

1)
a)

G	←	←
+1000	847.65	732.35
W	↑	↑
-1000	570.67	634.78
→	↑	↑
288.44	490.95	612.32

Begin by solving for the two locations adjacent to our locations of known utility:

$$U([2,3]) = R([2,3]) + (0.9)*[(0.8)*U([1,3]) + (0.1)*U([2,3]) + (0.1)*U([2,2])]$$

$$U([2,3]) = (-1) + (0.72)*(1000) + (0.09)*U([2,3]) + (0.09)*U([2,2])$$

$$U([2,3]) = (720) + (0.09)*U([2,3]) + (0.09)*U([2,2])$$

$$0 = (720) + (-0.91)*U([2,3]) + (0.09)*U([2,2])$$

$$U([2,2]) = R([2,2]) + (0.9)*[(0.8)*U([2,3]) + (0.1)*U([2,2]) + (0.1)*U([1,2])]$$

$$U([2,2]) = (-1) + (0.72)*U([2,3]) + (0.09)*U([2,2]) + (0.09)*(-1000)$$

$$U([2,2]) = (-90.04) + (0.72)*U([2,3]) + (0.09)*U([2,2])$$

$$0 = (-91) + (0.72)*U([2,3]) + (-0.91)*U([2,2])$$

2 equations, 2 unknowns, set the two equal to each other, solve for one term, substitute that solution into one of the equations to solve for the second term, use that answer to solve the first term:

$$(720) + (-0.91)*U([2,3]) + (0.09)*U([2,2]) = (-91) + (0.72)*U([2,3]) + (-0.91)*U([2,2])$$

$$(720) + (-0.91)*U([2,3]) + (0.09)*U([2,2]) + (91) - (0.72)*U([2,3]) + (0.91)*U([2,2]) = 0$$

$$(811) + (-1.63)*U([2,3]) + (1)*U([2,2]) = 0$$

Solve for U([2,2]):

$$U([2,2]) = (-811) + (1.63)*U([2,3])$$

Substitute into either of the equations to solve for U([2,3]):

$$0 = (720) + (-0.91)*U([2,3]) + (0.09)*[(-811) + (1.63)*U([2,3])]$$

$$0 = (720) + (-0.91)*U([2,3]) + (-72.99) + (0.1467)*U([2,3])$$

$$0 = (647.01) + (-0.7633)*U([2,3])$$

$$(0.7633)*U([2,3]) = (647.01)$$

$$U([2,3]) = (647.01) / (0.7633)$$

$$U([2,3]) = (847.648)$$

With $U([2,3])$ known, use either of the two original equations to solve for $U([2,2])$:

$$0 = (-91) + (0.72)*(847.648) + (-0.91)*U([2,2])$$

$$0 = (-91) + (610.307) + (-0.91)*U([2,2])$$

$$0 = (519.307) + (-0.91)*U([2,2])$$

$$(0.91)*U([2,2]) = (519.307)$$

$$U([2,2]) = (519.307) / (0.91)$$

$$U([2,2]) = (570.667)$$

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Next, use the next two adjacent locations to the two we have just calculated:

$$U([3,3]) = R([3,3]) + (0.9)*[(0.8)*U([2,3]) + (0.1)*U([3,3]) + (0.1)*U([3,2])]$$

$$U([3,3]) = (-1) + (0.72)*(847.648) + (0.09)*U([3,3]) + (0.09)*U([3,2])$$

$$U([3,3]) = (609.307) + (0.09)*U([3,3]) + (0.09)*U([3,2])$$

$$0 = (609.307) + (-0.91)*U([3,3]) + (0.09)*U([3,2])$$

$$U([3,2]) = R([3,2]) + (0.9)*[(0.8)*U([3,3]) + (0.1)*U([3,2]) + (0.1)*U([2,2])]$$

$$U([3,2]) = (-1) + (0.72)*U([3,3]) + (0.09)*U([3,2]) + (0.09)*(570.667)$$

$$U([3,2]) = (50.3603) + (0.72)*U([3,3]) + (0.09)*U([3,2])$$

$$0 = (50.3603) + (0.72)*U([3,3]) + (-0.91)*U([3,2])$$

$$(609.307) + (-0.91)*U([3,3]) + (0.09)*U([3,2]) = (50.3603) + (0.72)*U([3,3]) + (-0.91)*U([3,2])$$

$$(609.307) + (-0.91)*U([3,3]) + (0.09)*U([3,2]) + (-50.3603) - (0.72)*U([3,3]) + (0.91)*U([3,2]) = 0$$

$$(558.9467) + (-1.63)*U([3,3]) + (1)*U([3,2]) = 0$$

Solve for $U([3,3])$:

