Marcus Blaisdell
CptS 440
Homework #11
November 29, 2018
Dr. Holder

1.
a.

$$Entropy(S) \equiv -p_+ \log_2 p_+ - p_- \log_2 p_-$$

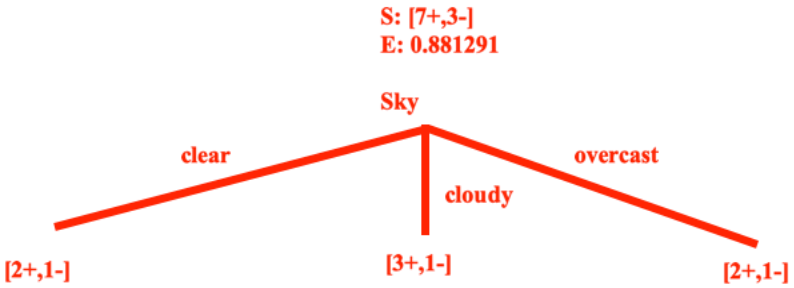
S = {7 yes(+), 3 no(-)}
Total = 10

$$Entropy(S) \equiv -\frac{7}{10}\log_2\frac{7}{10} - \frac{3}{10}\log_2\frac{3}{10} = 0.881291$$

Entropy(S) = 0.881291

b.

$$Gain(S, A) \equiv Entropy(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

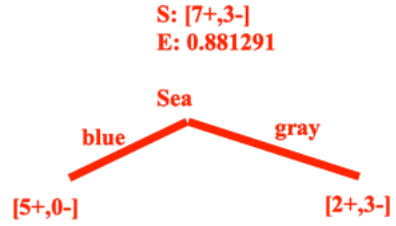


Entropy(S) = 0.881291
Entropy(S_{clear}) = $-\frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3} = 0.918296$
Entropy(S_{cloudy}) = $-\frac{3}{4}\log_2\frac{3}{4} - \frac{1}{4}\log_2\frac{1}{4} = 0.811278$
Entropy($S_{overcast}$) = $-\frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3} = 0.918296$

$$Gain(S, A) \equiv 0.881291 - \left(\left(\frac{3}{10}\right)(0.918296) + \left(\frac{4}{10}\right)(0.811278) + \left(\frac{3}{10}\right)(0.918296)\right)$$
$$= 0.005802$$

Information gain using Sky as root = 0.005802

c.



Marcus Blaisdell
CptS 440
Homework #11
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Dr. Holder

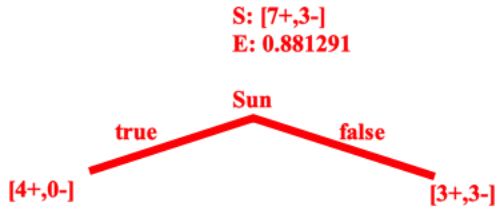
Entropy(S_{blue}) = $-\frac{5}{5}\log_2\frac{5}{5} - \frac{0}{5}\log_2\frac{0}{5} = 0$

Entropy(S_{gray}) = $-\frac{2}{5}\log_2\frac{2}{5} - \frac{3}{5}\log_2\frac{3}{5} = 0.970951$

$$Gain(S, A) \equiv 0.881291 - \left(\left(\frac{5}{10}\right)(0) + \left(\frac{5}{10}\right)(0.970951)\right) = 0.395816$$

Information gain using Sea as root = 0.395816

d.



