0.1 Tutorial 4 - Randomized algorithm

1. (a) probability that you hire exactly one person

$$\begin{aligned} P(\texttt{one person}) &= \sum_{i=1}^{n} \frac{1}{i} \\ &= O(n \log n) \end{aligned}$$

.

correct answer: $\frac{1}{n}$

(b) probability that you hire exactly n person

$$\begin{aligned} P(\texttt{n person}) &= \prod_{i=1}^{n} \frac{1}{i} \\ &= \frac{1}{n!} \end{aligned}$$

.

2. Let $x_i = 1$ when the hat is correct

$$\begin{aligned} E[x_i] &= P[x_i] \\ &= \frac{n-i}{n} && \texttt{correct answer: } \frac{1}{n} \\ E[x] &= \sum_{i=1}^{n} E[x_i] \\ &= O(n \log n) && \texttt{correct answer: 1} \end{aligned}$$

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3. Let $x_{i,j} = 1$ when $A[i] > A[j]$

$$\begin{aligned} E[x_{i,j}] &= \frac{1}{2} \\ E[x] &= \frac{\binom{n}{2}}{2} \\ &= \frac{n(n-1)}{4} \end{aligned}$$

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