

1.

- a. $\text{Color}(\text{Sky}, \text{Blue}) \Rightarrow \text{Shining}(\text{Sun})$
- b. $\text{Color}(\text{Sky}, \text{Gray}) \Rightarrow \neg \text{Shining}(\text{Sun})$
- c. $(\neg \text{Color}(\text{Sky}, \text{Blue}) \wedge \neg \text{Color}(\text{Sky}, \text{Gray})) \Rightarrow (\neg \text{Shining}(\text{Sun}) \wedge \text{Shining}(\text{Moon}))$

2.

- i. $\text{Cloudy}(\text{Sky}) \Rightarrow \text{Color}(\text{Sky}, \text{Gray})$
- ii. $\text{Color}(\text{Sky}, \text{Gray}) \Rightarrow \neg \text{Shining}(\text{Moon})$
- iii. $\text{Shining}(\text{Moon}) \Leftrightarrow \neg \text{DayTime}$
- iv. $\text{Cloudy}(\text{Sky})$

a.

Step 1: Eliminate Implications:

- i. $\neg \text{Cloudy}(\text{Sky}) \vee \text{Color}(\text{Sky}, \text{Gray})$
- ii. $\neg \text{Color}(\text{Sky}, \text{Gray}) \vee \neg \text{Shining}(\text{Moon})$
- iii. $(\text{Shining}(\text{Moon}) \vee \text{DayTime}) \wedge (\neg \text{Shining}(\text{Moon}) \vee \neg \text{DayTime})$
- iv. $\text{Cloudy}(\text{Sky})$

Step 2: Move \neg inwards: (no action required)

- R1: $\neg \text{Cloudy}(\text{Sky}) \vee \text{Color}(\text{Sky}, \text{Gray})$
- R2: $\neg \text{Color}(\text{Sky}, \text{Gray}) \vee \neg \text{Shining}(\text{Moon})$
- R3: $(\text{Shining}(\text{Moon}) \vee \text{DayTime}) \wedge (\neg \text{Shining}(\text{Moon}) \vee \neg \text{DayTime})$
- R4: $\text{Cloudy}(\text{Sky})$

b.

Proof by refutation. Insert the negation of what we want to prove into the knowledge base and see if it results in a contradiction:

- R1: $\neg \text{Cloudy}(\text{Sky}) \vee \text{Color}(\text{Sky}, \text{Gray})$
- R2: $\neg \text{Color}(\text{Sky}, \text{Gray}) \vee \neg \text{Shining}(\text{Moon})$
- R3: $(\text{Shining}(\text{Moon}) \vee \text{DayTime}) \wedge (\neg \text{Shining}(\text{Moon}) \vee \neg \text{DayTime})$
- R4: $\text{Cloudy}(\text{Sky})$
- R5: $\neg \text{DayTime}$

R1: $\neg \text{Cloudy}(\text{Sky}) \vee \text{Color}(\text{Sky}, \text{Gray})$

R4: $\text{Cloudy}(\text{Sky})$

R6: $\text{Color}(\text{Sky}, \text{Gray})$

R2: $\neg \text{Color}(\text{Sky}, \text{Gray}) \vee \neg \text{Shining}(\text{Moon})$

R6: $\text{Color}(\text{Sky}, \text{Gray})$

R7: $\neg \text{Shining}(\text{Moon})$

R3: $(\text{Shining}(\text{Moon}) \vee \text{DayTime}) \wedge (\neg \text{Shining}(\text{Moon}) \vee \neg \text{DayTime})$

And elimination:

R8: $(\text{Shining}(\text{Moon}) \vee \text{DayTime})$

R8: $(\text{Shining}(\text{Moon}) \vee \text{DayTime})$

R7: $\neg \text{Shining}(\text{Moon})$

R9: DayTime

R9: DayTime conflicts with R5: $\neg \text{DayTime}$ therefore, DayTime

□

3.

- a. $\exists t \text{ Shining}(\text{Sun}, t) \wedge \text{Shining}(\text{Moon}, t)$
- b. $\forall t \text{ Color}(\text{Sea}, \text{Blue}) \Rightarrow \text{Shining}(\text{Sun}, t)$
- c. $\forall t \text{ DayTime}(t) \Rightarrow (\text{Color}(\text{Sky}, \text{Blue}) \vee \text{Color}(\text{Sky}, \text{Gray}))$
- d. $\forall t \text{ Shining}(\text{Moon}, t) \Rightarrow \text{NightTime}(t)$

4.

