Marcus Blaisdell
CptS 440
Homework #7
October 18, 2018
Dr. Holder

1.
a.
P(Sea=blue, Sun=true, Sky=clear) is given as 0.12

b.
P(Sky=clear) Sum the probabilities that the sky is clear over all four possible ways it can happen:
0.12 + 0.07 + 0.06 + 0.08 = 0.33
P(Sky=clear) = 0.33

c.
P(Sun=true) Sum the probabilities that the Sun=true over all six possible ways it can happen:
0.12 + 0.07 + 0.05 + 0.06 + 0.07 + 0.08 = 0.45
P(Sun=true) = 0.45

d.
P(Sun=true | Sky=clear)
P(A|B) = P(A^B) / P(B)
P(Sun=true | Sky=clear) = P(Sun=true ^ Sky=clear) / P(Sky=clear)
P(Sun=true) = 0.45
P(Sky=clear) = 0.33
P(Sun=true ^ Sky=clear) = sum of all possible ways this combination can occur:
    0.12 + 0.06 = 0.18
0.18 / 0.33 = 0.545 ≈ 0.55
P(Sun=true | Sky=clear) = 0.55

e.
P(Sea=blue| Sky=clear v Sky=cloudy)
P(Sea=blue) = sum of all possible ways this can occur:
0.12 + 0.07 + 0.07 + 0.10 + 0.05 + 0.09 = 0.5
P(Sky=clear v Sky=cloudy) = sum of all possible ways the sky can be clear or cloudy:
P(Sky=clear): 0.12 + 0.07 + 0.06 + 0.08 = 0.33
P(Sky=cloudy): 0.07 + 0.10 + 0.07 + 0.09 = 0.33
P(Sky=clear) + P(Sky=cloudy) = 0.33 + 0.33 = 0.66
P(Sea=blue ^ (Sky=clear v Sky=cloudy)) = sum of all possibilities:
    0.12 + 0.07 + 0.07+ 0.10 = 0.36
P(Sea=blue| Sky=clear v Sky=cloudy)
    = P(Sea=blue ^ (Sky=clear v Sky=cloudy)) / P(Sky=clear v Sky=cloudy)
    = 0.36 / 0.66 = 0.545 ≈ 0.55
P(Sea=blue| Sky=clear v Sky=cloudy) = 0.55

2.
Let EatRight=true = R
Let Exercise=true = E
Let Healthy=true = H

Marcus Blaisdell
CptS 440
Homework #7
October 18, 2018
Dr. Holder

$P(H) = 0.8$
$P(R \wedge E \mid H) = 0.6$
$P(R \wedge E \mid (1 - H)) = 0.3$

By marginalization,
$P(R \wedge E) = P(R \wedge E|H)*P(H) + P(R \wedge E|(1 - H))*P(1 - H)$
$P(R \wedge E) = 0.6 * 0.8 + 0.3 * 0.2 = 0.48 + 0.06 = 0.54$
$P(R \wedge E) = 0.54$

$P(H|R \wedge E) = P(R \wedge E|H)P(H) / P(R \wedge E) = 0.6 * 0.8 / 0.54 = 0.888 \approx 0.89$
$P((1 - H)|R \wedge E) = P(R \wedge E|(1 – H))P((1 – H)) / P(R \wedge E) = 0.3 * 0.2 / 0.54 = 0.1111 \approx 0.11$

Let H' = Healthy

$P(H'|R \wedge E) =$ {0.89 H'=true
{0.11 H'=false

3.
Given:
P(Healthy=true) = 0.8
P(EatRight=true | Healthy=true) = 0.6
P(EatRight=true | Healthy=false) = 0.2
P(Exercise=true | Healthy=true) = 0.8
P(Exercise=true | Healthy=false) = 0.3

Compute:
P(Healthy | EatRight=true ^ Exercise=true)

Simplify terms:
Let Healthy be H
Let Healthy=true be H1
Let Healthy=false be H2
Let EatRight=true = R
Let Exercise=true = E

Solve as:
$P(H \mid R \wedge E) = (P(R \wedge E \mid H) * P(H)) / P(R \wedge E)$
=>
$P(H \mid R \wedge E) = [P(H1 \wedge R \wedge E) / P(R \wedge E)] + [P(H2 \wedge R \wedge E) / P(R \wedge E)]$

