1.
a.

$$Entropy(S) \equiv -p_+ \log_2 p_+ - p_- \log_2 p_-$$

S = {7 yes(+), 3 no(-)}
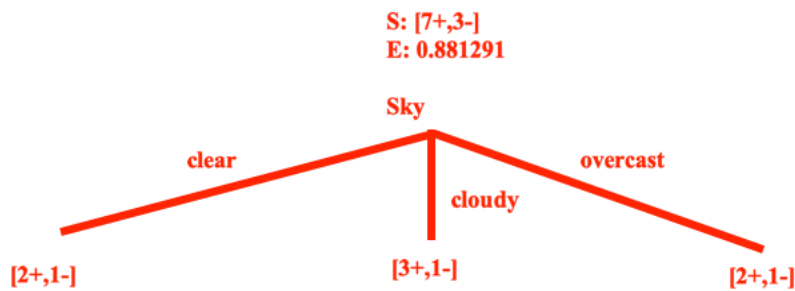Total = 10

$$Entropy(S) \equiv -\frac{7}{10}\log_2\frac{7}{10} - \frac{3}{10}\log_2\frac{3}{10} = 0.881291$$

Entropy(S) = 0.881291

b.

$$Gain(S, A) \equiv Entropy(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

S: [7+,3-]
E: 0.881291

Sky

clear          overcast

cloudy

[2+,1-]          [3+,1-]          [2+,1-]

Entropy(S) = 0.881291
Entropy(S$_{clear}$) = $-\frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3} = 0.918296$
Entropy(S$_{cloudy}$) = $-\frac{3}{4}\log_2\frac{3}{4} - \frac{1}{4}\log_2\frac{1}{4} = 0.811278$
Entropy(S$_{overcast}$) = $-\frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3} = 0.918296$

$$Gain(S, A) \equiv 0.881291 - \left(\left(\frac{3}{10}\right)(0.918296) + \left(\frac{4}{10}\right)(0.811278) + \left(\frac{3}{10}\right)(0.918296)\right)$$
$$= 0.005802$$

Information gain using Sky as root = 0.005802

c.

S: [7+,3-]
E: 0.881291

Sea

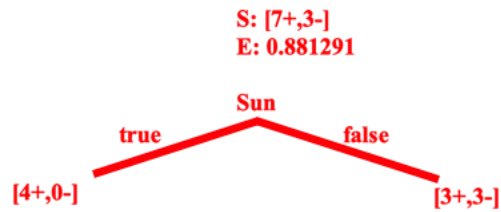blue          gray

[5+,0-]          [2+,3-]

$$\text{Entropy}(S_{\text{blue}}) = -\frac{5}{5}\log_2\frac{5}{5} - \frac{0}{5}\log_2\frac{0}{5} = 0$$

$$\text{Entropy}(S_{\text{gray}}) = -\frac{2}{5}\log_2\frac{2}{5} - \frac{3}{5}\log_2\frac{3}{5} = 0.970951$$

$$Gain(S,A) \equiv 0.881291 - \left(\left(\frac{5}{10}\right)(0) + \left(\frac{5}{10}\right)(0.970951)\right) = 0.395816$$

Information gain using Sea as root = 0.395816

d.

**S: [7+,3-]**
**E: 0.881291**

**Sun**

true          false
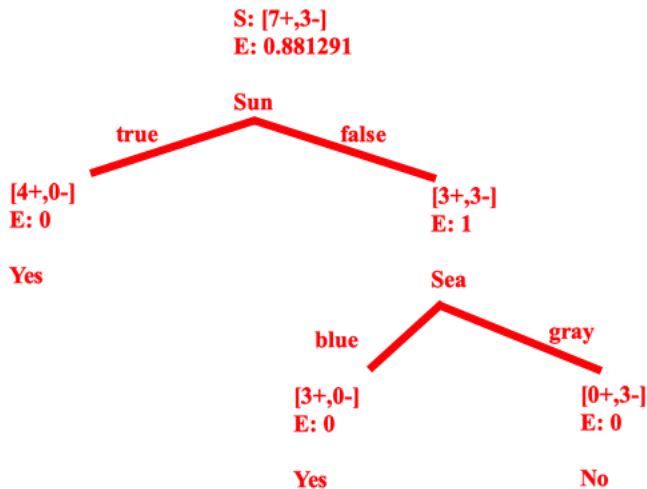
**[4+,0-]**          **[3+,3-]**

$$\text{Entropy}(S_{\text{true}}) = -\frac{4}{4}\log_2\frac{4}{4} - \frac{0}{4}\log_2\frac{0}{4} = 0$$

$$\text{Entropy}(S_{\text{false}}) = -\frac{3}{6}\log_2\frac{3}{6} - \frac{3}{6}\log_2\frac{3}{3} = 0$$

$$Gain(S,A) \equiv 0.881291 - \left(\left(\frac{4}{10}\right)(0) + \left(\frac{6}{10}\right)(0)\right) = 0.881291$$

Information gain using Sun as root = 0.881291

e.