- i. $\forall t \text{ Shining}(\text{Sun}, t) \Rightarrow \text{Color}(\text{Sky}, \text{Blue}, t)$
- ii. $\forall t (\text{Color}(\text{Sea}, \text{Gray}, t) \vee \text{Color}(\text{Sea}, \text{Blue}, t)) \Rightarrow \neg \text{NightTime}(t)$
- iii. $\forall c, t \text{ Color}(\text{Sea}, c, t) \Leftrightarrow \text{Color}(\text{Sky}, c, t)$
- iv. $\forall t \neg \text{DayTime}(t) \Rightarrow \text{NightTime}(t)$
- v. $\exists t \text{ Shining}(\text{Sun}, t)$

a. Convert to Conjunctive Normal Form

Step 1: Eliminate Implications:

- i. $\forall t \neg \text{Shining}(\text{Sun}, t) \vee \text{Color}(\text{Sky}, \text{Blue}, t)$
- ii. $\forall t \neg (\text{Color}(\text{Sea}, \text{Gray}, t) \vee \text{Color}(\text{Sea}, \text{Blue}, t)) \vee \neg \text{NightTime}(t)$
- iii. $\forall c, t (\text{Color}(\text{Sea}, c, t) \vee \neg \text{Color}(\text{Sky}, c, t)) \wedge (\neg \text{Color}(\text{Sea}, c, t) \vee \text{Color}(\text{Sky}, c, t))$
- iv. $\forall t \text{ DayTime}(t) \vee \text{NightTime}(t)$
- v. $\exists t \text{ Shining}(\text{Sun}, t)$

Step 2: Move \neg inwards

- i. $\forall t \neg \text{Shining}(\text{Sun}, t) \vee \text{Color}(\text{Sky}, \text{Blue}, t)$

- ii. $\forall t (\neg \text{Color}(\text{Sea}, \text{Gray}, t) \wedge \neg \text{Color}(\text{Sea}, \text{Blue}, t)) \vee \neg \text{NightTime}(t)$
- iii. $\forall c, t (\text{Color}(\text{Sea}, c, t) \vee \neg \text{Color}(\text{Sky}, c, t)) \wedge (\neg \text{Color}(\text{Sea}, c, t) \vee \text{Color}(\text{Sky}, c, t))$
- iv. $\forall t \text{DayTime}(t) \vee \text{NightTime}(t)$
- v. $\exists t \text{Shining}(\text{Sun}, t)$

Step 3: Standardize variables:

- i. $\forall t_1 \neg \text{Shining}(\text{Sun}_1, t_1) \vee \text{Color}(\text{Sky}_2, \text{Blue}_2, t_2)$
- ii. $\forall t_1 (\neg \text{Color}(\text{Sea}_1, \text{Gray}_1, t_1) \wedge \neg \text{Color}(\text{Sea}_2, \text{Blue}_2, t_2)) \vee \neg \text{NightTime}(t_3)$
- iii. $\forall c_1, t_1 (\text{Color}(\text{Sea}_1, c_1, t_1) \vee \neg \text{Color}(\text{Sky}_2, c_2, t_2)) \wedge (\neg \text{Color}(\text{Sea}_3, c_3, t_3) \vee \text{Color}(\text{Sky}_4, c_4, t_4))$
- iv. $\forall t_1 \text{DayTime}(t_1) \vee \text{NightTime}(t_2)$
- v. $\exists t_1 \text{Shining}(\text{Sun}_1, t_1)$

Step 4: Skolemize:

- i. $\forall t_1 \neg \text{Shining}(\text{Sun}_1, t_1) \vee \text{Color}(\text{Sky}_2, \text{Blue}_2, t_2)$
- ii. $\forall t_1 (\neg \text{Color}(\text{Sea}_1, \text{Gray}_1, t_1) \wedge \neg \text{Color}(\text{Sea}_2, \text{Blue}_2, t_2)) \vee \neg \text{NightTime}(t_3)$
- iii. $\forall c_1, t_1 (\text{Color}(\text{Sea}_1, c_1, t_1) \vee \neg \text{Color}(\text{Sky}_2, c_2, t_2)) \wedge (\neg \text{Color}(\text{Sea}_3, c_3, t_3) \vee \text{Color}(\text{Sky}_4, c_4, t_4))$
- iv. $\forall t_1 \text{DayTime}(t_1) \vee \text{NightTime}(t_2)$
- v. $\text{Shining}(\text{Sun}_1, F(\text{Sun}))$

Step 5: Drop Universal Quantifiers:

- i. $\neg \text{Shining}(\text{Sun}_1, t_1) \vee \text{Color}(\text{Sky}_2, \text{Blue}_2, t_2)$
- ii. $(\neg \text{Color}(\text{Sea}_1, \text{Gray}_1, t_1) \wedge \neg \text{Color}(\text{Sea}_2, \text{Blue}_2, t_2)) \vee \neg \text{NightTime}(t_3)$
- iii. $(\text{Color}(\text{Sea}_1, c_1, t_1) \vee \neg \text{Color}(\text{Sky}_2, c_2, t_2)) \wedge (\neg \text{Color}(\text{Sea}_3, c_3, t_3) \vee \text{Color}(\text{Sky}_4, c_4, t_4))$
- iv. $\text{DayTime}(t_1) \vee \text{NightTime}(t_2)$
- v. $\text{Shining}(\text{Sun}_1, F(\text{Sun}))$