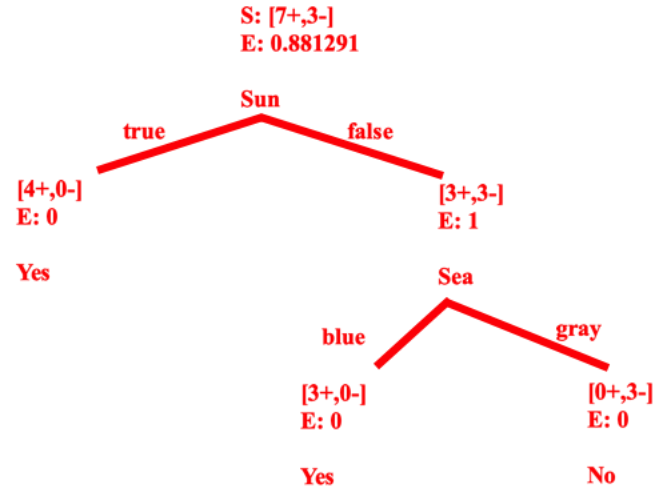
Entropy(S_{true}) = $-\frac{4}{4}\log_2\frac{4}{4} - \frac{0}{4}\log_2\frac{0}{4} = 0$

Entropy(S_{false}) = $-\frac{3}{6}\log_2\frac{3}{6} - \frac{3}{6}\log_2\frac{3}{3} = 0$

$$Gain(S, A) \equiv 0.881291 - \left(\left(\frac{4}{10}\right)(0) + \left(\frac{6}{10}\right)(0)\right) = 0.881291$$

Information gain using Sun as root = 0.881291

e.



f.

Sail = yes

Marcus Blaisdell
CptS 440
Homework #11
November 29, 2018
Dr. Holder

2.
a.

Sky	Sea	Sun	Sail
1	1	1	1
1	2	2	1
1	2	1	0
2	1	2	1
2	1	1	1
2	2	2	1
2	2	1	0
3	1	2	1
3	1	1	1
3	2	1	0

b.
Initial weights: <1.0, 1.0, 1.0, 1.0>

Update rule:
$w_i \leftarrow w_i + \alpha(y - h_w(X)) * x_i$

where $h_w(X) = \{1$ if $w.x \geq 0$
$\{0$ otherwise

pass: 1

$weight_0$: [1.0, 1.0, 1.0, 1.0]
h_0 ([1.0, 1.0, 1.0, 1.0]) = [1.0, 1.0, 1.0, 1.0] * [1.0, 1.0, 1.0, 1.0] = 4.0 = 1.0
$weight_1$ = [1.0, 1.0, 1.0, 1.0] + 0.5 * (1.0 - 1.0) * [1.0, 1.0, 1.0, 1.0] = [1.0, 1.0, 1.0, 1.0]
no update

$weight_1$: [1.0, 1.0, 1.0, 1.0]
h_1 ([1.0, 1.0, 2.0, 2.0]) = [1.0, 1.0, 1.0, 1.0] * [1.0, 1.0, 2.0, 2.0] = 6.0 = 1.0
$weight_2$ = [1.0, 1.0, 1.0, 1.0] + 0.5 * (1.0 - 1.0) * [1.0, 1.0, 2.0, 2.0] = [1.0, 1.0, 1.0, 1.0]
no update

$weight_2$: [0.5, 0.5, 0.0, 0.5]
h_2 ([1.0, 1.0, 2.0, 1.0]) = [0.5, 0.5, 0.0, 0.5] * [1.0, 1.0, 2.0, 1.0] = 5.0 = 1.0
$weight_3$ = [0.5, 0.5, 0.0, 0.5] + 0.5 * (0.0 - 1.0) * [1.0, 1.0, 2.0, 1.0] = [0.5, 0.5, 0.0, 0.5]
updated

$weight_3$: [0.5, 0.5, 0.0, 0.5]
h_3 ([1.0, 2.0, 1.0, 2.0]) = [0.5, 0.5, 0.0, 0.5] * [1.0, 2.0, 1.0, 2.0] = 2.5 = 1.0
weight 4 = [0.5, 0.5, 0.0, 0.5] + 0.5 * (1.0 - 1.0) * [1.0, 2.0, 1.0, 2.0] = [0.5, 0.5, 0.0, 0.5]
no update