Since EatRight and Exercise are conditionally independent given Healthy:
$P(R \wedge E \mid H1) = P(R \mid H1) * P(E \mid H1) = 0.6 * 0.8 = 0.48$
$P(R \wedge E \mid H2) = P(R \mid H2) * P(E \mid H2) = 0.2 * 0.3 = 0.06$

Marcus Blaisdell
CptS 440
Homework #7
October 18, 2018
Dr. Holder

$P(R \wedge E \mid H1) = P(R \wedge E \wedge H1) / P(H1) \Rightarrow$ **$P(R \wedge E \wedge H1)$** $= P(R \wedge E \mid H1) * P(H1) = 0.48 * 0.8 = 0.384$

$P(R \wedge E \mid H2) = P(R \wedge E \wedge H2) / P(H2) \Rightarrow$ **$P(R \wedge E \wedge H2)$** $= P(R \wedge E \mid H2) * P(H2) = 0.06 * 0.8 = 0.096$

By Commutativity:
$P(A \wedge B \wedge C) = P(A) * P(B) * P(C) = P(C) * P(A) * P(B) = P(C \wedge A \wedge B)$
Therefore:
$P(R \wedge E \wedge H1) = P(H1 \wedge R \wedge E)$ and $P(R \wedge E \wedge H2) = P(H2 \wedge R \wedge E)$

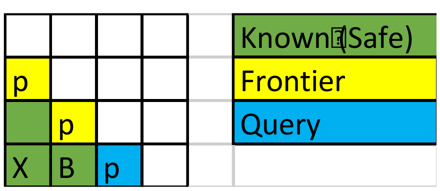
By Normalization:
$P(H1 \mid R \wedge E) = \alpha P(H1 \wedge R \wedge E) = \alpha 0.384$
$P(H2 \mid R \wedge E) = \alpha P(H2 \wedge R \wedge E) = \alpha 0.096$

And since $P(H1 \mid R \wedge E) + P(H2 \mid R \wedge E)$ must equal 1:
$\alpha 0.384 + \alpha 0.096 = 1$
$\alpha 0.48 = 1$
$\alpha = 2.08 \Rightarrow H = \{0.8, 0.2\}$

P(H | EatRight=true ^ Exercise=true) = {0.8 H = true
{0.2 H = false

4.

a.



Let B = Breeze
Let K = Known
Let F = Frontier
Let O = Other
Let p = Pit
Let P = Probability

Breeze = $\{B_{2,1}\}$

Marcus Blaisdell
CptS 440
Homework #7
October 18, 2018
Dr. Holder

Known = $\{\neg p_{1,1}, \neg p_{2,1}, \neg p_{1,2}\}$ (no breeze in 1,1 implies no pit in 1,2)
Frontier = $\{p_{1,3}, p_{2,2}, p_{3,1}\}$
Other = $\{p_{1,3}, p_{1,4}, p_{2,3}, p_{2,4}, p_{3,2}, p_{3,3}, p_{3,4}, p_{4,1}, p_{4,2}, p_{4,3}, p_{4,4}\}$

b.
compute:
$\mathbf{P}(p_{3,1} \mid B, K)$

Use the General Rule for Probabilistic Inference:

$$P(X|e) = \alpha P(X,e) = \alpha \sum_{y} P(X,e,y)$$

and the rule for conditional probability:

$$P(A|B) = \frac{P(A,B)}{P(B)}$$

so:

$$\boldsymbol{P}\left(p_{3,1}\middle|K,B\right) = \frac{\boldsymbol{P}\left(p_{3,1},K,B\right)}{\boldsymbol{P}(K,B)}$$
$$= P(p_{3,1},K,B)$$
$$= \alpha \sum_{F} \sum_{O} P(p_{3,1},K,B,F,O)$$

note:
P(a,b,c) = P(b|a,c)P(a,c) so we can rewrite with Breeze, given pit location:

$$= \alpha \sum_{F} \sum_{O} P(B|p_{3,1},K,F,O) P(p_{3,1},K,F,O)$$

A Breeze is independent of Other, given Frontier which means that P(B,O) = P(B) * P(O), that is, Other does not affect if there is a Breeze in Known, so we can rewrite with Other in just the second part of the product:

$$= \alpha \sum_{F} \sum_{O} P(B|p_{3,1},K,F) P(p_{3,1},K,F,O)$$

The Probability of a pit in any location is independent, that is, it is not affected by a pit in any other location so we can rewrite with the probabilities of the pit locations as independent variables:

$$= \alpha \sum_{F} \sum_{O} P(B|p_{3,1},K,F) P(p_{3,1}) P(K) P(F) P(O)$$

and since:

$$\sum_{O} P(O) = 1$$

It will have no affect on the probability of the equation (1 * x = x) so we can remove it from the equation:

Marcus Blaisdell
CptS 440
Homework #7
October 18, 2018
Dr. Holder

$$= \alpha \sum_F P(B|p_{3,1}, K, F)P(p_{3,1})P(K)P(F)$$

The probability of a pit in 3,1 is constant and will not affect the probability of the equation as is the known locations so they can be moved outside of the summations:

$$= \alpha P(p_{3,1})P(K) \sum_F P(B|p_{3,1}, K, F)P(F)$$

Calculate P(K):
Known = $\{\neg p_{1,1}, \neg p_{2,1}, \neg p_{1,2}\}$
$P(\neg p_{1,1}, \neg p_{2,1}, \neg p_{1,2})$
$P(\neg p_{1,1}) = 1$ since it is given that there will never be a pit in location 1,1
since it is given that the probability of a pit in any location is 0.2 as a rule of the game so the probability of no pit is 1 – 0.2 = 0.8
$P(\neg p_{2,1}) = 0.8$
$P(\neg p_{1,2}) = 0.8$
So:
$P(K) = P(\neg p_{1,1}, \neg p_{2,1}, \neg p_{1,2}) = (1 * 0.8 * 0.8) = 0.64$

And:
$P(p_{3,1}) = 0.2$

Calculate P(F) as $P(B|p_{3,1}, K,F)$ (iterate all possible P(F)):
Frontier = $\{p_{1,3}, p_{2,2}, p_{3,1}\}$
The query is whether or not there is a pit in 3,1 so remove it from the frontier:
If there is a pit in 3,1, then the possibility of a pit in 2,2 and 1,3 are valid for all possible scenarios so we need to articulate all of them:

P_{13}	P_{22}	
T	T	0.2 * 0.2 = 0.04
T	F	0.2 * 0.8 = 0.16
F	T	0.8 * 0.2 = 0.16
F	F	0.8 * 0.8 = 0.64

If there is not a pit in 3,1, then there must be a pit in 2,2 given there is a breeze in 2,1 and it is known that there is no pit in 1,1:

$$= \alpha' < P(p_{3,1}[P\big(B\big|p_{3,1}, K, p_{2,2}, p_{1,3}\big)P\big(p_{2,2}\big)P\big(p_{1,3}\big) + P\big(B\big|p_{3,1}, K, \neg p_{2,2}, p_{1,3}\big)P\big(\neg p_{2,2}, p_{1,3}\big)$$
$$+ P\big(B\big|p_{3,1}, K, p_{2,2}, \neg p_{1,3}\big)P\big(p_{2,2}, \neg p_{1,3}\big)$$
$$+ P\big(B\big|p_{3,1}, K, \neg p_{2,2}, \neg p_{1,3}\big)P\big(\neg p_{2,2}, \neg p_{1,3}\big)\big], P(\neg p_{3,1})[P(B|\neg p_{3,1}, K, p_{2,2})(p_{2,2})$$
$$+ P(\neg p_{3,1})[P(B|\neg p_{3,1}, K, \neg p_{2,2})(\neg p_{2,2})] >$$

=<0.2 * (0.2((0.2)(0.2) + (0.2)(0.8) + (0.8)(0.2) + (0.8)(0.8)), 0.8(0.2 + 0.8))>
=<0.04, 0.16>
must sum to 1 so:
$\alpha 0.04 + \alpha 0.16 = 1$
$\alpha 0.2 = 1$
$\alpha = 5$

$=\langle 0.2, 0.8\rangle$

so:

$\mathbf{P}(p_{3,1} \mid B, K) =$ { 0.2 there is a pit in 3,1
{ 0.8 there is not a pit in 3,1

The knowledge base did not provide enough new information to reduce or increase the probability of a pit in location 3,1