, t*=t/$\sqrt{L/g}$

N. A. Dudukovic *et al.*, “Cellular fluidics,” *Nature*, vol. 595, no. 7865, pp. 58–65, Jul. 2021, doi: 10.1038/s41586-021-03603-2.

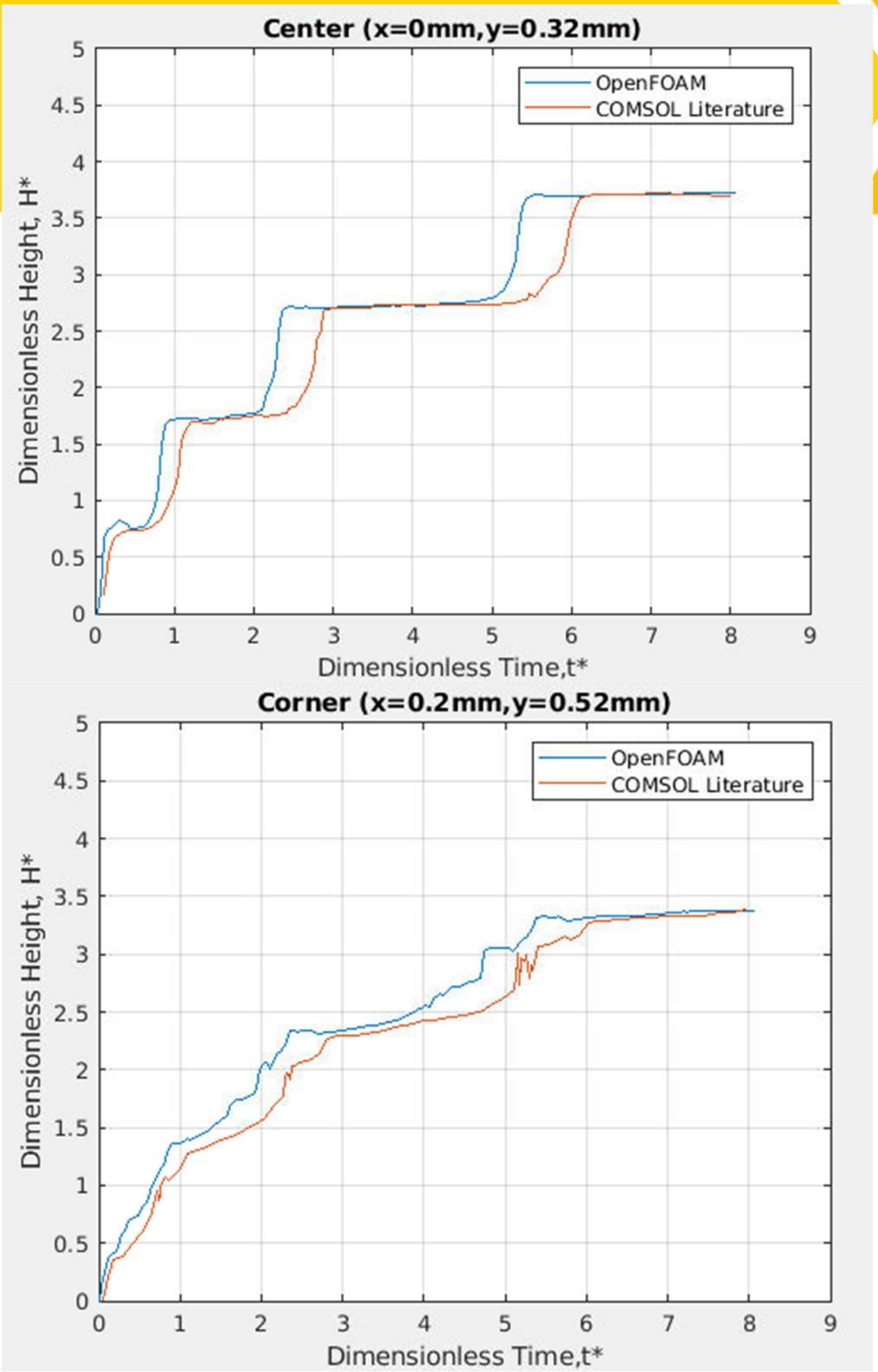
Boundary Conditions



Results



Time: 0.000000
1.0e+00
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0e+00
alpha.water



Center (x=0mm,y=0.32mm)
OpenFOAM
COMSOL Literature
Dimensionless Height, H*
Dimensionless Time,t*
Corner (x=0.2mm,y=0.52mm)
OpenFOAM
COMSOL Literature
Dimensionless Height, H*
Dimensionless Time,t*

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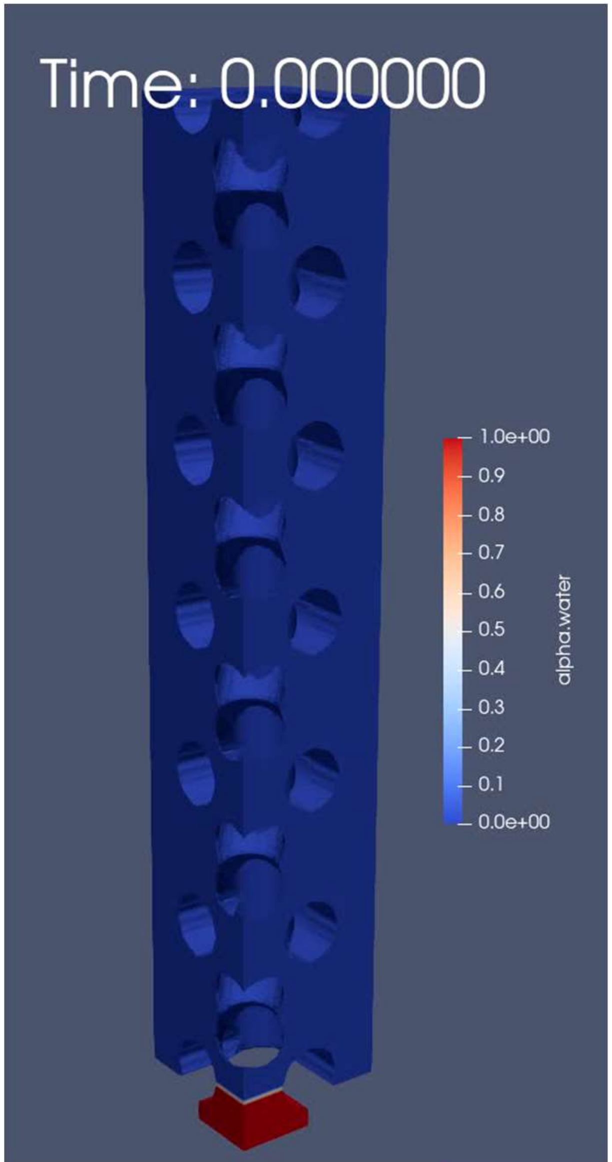
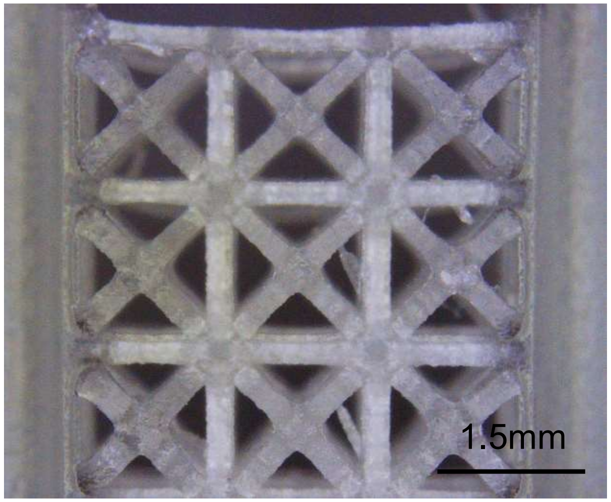
Experimental Validation of Uniform and Non-Uniform Lattice

