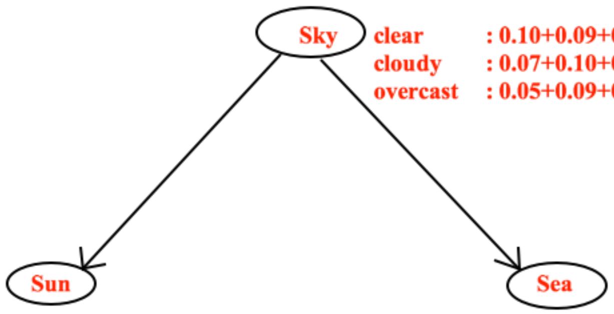
Marcus Blaisdell
CptS 440
Homework #8
October 25, 2018
Dr. Holder

1.



clear : 0.10+0.09+0.08+0.06 = 0.33
cloudy : 0.07+0.10+0.07+0.09 = 0.33
overcast : 0.05+0.09+0,08+0.12 = 0.34

Sun	Sky	P(Sun)
True	clear	0.10+0.08 = 0.18
True	cloudy	0.07+0.07 = 0.14
True	overcast	0.05+0.08 = 0.13
False	clear	0.09+0.06 = 0.15
False	cloudy	0.10+0.09 = 0.19
False	overcast	0.09+0.12 = 0.21

Sea	Sky	P(Sea)
Blue	clear	0.10+0.09 = 0.19
Blue	cloudy	0.07+0.10 = 0.17
Blue	overcast	0.05+0.09 = 0.14
Grey	clear	0.08+0.06 = 0.14
Grey	cloudy	0.07+0.09 = 0.16
Grey	overcast	0.08+0.12 = 0.20

2.

For all parts a through e of problem 2:
Let t be Time = day and ¬t be Time = night
Let sk1 be Sky = clear
Let sk2 be Sky = cloudy
Let sk3 be Sky = overcast
Let sn be Sun=true and ¬sn be Sun=false
Let mn be Moon=true and ¬mn be Moon=false
Let se be Sea=blue and ¬se be Sea=gray

a. P(Time=day, Sky=clear, Sun=true, Moon=false, Sea=blue)
$P(t \wedge sk1 \wedge sn \wedge \neg mn \wedge se)$
Given:
$P(t \wedge sk1 \wedge sn) = 0.9$
$P(t \wedge sk1 \wedge \neg mn) = 0.8$
$P(sn \wedge se) = 0.8$

$P(t \wedge sk1 \wedge sn \wedge \neg mn \wedge se) = P(t \wedge sk1 \wedge sn)\ P(t \wedge sk1 \wedge \neg mn)\ P(sn \wedge se) = 0.9 * 0.8 * 0.8 = 0.576$

b. P(Moon=true | Time=night, Sky=cloudy)
This is given on the Bayesian network as 0.5

c. P(Time=day | Moon=true)

Marcus Blaisdell
CptS 440
Homework #8
October 25, 2018
Dr. Holder

Choose the minimal set of parents.
Time has no parent.
Moon has two parents: Time and Sky.
Moon and Time are independent of Sun.
Moon and Time are independent of Sea.

$$P(t|mn) = \alpha P(t, mn) = \alpha P(t, mn, sk)$$

P(mn) is not given, solve by normalization:

$$\boldsymbol{P}(t|mn) = \alpha < P(t, mn), P(\neg t, mn) >$$

P(t,mn) = sum of all possible ways this can be true: 0.2 + 0.1 + 0.0 = 0.3
P(¬t,mn) = sum of all possible ways this can be true: 0.9 + 0.5 + 0.1 = 1.5

P(t|mn) = α<0.3,1.5>
α = 1 / 1.8 = 0.555
P(t|mn) = <0.166, 0.833> => <0.17, 0.83>
P(t|mn) = 0.17

d. P(Sea=blue | Time=day, Sky=clear)

Choose a minimal set of parents:
Sea is independent of Moon.
Sea has a parent Sun.
Sun has parents Time, and Sky

Calculate the distribution of P(Sea=blue):

