Marcus Blaisdell
CptS 440
Homework #6
October 11, 2018
Dr. Holder

1.

a. Color(Sky, Blue)⇒Shining(Sun)
b. Color(Sky, Gray)⇒¬Shining(Sun)
c. (¬Color(Sky, Blue) ^ ¬Color(Sky, Gray)) ⇒(¬Shining(Sun) ^ Shining(Moon))

2.

i. Cloudy(Sky) ⇒ Color(Sky,Gray)
ii. Color(Sky, Gray) ⇒ ¬Shining(Moon)
iii. Shining(Moon) ⇔ ¬ DayTime
iv. Cloudy(Sky)

a.

Step 1: Eliminate Implications:

i. ¬Cloudy(Sky) v Color(Sky,Gray)
ii. ¬Color(Sky, Gray) v ¬Shining(Moon)
iii. (Shining(Moon) v DayTime) ^ (¬Shining(Moon) v ¬ DayTime)
iv. Cloudy(Sky)

Step 2: Move ¬ inwards: (no action required)

R1: ¬Cloudy(Sky) v Color(Sky,Gray)
R2: ¬Color(Sky, Gray) v ¬Shining(Moon)
R3: (Shining(Moon) v DayTime) ^ (¬Shining(Moon) v ¬ DayTime)
R4: Cloudy(Sky)

b.

Proof by refutation. Insert the negation of what we want to prove into the knowledge base and see if it results in a contradiction:

R1: ¬Cloudy(Sky) v Color(Sky,Gray)
R2: ¬Color(Sky, Gray) v ¬Shining(Moon)
R3: (Shining(Moon) v DayTime) ^ (¬Shining(Moon) v ¬ DayTime)
R4: Cloudy(Sky)
R5: ¬ DayTime

R1: ¬Cloudy(Sky) v Color(Sky,Gray)
R4: Cloudy(Sky)

R6: Color(Sky,Gray)

R2: ¬Color(Sky, Gray) v ¬Shining(Moon)

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R6: Color(Sky,Gray)

R7: ¬Shining(Moon)

R3: (Shining(Moon) v DayTime) ^ (¬Shining(Moon) v ¬ DayTime)
And elimination:

R8: (Shining(Moon) v DayTime)

R8: (Shining(Moon) v DayTime)
R7: ¬Shining(Moon)

R9: DayTime

R9: DayTime conflicts with R5: ¬ DayTime therefore, DayTime
□

3.

a. ∃t Shining (Sun, t) ^ Shining (Moon, t)
b. ∀t Color (Sea, Blue) ⇒ Shining (Sun, t)
c. ∀t DayTime(t) ⇒ (Color(Sky, Blue) v Color(Sky, Gray))
d. ∀t Shining(Moon, t) ⇒ NightTime(t)

4.

i. ∀t Shining(Sun,t) ⇒ Color(Sky, Blue, t)
ii. ∀t (Color(Sea, Gray, t) v Color(Sea, Blue, t)) ⇒ ¬NightTime(t)
iii. ∀c,t Color(Sea, c, t) ⇔ Color(Sky, c, t)
iv. ∀t ¬DayTime(t) ⇒ NightTime(t)
v. ∃t Shining (Sun, t)

a. Convert to Conjunctive Normal Form

Step 1: Eliminate Implications:

i. ∀t ¬Shining(Sun,t) v Color(Sky, Blue, t)
ii. ∀t ¬(Color(Sea, Gray, t) v Color(Sea, Blue, t)) v ¬NightTime(t)
iii. ∀c,t (Color(Sea, c, t) v ¬Color(Sky, c, t)) ^ (¬Color(Sea, c, t) v Color(Sky, c, t))
iv. ∀t DayTime(t) v NightTime(t)
v. ∃t Shining (Sun, t)

Step 2: Move ¬ inwards

i. ∀t ¬Shining(Sun,t) v Color(Sky, Blue, t)

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ii. $\forall t\ (\neg Color(Sea, Gray, t) \wedge \neg Color(Sea, Blue, t)) \vee \neg NightTime(t)$
iii. $\forall c,t\ (Color(Sea, c, t) \vee \neg Color(Sky, c, t)) \wedge (\neg Color(Sea, c, t) \vee Color(Sky, c, t))$
iv. $\forall t\ DayTime(t) \vee NightTime(t)$
v. $\exists t\ Shining(Sun, t)$

Step 3: Standardize variables:

i. $\forall t_1\ \neg Shining(Sun_1,t_1) \vee Color(Sky_2, Blue_2, t_2)$
ii. $\forall t_1\ (\neg Color(Sea_1, Gray_1, t_1) \wedge \neg Color(Sea_2, Blue_2, t_2)) \vee \neg NightTime(t_3)$
iii. $\forall c_1,t_1\ (Color(Sea_1, c_1, t_1) \vee \neg Color(Sky_2, c_2, t_2)) \wedge (\neg Color(Sea_3, c_3, t_3) \vee Color(Sky_4, c_4, t_4))$
iv. $\forall t_1\ DayTime(t_1) \vee NightTime(t_2)$
v. $\exists t_1\ Shining(Sun_1, t_1)$