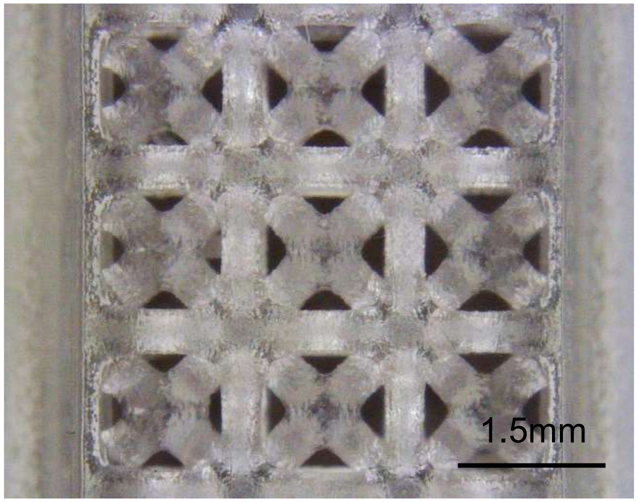


- Similar to BCC structure but vary strut diameter
- Reduction of pore size leads to air bubble entrapment. Mathematically, this is possible however, it is unlikely to appear in physical experiments
- More research on the limitation ongoing with in-house experiments
  - Effects of pore size
  - Effects of periodic versus atmospheric B.C.

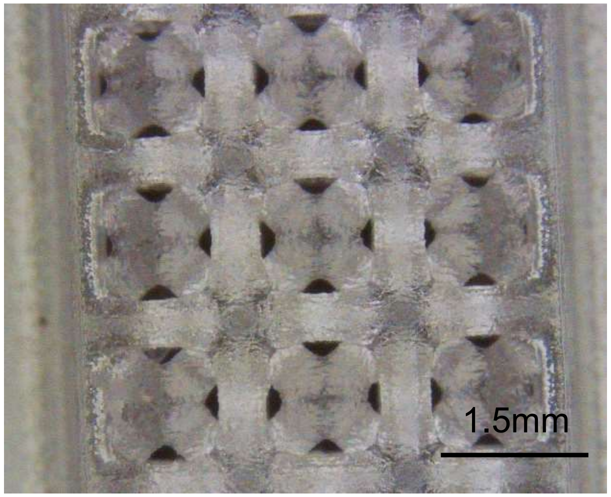
Experimental Validation of Uniform and Non-Uniform Lattice



150um



300um



400um

Rate-of-Rise Test: Working Principle

Hydrostatic Pressure

$$\Delta p_{hs} = \rho g h$$

Pressure due to friction (Darcy's Law)

$$\Delta p_f = \frac{\varepsilon\mu}{K} h \frac{dh}{dt}$$

Capillary pressure (Young-Laplace Equation)

$$\Delta p_{cap} = \frac{2\sigma\cos\theta}{r_p} = \frac{2\sigma}{r_{eff}}$$

$$h\frac{dh}{dt} = \frac{K}{\varepsilon\mu}\left(\frac{2\sigma}{r_{eff}} - \rho g h\right)$$