Step 6: Distribute \vee over \wedge

- i. $\neg \text{Shining}(\text{Sun}_1, t_1) \vee \text{Color}(\text{Sky}_2, \text{Blue}_2, t_2)$
- ii. $((\neg \text{Color}(\text{Sea}_1, \text{Gray}_1, t_1) \vee \neg \text{NightTime}(t_3)) \wedge (\neg \text{Color}(\text{Sea}_2, \text{Blue}_2, t_2) \vee \neg \text{NightTime}(t_3)))$
- iii. $(\text{Color}(\text{Sea}_1, c_1, t_1) \vee \neg \text{Color}(\text{Sky}_2, c_2, t_2)) \wedge (\neg \text{Color}(\text{Sea}_3, c_3, t_3) \vee \text{Color}(\text{Sky}_4, c_4, t_4))$
- iv. $\text{DayTime}(t_1) \vee \text{NightTime}(t_2)$
- v. $\text{Shining}(\text{Sun}_1, F(\text{Sun}))$

b. Show a resolution proof by refutation that $\exists t \text{DayTime}(t)$ is true.

Add the negation of what we want to prove to the knowledge base and test if it results in a contradiction:

- R1: $\neg \text{Shining}(\text{Sun}_1, t_1) \vee \text{Color}(\text{Sky}_2, \text{Blue}_2, t_2)$
- R2: $((\neg \text{Color}(\text{Sea}_1, \text{Gray}_1, t_1) \vee \neg \text{NightTime}(t_3)) \wedge (\neg \text{Color}(\text{Sea}_2, \text{Blue}_2, t_2) \vee \neg \text{NightTime}(t_3)))$

R3: $(\text{Color}(\text{Sea}_1, c_1, t_1) \vee \neg \text{Color}(\text{Sky}_2, c_2, t_2)) \wedge (\neg \text{Color}(\text{Sea}_3, c_3, t_3) \vee \text{Color}(\text{Sky}_4, c_4, t_4))$

R4: $\text{DayTime}(t_1) \vee \text{NightTime}(t_2)$

R5: Shining (Sun₁, F1)

R6: $\neg \exists t \text{ DayTime}(t)$ becomes: $\neg \text{DayTime}(F(\text{Sun}))$

R4: $\text{DayTime}(t_1) \vee \text{NightTime}(t_2)$

R6: $\neg \text{DayTime}(F(\text{Sun}))$

R7: $\text{NightTime}(t_2)$

R2: $((\neg \text{Color}(\text{Sea}_1, \text{Gray}_1, t_1) \vee \neg \text{NightTime}(t_3)) \wedge (\neg \text{Color}(\text{Sea}_2, \text{Blue}_2, t_2) \vee \neg \text{NightTime}(t_3)))$

And Elimination:

R8: $\neg \text{Color}(\text{Sea}_2, \text{Blue}_2, t_2) \vee \neg \text{NightTime}(t_3)$

R8: $\neg \text{Color}(\text{Sea}_2, \text{Blue}_2, t_2) \vee \neg \text{NightTime}(t_3)$

Commutativity:

R9: $\neg \text{NightTime}(t_3) \vee \neg \text{Color}(\text{Sea}_2, \text{Blue}_2, t_2)$

R9: $\neg \text{NightTime}(t_3) \vee \neg \text{Color}(\text{Sea}_2, \text{Blue}_2, t_2)$

R7: $\text{NightTime}(t_2)$

R10: $\neg \text{Color}(\text{Sea}_2, \text{Blue}_2, t_2)$

R1: $\neg \text{Shining}(\text{Sun}_1, t_1) \vee \text{Color}(\text{Sky}_2, \text{Blue}_2, t_2)$

Commutativity:

R11: $\text{Color}(\text{Sky}_2, \text{Blue}_2, t_2) \vee \neg \text{Shining}(\text{Sun}_1, t_1)$

R11: $\text{Color}(\text{Sky}_2, \text{Blue}_2, t_2) \vee \neg \text{Shining}(\text{Sun}_1, t_1)$

R10: $\neg \text{Color}(\text{Sea}_2, \text{Blue}_2, t_2)$

R12: $\neg \text{Shining}(\text{Sun}_1, t_1)$ conflicts with R5: Shining (Sun₁, F1) therefore $\exists t \text{ DayTime}(t)$ is true

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