1.

a. Color(Sky, Blue)⇒Shining(Sun)

b. Color(Sky, Gray)⇒¬Shining(Sun)

c. (¬Color(Sky, Blue) ^ ¬Color(Sky, Gray) ) ⇒(¬Shining(Sun) ^ Shining(Moon) )

2.

i. Cloudy(Sky) ⇒ Color(Sky,Gray)

ii. Color(Sky, Gray) ⇒ ¬Shining(Moon)

iii. Shining(Moon) ⇔ ¬ DayTime

iv. Cloudy(Sky)

a.

Step 1: Eliminate Implications:

i. ¬Cloudy(Sky) v Color(Sky,Gray)

ii. ¬Color(Sky, Gray) v ¬Shining(Moon)

iii. (Shining(Moon) v DayTime) ^ (¬Shining(Moon) v ¬ DayTime)

iv. Cloudy(Sky)

Step 2: Move ¬ inwards: (no action required)

R1: ¬Cloudy(Sky) v Color(Sky,Gray)

R2: ¬Color(Sky, Gray) v ¬Shining(Moon)

R3: (Shining(Moon) v DayTime) ^ (¬Shining(Moon) v ¬ DayTime)

R4: Cloudy(Sky)

b.

Proof by refutation. Insert the negation of what we want to prove into the knowledge base and see if it results in a contradiction:

R1: ¬Cloudy(Sky) v Color(Sky,Gray)

R2: ¬Color(Sky, Gray) v ¬Shining(Moon)

R3: (Shining(Moon) v DayTime) ^ (¬Shining(Moon) v ¬ DayTime)

R4: Cloudy(Sky)

R5: ¬ DayTime

R1: ¬Cloudy(Sky) v Color(Sky,Gray)

R4: Cloudy(Sky)

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R6: Color(Sky,Gray)

R2: ¬Color(Sky, Gray) v ¬Shining(Moon)

R6: Color(Sky,Gray)

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R7: ¬Shining(Moon)

R3: (Shining(Moon) v DayTime) ^ (¬Shining(Moon) v ¬ DayTime)

And elimination:

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R8: (Shining(Moon) v DayTime)

R8: (Shining(Moon) v DayTime)

R7: ¬Shining(Moon)

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R9: DayTime

R9: DayTime conflicts with R5: ¬ DayTime therefore, DayTime

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3.

a. ∃t Shining (Sun, t) ^ Shining (Moon, t)

b. ∀t Color (Sea, Blue) ⇒ Shining (Sun, t)

c. ∀t DayTime(t) ⇒ (Color(Sky, Blue) v Color(Sky, Gray))

d. ∀t Shining(Moon, t) ⇒ NightTime(t)

4.

i. ∀t Shining(Sun,t) ⇒ Color(Sky, Blue, t)

ii. ∀t (Color(Sea, Gray, t) v Color(Sea, Blue, t)) ⇒ ¬NightTime(t)

iii. ∀c,t Color(Sea, c, t) ⇔ Color(Sky, c, t)

iv. ∀t ¬DayTime(t) ⇒ NightTime(t)

v. ∃t Shining (Sun, t)

a. Convert to Conjunctive Normal Form

Step 1: Eliminate Implications:

i. ∀t ¬Shining(Sun,t) v Color(Sky, Blue, t)

ii. ∀t ¬(Color(Sea, Gray, t) v Color(Sea, Blue, t)) v ¬NightTime(t)

iii. ∀c,t (Color(Sea, c, t) v ¬Color(Sky, c, t)) ^ (¬Color(Sea, c, t) v Color(Sky, c, t))

iv. ∀t DayTime(t) v NightTime(t)

v. ∃t Shining (Sun, t)

Step 2: Move ¬ inwards

i. ∀t ¬Shining(Sun,t) v Color(Sky, Blue, t)

ii. ∀t (¬Color(Sea, Gray, t) ^ ¬Color(Sea, Blue, t)) v ¬NightTime(t)

iii. ∀c,t (Color(Sea, c, t) v ¬Color(Sky, c, t)) ^ (¬Color(Sea, c, t) v Color(Sky, c, t))

iv. ∀t DayTime(t) v NightTime(t)

v. ∃t Shining (Sun, t)