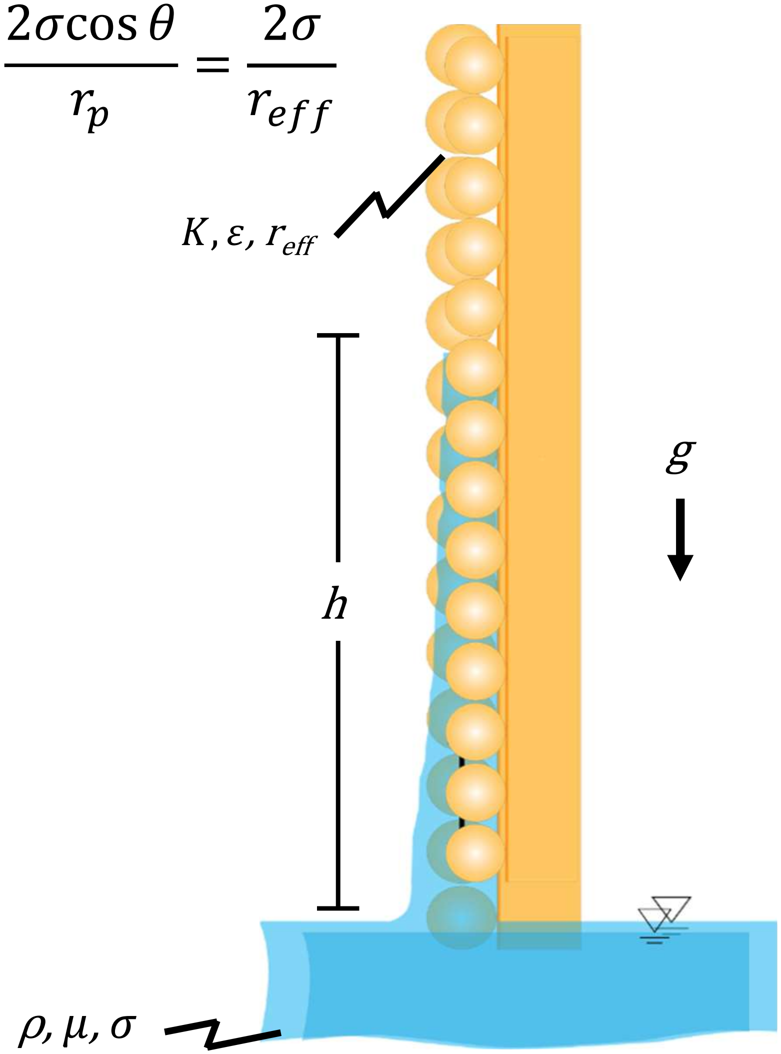
μ, σ, ρ ➡ Known fluid properties

ε ➡ Porosity

r_{eff} ➡ Effective Pore Radius

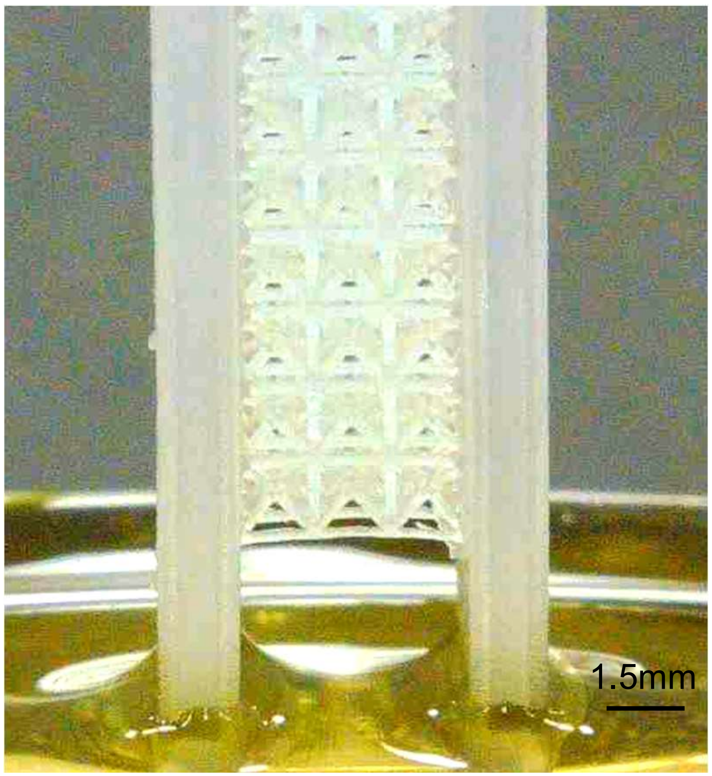
h ➡ Fluid height

K ➡ Permeability

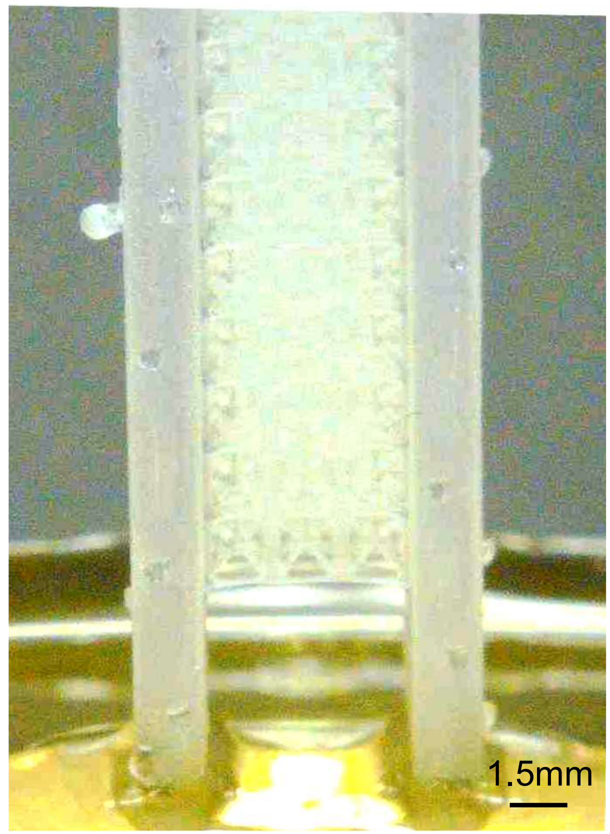


B. Holley and A. Faghri, "Permeability and effective pore radius measurements for heat pipe and fuel cell applications," *Applied Thermal Engineering*, vol. 26, no. 4, pp. 448–462, Mar. 2006.

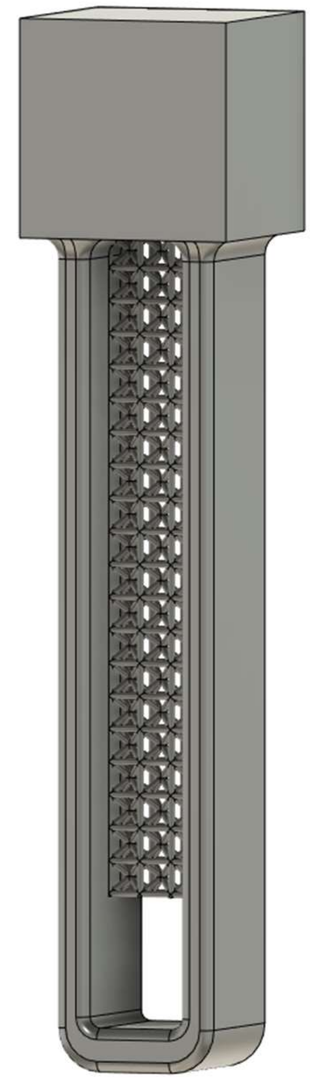
Experimental Validation of Uniform and Non-Uniform Lattice