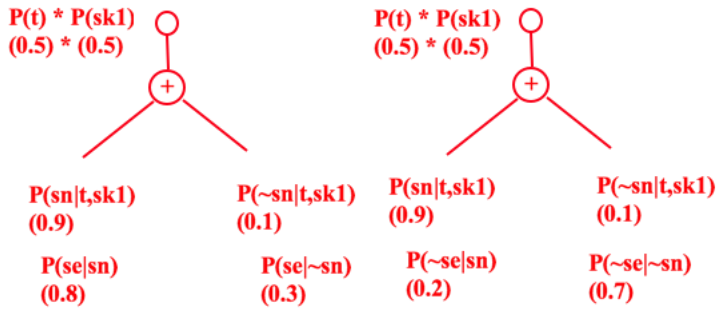
$$P(se|t, sk1) = \alpha P(se, t, sk1) = \alpha \sum_{sn} P(se, t, sk1, sn)$$
$$P(se|t, sk1) = \alpha \sum_{sn} P(se|sn)P(sn|t, sk1)P(t)P(sk1)$$
$$P(se|t, sk1) = \alpha P(t)P(sk1) \sum_{sn} P(sn|t, sk1)P(se|sn)$$

and:

$$P(\neg se|t, sk1) = \alpha P(se, t, sk1) = \alpha \sum_{sn} P(\neg se, t, sk1, sn)$$
$$P(\neg se|t, sk1) = \alpha \sum_{sn} P(\neg se|sn)P(sn|t, sk1)P(t)P(sk1)$$
$$P(\neg se|t, sk1) = \alpha P(t)P(sk1) \sum_{sn} P(sn|t, sk1)P(\neg se|sn)$$

Bayesian Networks:

Marcus Blaisdell
CptS 440
Homework #8
October 25, 2018
Dr. Holder



=>
P(se|t,sk1)= α (0.5)(0.5) * ((0.9 * 0.8) + (0.1 * 0.2)) = α0.1875
P(¬se|t,sk1)= α (0.5)(0.5) * ((0.9 * 0.2) + (0.1 * 0.7)) = α0.0625

=> **P**(se|t,sk1) = α<0.1875,0.0625> => <0.75,0.25>

P(se|t,sk1) = 0.75

e. P(Time=day | Sea=blue, Moon=false)

Choose minimal set:
Time has no parent
Sea has parent Sun
Moon has parents Time and Sky

$$P(t|se,\neg mn) = \alpha P(t,se,\neg mn) = \alpha \sum_{sk}\sum_{sn} P(t,se,\neg mn,sk,sn)$$

$$P(t|se,\neg mn) = \alpha \sum_{sk}\sum_{sn} P(t)P(se|sn)P(\neg mn|t,sk)P(sn|t,sk)$$

Moon is independent of Sun so it moves in front of Sun summation,
Time is independent of Sun and Sky so it moves in front of each summation:

$$P(t|se,\neg mn) = \alpha P(t)\sum_{sk} P(\neg mn|t,sk)P(sn|t,sk)\sum_{sn} P(se|sn)$$

Marcus Blaisdell
CptS 440
Homework #8
October 25, 2018
Dr. Holder

P(time=day)
(0.5)

+

P(moon=false | time=day, sky=clear)
(0.8)

+

P(sun=true | day=t, sky=clear)
(0.9)

P(seq=blue | sun=t)
(0.8)

$[(0.9*0.8) + (0.1*0.3)]*0.8$ (0.72, 0.03)

$= 0.6$

P(s=f | d=t, sk=clear)
(0.1)

P(seq=blue | s=f)
(0.3)

+

P(m=f, ..., cloudy)
(0.9)

+

P(s=t, d=t, cloudy)
(0.6) (0.4)

(0.8) (0.3)

$[(0.6*0.8) + (0.4*0.3)]*0.9$ (0.48, 0.12)

0.54

+

overcast
(1.0)

+

s=t, d=t, overcast
(0.2) (0.8)

