1.

a.

P(Sea=blue, Sun=true, Sky=clear) is given as 0.12

b.

P(Sky=clear) Sum the probabilities that the sky is clear over all four possible ways it can happen: 0.12 + 0.07 + 0.06 + 0.08 = 0.33

P(Sky=clear) = 0.33

c.

P(Sun=true) Sum the probabilities that the Sun=true over all six possible ways it can happen: 0.12 + 0.07 + 0.05 + 0.06 + 0.07 + 0.08 = 0.45

P(Sun=true) = 0.45

d.

P(Sun=true | Sky=clear)

P(A|B) = P(A^B) / P(B)

P(Sun=true | Sky=clear) = P(Sun=true ^ Sky=clear) / P(Sky=clear)

P(Sun=true) = 0.45

P(Sky=clear) = 0.33

P(Sun=true ^ Sky=clear) = sum of all possible ways this combination can occur:

0.12 + 0.06 = 0.18

0.18 / 0.33 = 0.545 ≈ 0.55

P(Sun=true | Sky=clear) = 0.55

e.

P(Sea=blue| Sky=clear v Sky=cloudy)

P(Sea=blue) = sum of all possible ways this can occur:

0.12 + 0.07 + 0.07 + 0.10 + 0.05 + 0.09 = 0.5

P(Sky=clear v Sky=cloudy) = sum of all possible ways the sky can be clear or cloudy:

P(Sky=clear): 0.12 + 0.07 + 0.06 + 0.08 = 0.33

P(Sky=cloudy): 0.07 + 0.10 + 0.07 + 0.09 = 0.33

P(Sky=clear) + P(Sky=cloudy) = 0.33 + 0.33 = 0.66

P(Sea=blue ^ (Sky=clear v Sky=cloudy)) = sum of all possibilities:

0.12 + 0.07 + 0.07+ 0.10 = 0.36

P(Sea=blue| Sky=clear v Sky=cloudy)

= P(Sea=blue ^ (Sky=clear v Sky=cloudy)) / P(Sky=clear v Sky=cloudy)

= 0.36 / 0.66 = 0.545 ≈ 0.55

P(Sea=blue| Sky=clear v Sky=cloudy) = 0.55

2.

Let EatRight=true = R

Let Exercise=true = E

Let Healthy=true = H

P(H) = 0.8

P(R ^ E | H) = 0.6

P(R ^ E | (1 - H)) = 0.3

By marginalization,

P(R^E) = P(R^E|H)\*P(H) + P(R^E|(1 - H))\*P(1 - H)

P(R^E) = 0.6 \* 0.8 + 0.3 \* 0.2 = 0.48 + 0.06 = 0.54

P(R^E) = 0.54

P(H|R^E) = P(R^E|H)P(H) / P(R^E) = 0.6 \* 0.8 / 0.54 = 0.888 ≈ 0.89

P((1 - H)|R^E) = P(R^E|(1 – H))P((1 – H)) / P(R^E) = 0.3 \* 0.2 / 0.54 = 0.1111 ≈ 0.11

Let H’ = Healthy

P(H’|R^E) = {0.89 H’=true

{0.11 H’=false

3.

**Given:**

P(Healthy=true) = 0.8

P(EatRight=true | Healthy=true) = 0.6

P(EatRight=true | Healthy=false) = 0.2

P(Exercise=true | Healthy=true) = 0.8

P(Exercise=true | Healthy=false) = 0.3

**Compute:**

**P**(Healthy | EatRight=true ^ Exercise=true)

**Simplify terms:**

Let Healthy be H

Let Healthy=true be H1

Let Healthy=false be H2

Let EatRight=true = R

Let Exercise=true = E

**Solve as:**

P(H | R ^ E) = (P(R ^ E | H) \* P(H) ) / P(R ^ E)

=>

P(H | R ^ E) = [P(H1 ^ R ^ E) / P(R ^ E)] + [P(H2 ^ R ^ E) / P(R ^ E)]