**S: [7+,3-]**
**E: 0.881291**

**Sun**

true          false

**[4+,0-]**          **[3+,3-]**
**E: 0**            **E: 1**

**Yes**

**Sea**

blue          gray

**[3+,0-]**          **[0+,3-]**
**E: 0**            **E: 0**

**Yes**          **No**

f.
Sail = yes

2.
a.

| Sky | Sea | Sun | Sail |
|-----|-----|-----|------|
| 1 | 1 | 1 | 1 |
| 1 | 2 | 2 | 1 |
| 1 | 2 | 1 | 0 |
| 2 | 1 | 2 | 1 |
| 2 | 1 | 1 | 1 |
| 2 | 2 | 2 | 1 |
| 2 | 2 | 1 | 0 |
| 3 | 1 | 2 | 1 |
| 3 | 1 | 1 | 1 |
| 3 | 2 | 1 | 0 |

b.
Initial weights: <1.0, 1.0, 1.0, 1.0>

Update rule:
$w_i <- w_i + \alpha(y - h_w(X)) * x_i$

where $h_w(X) = $ {1 if w.x $\geq$ 0
        {0 otherwise

 ==pass:  1==
weight 1 :  [1.0, 1.0, 1.0, 1.0]
h 1 ( [1.0, 1.0, 1.0, 1.0] ) = [1.0, 1.0, 1.0, 1.0] * [1.0, 1.0, 1.0, 1.0] = 4.0 = 1.0
weight 2 = [1.0, 1.0, 1.0, 1.0] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 1.0, 1.0, 1.0] = [1.0, 1.0, 1.0, 1.0]
no update

weight 2 :  [1.0, 1.0, 1.0, 1.0]
h 2 ( [1.0, 1.0, 2.0, 2.0] ) = [1.0, 1.0, 1.0, 1.0] * [1.0, 1.0, 2.0, 2.0] = 6.0 = 1.0
weight 3 = [1.0, 1.0, 1.0, 1.0] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 1.0, 2.0, 2.0] = [1.0, 1.0, 1.0, 1.0]
no update

weight 3 :  [1.0, 1.0, 1.0, 1.0]
h 3 ( [1.0, 1.0, 2.0, 1.0] ) = [1.0, 1.0, 1.0, 1.0] * [1.0, 1.0, 2.0, 1.0] = 5.0 = 1.0
weight 4 = [1.0, 1.0, 1.0, 1.0] + 0.5 * ( 0.0 - 1.0 ) * [1.0, 1.0, 2.0, 1.0] = [0.5, 0.5, 0.0, 0.5]
updated

weight 4 :  [0.5, 0.5, 0.0, 0.5]
h 4 ( [1.0, 2.0, 1.0, 2.0] ) = [0.5, 0.5, 0.0, 0.5] * [1.0, 2.0, 1.0, 2.0] = 2.5 = 1.0
weight 5 = [0.5, 0.5, 0.0, 0.5] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 2.0, 1.0, 2.0] = [0.5, 0.5, 0.0, 0.5]
no update

weight 5 :  [0.5, 0.5, 0.0, 0.5]
h 5 ( [1.0, 2.0, 1.0, 1.0] ) = [0.5, 0.5, 0.0, 0.5] * [1.0, 2.0, 1.0, 1.0] = 2.0 = 1.0
weight 6 = [0.5, 0.5, 0.0, 0.5] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 2.0, 1.0, 1.0] = [0.5, 0.5, 0.0, 0.5]
no update

weight 6 :  [0.5, 0.5, 0.0, 0.5]
h 6 ( [1.0, 2.0, 2.0, 2.0] ) = [0.5, 0.5, 0.0, 0.5] * [1.0, 2.0, 2.0, 2.0] = 2.5 = 1.0
weight 7 = [0.5, 0.5, 0.0, 0.5] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 2.0, 2.0, 2.0] = [0.5, 0.5, 0.0, 0.5]
no update