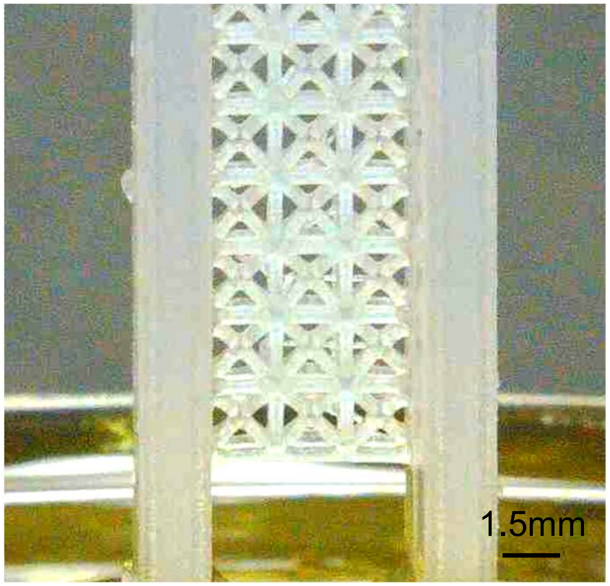
3x3 Lattice. Strut diameter=150um



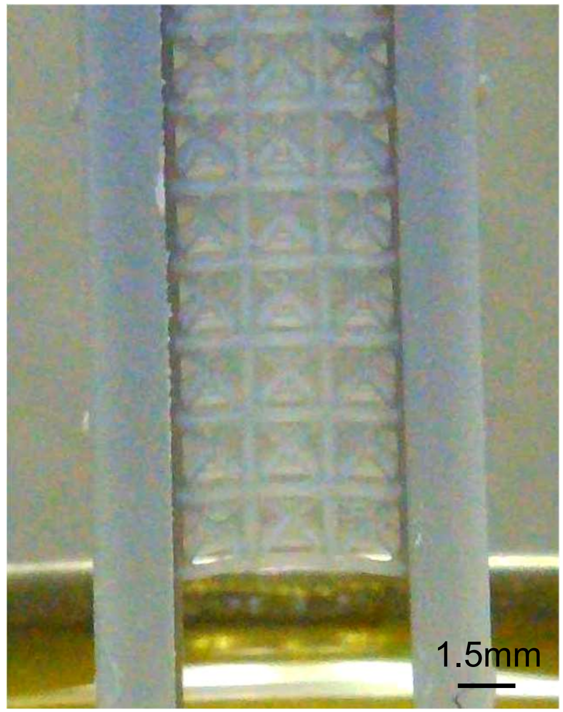
3x3 Lattice. Strut diameter=300um



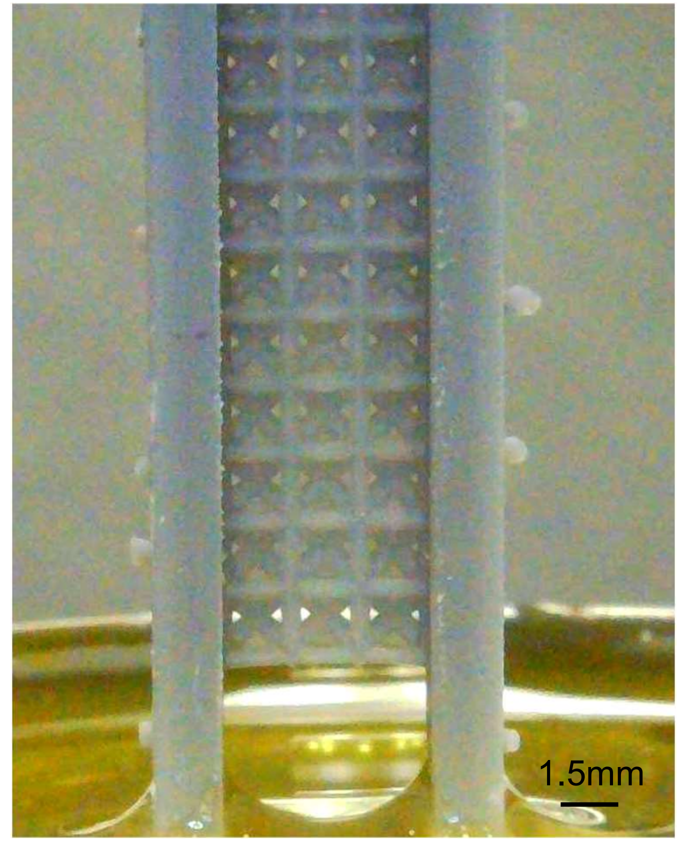
Experimental Validation of Uniform and Non-Uniform Lattice