$$(1.63)*U([3,3]) = (558.9467) + (1)*U([3,2])$$

$$U([3,3]) = (558.9467) + (1)*U([3,2]) / (1.63)$$

Substitute into either equation:

$$0 = (609.307) + (-0.91)*[(558.9467) + (1)*U([3,2]) / (1.63)] + (0.09)*U([3,2])$$

$$0 = (609.307) + [(-508.6415) + (-0.91)*U([3,2]) / (1.63)] + (0.09)*U([3,2])$$

$$0 = (993.17) + (-508.6415) + (-0.91)*U([3,2]) + (0.1467)*U([3,2]) / (1.63)$$

$$0 = (993.17) + (-508.6415) + (-0.91)*U([3,2]) + (0.1467)*U([3,2])$$

$$0 = (484.52891) + (-0.7633)*U([3,2])$$

$$(0.7633)*U([3,2]) = (484.52891)$$

$$U([3,2]) = (484.52891) / (0.7633)$$

$$U([3,2]) = (634.782)$$

$$0 = (50.3603) + (0.72)*U([3,3]) + (-0.91)*(634.782)$$

$$0 = (-527.29) + (0.72)*U([3,3])$$

$$(0.72)*U([3,3]) = (527.29)$$

$$U([3,3]) = (527.29) / (0.72)$$

$$U([3,3]) = (732.349)$$

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$$U([1,1]) = R([1,1]) + (0.9)*[(0.8)*U([2,1]) + (0.1)*U([1,1]) + (0.1)*U([1,2])]$$

$$U([1,1]) = (-1) + (0.72)*U([2,1]) + (0.09)*U([1,1]) + (0.09)*(-1000)$$

$$U([1,1]) = (-90.04) + (0.72)*U([2,1]) + (0.09)*U([1,1])$$

$$0 = (-91) + (0.72)*U([2,1]) + (-0.91)*U([1,1])$$

Neither $U([2,1])$ or $U([1,1])$ is known, solve for them first:

$$U([2,1]) = R([2,1]) + (0.9)*[(0.8)*U([2,2]) + (0.1)*U([1,1]) + (0.1)*U([3,1])]$$

$$U([2,1]) = (-1) + (0.72)*(570.667) + (0.09)*U([1,1]) + (0.09)*U([3,1])$$

$$U([2,1]) = (409.88) + (0.09)*U([1,1]) + (0.09)*U([3,1])$$

$$0 = (409.88) + (0.09)*U([1,1]) + (0.09)*U([3,1]) - (1)U([2,1])$$

Three unknowns

$$U([3,1]) = R([3,1]) + (0.9)*[(0.8)*U([3,2]) + (0.1)*U([3,1]) + (0.1)*U([2,1])]$$

$$U([3,1]) = (-1) + (0.72)*(634.782) + (0.09)*U([3,1]) + (0.09)*U([2,1])$$

$$U([3,1]) = (457.043) + (0.09)*U([3,1]) + (0.09)*U([2,1])$$

$$0 = (457.043) + (-0.91)*U([3,1]) + (0.09)*U([2,1])$$

2 unknowns, no complementary equation, use these three to solve for 3 equations, 3 unknowns:

Solve for $U([1,1])$:

$$(0.91)*U([1,1]) = (-91) + (0.72)*U([2,1])$$

$$U([1,1]) = (-91) + (0.72)*U([2,1]) / (0.91)$$

Use to eliminate $U([1,1])$ in the only other equation that has it:

$$0 = (409.88) + (0.09)*[(-91) + (0.72)*U([2,1]) / (0.91)] + (0.09)*U([3,1]) - (1)U([2,1])$$

$$0 = (409.88) + [(-8.19) + (0.0648)*U([2,1]) / (0.91)] + (0.09)*U([3,1]) - (1)U([2,1])$$

$$0 = [(372.99) + (-8.19) + (0.0648)*U([2,1]) + (0.0819)*U([3,1]) - (0.91)U([2,1])] / (0.91)$$

$$0 = (364.8008) + (-0.8452)*U([2,1]) + (0.0819)*U([3,1])$$

We now have 2 equations with 2 complementary unknowns:

$$(457.043) + (-0.91)*U([3,1]) + (0.09)*U([2,1]) = (364.8008) + (-0.8452)*U([2,1]) + (0.0819)*U([3,1])$$

$$(457.043) + (-0.91)*U([3,1]) + (0.09)*U([2,1]) - (364.8008) + (0.8452)*U([2,1]) - (0.0819)*U([3,1]) = 0$$

$$(92.2422) + (-0.9919)*U([3,1]) + (0.9352)*U([2,1]) = 0$$

Solve for $U([3,1])$:

$$(0.9919)*U([3,1]) = (92.2422) + (0.9352)*U([2,1])$$

$$U([3,1]) = [(92.2422) + (0.9352)*U([2,1])] / (0.9919)$$

Substitute into either equation:

$$0 = (457.043) + (-0.91)*([(92.2422) + (0.9352)*U([2,1])] / (0.9919)) + (0.09)*U([2,1])$$

$$0 = (458.043) + ([(-83.94) + (-0.851)*U([2,1])] / (0.9919)) + (0.09)*U([2,1])$$

$$0 = [(454.878) + (-83.81956) + (-0.851)*U([2,1]) + (0.089)*U([2,1])] / (0.9919)$$

$$0 = (374.103) + (-0.762)*U([2,1])$$

$$(0.762)*U([2,1]) = (374.103)$$

$$U([2,1]) = (374.103) / (0.762)$$

$$U([2,1]) = (490.949)$$

Use to solve:

$$0 = (364.8008) + (-0.8452)*U([2,1]) + (0.0819)*U([3,1])$$

$$0 = (364.8008) + (-0.8452)*(490.949) + (0.0819)*U([3,1])$$

$$0 = (-50.149) + (0.0819)*U([3,1])$$

$$U([3,1]) = (50.149) / (0.0819)$$

$$U([3,1]) = (612.32)$$

And:

$$0 = (-91) + (0.72)*U([2,1]) + (-0.91)*U([1,1])$$

$$0 = (-91) + (0.72)*(490.949) + (-0.91)*U([1,1])$$

$$0 = (262.483) + (-0.91)*U([1,1])$$

$$(0.91)*U([1,1]) = (262.483)$$

$$U([1,1]) = (262.483) / (0.91)$$

$$U([1,1]) = (288.443)$$

b)

Temporal difference Q-learning:

$$U^\pi(s) = U^\pi(s) + \alpha(R(s) + \gamma U^\pi(s') - U^\pi(s))$$

$$\alpha = 1$$

$$\gamma = 0.9$$

5 executions of sequence: Right, Up, Up, Left (starting from [1,1])

Pass 1:

[1,1]:

$$U^\pi([1,1]) = U^\pi([1,1]) + (1)((-1) + (0.9)* U^\pi([2,1]) - U^\pi([1,1]))$$

$$U^\pi([1,1]) = 0 + (1)((-0.04) + (0.9)* (0) - (0))$$

$$U^\pi([1,1]) = (-1)$$

[2,1]:

$$U^\pi([2,1]) = U^\pi([2,1]) + (1)((-1) + (0.9)* U^\pi([2,2]) - U^\pi([2,1]))$$

$$U^\pi([2,1]) = (0) + (1)((-0.04) + (0.9)*(0) - (0))$$

$$U^\pi([2,1]) = (-1)$$

[2,2]:

$$U^\pi([2,2]) = U^\pi([2,2]) + (1)(-1) + (0.9) * U^\pi([2,3]) - U^\pi([2,2])$$

$$U^\pi([2,2]) = (0) + (1)(-1) + (0.9) * (0) - (0)$$

$$U^\pi([2,2]) = 0 + (1)(-1) + (0.9) * (0) - (0)$$

$$U^\pi([2,2]) = (-1)$$

[2,3]:

$$U^\pi([2,3]) = U^\pi([2,3]) + (1)(-1) + (0.9) * U^\pi([3,3]) - U^\pi([2,3])$$

$$U^\pi([2,3]) = (0) + (1)(-1) + (0.9) * (1000) - (0)$$

$$U^\pi([2,3]) = 0 + (1)(-1) + (900) - (0)$$

$$U^\pi([2,3]) = (899)$$

[1,3]:

Terminal state, $U([3,3]) = 1000$

$$U^\pi[1,1] = -1$$

$$U^\pi[2,1] = -1$$

$$U^\pi[2,2] = -1$$

$$U^\pi[2,3] = 899$$

$$U^\pi[1,3] = 1000$$

Pass 2:

[1,1]:

$$U^\pi([1,1]) = U^\pi([1,1]) + (1)(-1) + (0.9) * U^\pi([2,1]) - U^\pi([1,1])$$

$$U^\pi([1,1]) = (-1) + (1)(-1) + (0.9) * (-1) - (-1)$$

$$U^\pi([1,1]) = -1.9$$

[2,1]:

$$U^\pi([2,1]) = U^\pi([2,1]) + (1)(-1) + (0.9) * U^\pi([2,2]) - U^\pi([2,1])$$

$$U^\pi([2,1]) = (-1) + (1)(-1) + (0.9) * (-1) - (-1)$$

$$U^\pi([2,1]) = -1.9$$

[2,2]:

$$U^\pi([2,2]) = U^\pi([2,2]) + (1)(-1) + (0.9) * U^\pi([2,3]) - U^\pi([2,2])$$

$$U^\pi([2,2]) = (-1) + (1)(-1) + (0.9) * (899) - (-1)$$

$$U^\pi([2,2]) = 808.1$$

[2,3]:

$$U^\pi([2,3]) = U^\pi([2,3]) + (1)(-1) + (0.9) * U^\pi([3,3]) - U^\pi([2,3])$$

$$U^\pi([2,3]) = (899) + (1)(-1) + (0.9) * (1000) - (899)$$

$$U^\pi([2,3]) = (899)$$

[1,3]:

Terminal state, $U([3,3]) = 1000$

$$U^\pi[1,1] = -1.9$$

$$U^\pi[2,1] = -1.9$$

$$U^\pi[2,2] = 808.1$$

$$U^\pi[2,3] = 899$$

$$U^\pi[1,3] = 1000$$

Pass 3:

[1,1]:

$$U^\pi([1,1]) = U^\pi([1,1]) + (1)(-1) + (0.9) * U^\pi([2,1]) - U^\pi([1,1])$$

$$U^\pi([1,1]) = (-1.9) + (1)(-1) + (0.9) * (-1.9) - (-1.9)$$

$$U^\pi([1,1]) = -2.71$$

[2,1]:

$$U^\pi([2,1]) = U^\pi([2,1]) + (1)(-1) + (0.9) * U^\pi([2,2]) - U^\pi([2,1])$$

$$U^\pi([2,1]) = (-1.9) + (1)(-1) + (0.9) * (808.1) - (-1.9)$$

$$U^\pi([2,1]) = 726.29$$

[2,2]:

$$U^\pi([2,2]) = U^\pi([2,2]) + (1)(-1) + (0.9) * U^\pi([2,3]) - U^\pi([2,2])$$

$$U^\pi([2,2]) = (808.1) + (1)(-1) + (0.9) * (899) - (808.1)$$

$$U^\pi([2,2]) = 808.1$$

[2,3]:

$$U^\pi([2,3]) = U^\pi([2,3]) + (1)(-1) + (0.9) * U^\pi([3,3]) - U^\pi([2,3])$$

$$U^\pi([2,3]) = (899) + (1)(-1) + (0.9) * (1000) - (899)$$

$$U^\pi([2,3]) = 899$$

[1,3]:

Terminal state, $U([3,3]) = 1000$

$$U^\pi[1,1] = -2.71$$

$$U^\pi[2,1] = 726.29$$

$$U^\pi[2,2] = 808.1$$

$$U^\pi[2,3] = 899$$

$$U^\pi[1,3] = 1000$$

Pass 4:

[1,1]:

$$U^\pi([1,1]) = U^\pi([1,1]) + (1)(-1) + (0.9) * U^\pi([2,1]) - U^\pi([1,1])$$

$$U^\pi([1,1]) = (-2.71) + (1)(-1) + (0.9) * (726.29) - (-2.71)$$

$$U^\pi([1,1]) = 652.661$$

[2,1]:

$$U^\pi([2,1]) = U^\pi([2,1]) + (1)(-1) + (0.9) * U^\pi([2,2]) - U^\pi([2,1])$$

$$U^\pi([2,1]) = (726.29) + (1)(-1) + (0.9) * (808.1) - (726.29)$$

$$U^\pi([2,1]) = 726.29$$

[2,2]:

$$U^\pi([2,2]) = U^\pi([2,2]) + (1)(-1) + (0.9) * U^\pi([2,3]) - U^\pi([2,2])$$

$$U^\pi([2,2]) = (808.1) + (1)(-1) + (0.9) * (899) - (808.1)$$

$$U^\pi([2,2]) = 808.1$$

[2,3]:

$$U^\pi([2,3]) = U^\pi([2,3]) + (1)(-1) + (0.9) * U^\pi([3,3]) - U^\pi([2,3])$$

$$U^\pi([2,3]) = (899) + (1)(-1) + (0.9) * (1000) - (899)$$

$$U^\pi([2,3]) = 899$$

[1,3]:

Terminal state, $U([3,3]) = 1000$

$$U^\pi[1,1] = 652.661$$

$$U^\pi[2,1] = 726.29$$

$$U^\pi[2,2] = 808.1$$

$$U^\pi[2,3] = 899$$

$$U^\pi[1,3] = 1000$$

Pass 5:

[1,1]:

$$U^\pi([1,1]) = U^\pi([1,1]) + (1)(-1) + (0.9) * U^\pi([2,1]) - U^\pi([1,1])$$

$$U^\pi([1,1]) = (652.661) + (1)(-1) + (0.9) * (726.29) - (652.661)$$

$$U^\pi([1,1]) = 652.661$$

[2,1]:

$$U^\pi([2,1]) = U^\pi([2,1]) + (1)(-1) + (0.9) * U^\pi([2,2]) - U^\pi([2,1])$$

$$U^\pi([2,1]) = (726.29) + (1)(-1) + (0.9) * (808.1) - (726.29)$$

$$U^\pi([2,1]) = 726.29$$

[2,2]:

$$U^\pi([2,2]) = U^\pi([2,2]) + (1)(-1) + (0.9) * U^\pi([2,3]) - U^\pi([2,2])$$

$$U^\pi([2,2]) = (808.1) + (1)(-1) + (0.9) * (899) - (808.1)$$

$$U^\pi([2,2]) = 808.1$$

[2,3]:

$$U^\pi([2,3]) = U^\pi([2,3]) + (1)(-1) + (0.9) * U^\pi([3,3]) - U^\pi([2,3])$$

$$U^\pi([2,3]) = (899) + (1)(-1) + (0.9) * (1000) - (899)$$

$$U^\pi([2,3]) = 899$$

[1,3]:

Terminal state, $U([3,3]) = 1000$

$$U^\pi[1,1] = -0.13756$$

$$U^\pi[2,1] = -0.13756$$

$$U^\pi[2,2] = 809.924$$

$$U^\pi[2,3] = 899.96$$

$$U^\pi[1,3] = 1000$$

Cell	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5
[1,1]	-1	-1.9	-2.71	652.66	652.66
[2,1]	-1	-1.9	726.29	726.29	726.29
[2,2]	-1	808.1	808.1	808.1	808.1
[2,3]	899	899	899	899	899

[3,3]	1000	1000	1000	1000	1000
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2)

a)

$P(\text{Class}) = \# \text{ times hungry/not hungry appears as the classification out of the total number of classifications:}$

$$P(\text{Class} = \text{hungry}) = 600/1000 = 0.6$$

$$P(\text{Class} = \text{not hungry}) = 400/1000 = 0.4$$

b)

$m = \text{"the wumpus is not hungry"}$

bigrams:

the wumpus

wumpus is

is not

not hungry

By Bayes rule:

$$\begin{aligned} P(\text{Class} = \text{hungry} \mid \text{Message} = m) &= P(\text{Message} = m \mid \text{Class} = \text{hungry}) / P(\text{Class} = \text{hungry}) \\ &= P(P(\text{wumpus}|\text{the}) \mid \text{class} = \text{hungry} * P(\text{is}|\text{wumpus}) \mid \text{class} = \text{hungry} * P(\text{not}|\text{is}) \mid \text{class} = \text{hungry} * \\ &P(\text{hungry}|\text{not}) \mid \text{class} = \text{hungry}) / P(\text{Class} = \text{hungry}) \end{aligned}$$

