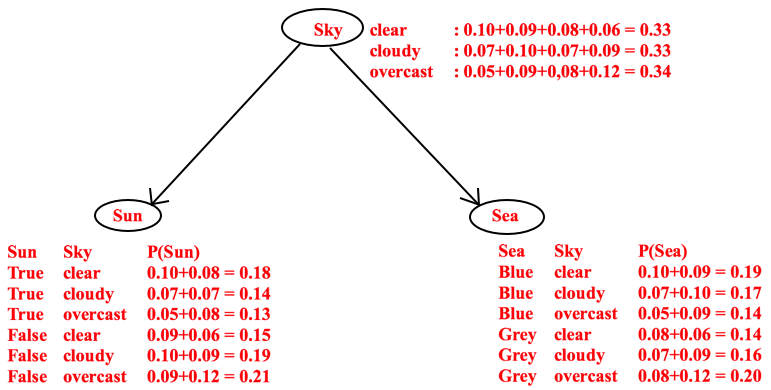
1.



2.

For all parts a through e of problem 2:

Let t be Time = day and ¬t be Time = night

Let sk1 be Sky = clear

Let sk2 be Sky = cloudy

Let sk3 be Sky = overcast

Let sn be Sun=true and ¬sn be Sun=false

Let mn be Moon=true and ¬mn be Moon=false

Let se be Sea=blue and ¬se be Sea=gray

a. P(Time=day, Sky=clear, Sun=true, Moon=false, Sea=blue)

P(t ^ sk1 ^ sn ^ ¬mn ^ se)

Given:

P(t ^ sk1 ^ sn) = 0.9

P(t ^ sk1 ^ ¬mn) = 0.8

P(sn ^ se) = 0.8

P(t ^ sk1 ^ sn ^ ¬mn ^ se) = P(t ^ sk1 ^ sn) P(t ^ sk1 ^ ¬mn) P(sn ^ se) = 0.9 \* 0.8 \* 0.8 = 0.576

b. P(Moon=true | Time=night, Sky=cloudy)

This is given on the Bayesian network as 0.5

c. P(Time=day | Moon=true)

Choose the minimal set of parents.

Time has no parent.

Moon has two parents: Time and Sky.

Moon and Time are independent of Sun.

Moon and Time are independent of Sea.

P(mn) is not given, solve by normalization:

P(t,mn) = sum of all possible ways this can be true: 0.2 + 0.1 + 0.0 = 0.3

P(¬t,mn) = sum of all possible ways this can be true: 0.9 + 0.5 + 0.1 = 1.5

**P**(t|mn) = α<0.3,1.5>

α = 1 / 1.8 = 0.555

**P**(t|mn) = <0.166, 0.833> => <0.17, 0.83>

P(t|mn) = 0.17

d. P(Sea=blue | Time=day, Sky=clear)

Choose a minimal set of parents:

Sea is independent of Moon.

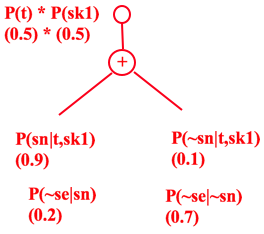
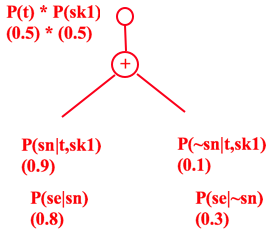
Sea has a parent Sun.

Sun has parents Time, and Sky

Calculate the distribution of P(Sea=blue):

and:

Bayesian Networks:



=>

P(se|t,sk1)= α (0.5)(0.5) \* ((0.9 \* 0.8) + (0.1 \* 0.2)) = α0.1875

P(¬se|t,sk1)= α (0.5)(0.5) \* ((0.9 \* 0.2) + (0.1 \* 0.7)) = α0.0625

=> **P**(se|t,sk1) = α<0.1875,0.0625> => <0.75,0.25>

P(se|t,sk1) = 0.75

e. P(Time=day | Sea=blue, Moon=false)

Choose minimal set:

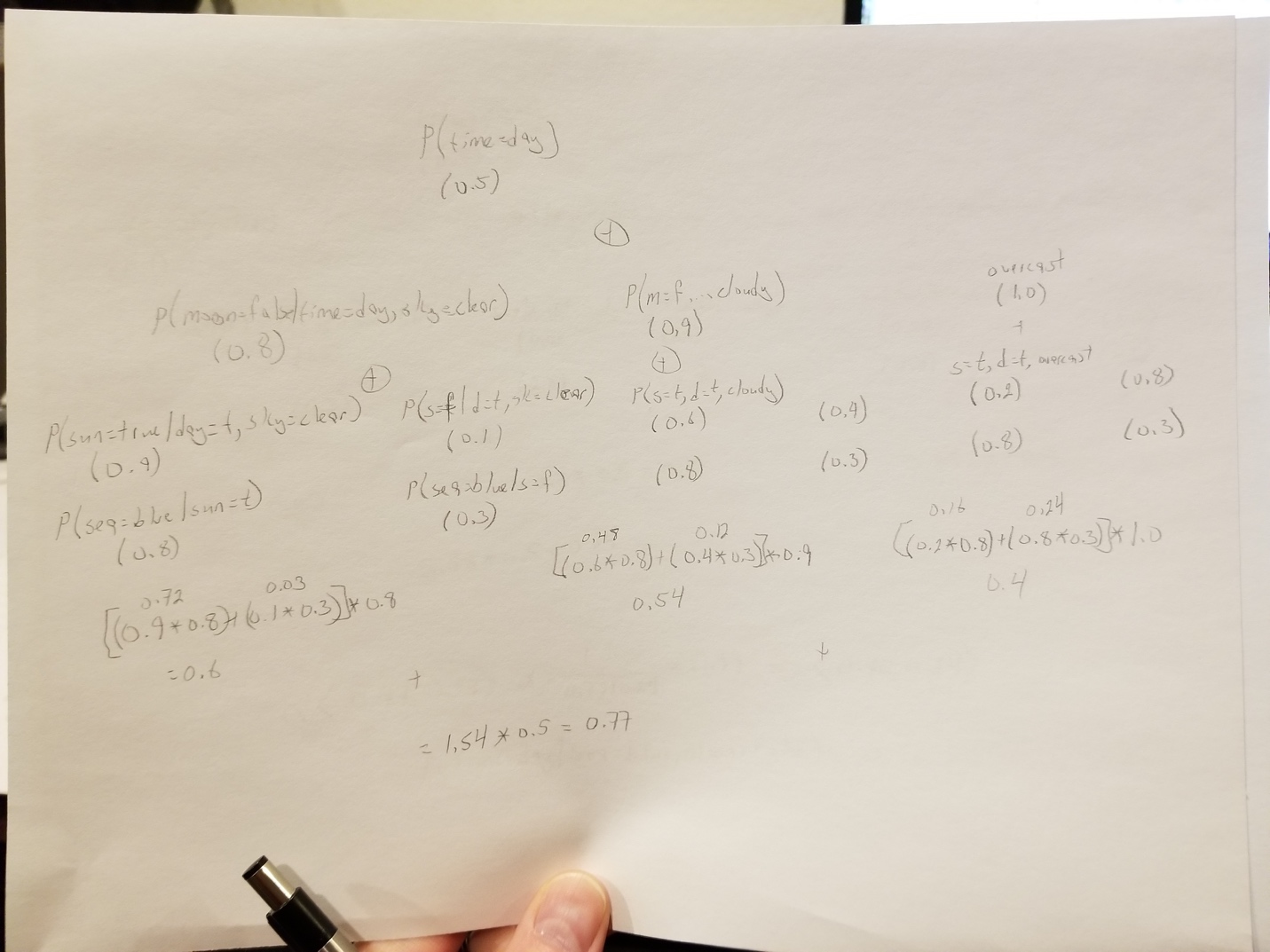
Time has no parent

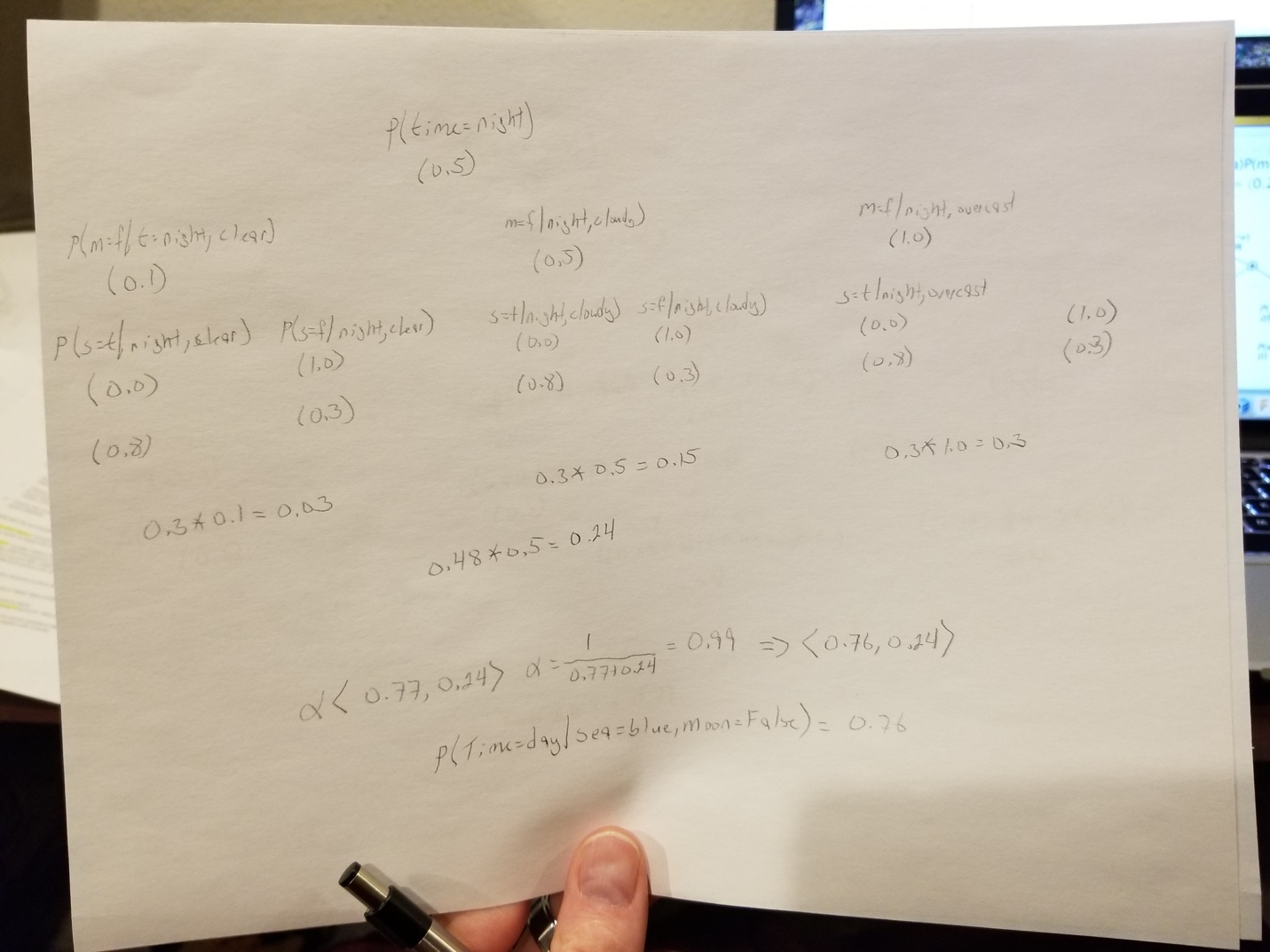
Sea has parent Sun

Moon has parents Time and Sky

Moon is independent of Sun so it moves in front of Sun summation,

Time is independent of Sun and Sky so it moves in front of each summation:





=> α<0.77,0.24> => <0.76,0.24>

P(t|se, ¬mn) = 0.76

3.

a.

Given T=day:

Sky is independent of T, choose highest probability, 0.5, Clear

Sun is dependent on T and Sky, if T=Day and Sky=Clear, highest probability is 0.9, True

Moon is dependent on T and Sky, if T=Day and Sky=Clear, highest probability is 0.8, False

Sea is dependent on Sun, if Sun=True, highest probability is 0.8, Blue

So:

Sky=Clear

Sun=True

Moon=False

Sea=Blue

b.

Given:

Time=Night

Sky=Overcast

Sun is dependent on T and Sky, if T=Night and Sky=Overcast, highest probability is 1.0, False

Moon is dependent on T and Sky, if T=Night and Sky=Overcast, highest probability is 0.9, False

Sea is dependent on Sun, if Sun=False, highest probability is 0.7, Gray

So:

Sun=False

Moon=False

Sea=Gray