1x3 Lattice. Strut diameter=150um

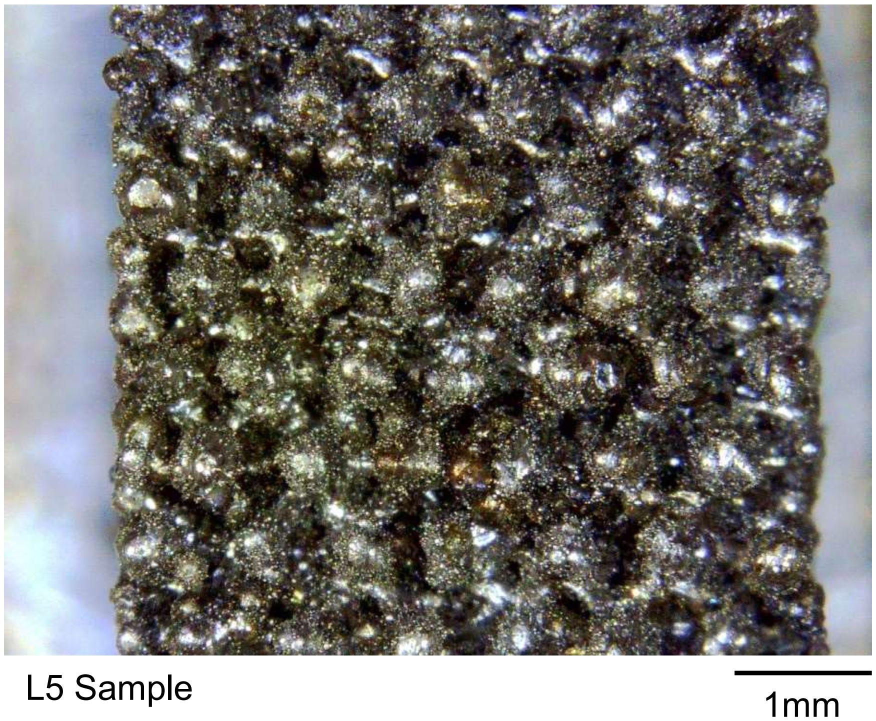


1x3 Lattice. Strut diameter=300um

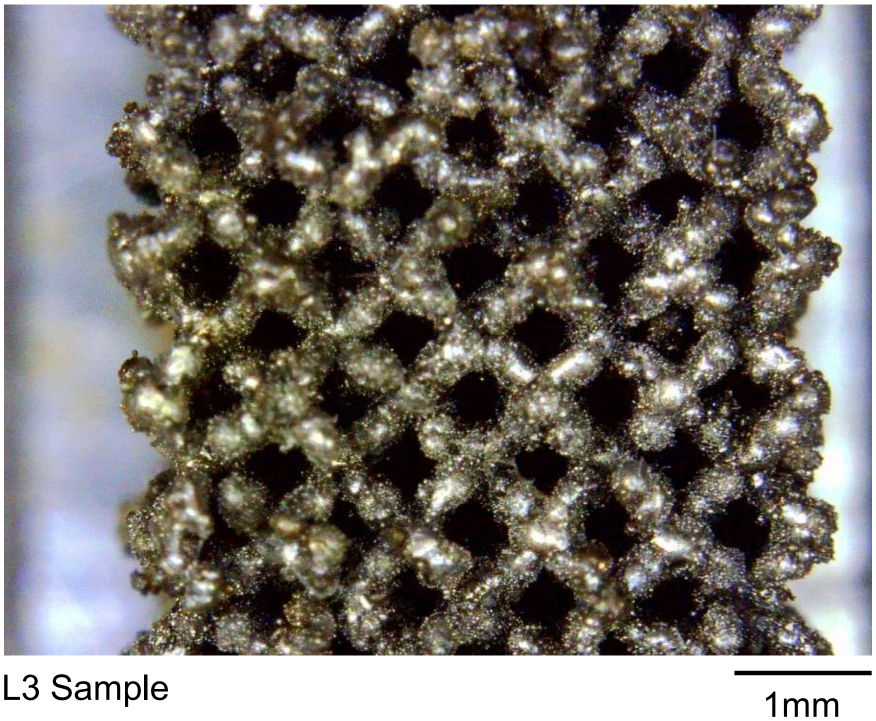


1x3 Lattice. Strut diameter=400um

Experimental Validation of Additively Manufactured Wicks

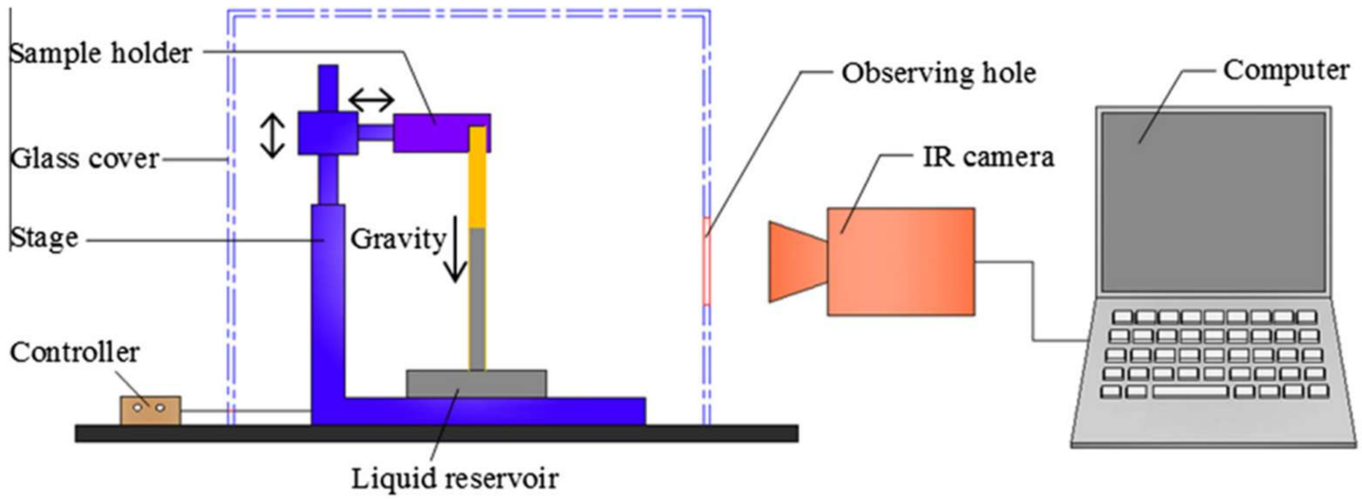


L5 Sample



L3 Sample

Experimental Validation of Additively Manufactured Wicks

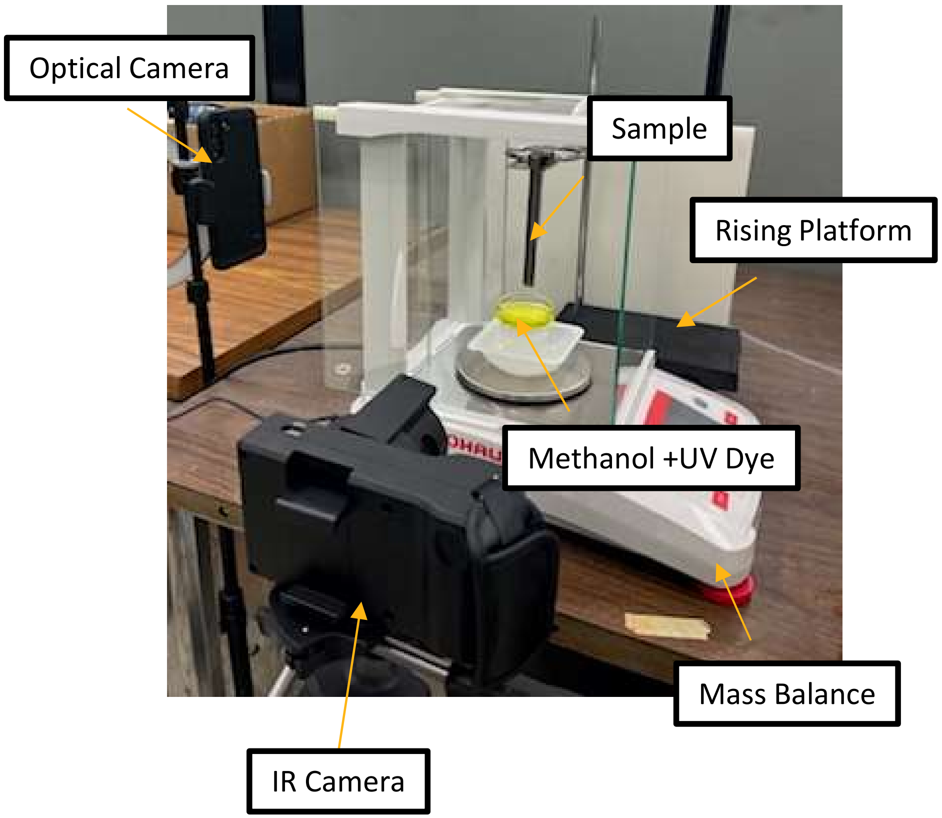


- We cannot measure optically for most metal wicks
- Infrared camera can measure the difference in emissivity (and temperature) difference between dry area and wetted area
- However, camera must be extremely sensitive otherwise the interface will appear smeared

Y. Tang, D. Deng, L. Lu, M. Pan, and Q. Wang, "Experimental investigation on capillary force of Composite Wick structure by IR thermal imaging camera," *Experimental Thermal and Fluid Science*, vol. 34, no. 2, pp. 190–196, 2010. doi:10.1016/j.expthermflusci.2009.10.016