weight 7 :  [0.5, 0.5, 0.0, 0.5]
h 7 ( [1.0, 2.0, 2.0, 1.0] ) = [0.5, 0.5, 0.0, 0.5] * [1.0, 2.0, 2.0, 1.0] = 2.0 = 1.0
weight 8 = [0.5, 0.5, 0.0, 0.5] + 0.5 * ( 0.0 - 1.0 ) * [1.0, 2.0, 2.0, 1.0] = [0.0, -0.5, -1.0, 0.0]
updated

weight 8 :  [0.0, -0.5, -1.0, 0.0]
h 8 ( [1.0, 3.0, 1.0, 2.0] ) = [0.0, -0.5, -1.0, 0.0] * [1.0, 3.0, 1.0, 2.0] = -2.5 = 0.0
weight 9 = [0.0, -0.5, -1.0, 0.0] + 0.5 * ( 1.0 - 0.0 ) * [1.0, 3.0, 1.0, 2.0] = [0.5, 1.0, -0.5, 1.0]
updated

weight 9 :  [0.5, 1.0, -0.5, 1.0]
h 9 ( [1.0, 3.0, 1.0, 1.0] ) = [0.5, 1.0, -0.5, 1.0] * [1.0, 3.0, 1.0, 1.0] = 4.0 = 1.0
weight 10 = [0.5, 1.0, -0.5, 1.0] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 3.0, 1.0, 1.0] = [0.5, 1.0, -0.5, 1.0]
no update

weight 10 :  [0.5, 1.0, -0.5, 1.0]
h 10 ( [1.0, 3.0, 2.0, 1.0] ) = [0.5, 1.0, -0.5, 1.0] * [1.0, 3.0, 2.0, 1.0] = 3.5 = 1.0
weight 11 = [0.5, 1.0, -0.5, 1.0] + 0.5 * ( 0.0 - 1.0 ) * [1.0, 3.0, 2.0, 1.0] = [0.0, -0.5, -1.5, 0.5]
updated

pass:  2
weight 11 :  [0.0, -0.5, -1.5, 0.5]
h 1 ( [1.0, 1.0, 1.0, 1.0] ) = [0.0, -0.5, -1.5, 0.5] * [1.0, 1.0, 1.0, 1.0] = -1.5 = 0.0
weight 12 = [0.0, -0.5, -1.5, 0.5] + 0.5 * ( 1.0 - 0.0 ) * [1.0, 1.0, 1.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
updated

weight 12 :  [0.5, 0.0, -1.0, 1.0]
h 2 ( [1.0, 1.0, 2.0, 2.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 1.0, 2.0, 2.0] = 0.5 = 1.0
weight 13 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 1.0, 2.0, 2.0] = [0.5, 0.0, -1.0, 1.0]
no update

weight 13 :  [0.5, 0.0, -1.0, 1.0]
h 3 ( [1.0, 1.0, 2.0, 1.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 1.0, 2.0, 1.0] = -0.5 = 0.0

weight 14 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 0.0 - 0.0 ) * [1.0, 1.0, 2.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

weight 14 : [0.5, 0.0, -1.0, 1.0]
h 4 ( [1.0, 2.0, 1.0, 2.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 2.0, 1.0, 2.0] = 1.5 = 1.0
weight 15 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 2.0, 1.0, 2.0] = [0.5, 0.0, -1.0, 1.0]
no update

weight 15 : [0.5, 0.0, -1.0, 1.0]
h 5 ( [1.0, 2.0, 1.0, 1.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 2.0, 1.0, 1.0] = 0.5 = 1.0
weight 16 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 2.0, 1.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

weight 16 : [0.5, 0.0, -1.0, 1.0]
h 6 ( [1.0, 2.0, 2.0, 2.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 2.0, 2.0, 2.0] = 0.5 = 1.0
weight 17 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 2.0, 2.0, 2.0] = [0.5, 0.0, -1.0, 1.0]
no update

weight 17 : [0.5, 0.0, -1.0, 1.0]
h 7 ( [1.0, 2.0, 2.0, 1.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 2.0, 2.0, 1.0] = -0.5 = 0.0
weight 18 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 0.0 - 0.0 ) * [1.0, 2.0, 2.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

weight 18 : [0.5, 0.0, -1.0, 1.0]
h 8 ( [1.0, 3.0, 1.0, 2.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 3.0, 1.0, 2.0] = 1.5 = 1.0
weight 19 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 3.0, 1.0, 2.0] = [0.5, 0.0, -1.0, 1.0]
no update