Marcus Blaisdell
CptS 440
Homework #11
November 29, 2018
Dr. Holder

$weight_4$: [0.5, 0.5, 0.0, 0.5]
h_4 ([1.0, 2.0, 1.0, 1.0]) = [0.5, 0.5, 0.0, 0.5] * [1.0, 2.0, 1.0, 1.0] = 2.0 = 1.0
$weight_5$ = [0.5, 0.5, 0.0, 0.5] + 0.5 * (1.0 - 1.0) * [1.0, 2.0, 1.0, 1.0] = [0.5, 0.5, 0.0, 0.5]
no update

$weight_5$: [0.5, 0.5, 0.0, 0.5]
h_5 ([1.0, 2.0, 2.0, 2.0]) = [0.5, 0.5, 0.0, 0.5] * [1.0, 2.0, 2.0, 2.0] = 2.5 = 1.0
$weight_6$ = [0.5, 0.5, 0.0, 0.5] + 0.5 * (1.0 - 1.0) * [1.0, 2.0, 2.0, 2.0] = [0.5, 0.5, 0.0, 0.5]
no update

$weight_6$: [0.0, -0.5, -1.0, 0.0]
h_6 ([1.0, 2.0, 2.0, 1.0]) = [0.0, -0.5, -1.0, 0.0] * [1.0, 2.0, 2.0, 1.0] = 2.0 = 1.0
$weight_7$ = [0.0, -0.5, -1.0, 0.0] + 0.5 * (0.0 - 1.0) * [1.0, 2.0, 2.0, 1.0] = [0.0, -0.5, -1.0, 0.0]
updated

$weight_7$: [0.5, 1.0, -0.5, 1.0]
h_7 ([1.0, 3.0, 1.0, 2.0]) = [0.5, 1.0, -0.5, 1.0] * [1.0, 3.0, 1.0, 2.0] = -2.5 = 0.0
$weight_8$ = [0.5, 1.0, -0.5, 1.0] + 0.5 * (1.0 - 0.0) * [1.0, 3.0, 1.0, 2.0] = [0.5, 1.0, -0.5, 1.0]
updated

$weight_8$: [0.5, 1.0, -0.5, 1.0]
h_8 ([1.0, 3.0, 1.0, 1.0]) = [0.5, 1.0, -0.5, 1.0] * [1.0, 3.0, 1.0, 1.0] = 4.0 = 1.0
weight 9 = [0.5, 1.0, -0.5, 1.0] + 0.5 * (1.0 - 1.0) * [1.0, 3.0, 1.0, 1.0] = [0.5, 1.0, -0.5, 1.0]
no update

$weight_9$: [0.0, -0.5, -1.5, 0.5]
h_9 ([1.0, 3.0, 2.0, 1.0]) = [0.0, -0.5, -1.5, 0.5] * [1.0, 3.0, 2.0, 1.0] = 3.5 = 1.0
$weight_{10}$ = [0.0, -0.5, -1.5, 0.5] + 0.5 * (0.0 - 1.0) * [1.0, 3.0, 2.0, 1.0] = [0.0, -0.5, -1.5, 0.5]
updated

pass: 2

$weight_{10}$: [0.5, 0.0, -1.0, 1.0]
h_{10} ([1.0, 1.0, 1.0, 1.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 1.0, 1.0, 1.0] = -1.5 = 0.0
$weight_{11}$ = [0.5, 0.0, -1.0, 1.0] + 0.5 * (1.0 - 0.0) * [1.0, 1.0, 1.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
updated

$weight_{11}$: [0.5, 0.0, -1.0, 1.0]
h_{11} ([1.0, 1.0, 2.0, 2.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 1.0, 2.0, 2.0] = 0.5 = 1.0
$weight_{12}$ = [0.5, 0.0, -1.0, 1.0] + 0.5 * (1.0 - 1.0) * [1.0, 1.0, 2.0, 2.0] = [0.5, 0.0, -1.0, 1.0]
no update

$weight_{12}$: [0.5, 0.0, -1.0, 1.0]
h_{12} ([1.0, 1.0, 2.0, 1.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 1.0, 2.0, 1.0] = -0.5 = 0.0