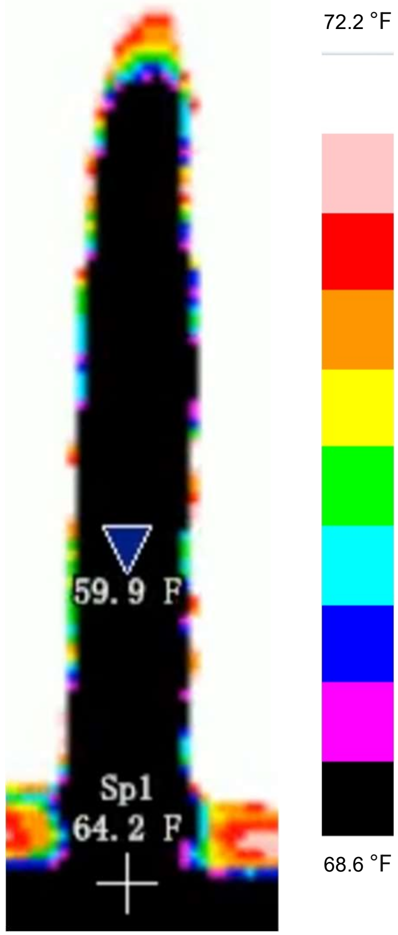
Experimental Validation of Additively Manufactured Wicks

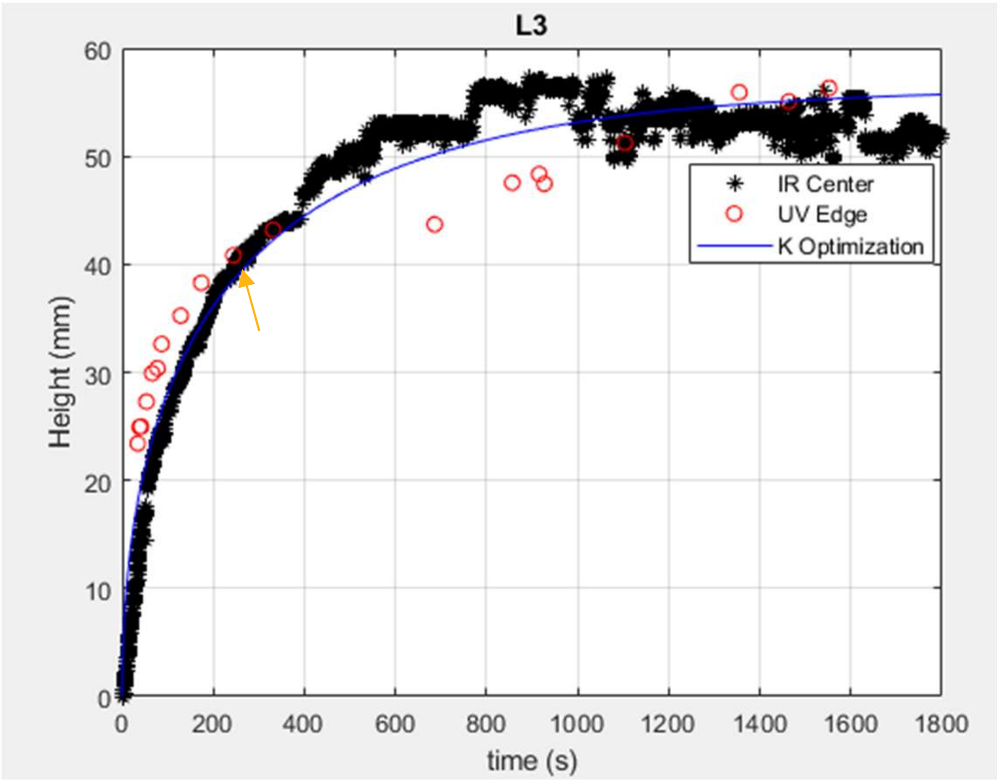
- Methanol-UV solution shows the interface clearly at the edges, but cannot be seen in the center which is most important.
- IR camera shows the center of the wick but the interface is not clear.
- A hybrid approach is used where the optical camera corrects the temperature range of IR data, accurate within 0.3°F



Experimental Validation of Additively Manufactured Wicks

At t=245s





Conclusion

- Non-uniform wick structures facilitates high wickability for efficient high heat flux thermal management system.
- Modelling surface tension driven two phase flow is computationally challenging and modelling errors can have a large impact to the predicted equilibrium height.
- The standard OpenFOAM solver should not be applied for this application, instead twoPhaseFlow library should be used.
- CFD results are comparable to analytical solutions and literature numerical studies, however it is difficult the initial transient dynamics because of the assumption of static contact angle.
- Validation with AM wicks are needed, and the improved rate-of-rise setup have promising accuracy.

Questions?

Progress on Computational Modelling of Surface Tension Driven Flow and Experimental Measurements of Metallic Wicks

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Wichita State University

12/1/2023

Supplemental Materials

We will assume 2 incompressible fluid in laminar flow with no phase change. α is the volume fraction of the tracked phase in a control volume, for simplicity, liquid fluid will be water (α=1) and gaseous fluid will be air (α=0).

Continuity Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{u} = 0 \tag{1}$$

where $\mathbf{u}$ is the fluid velocity

Momentum Equation

$$\frac{\partial(\rho \mathbf{u})}{\partial t} + \mathbf{u} \cdot \nabla(\rho \mathbf{u}) - \nabla \cdot (\mu \nabla \mathbf{u}) = -\nabla p + \rho(\mathbf{g} \cdot \mathbf{x}) + \mathbf{f} \tag{2}$$

where
ρ is the weighted density given as $\rho = \alpha\rho_{water} + (1-\alpha)\rho_{air}$
μ is the weighted dynamic viscosity given as $\mu = \alpha\mu_{water} + (1-\alpha)\mu_{air}$
p is the pressure
$\mathbf{g}$ is the gravity vector
$\mathbf{x}$ is the position vector
$\mathbf{f}$ accounts for additional source terms such as surface tension

Numerically, it is easier to define boundary condition of pressure by considering an auxillary quantity p_{rgh} which excludes the hydrostatic contribution. It is defined as,

$$p_{rgh} = p - \rho(\mathbf{g} \cdot \mathbf{x}) \tag{3}$$

It can be shown that this leads to the momentum equation in the form,

$$\frac{\partial(\rho \mathbf{u})}{\partial t} + \mathbf{u} \cdot \nabla(\rho \mathbf{u}) - \nabla \cdot (\mu \nabla \mathbf{u}) = -\nabla p_{rgh} - \mathbf{g} \cdot \mathbf{x} \nabla \rho + \mathbf{f} \tag{4}$$