weight 19 : [0.5, 0.0, -1.0, 1.0]
h 9 ( [1.0, 3.0, 1.0, 1.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 3.0, 1.0, 1.0] = 0.5 = 1.0
weight 20 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 3.0, 1.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

weight 20 : [0.5, 0.0, -1.0, 1.0]
h 10 ( [1.0, 3.0, 2.0, 1.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 3.0, 2.0, 1.0] = -0.5 = 0.0
weight 21 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 0.0 - 0.0 ) * [1.0, 3.0, 2.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

pass: 3
weight 21 : [0.5, 0.0, -1.0, 1.0]
h 1 ( [1.0, 1.0, 1.0, 1.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 1.0, 1.0, 1.0] = 0.5 = 1.0
weight 22 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 1.0, 1.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

weight 22 : [0.5, 0.0, -1.0, 1.0]
h 2 ( [1.0, 1.0, 2.0, 2.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 1.0, 2.0, 2.0] = 0.5 = 1.0
weight 23 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 1.0, 2.0, 2.0] = [0.5, 0.0, -1.0, 1.0]
no update

weight 23 : [0.5, 0.0, -1.0, 1.0]
h 3 ( [1.0, 1.0, 2.0, 1.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 1.0, 2.0, 1.0] = -0.5 = 0.0
weight 24 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 0.0 - 0.0 ) * [1.0, 1.0, 2.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

weight 24 : [0.5, 0.0, -1.0, 1.0]
h 4 ( [1.0, 2.0, 1.0, 2.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 2.0, 1.0, 2.0] = 1.5 = 1.0
weight 25 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 2.0, 1.0, 2.0] = [0.5, 0.0, -1.0, 1.0]
no update

weight 25 : [0.5, 0.0, -1.0, 1.0]
h 5 ( [1.0, 2.0, 1.0, 1.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 2.0, 1.0, 1.0] = 0.5 = 1.0
weight 26 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 2.0, 1.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

weight 26 : [0.5, 0.0, -1.0, 1.0]
h 6 ( [1.0, 2.0, 2.0, 2.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 2.0, 2.0, 2.0] = 0.5 = 1.0
weight 27 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 2.0, 2.0, 2.0] = [0.5, 0.0, -1.0, 1.0]
no update

weight 27 : [0.5, 0.0, -1.0, 1.0]
h 7 ( [1.0, 2.0, 2.0, 1.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 2.0, 2.0, 1.0] = -0.5 = 0.0
weight 28 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 0.0 - 0.0 ) * [1.0, 2.0, 2.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

weight 28 : [0.5, 0.0, -1.0, 1.0]
h 8 ( [1.0, 3.0, 1.0, 2.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 3.0, 1.0, 2.0] = 1.5 = 1.0
weight 29 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 3.0, 1.0, 2.0] = [0.5, 0.0, -1.0, 1.0]
no update

weight 29 : [0.5, 0.0, -1.0, 1.0]
h 9 ( [1.0, 3.0, 1.0, 1.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 3.0, 1.0, 1.0] = 0.5 = 1.0
weight 30 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 1.0 - 1.0 ) * [1.0, 3.0, 1.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

weight 30 : [0.5, 0.0, -1.0, 1.0]
h 10 ( [1.0, 3.0, 2.0, 1.0] ) = [0.5, 0.0, -1.0, 1.0] * [1.0, 3.0, 2.0, 1.0] = -0.5 = 0.0
weight 31 = [0.5, 0.0, -1.0, 1.0] + 0.5 * ( 0.0 - 0.0 ) * [1.0, 3.0, 2.0, 1.0] = [0.5, 0.0, -1.0, 1.0]
no update

c.
<Sky=overcast, Sea=gray, Sun=true> = (3.0, 2.0, 2.0) => (1.0, 3.0, 2.0, 2.0)

weight = (0.5, 0.0, -1.0, 1.0)

classification: $h_w(X)$ = {1 if w.x ≥ 0
                   {0 otherwise


w.x = (0.5, 0.0, -1.0, 1.0) . (1.0, 3.0, 2.0, 2.0) = 0.5 + 0 + (-2.0) + (2.0) = 0.5

0.5 ≥ 0 therefore the classification is 1 which is sail = yes.

The learned perceptron will classify <Sky=overcast, Sea=gray, Sun=true> as sail = yes.