1.

a.

P(Sea=blue, Sun=true, Sky=clear) is given as 0.12

b.

P(Sky=clear) Sum the probabilities that the sky is clear over all four possible ways it can happen:

0.12 + 0.07 + 0.06 + 0.08 = 0.33

P(Sky=clear) = 0.33

c.

P(Sun=true) Sum the probabilities that the Sun=true over all six possible ways it can happen:

0.12 + 0.07 + 0.05 + 0.06 + 0.07 + 0.08 = 0.45

P(Sun=true) = 0.45

d.

P(Sun=true | Sky=clear)

P(A|B) = P(A^B) / P(B)

P(Sun=true | Sky=clear) = P(Sun=true ^ Sky=clear) / P(Sky=clear)

P(Sun=true) = 0.45

P(Sky=clear) = 0.33

P(Sun=true ^ Sky=clear) = sum of all possible ways this combination can occur:

      0.12 + 0.06 = 0.18

0.18 / 0.33 = 0.545 ≈ 0.55

P(Sun=true | Sky=clear) = 0.55

e.

P(Sea=blue| Sky=clear v Sky=cloudy)

P(Sea=blue) = sum of all possible ways this can occur:

0.12 + 0.07 + 0.07 + 0.10 + 0.05 + 0.09 = 0.5

P(Sky=clear v Sky=cloudy) = sum of all possible ways the sky can be clear or cloudy:

P(Sky=clear): 0.12 + 0.07 + 0.06 + 0.08 = 0.33

P(Sky=cloudy): 0.07 + 0.10 + 0.07 + 0.09 = 0.33

P(Sky=clear) + P(Sky=cloudy) = 0.33 + 0.33 = 0.66

P(Sea=blue ^ (Sky=clear v Sky=cloudy)) = sum of all possibilities:

      0.12 + 0.07 + 0.07+ 0.10 = 0.36

P(Sea=blue| Sky=clear v Sky=cloudy)

      = P(Sea=blue ^ (Sky=clear v Sky=cloudy)) / P(Sky=clear v Sky=cloudy)

      = 0.36 / 0.66 = 0.545 ≈ 0.55

P(Sea=blue| Sky=clear v Sky=cloudy) = 0.55

2.

Let EatRight=true = R

Let Exercise=true = E

Let Healthy=true = H

P(H) = 0.8
P(R ^ E | H) = 0.6
P(R ^ E | (1 - H)) = 0.3

By marginalization,
P(R^E) = P(R^E|H)*P(H) + P(R^E|(1 - H))*P(1 - H)
P(R^E) = 0.6 * 0.8 + 0.3 * 0.2 = 0.48 + 0.06 = 0.54
P(R^E) = 0.54

P(H|R^E) = P(R^E|H)P(H) / P(R^E) = 0.6 * 0.8 / 0.54 = 0.888 ≈ 0.89
P((1 - H)|R^E) = P(R^E|(1 − H))P((1 − H)) / P(R^E) = 0.3 * 0.2 / 0.54 = 0.1111 ≈ 0.11

Let H' = Healthy

P(H'|R^E) =   {0.89   H'=true
                {0.11   H'=false

3.
Given:
P(Healthy=true) = 0.8
P(EatRight=true | Healthy=true) = 0.6
P(EatRight=true | Healthy=false) = 0.2
P(Exercise=true | Healthy=true) = 0.8
P(Exercise=true | Healthy=false) = 0.3

Compute:
P(Healthy | EatRight=true ^ Exercise=true)

Simplify terms:
Let Healthy be H
Let Healthy=true be H1
Let Healthy=false be H2
Let EatRight=true = R
Let Exercise=true = E

Solve as:
P(H | R ^ E) = (P(R ^ E | H) * P(H)) / P(R ^ E)
=>
P(H | R ^ E) = [P(H1 ^ R ^ E) / P(R ^ E)] + [P(H2 ^ R ^ E) / P(R ^ E)]

Since EatRight and Exercise are conditionally independent given Healthy:
P(R ^ E | H1) = P(R | H1) * P(E | H1) = 0.6 * 0.8 = 0.48
P(R ^ E | H2) = P(R | H2) * P(E | H2) = 0.2 * 0.3 = 0.06

$P(R \wedge E \mid H1) = P(R \wedge E \wedge H1) / P(H1) \Rightarrow \mathbf{P(R \wedge E \wedge H1)} = P(R \wedge E \mid H1) * P(H1) = 0.48 * 0.8 = 0.384$

$P(R \wedge E \mid H2) = P(R \wedge E \wedge H2) / P(H2) \Rightarrow \mathbf{P(R \wedge E \wedge H2)} = P(R \wedge E \mid H2) * P(H2) = 0.06 * 0.8 = 0.096$

By Commutativity:
$P(A \wedge B \wedge C) = P(A) * P(B) * P(C) = P(C) * P(A) * P(B) = P(C \wedge A \wedge B)$
Therefore:
$P(R \wedge E \wedge H1) = P(H1 \wedge R \wedge E)$ and $P(R \wedge E \wedge H2) = P(H2 \wedge R \wedge E)$

By Normalization:
$P(H1 \mid R \wedge E) = \alpha P (H1 \wedge R \wedge E) = \alpha 0.384$
$P(H2 \mid R \wedge E) = \alpha P (H2 \wedge R \wedge E) = \alpha 0.096$

And since P(H1 | R ^ E) + P(H2 | R ^ E) must equal 1:
$\alpha 0.384 + \alpha 0.096 = 1$
$\alpha 0.48 = 1$
$\alpha = 2.08 \Rightarrow H = \{0.8, 0.2\}$

$\mathbf{P}(H \mid \text{EatRight=true} \wedge \text{Exercise=true}) = \quad \{0.8\ H = \text{true}$
$\{0.2\ H = \text{false}$

4.

a.