Step 4: Skolemize:

i. $\forall t_1\ \neg Shining(Sun_1,t_1) \vee Color(Sky_2, Blue_2, t_2)$
ii. $\forall t_1\ (\neg Color(Sea_1, Gray_1, t_1) \wedge \neg Color(Sea_2, Blue_2, t_2)) \vee \neg NightTime(t_3)$
iii. $\forall c_1,t_1\ (Color(Sea_1, c_1, t_1) \vee \neg Color(Sky_2, c_2, t_2)) \wedge (\neg Color(Sea_3, c_3, t_3) \vee Color(Sky_4, c_4, t_4))$
iv. $\forall t_1\ DayTime(t_1) \vee NightTime(t_2)$
v. $Shining(Sun_1, F(Sun_1))$

Step 5: Drop Universal Quantifiers:
i. $\neg Shining(Sun_1,t_1) \vee Color(Sky_2, Blue_2, t_2)$
ii. $(\neg Color(Sea_1, Gray_1, t_1) \wedge \neg Color(Sea_2, Blue_2, t_2)) \vee \neg NightTime(t_3)$
iii. $(Color(Sea_1, c_1, t_1) \vee \neg Color(Sky_2, c_2, t_2)) \wedge (\neg Color(Sea_3, c_3, t_3) \vee Color(Sky_4, c_4, t_4))$
iv. $DayTime(t_1) \vee NightTime(t_2)$
v. $Shining(Sun_1, F(Sun_1))$

Step 6: Distribute v over ^
i. $\neg Shining(Sun_1,t_1) \vee Color(Sky_2, Blue_2, t_2)$
ii. $((\neg Color(Sea_1, Gray_1, t_1) \vee \neg NightTime(t_3)) \wedge (\neg Color(Sea_2, Blue_2, t_2) \vee \neg NightTime(t_3))$
iii. $(Color(Sea_1, c_1, t_1) \vee \neg Color(Sky_2, c_2, t_2)) \wedge (\neg Color(Sea_3, c_3, t_3) \vee Color(Sky_4, c_4, t_4))$
iv. $DayTime(t_1) \vee NightTime(t_2)$
v. $Shining(Sun_1, F(Sun_1))$

b. Show a resolution proof by refutation that $\exists t\ DayTime(t)$ is true.

Add the negation of what we want to prove to the knowledge base and test if it results in a contradiction:

R1: $\neg Shining(Sun_1,t_1) \vee Color(Sky_2, Blue_2, t_2)$
R2: $((\neg Color(Sea_1, Gray_1, t_1) \vee \neg NightTime(t_3)) \wedge (\neg Color(Sea_2, Blue_2, t_2) \vee \neg NightTime(t_3))$

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R3: (Color(Sea_1, c_1, t_1) v ¬Color(Sky_2, c_2, t_2)) ^ (¬Color(Sea_3, c_3, t_3) v Color(Sky_4, c_4, t_4))
R4: DayTime(t_1) v NightTime(t_2)
R5: Shining (Sun_1, F(Sun_1))
R6: ¬∃t DayTime(t) becomes: ¬DayTime(F(Sun_1))

R5: Shining (Sun_1, F(Sun_1))
{t_1, F(Sun_1)}

R5: Shining (Sun_1, t_1)

R6: ¬DayTime(F(Sun_1))
{t_1, F(Sun_1)}

R6: ¬DayTime(t_1)

R4: DayTime(t_1) v NightTime(t_2)
R6: ¬DayTime(t_1)

R7: NightTime(t_2)

R2: ((¬Color(Sea_1, $Gray_1$, t_1) v ¬NightTime(t_3)) ^ (¬Color(Sea_2, $Blue_2$, t_2) v ¬NightTime(t_3))
And Elimination:

R8: ¬Color(Sea_2, $Blue_2$, t_2) v ¬NightTime(t_3)

R8: ¬Color(Sea_2, $Blue_2$, t_2) v ¬NightTime(t_3)
Commutativity:

R9: ¬NightTime(t_3) v ¬Color(Sea_2, $Blue_2$, t_2)

R7: NightTime(t_2)
{t_3, t_2}

R7: NightTime(t_3)

R9: ¬NightTime(t_3) v ¬Color(Sea_2, $Blue_2$, t_2)
R7: NightTime(t_3)

R10: ¬Color(Sea_2, $Blue_2$, t_2)

R1: ¬Shining(Sun_1,t_1) v Color(Sky_2, $Blue_2$, t_2)
Commutativity:

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R11: Color(Sky_2, $Blue_2$, t_2) v ¬Shining(Sun_1,t_1)

R11: Color(Sky_2, $Blue_2$, t_2) v ¬Shining(Sun_1,t_1)
R10: ¬Color(Sea_2, $Blue_2$, t_2)

R12: ¬Shining(Sun_1,t_1) conflicts with R5: Shining (Sun_1, t_1) therefore ∃t DayTime(t) is true
□