To transport the volume fraction, an additional advection equation is required,

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\mathbf{u}\alpha) = 0 \tag{5}$$

Interface Advection

Consider a pure advection problem, momentum equation simplifies to

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = 0 \tag{6}$$

The solution does not depend of the density of the individual fluid. We will define a continuous indicator field based on the density of the fluids such that H=1 when cell is in water and H=0 when cell is in air

$$H(\mathbf{x}, t) = \frac{\rho(\mathbf{x}, t) - \rho_{air}}{\rho_{water} - \rho_{air}} \tag{7}$$

Based on this, we can define the volume fraction as,

$$\alpha_i(t) = \frac{1}{V_i} \int_{C_i} H(\mathbf{x}, t)\, dV \tag{8}$$

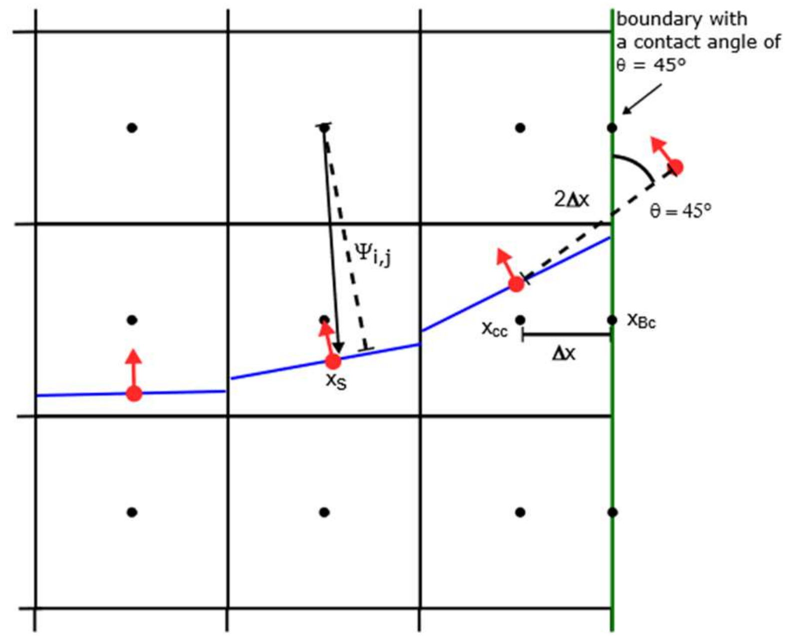
where i is the cell number, V is the volume and C_i is a single cell.
Substituting Eq. 8 into the discretized advection equation, we obtain

$$\alpha_i(t + \Delta t) = \alpha_i(t) - \frac{1}{V_i} \sum_{j \epsilon B_i} s_{ij} \int_t^{t+\Delta t} \int_{F_j} H(\mathbf{x}, t) \mathbf{u}(\mathbf{x}, t) d\mathbf{S} dt \tag{9}$$

where B_i is the list of all faces F_j belonging to cell i, s_{ij} is used to orient the flux going out relative from cell i, dS is the unit normal surface vector

Reconstructed Distance Function

- At this point, we know have marched 1 iteration and we know what the interface looks like.
- To begin the next iteration, we need predict the surface tension force.
- Signed distance function is calculated for each cell close to the interface, Ψ_{cc}
- It is piecewise continuous function that calculates the distance nearest to a narrow band around the interface and changes sign when across interface
- Similar to idea with diagram except we are only going consider the closest interface centroid, $\mathbf{x}_s$



Reconstructed Distance Function

The value of ψ is calculated for each cell center (cc) near the interface

$$\psi_{cc} = \hat{\mathbf{n_s}} \cdot (\mathbf{x}_{cc} - \mathbf{x}_s) \tag{12}$$

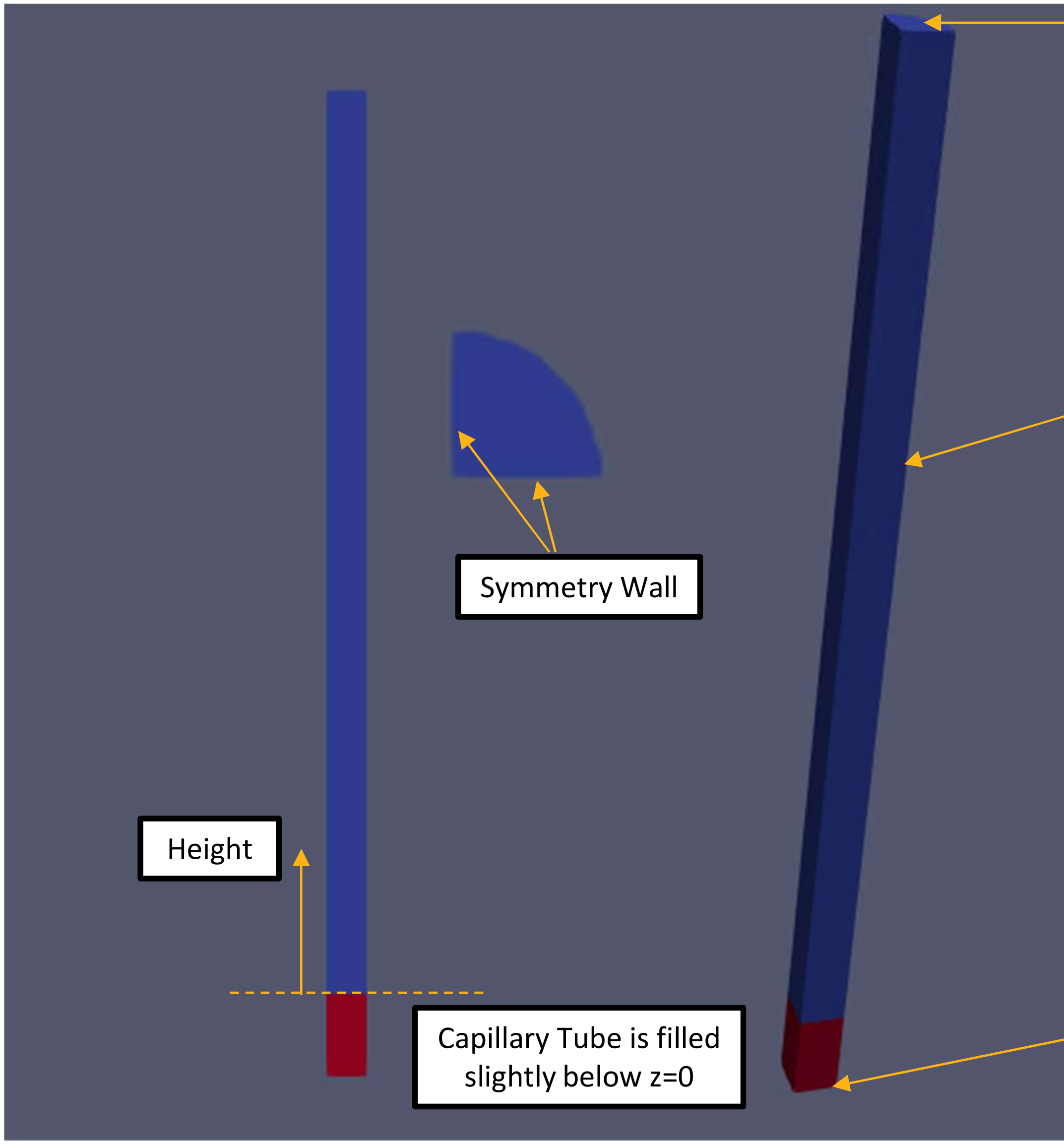
where $\mathbf{x}_s$ is the position of the centroid of the nearest interface segment
Then we calculate the gradient to find the unit vector perpendicular to the RDF