$$\begin{aligned} P(\text{wumpus}|\text{the}) &= 300/1000 = 0.3 \\ P(\text{is}|\text{wumpus}) &= 90/1000 = 0.09 \\ P(\text{not}|\text{is}) &= 60/1000 = 0.06 \\ P(\text{hungry}|\text{not}) &= 100/1000 = 0.1 \end{aligned}$$

$$0.3 * 0.09 * 0.06 * 0.1 = 0.000162$$

$$P(\text{message} = m \mid \text{class} = \text{hungry}) = 0.000162$$

$$P(\text{class} = \text{hungry}) = 0.6$$

$$0.000162 / 0.6 = 0.00027$$

$$P(\text{class} = \text{hungry} \mid \text{message} = m) = 0.00027$$

c)

$$P(\text{Class} = \text{not hungry} \mid \text{Message} = m) = P(\text{Message} = m \mid \text{Class} = \text{not hungry}) / P(\text{Class} = \text{not hungry})$$

$$= P(P(\text{wumpus}|\text{the}) \mid \text{class} = \text{not hungry} * P(\text{is}|\text{wumpus}) \mid \text{class} = \text{not hungry} * P(\text{not}|\text{is}) \mid \text{class} = \text{not hungry} * P(\text{hungry}|\text{not}) \mid \text{class} = \text{not hungry}) / P(\text{Class} = \text{not hungry})$$

$P(\text{wumpus} \text{the})$	$= 150/1000$	$= 0.15$
$P(\text{is} \text{wumpus})$	$= 40/1000$	$= 0.04$
$P(\text{not} \text{is})$	$= 60/1000$	$= 0.06$
$P(\text{hungry} \text{not})$	$= 200/1000$	$= 0.2$

$$0.15 * 0.04 * 0.06 * 0.2 = 0.000072$$

$$P(\text{message} = m \mid \text{class} = \text{not hungry}) = 0.000072$$

$$P(\text{class} = \text{not hungry}) = 0.4$$

$$0.000072 / 0.4 = 0.00018$$

$$P(\text{class} = \text{not hungry} \mid \text{message} = m) = 0.00018$$

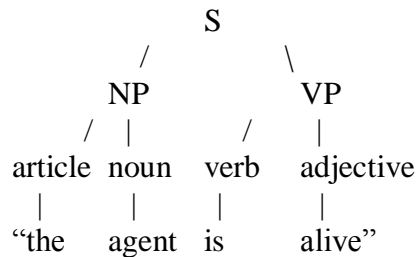
d)

$$P(\text{class} = \text{hungry} \mid \text{message} = m) = 0.00027 > P(\text{class} = \text{not hungry} \mid \text{message} = m) = 0.00018$$

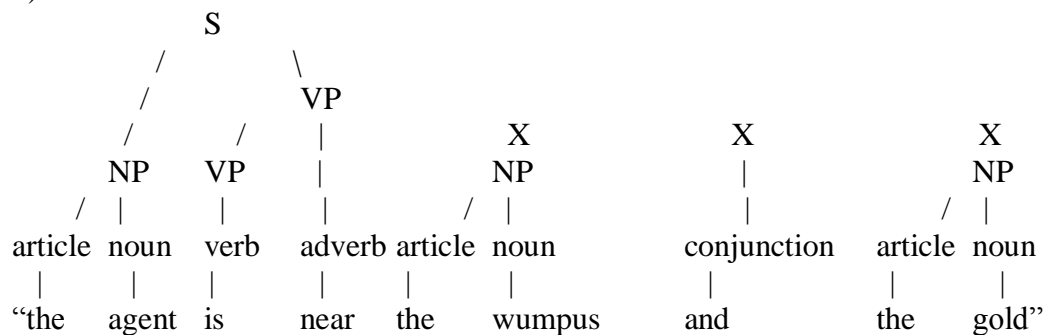
The more likely class, given this corpus, is the classification is “not hungry” for message m.