Step 3: Standardize variables:

i. ∀t1 ¬Shining(Sun1,t1) v Color(Sky2, Blue2, t2)

ii. ∀t1 (¬Color(Sea1, Gray1, t1) ^ ¬Color(Sea2, Blue2, t2)) v ¬NightTime(t3)

iii. ∀c1,t1 (Color(Sea1, c1, t1) v ¬Color(Sky2, c2, t2)) ^ (¬Color(Sea3, c3, t3) v Color(Sky4, c4, t4))

iv. ∀t1 DayTime(t1) v NightTime(t2)

v. ∃t1 Shining (Sun1, t1)

Step 4: Skolemize:

i. ∀t1 ¬Shining(Sun1,t1) v Color(Sky2, Blue2, t2)

ii. ∀t1 (¬Color(Sea1, Gray1, t1) ^ ¬Color(Sea2, Blue2, t2)) v ¬NightTime(t3)

iii. ∀c1,t1 (Color(Sea1, c1, t1) v ¬Color(Sky2, c2, t2)) ^ (¬Color(Sea3, c3, t3) v Color(Sky4, c4, t4))

iv. ∀t1 DayTime(t1) v NightTime(t2)

v. Shining (Sun1, F(Sun1))

Step 5: Drop Universal Quantifiers:

i. ¬Shining(Sun1,t1) v Color(Sky2, Blue2, t2)

ii. (¬Color(Sea1, Gray1, t1) ^ ¬Color(Sea2, Blue2, t2)) v ¬NightTime(t3)

iii. (Color(Sea1, c1, t1) v ¬Color(Sky2, c2, t2)) ^ (¬Color(Sea3, c3, t3) v Color(Sky4, c4, t4))

iv. DayTime(t1) v NightTime(t2)

v. Shining (Sun1, F(Sun1))

Step 6: Distribute v over ^

i. ¬Shining(Sun1,t1) v Color(Sky2, Blue2, t2)

ii. ((¬Color(Sea1, Gray1, t1) v ¬NightTime(t3)) ^ (¬Color(Sea2, Blue2, t2) v ¬NightTime(t3))

iii. (Color(Sea1, c1, t1) v ¬Color(Sky2, c2, t2)) ^ (¬Color(Sea3, c3, t3) v Color(Sky4, c4, t4))

iv. DayTime(t1) v NightTime(t2)

v. Shining (Sun1, F(Sun1))

b. Show a resolution proof by refutation that ∃t DayTime(t) is true.

Add the negation of what we want to prove to the knowledge base and test if it results in a contradiction:

R1: ¬Shining(Sun1,t1) v Color(Sky2, Blue2, t2)

R2: ((¬Color(Sea1, Gray1, t1) v ¬NightTime(t3)) ^ (¬Color(Sea2, Blue2, t2) v ¬NightTime(t3))

R3: (Color(Sea1, c1, t1) v ¬Color(Sky2, c2, t2)) ^ (¬Color(Sea3, c3, t3) v Color(Sky4, c4, t4))

R4: DayTime(t1) v NightTime(t2)

R5: Shining (Sun1, F(Sun1))

R6: ¬∃t DayTime(t) becomes: ¬DayTime(F(Sun1))

R5: Shining (Sun1, F(Sun1))

{t1, F(Sun1)}

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R5: Shining (Sun1, t1)

R6: ¬DayTime(F(Sun1))

{t1, F(Sun1)}

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R6: ¬DayTime(t1)

R4: DayTime(t1) v NightTime(t2)

R6: ¬DayTime(t1)

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R7: NightTime(t2)

R2: ((¬Color(Sea1, Gray1, t1) v ¬NightTime(t3)) ^ (¬Color(Sea2, Blue2, t2) v ¬NightTime(t3))

And Elimination:

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R8: ¬Color(Sea2, Blue2, t2) v ¬NightTime(t3)

R8: ¬Color(Sea2, Blue2, t2) v ¬NightTime(t3)

Commutativity:

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R9: ¬NightTime(t3) v ¬Color(Sea2, Blue2, t2)

R7: NightTime(t2)

{t3, t2}

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R7: NightTime(t3)

R9: ¬NightTime(t3) v ¬Color(Sea2, Blue2, t2)

R7: NightTime(t3)

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R10: ¬Color(Sea2, Blue2, t2)

R1: ¬Shining(Sun1,t1) v Color(Sky2, Blue2, t2)

Commutativity:

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R11: Color(Sky2, Blue2, t2) v ¬Shining(Sun1,t1)

R11: Color(Sky2, Blue2, t2) v ¬Shining(Sun1,t1)

R10: ¬Color(Sea2, Blue2, t2)

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R12: ¬Shining(Sun1,t1) conflicts with R5: Shining (Sun1, t1) therefore ∃t DayTime(t) is true

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