				Known (Safe)
p				Frontier
	p			Query
X	B	p		

Let B = Breeze
Let K = Known
Let F = Frontier
Let O = Other
Let p = Pit
Let P = Probability

Breeze = $\{B_{2,1}\}$

Known = $\{\neg p_{1,1}, \neg p_{2,1}, \neg p_{1,2}\}$ (no breeze in 1,1 implies no pit in 1,2)
Frontier = $\{p_{1,3}, p_{2,2}, p_{3,1}\}$
Other = $\{p_{1,3}, p_{1,4}, p_{2,3}, p_{2,4}, p_{3,2}, p_{3,3}, p_{3,4}, p_{4,1}, p_{4,2}, p_{4,3}, p_{4,4}\}$

b.
compute:
$\mathbf{P}(p_{3,1} \mid B, K)$

Use the General Rule for Probabilistic Inference:

$$P(X|e) = \alpha P(X, e) = \alpha \sum_y P(X, e, y)$$

and the rule for conditional probability:

$$P(A|B) = \frac{P(A, B)}{P(B)}$$

so:

$$\mathbf{P}(p_{3,1}|K, B) = \frac{\mathbf{P}(p_{3,1}, K, B)}{\mathbf{P}(K, B)}$$
$$= P(p_{3,1}, K, B)$$
$$= \alpha \sum_F \sum_O P(p_{3,1}, K, B, F, O)$$

note:
$P(a,b,c) = P(b|a,c)P(a,c)$ so we can rewrite with Breeze, given pit location:

$$= \alpha \sum_F \sum_O P(B|p_{3,1}, K, F, O)P(p_{3,1}, K, F, O)$$

A Breeze is independent of Other, given Frontier which means that $P(B,O) = P(B) * P(O)$, that is, Other does not affect if there is a Breeze in Known, so we can rewrite with Other in just the second part of the product:

$$= \alpha \sum_F \sum_O P(B|p_{3,1}, K, F)P(p_{3,1}, K, F, O)$$

The Probability of a pit in any location is independent, that is, it is not affected by a pit in any other location so we can rewrite with the probabilities of the pit locations as independent variables:

$$= \alpha \sum_F \sum_O P(B|p_{3,1}, K, F)P(p_{3,1})P(K)P(F)P(O)$$

and since:

$$\sum_O P(O) = 1$$

It will have no affect on the probability of the equation (1 * x = x) so we can remove it from the equation:

$$= \alpha \sum_F P(B|p_{3,1}, K, F)P(p_{3,1})P(K)P(F)$$

The probability of a pit in 3,1 is constant and will not affect the probability of the equation as is the known locations so they can be moved to the front of the summations:

$$= \alpha P(p_{3,1})P(K) \sum_F P(B|p_{3,1}, K, F)P(F)$$

Calculate P(K):

Known = $\{\neg p_{1,1}, \neg p_{2,1}, \neg p_{1,2}\}$

$P(\neg p_{1,1}, \neg p_{2,1}, \neg p_{1,2})$

$P(\neg p_{1,1}) = 1$ since it is given that there will never be a pit in location 1,1

since it is given that the probability of a pit in any location is 0.2 as a rule of the game so the probability of no pit is $1 - 0.2 = 0.8$

$P(\neg p_{2,1}) = 0.8$

$P(\neg p_{1,2}) = 0.8$

So:

$P(K) = P(\neg p_{1,1}, \neg p_{2,1}, \neg p_{1,2}) = (1 * 0.8 * 0.8) = 0.64$

And:

$P(p_{3,1}) = 0.2$

Calculate P(F) as P(B|p_{3,1}, K,F) (iterate all possible P(F)):

Frontier = $\{p_{1,3}, p_{2,2}, p_{3,1}\}$

The query is whether or not there is a pit in 3,1 so remove it from the frontier:

If there is a pit in 3,1 and a breeze in 2, 1 and no known breeze in another location, then location 1,3 has a constant probability of 0.2 for having a pit and will not affect the probability of a pit in 2,2 or 3,1 and so does not need to be articulated in this equation, so:

$$= \alpha' < P(p_{3,1}[P(B|p_{3,1}, K, p_{2,2})P(p_{2,2})$$
$$+ P(B|p_{3,1}, K, \neg p_{2,2})P(\neg p_{2,2})], P(\neg p_{3,1})[P(B|\neg p_{3,1}, K, p_{2,2})(p_{2,2})$$
$$+ P(\neg p_{3,1})[P(B|\neg p_{3,1}, K, \neg p_{2,2})(\neg p_{2,2})] >$$

$=<0.2 * (0.2(0.2 + 0.8), 0.8(0.2 + 0.8))>$

$=<0.04, 0.16>$

must sum to 1 so:

$\alpha 0.04 + \alpha 0.16 = 1$

$\alpha 0.2 = 1$

$\alpha = 5$

$=<0.2, 0.8>$

so:

$P(p_{3,1} | B, K) =$ { 0.2 there is a pit in 3,1
                                      { 0.8 there is not a pit in 3,1

The knowledge base did not provide enough new information to reduce or increase the probability of a pit in location 3,1

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