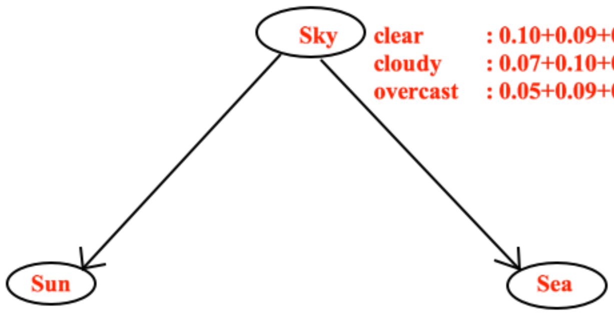
Marcus Blaisdell
CptS 440
Homework #8
October 25, 2018
Dr. Holder

1.



clear	: 0.10+0.09+0.08+0.06 = 0.33
cloudy	: 0.07+0.10+0.07+0.09 = 0.33
overcast	: 0.05+0.09+0,08+0.12 = 0.34

Sun	Sky	P(Sun)
True	clear	0.10+0.08 = 0.18
True	cloudy	0.07+0.07 = 0.14
True	overcast	0.05+0.08 = 0.13
False	clear	0.09+0.06 = 0.15
False	cloudy	0.10+0.09 = 0.19
False	overcast	0.09+0.12 = 0.21

Sea	Sky	P(Sea)
Blue	clear	0.10+0.09 = 0.19
Blue	cloudy	0.07+0.10 = 0.17
Blue	overcast	0.05+0.09 = 0.14
Grey	clear	0.08+0.06 = 0.14
Grey	cloudy	0.07+0.09 = 0.16
Grey	overcast	0.08+0.12 = 0.20

2.

For all parts a through e of problem 2:
Let t be Time = day and ¬t be Time = night
Let sk1 be Sky = clear
Let sk2 be Sky = cloudy
Let sk3 be Sky = overcast
Let sn be Sun=true and ¬sn be Sun=false
Let mn be Moon=true and ¬mn be Moon=false
Let se be Sea=blue and ¬se be Sea=gray

a. P(Time=day, Sky=clear, Sun=true, Moon=false, Sea=blue)
P(t ^ sk1 ^ sn ^ ¬mn ^ se)
Given:
P(t ^ sk1 ^ sn) = 0.9
P(t ^ sk1 ^ ¬mn) = 0.8
P(sn ^ se) = 0.8

P(t ^ sk1 ^ sn ^ ¬mn ^ se) = P(t ^ sk1 ^ sn) P(t ^ sk1 ^ ¬mn) P(sn ^ se) = 0.9 * 0.8 * 0.8 = 0.576

b. P(Moon=true | Time=night, Sky=cloudy)
This is given on the Bayesian network as 0.5

c. P(Time=day | Moon=true)

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Choose the minimal set of parents.
Time has no parent.
Moon has two parents: Time and Sky.
Moon and Time are independent of Sun.
Moon and Time are independent of Sea.

$$P(t|mn) = \alpha P(t, mn) = \alpha P(t, mn, sk)$$

P(mn) is not given, solve by normalization:

$$\boldsymbol{P}(t|mn) = \alpha < P(t, mn), P(\neg t, mn) >$$

P(t,mn) = sum of all possible ways this can be true: 0.2 + 0.1 + 0.0 = 0.3
P(¬t,mn) = sum of all possible ways this can be true: 0.9 + 0.5 + 0.1 = 1.5

P(t|mn) = α<0.3,1.5>
α = 1 / 1.8 = 0.555
P(t|mn) = <0.166, 0.833> => <0.17, 0.83>
P(t|mn) = 0.17

d. P(Sea=blue | Time=day, Sky=clear)

Choose a minimal set of parents:
Sea is independent of Moon.
Sea has a parent Sun.
Sun has parents Time, and Sky

Calculate the distribution of P(Sea):

$$P(se|t, sk1) = \alpha P(se, t, sk1) = \alpha \sum_{sn} P(se, t, sk1, sn)$$
$$= \alpha \sum_{sn} P(se|sn) P(t) P(sk1)$$
$$= \alpha P(t) P(sk1) \sum_{sn} P(se|sn)$$
$$P(\neg se|t, sk1) = \alpha P(se, t, sk1) = \alpha \sum_{sn} P(\neg se, t, sk1, sn)$$
$$= \alpha \sum_{sn} P(\neg se|sn) P(t) P(sk1)$$
$$= \alpha P(t) P(sk1) \sum_{sn} P(\neg se|sn)$$

Bayesian Network (se, substitute ¬se|sn = 0.2 and ¬se|¬sn = 0.7 for ¬se):

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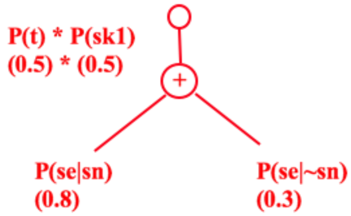

P(se|t,sk1) = α<0.275,0.225> => <0.55,0.45>

P(se|t,sk1) = 0.55

e. P(Time=day | Sea=blue, Moon=false)

Choose minimal set:
Time has no parent
Sea has parent Sun
Moon has parents Time and Sky

$$P(t|se,\neg mn) = \alpha P(t,se,\neg mn) = \alpha \sum_{sk} \sum_{sn} P(t,se,\neg mn,sk,sn)$$

$$P(t|se,\neg mn) = \alpha \sum_{sk} \sum_{sn} P(t)P(se|sn)P(\neg mn|t,sk)P(sn|t,sk)$$

Moon is independent of Sun so it moves in front of Sun summation,
Time is independent of Sun and Sky so it moves in front of each summation:

$$P(t|se,\neg mn) = \alpha P(t) \sum_{sk} P(\neg mn|t,sk)P(sn|t,sk) \sum_{sn} P(se|sn)$$

3.
a.

Given T=day:

Sky is independent of T, choose highest probability, 0.5, Clear
Sun is dependent on T and Sky, if T=Day and Sky=Clear, highest probability is 0.9, True
Moon is dependent on T and Sky, if T=Day and Sky=Clear, highest probability is 0.8, False
Sea is dependent on Sun, if Sun=True, highest probability is 0.8, Blue

So:

Sky=Clear
Sun=True
Moon=False
Sea=Blue

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b.

Given:
Time=Night
Sky=Overcast

Sun is dependent on T and Sky, if T=Night and Sky=Overcast, highest probability is 1.0, False
Moon is dependent on T and Sky, if T=Night and Sky=Overcast, highest probability is 0.9, False
Sea is dependent on Sun, if Sun=False, highest probability is 0.7, Gray

So:

Sun=False
Moon=False
Sea=Gray