$$\hat{\mathbf{n}_\psi} = \frac{\nabla\psi}{|\nabla\psi|} \tag{13}$$

where $\hat{\mathbf{n}_\psi}$ is the normal vector of the RDF.
The curvature is computed as

$$\kappa = trace(\nabla\hat{\mathbf{n}_\psi}) \tag{14}$$

Boundary Conditions



Atmosphere

	alpha
type	zeroGradient;
	P_rgh
type	fixedValue;
value	uniform 0;
	U
type	pressureInletOutletVelocity;
value	uniform (0 0 0);

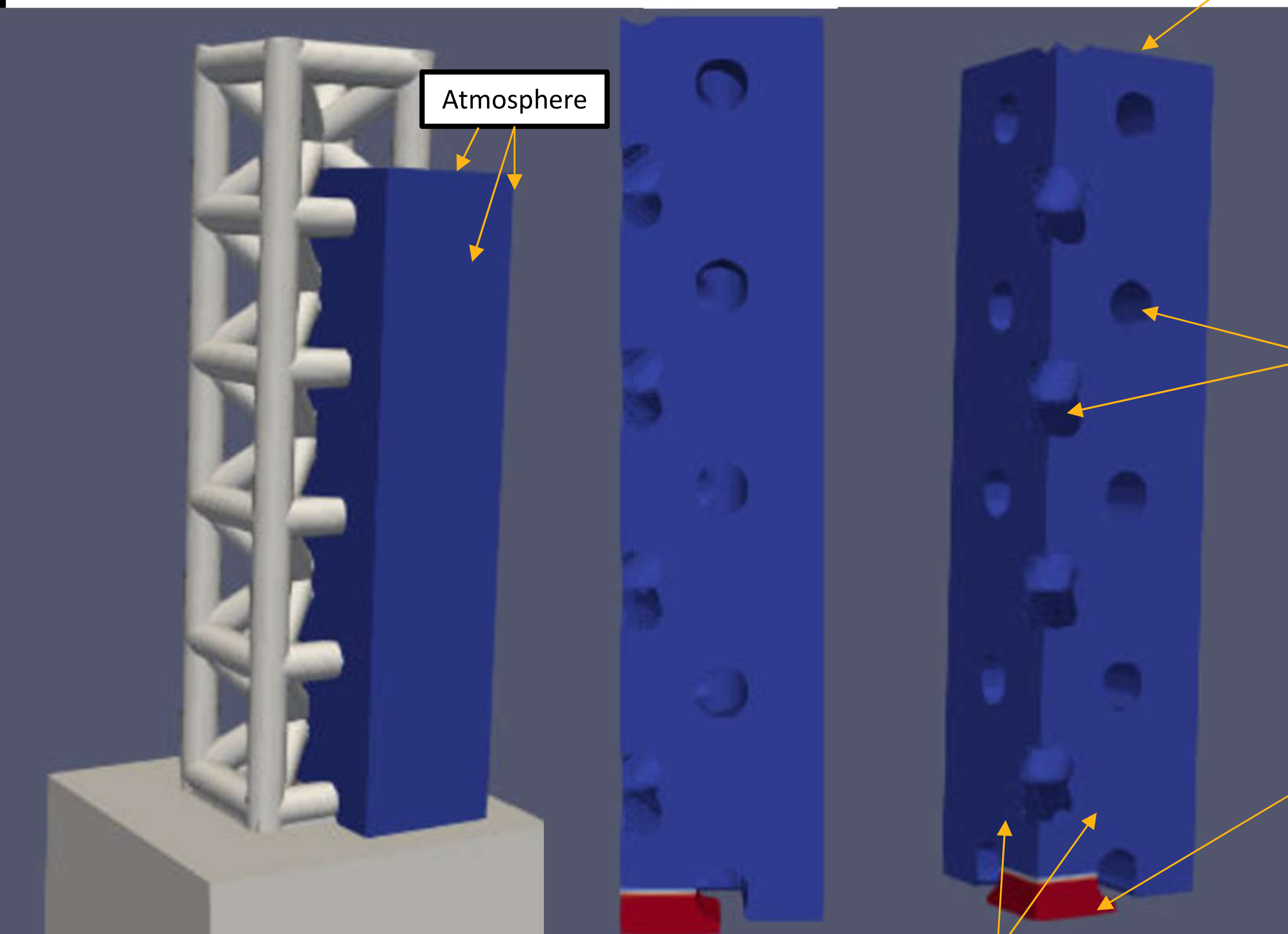
Capillary Tube

	alpha
type	constantAlphaContactAngle;
theta0	64;
limit	gradient;
value	uniform 0;
	P_rgh
type	fixedFluxPressure;
	U
type	noSlip;

Inlet

	alpha
type	inletOutlet;
value	uniform 1;
inletValue	uniform 1;
	P_rgh
type	fixedValue;
value	uniform 0;
	U
type	pressureInletOutletVelocity;
value	uniform (0 0 0);

Boundary Conditions



Atmosphere

	alpha
type	zeroGradient;
	P_rgh
type	fixedValue;
value	uniform 0;
	U
type	pressureInletOutletVelocity;
value	uniform (0 0 0);

Lattice

	alpha
type	constantAlphaContactAngle;
theta0	22.5;
limit	gradient;
value	uniform 0;
	P_rgh
type	fixedFluxPressure;
	U
type	noSlip;

Inlet

	alpha
type	inletOutlet;
value	uniform 1;
inletValue	uniform 1;
	P_rgh
type	fixedValue;
value	uniform 0;
	U
type	pressureInletOutletVelocity;
value	uniform (0 0 0);