2)

a)

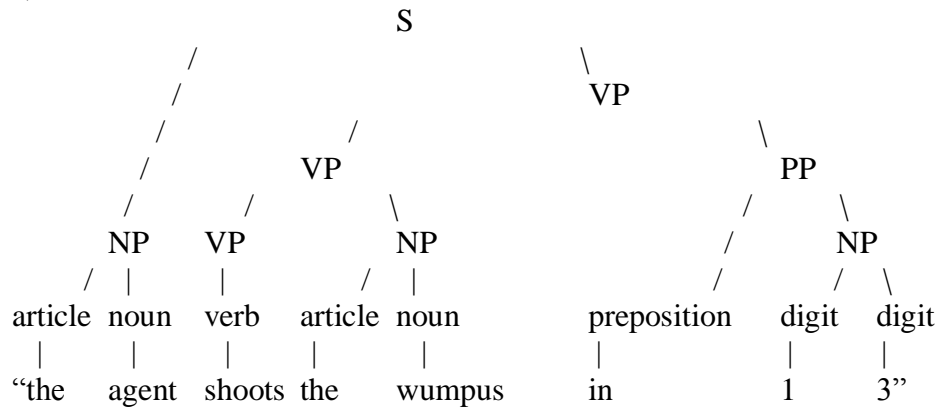


b)

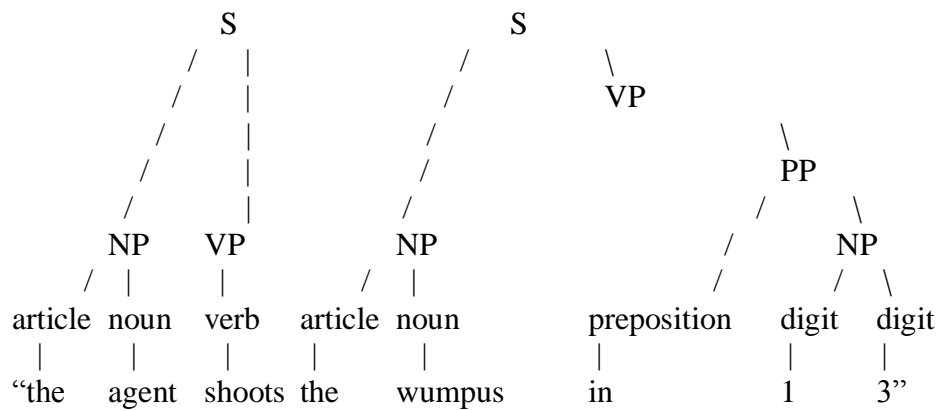


Sentence cannot be parsed, it is not consistent with (accepted) by the grammar.

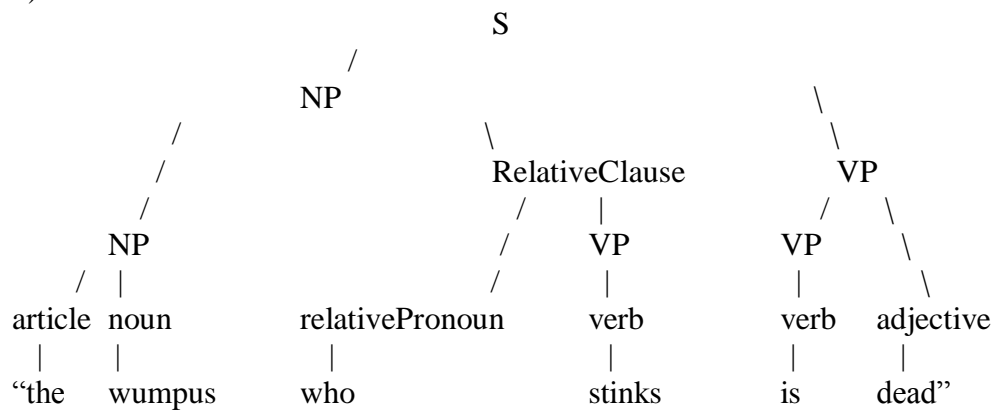
c)



and:



d)



4)

input sequence: [C₂, C₃, C₃, C₄, C₅, C₆, C₇]

$$\begin{aligned} V1, \text{onset} &= P(C_2|\text{onset})P(\text{onset}) \\ &= (0.5)(1.0) = \mathbf{0.5} \\ V1, \text{mid} &= P(C_2|\text{mid}) = 0 \\ V1, \text{end} &= P(C_2|\text{end}) = 0 \\ V1, \text{final} &= 0 \end{aligned}$$