Marcus Blaisdell
CptS 440
Homework #11
November 29, 2018
Dr. Holder

$weight_{13}$ = [0.5, 0.0, -1.0, 1.0] + 0.5 * (0.0 - 0.0) * [1.0, 1.0, 2.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

$weight_{13}$: [0.5, 0.0, -1.0, 1.0]
h_{13} ([1.0, 2.0, 1.0, 2.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 2.0, 1.0, 2.0] = 1.5 = 1.0
$weight_{14}$ = [0.5, 0.0, -1.0, 1.0] + 0.5 * (1.0 - 1.0) * [1.0, 2.0, 1.0, 2.0] = [0.5, 0.0, -1.0, 1.0]
no update

$weight_{14}$: [0.5, 0.0, -1.0, 1.0]
h_{14} ([1.0, 2.0, 1.0, 1.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 2.0, 1.0, 1.0] = 0.5 = 1.0
$weight_{15}$ = [0.5, 0.0, -1.0, 1.0] + 0.5 * (1.0 - 1.0) * [1.0, 2.0, 1.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

$weight_{15}$: [0.5, 0.0, -1.0, 1.0]
h_{15} ([1.0, 2.0, 2.0, 2.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 2.0, 2.0, 2.0] = 0.5 = 1.0
$weight_{16}$ = [0.5, 0.0, -1.0, 1.0] + 0.5 * (1.0 - 1.0) * [1.0, 2.0, 2.0, 2.0] = [0.5, 0.0, -1.0, 1.0]
no update

$weight_{16}$: [0.5, 0.0, -1.0, 1.0]
h_{16} ([1.0, 2.0, 2.0, 1.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 2.0, 2.0, 1.0] = -0.5 = 0.0
$weight_{17}$ = [0.5, 0.0, -1.0, 1.0] + 0.5 * (0.0 - 0.0) * [1.0, 2.0, 2.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

$weight_{17}$: [0.5, 0.0, -1.0, 1.0]
h_{17} ([1.0, 3.0, 1.0, 2.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 3.0, 1.0, 2.0] = 1.5 = 1.0
$weight_{18}$ = [0.5, 0.0, -1.0, 1.0] + 0.5 * (1.0 - 1.0) * [1.0, 3.0, 1.0, 2.0] = [0.5, 0.0, -1.0, 1.0]
no update

$weight_{18}$: [0.5, 0.0, -1.0, 1.0]
h_{18} ([1.0, 3.0, 1.0, 1.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 3.0, 1.0, 1.0] = 0.5 = 1.0
$weight_{19}$ = [0.5, 0.0, -1.0, 1.0] + 0.5 * (1.0 - 1.0) * [1.0, 3.0, 1.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

$weight_{19}$: [0.5, 0.0, -1.0, 1.0]
h_{19} ([1.0, 3.0, 2.0, 1.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 3.0, 2.0, 1.0] = -0.5 = 0.0
$weight_{20}$ = [0.5, 0.0, -1.0, 1.0] + 0.5 * (0.0 - 0.0) * [1.0, 3.0, 2.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

pass: 3

$weight_{20}$: [0.5, 0.0, -1.0, 1.0]
h_{20} ([1.0, 1.0, 1.0, 1.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 1.0, 1.0, 1.0] = 0.5 = 1.0
$weight_{21}$ = [0.5, 0.0, -1.0, 1.0] + 0.5 * (1.0 - 1.0) * [1.0, 1.0, 1.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

Marcus Blaisdell
CptS 440
Homework #11
November 29, 2018
Dr. Holder

$weight_{21}$: [0.5, 0.0, -1.0, 1.0]
h_{21} ([1.0, 1.0, 2.0, 2.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 1.0, 2.0, 2.0] = 0.5 = 1.0
$weight_{21}$ = [0.5, 0.0, -1.0, 1.0] + 0.5 * (1.0 - 1.0) * [1.0, 1.0, 2.0, 2.0] = [0.5, 0.0, -1.0, 1.0]
no update

