

1.

a.

$$P(\text{Sail}=\text{yes}) = 7/10 = 0.7$$

$$P(\text{Sail}=\text{no}) = 3/10 = 0.3$$

b.

$P(\text{Sky}|\text{Sail})$ for all values of $\text{Sky} \in \{\text{clear, cloudy, overcast}\}$ and $\text{Sail} \in \{\text{yes, no}\}$

$$P(\text{Sky}|\text{Sail}) = \frac{P(\text{Sky} \wedge \text{Sail})}{P(\text{Sail})}$$

$$P(\text{clear}|\text{yes}) = (2/10)/(7/10) = 2/7 = 0.2857$$

$$P(\text{cloudy}|\text{yes}) = (3/10)/(7/10) = 3/7 = 0.4286$$

$$P(\text{overcast}|\text{yes}) = (2/10)/(7/10) = 2/7 = 0.2857$$

$$P(\text{clear}|\text{no}) = (1/10)/(3/10) = 1/3 = 0.3333$$

$$P(\text{cloudy}|\text{no}) = (1/10)/(3/10) = 1/3 = 0.3333$$

$$P(\text{overcast}|\text{no}) = (1/10)/(3/10) = 1/3 = 0.3333$$

c.

$P(\text{Sea}|\text{Sail})$ for all values of $\text{Sea} \in \{\text{blue, gray}\}$ and $\text{Sail} \in \{\text{yes, no}\}$

$$P(\text{Sea}|\text{Sail}) = \frac{P(\text{Sea} \wedge \text{Sail})}{P(\text{Sail})}$$

$$P(\text{blue}|\text{yes}) = (5/10)/(7/10) = 5/7 = 0.7143$$

$$P(\text{gray}|\text{yes}) = (2/10)/(7/10) = 2/7 = 0.2857$$

$$P(\text{blue}|\text{no}) = (0/10)/(3/10) = 0/3 \Rightarrow (0 + 1)/(3 + 2) = 1/5 = 0.2$$

$$P(\text{gray}|\text{no}) = (3/10)/(3/10) = 3/3 = 1$$

d.

$P(\text{Sun}|\text{Sail})$ for all values of $\text{Sun} \in \{\text{true, false}\}$ and $\text{Sail} \in \{\text{yes, no}\}$

$$P(\text{Sun}|\text{Sail}) = \frac{P(\text{Sun} \wedge \text{Sail})}{P(\text{Sail})}$$

$$P(\text{true}|\text{yes}) = (4/10)/(7/10) = 4/7 = 0.5714$$

$$P(\text{false}|\text{yes}) = (3/10)/(7/10) = 3/7 = 0.4286$$

$$P(\text{true}|\text{no}) = (0/10)/(3/10) = 0/3 \Rightarrow (0 + 1)/(3 + 2) = 1/5 = 0.2$$

$$P(\text{false}|\text{no}) = (3/10)/(3/10) = 3/3 = 1$$

e.

$$P(\text{Sail}=\text{yes} \mid \text{Sky}=\text{overcast}, \text{Sea}=\text{gray}, \text{Sun}=\text{true}) =$$

$$= P(\text{Sail}=\text{yes}) \Pi P(\text{Sky}=\text{overcast} \mid \text{Sail}=\text{yes}) P(\text{Sea}=\text{gray} \mid \text{Sail}=\text{yes}) P(\text{Sun}=\text{true} \mid \text{Sail}=\text{yes})$$

$$= (7/10) * (2/7) * (2/7) * (4/7) = \alpha 0.032653$$

$$\begin{aligned} P(\text{Sail=no} \mid \text{Sky=overcast, Sea=gray, Sun=true}) &= \\ &= P(\text{Sail=yes}) \prod P(\text{Sky=overcast} \mid \text{Sail=yes}) P(\text{Sea=gray} \mid \text{Sail=yes}) P(\text{Sun=true} \mid \text{Sail=yes}) \\ &= (3/10) * (2/7) * (2/7) * (4/7) = \alpha 0.013994 \end{aligned}$$

so:

$$P(\text{Sail} \mid \text{Sky=overcast, Sea=gray, Sun=true}) = \alpha \langle 0.032653, 0.013994 \rangle$$

$$\alpha = 1/(0.032653 + 0.013994) = 21.4376$$

$$P(\text{Sail} \mid \text{Sky=overcast, Sea=gray, Sun=true}) = \langle 0.7, 0.03 \rangle$$

therefore:

$$P(\text{Sail=yes} \mid \text{Sky=overcast, Sea=gray, Sun=true}) = 0.7$$

$$P(\text{Sail=no} \mid \text{Sky=overcast, Sea=gray, Sun=true}) = 0.3$$

f.

$$0.7 > 0.3$$

Naïve Bayes would choose Sail = yes

2.

a.

Sky	Sea	Sun	Sail
0	0	0	Yes
0	1	1	Yes
0	1	0	No
1	0	1	Yes
1	0	0	Yes
1	1	1	Yes
1	1	0	No
2	0	1	Yes
2	0	0	Yes
2	1	0	No

b.

New instance: <Sky=overcast, Sea=gray, Sun=true> => <Sky=2, Sea=1, Sun=1>

Euclidean distance between 3-dimensional points = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
(x_1, y_1, z_1) = (2, 1, 1) (the new instance values)

Line	Calculation	Distance	Class
1	$\sqrt{(2 - 0)^2 + (1 - 0)^2 + (1 - 0)^2} = \sqrt{6} = 2.45$	2.45	Yes
2	$\sqrt{(2 - 0)^2 + (1 - 1)^2 + (1 - 1)^2} = 2$	2	Yes
3	$\sqrt{(2 - 0)^2 + (1 - 1)^2 + (1 - 0)^2} = \sqrt{5} = 2.24$	2.24	No
4	$\sqrt{(2 - 1)^2 + (1 - 0)^2 + (1 - 1)^2} = \sqrt{2} = 1.41$	1.41	Yes
5	$\sqrt{(2 - 1)^2 + (1 - 0)^2 + (1 - 0)^2} = \sqrt{3} = 1.73$	1.73	Yes
6	$\sqrt{(2 - 1)^2 + (1 - 1)^2 + (1 - 1)^2} = 1$	1	Yes
7	$\sqrt{(2 - 1)^2 + (1 - 1)^2 + (1 - 0)^2} = \sqrt{2} = 1.41$	1.41	No
8	$\sqrt{(2 - 2)^2 + (1 - 0)^2 + (1 - 1)^2} = 1$	1	Yes
9	$\sqrt{(2 - 2)^2 + (1 - 0)^2 + (1 - 0)^2} = \sqrt{2} = 1.41$	1.41	Yes
10	$\sqrt{(2 - 2)^2 + (1 - 1)^2 + (1 - 0)^2} = 1$	1	No

c.

3 nearest neighbors, there are three points at distance 1, two are yes, one is no, majority of nearest neighbors are yes, so yes.

3NN would classify <Sky=overcast, Sea=gray, Sun=true> as Yes

d.

7 nearest neighbors: 1:yes, 1:yes, 1:no, 1.41:yes, 1.41:no, 1.41:yes, 1.73:yes
5 yes'es, 2 no's

7NN would classify <Sky=overcast, Sea=gray, Sun=true> as Yes