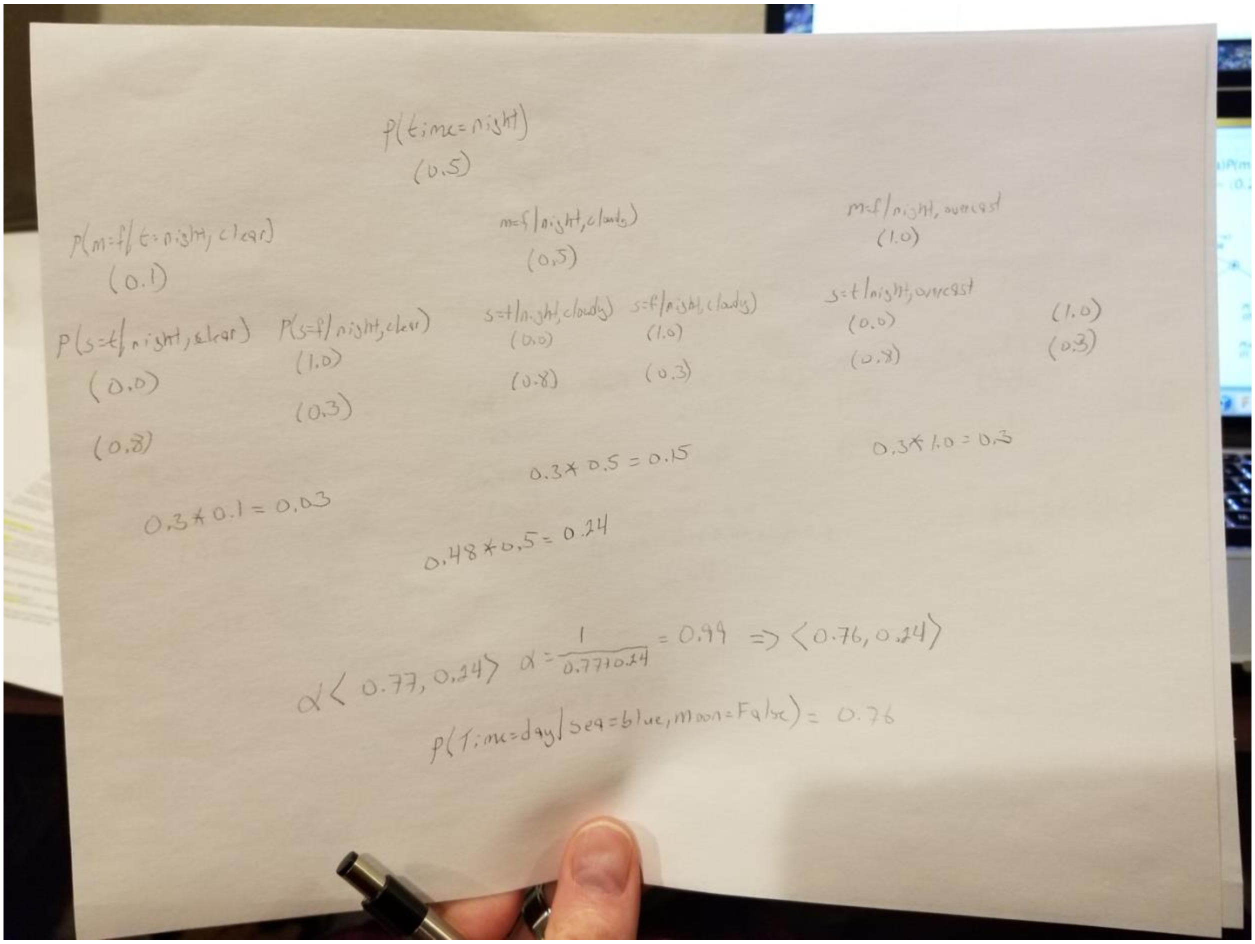
(0.8) (0.3)

$[(0.2*0.8) + (0.8*0.3)]*1.0$ (0.16, 0.24)

0.4

$= 1.54 * 0.5 = 0.77$

Marcus Blaisdell
CptS 440
Homework #8
October 25, 2018
Dr. Holder



=> α<0.77,0.24> => <0.76,0.24>

P(t|se, ¬mn) = 0.76

3.
a.

Given T=day:

Sky is independent of T, choose highest probability, 0.5, Clear
Sun is dependent on T and Sky, if T=Day and Sky=Clear, highest probability is 0.9, True
Moon is dependent on T and Sky, if T=Day and Sky=Clear, highest probability is 0.8, False
Sea is dependent on Sun, if Sun=True, highest probability is 0.8, Blue

So:

Sky=Clear

Marcus Blaisdell
CptS 440
Homework #8
October 25, 2018
Dr. Holder

Sun=True
Moon=False
Sea=Blue

b.

Given:
Time=Night
Sky=Overcast

Sun is dependent on T and Sky, if T=Night and Sky=Overcast, highest probability is 1.0, False
Moon is dependent on T and Sky, if T=Night and Sky=Overcast, highest probability is 0.9, False
Sea is dependent on Sun, if Sun=False, highest probability is 0.7, Gray

So:

Sun=False
Moon=False
Sea=Gray