**Since EatRight and Exercise are conditionally independent given Healthy:**

P(R ^ E | H1) = P(R | H1) \* P(E | H1) = 0.6 \* 0.8 = 0.48

P(R ^ E | H2) = P(R | H2) \* P(E | H2) = 0.2 \* 0.3 = 0.06

P(R ^ E | H1) = P(R ^ E ^ H1) / P(H1) => **P(R ^ E ^ H1)** = P(R ^ E | H1) \* P(H1) = 0.48 \* 0.8 = 0.384

P(R ^ E | H2) = P(R ^ E ^ H2) / P(H2) => **P(R ^ E ^ H2)** = P(R ^ E | H2) \* P(H2) = 0.06 \* 0.8 = 0.096

**By Commutativity:**

P(A ^ B ^ C) = P(A) \* P(B) \* P(C) = P(C) \* P(A) \* P(B) = P(C ^ A ^ B)

Therefore:

P(R ^ E ^ H1) = P(H1 ^ R ^ E) and P(R ^ E ^ H2) = P(H2 ^ R ^ E)

**By Normalization:**

P(H1 | R ^ E) = αP (H1 ^ R ^ E) = α0.384

P(H2 | R ^ E) = αP (H2 ^ R ^ E) = α0.096

**And since P(H1 | R ^ E) + P(H2 | R ^ E) must equal 1:**

α0.384 + α0.096 = 1

α0.48 = 1

α = 2.08 => H = {0.8, 0.2}

**P**(H | EatRight=true ^ Exercise=true) = {0.8 H = true

{0.2 H = false

4.

a.



Let B = Breeze

Let K = Known

Let F = Frontier

Let O = Other

Let p = Pit

Let P = Probability

Breeze = {B2,1}

Known = {¬p1,1, ¬p2,1, ¬p1,2} (no breeze in 1,1 implies no pit in 1,2)

Frontier = {p1,3, p2,2, p3,1}

Other = {p1,3, p1,4, p2,3, p2,4, p3,2, p3,3, p3,4, p4,1, p4,2, p4,3, p4,4}

b.

compute:

**P**(p3,1 | B, K)

Use the General Rule for Probabilistic Inference:

and the rule for conditional probability:

so:

note:

P(a,b,c) = P(b|a,c)P(a,c) so we can rewrite with Breeze, given pit location:

A Breeze is independent of Other, given Frontier which means that P(B,O) = P(B) \* P(O), that is, Other does not affect if there is a Breeze in Known, so we can rewrite with Other in just the second part of the product:

The Probability of a pit in any location is independent, that is, it is not affected by a pit in any other location so we can rewrite with the probabilities of the pit locations as independent variables:

and since:

It will have no affect on the probability of the equation (1 \* x = x) so we can remove it from the equation:

The probability of a pit in 3,1 is constant and will not affect the probability of the equation as is the known locations so they can be moved outside of the summations:

Calculate P(K):

Known = {¬p1,1, ¬p2,1, ¬p1,2}

P(¬p1,1, ¬p2,1, ¬p1,2)

P(¬p1,1) = 1 since it is given that there will never be a pit in location 1,1

since it is given that the probability of a pit in any location is 0.2 as a rule of the game so the probability of no pit is 1 – 0.2 = 0.8

P(¬p2,1) = 0.8

P(¬p1,2) = 0.8

So:

P(K) = P(¬p1,1, ¬p2,1, ¬p1,2) = (1 \* 0.8 \* 0.8) = 0.64

And:

P(p3,1) = 0.2

Calculate P(F) as P(B|p3,1, K,F) (iterate all possible P(F)):

Frontier = {p1,3, p2,2, p3,1}

The query is whether or not there is a pit in 3,1 so remove it from the frontier:

If there is a pit in 3,1, then the possibility of a pit in 2,2 and 1,3 are valid for all possible scenarios so we need to articulate all of them:

P13 P22

T T 0.2 \* 0.2 = 0.04

T F 0.2 \* 0.8 = 0.16

F T 0.8 \* 0.2 = 0.16

F F 0.8 \* 0.8 = 0.64

If there is not a pit in 3,1, then there must be a pit in 2,2 given there is a breeze in 2,1 and it is known that there is no pit in 1,1:

=<0.2 \* (0.2(), 0.8(0.2 + 0.8))>

=<0.04, 0.16>

must sum to 1 so:

α0.04 + α0.16 = 1

α0.2 = 1

α = 5

=<0.2, 0.8>

so:

**P**(p3,1 | B, K) = { 0.2 there is a pit in 3,1

{ 0.8 there is not a pit in 3,1

The knowledge base did not provide enough new information to reduce or increase the probability of a pit in location 3,1