-

$$\begin{aligned} V2, \text{onset} &= P(C_3|\text{onset})\max\{[P(\text{onset}|\text{onset})V1, \mathbf{\text{onset}}], 0, 0, 0\} \\ &= (0.3)[(0.6)(0.5)] = \mathbf{0.09} \\ V2, \text{mid} &= P(C_3|\text{mid})\max\{[P(\text{mid}|\text{onset})V1, \text{onset}, P(\text{mid}|\text{mid})V1, \text{mid}, 0, 0]\} \\ &= (0.3)\{[(0.4)(0.5)], [(0.5)(0)]\} = \{0.06, 0\} \\ V2, \text{end} &= P(C_3|\text{end}) = 0 \\ V2, \text{final} &= 0 \end{aligned}$$

-

$$\begin{aligned} V3, \text{onset} &= P(C_3|\text{onset})\max\{[P(\text{onset}|\text{onset})V2, \mathbf{\text{onset}}, 0, 0, 0]\} \\ &= (0.3)[(0.6)(0.09)] = \mathbf{0.0162} \\ V3, \text{mid} &= P(C_3|\text{mid})\max\{[P(\text{mid}|\text{onset})V2, \text{onset}, P(\text{mid}|\text{mid})V2, \text{mid}], 0, 0\} \\ &= (0.3)\{[(0.4)(0.09)], [(0.5)(0.06)]\} = \{0.0108, 0.009\} \\ V3, \text{end} &= P(C_3|\text{end}) = 0 \\ V3, \text{final} &= 0 \end{aligned}$$

-

$$\begin{aligned} V4, \text{onset} &= P(C_4|\text{onset}) = 0 \\ V4, \text{mid} &= P(C_4|\text{mid})\max\{[P(\text{mid}|\text{onset})V3, \mathbf{\text{onset}}, P(\text{mid}|\text{mid})V3, \text{mid}], 0, 0\} \\ &= (0.3)\{[(0.4)(0.0162)], [(0.5)(0.0108)]\} = \{\mathbf{0.00243}, 0.00162\} \\ V4, \text{end} &= P(C_4|\text{end})\max\{[P(\text{end}|\text{mid})V3, \text{mid}, P(\text{end}|\text{end})V3, \text{end}], 0\} \\ &= (0)\{[(0)], [(0)]\} \{0, 0\} \\ V4, \text{final} &= 0 \end{aligned}$$

-

$$\begin{aligned} V5, \text{onset} &= P(C_5|\text{onset}) = 0 \\ V5, \text{mid} &= P(C_5|\text{mid})\max\{[P(\text{mid}|\text{onset})V4, \text{onset}, P(\text{mid}|\text{mid})V4, \mathbf{\text{mid}}], 0, 0\} \\ &= (0.4)\{[(0.4)(0)], [(0.5)(0.00243)]\} = \{0, \mathbf{0.000486}\} \\ V5, \text{end} &= P(C_5|\text{end})\max\{[P(\text{end}|\text{mid})V4, \text{mid}, P(\text{end}|\text{end})V4, \text{end}], 0\} \\ &= (0.2)\{[(0.5)(0.00243)], [(0.3)(0)]\} = \{0.000243, 0\} \\ V5, \text{final} &= 0 \end{aligned}$$

-

$$V6, \text{onset} = P(C6|\text{onset}) = 0$$

$$V6, \text{mid} = P(C6|\text{mid}) = 0$$

$$V6, \text{end} = P(C6|\text{end})\max\{[P(\text{end}|\text{mid})V5, \text{mid}, P(\text{end}|\text{end})V5, \text{end}], 0\} \\ = (0.4)\{[(0.5)(0.000486)], [(0.3)(0.000243)]\} = \{0.0000972, 0.00002916\}$$

$$V6, \text{final} = 0$$

-

$$V7, \text{onset} = P(C7|\text{onset}) = 0$$

$$V7, \text{mid} = P(C7|\text{mid}) = 0$$

$$V7, \text{end} = P(C7|\text{end})\max\{[P(\text{end}|\text{mid})V6, \text{mid}, P(\text{end}|\text{end})V6, \text{end}], 0\} \\ = (0.4)\{[(0.5)(0)], [(0.3)(0.0000972)]\} = \{0, 0.000011664\}$$

$$V7, \text{final} = 0$$

$$P(\text{final}|\text{end})V7, \text{end} \\ = (0.7)(0.000011664) \\ = 0.0000081648 \\ = 8.16 \times 10^{-6}$$

$$P(\text{HMM}) = 8.16 \times 10^{-6}$$

Most probable path:

Onset, onset, onset, mid, mid, end, end, final