$weight_{22}$: [0.5, 0.0, -1.0, 1.0]
h_{22} ([1.0, 1.0, 2.0, 1.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 1.0, 2.0, 1.0] = -0.5 = 0.0
$weight_{23}$ = [0.5, 0.0, -1.0, 1.0] + 0.5 * (0.0 - 0.0) * [1.0, 1.0, 2.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

$weight_{23}$: [0.5, 0.0, -1.0, 1.0]
h_{23} ([1.0, 2.0, 1.0, 2.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 2.0, 1.0, 2.0] = 1.5 = 1.0
weight 4 = [0.5, 0.0, -1.0, 1.0] + 0.5 * (1.0 - 1.0) * [1.0, 2.0, 1.0, 2.0] = [0.5, 0.0, -1.0, 1.0]
no update

$weight_{24}$: [0.5, 0.0, -1.0, 1.0]
h_{24} ([1.0, 2.0, 1.0, 1.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 2.0, 1.0, 1.0] = 0.5 = 1.0
weight 5 = [0.5, 0.0, -1.0, 1.0] + 0.5 * (1.0 - 1.0) * [1.0, 2.0, 1.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

$weight_{25}$: [0.5, 0.0, -1.0, 1.0]
h_{25} ([1.0, 2.0, 2.0, 2.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 2.0, 2.0, 2.0] = 0.5 = 1.0
$weight_{26}$ = [0.5, 0.0, -1.0, 1.0] + 0.5 * (1.0 - 1.0) * [1.0, 2.0, 2.0, 2.0] = [0.5, 0.0, -1.0, 1.0]
no update

$weight_{26}$: [0.5, 0.0, -1.0, 1.0]
h_{26} ([1.0, 2.0, 2.0, 1.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 2.0, 2.0, 1.0] = -0.5 = 0.0
$weight_{27}$ = [0.5, 0.0, -1.0, 1.0] + 0.5 * (0.0 - 0.0) * [1.0, 2.0, 2.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

$weight_{27}$: [0.5, 0.0, -1.0, 1.0]
h_{27} ([1.0, 3.0, 1.0, 2.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 3.0, 1.0, 2.0] = 1.5 = 1.0
$weight_{28}$ = [0.5, 0.0, -1.0, 1.0] + 0.5 * (1.0 - 1.0) * [1.0, 3.0, 1.0, 2.0] = [0.5, 0.0, -1.0, 1.0]
no update

$weight_{28}$: [0.5, 0.0, -1.0, 1.0]
h_{28} ([1.0, 3.0, 1.0, 1.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 3.0, 1.0, 1.0] = 0.5 = 1.0
$weight_{29}$ = [0.5, 0.0, -1.0, 1.0] + 0.5 * (1.0 - 1.0) * [1.0, 3.0, 1.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

$weight_{29}$: [0.5, 0.0, -1.0, 1.0]
h_{29} ([1.0, 3.0, 2.0, 1.0]) = [0.5, 0.0, -1.0, 1.0] * [1.0, 3.0, 2.0, 1.0] = -0.5 = 0.0
$weight_{30}$ = [0.5, 0.0, -1.0, 1.0] + 0.5 * (0.0 - 0.0) * [1.0, 3.0, 2.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

Marcus Blaisdell
CptS 440
Homework #11
November 29, 2018
Dr. Holder

c.
<Sky=overcast, Sea=gray, Sun=true> = (3.0, 2.0, 2.0) => (1.0, 3.0, 2.0, 2.0)

weight = (0.5, 0.0, -1.0, 1.0)

classification: $h_w(X) = \{1 \text{ if } w.x \geq 0$
$\{0 \text{ otherwise}$

w.x = (0.5, 0.0, -1.0, 1.0) . (1.0, 3.0, 2.0, 2.0) = 0.5 + 0 + (-2.0) + (2.0) = 0.5

$0.5 \geq 0$ therefore the classification is 1 which is sail = yes.

The learned perceptron will classify <Sky=overcast, Sea=gray, Sun=true> as sail = yes.