0.1 Tutorial 4 - Randomized algorithm

1. (a) probability that you hire exactly one person

$$\begin{aligned} P(\texttt{one person}) &= \sum_{i=1}^{n} \frac{1}{i} \\ &= O(n \log n) \end{aligned}$$

.

correct answer: $\frac{1}{n}$

(b) probability that you hire exactly n person

$$\begin{aligned} P(\texttt{n person}) &= \prod_{i=1}^{n} \frac{1}{i} \\ &= \frac{1}{n!} \end{aligned}$$

.

2. Let $x_i = 1$ when the hat is correct

$$\begin{aligned} E[x_i] &= P[x_i] \\ &= \frac{n-i}{n} && \texttt{correct answer: } \frac{1}{n} \\ E[x] &= \sum_{i=1}^{n} E[x_i] \\ &= O(n \log n) && \texttt{correct answer: 1} \end{aligned}$$

.

3. Let $x_{i,j} = 1$ when $A[i] > A[j]$

$$\begin{aligned} E[x_{i,j}] &= \frac{1}{2} \\ E[x] &= \frac{\binom{n}{2}}{2} \\ &= \frac{n(n-1)}{4} \end{aligned}$$

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0.2 Tutorial 5 - Selection and quick sort

1. (a) when z_k is in between z_i and z_j, the number of elements $= j - i + 1$. So the probability that z_i or z_j is selected is $Pr[X_{ij} = 1] = \frac{2}{k-i+1}$

(b) i and j are on the left side of the decision tree, the number of elements $= k - i + 1$. So the probability that z_i or z_j is selected is $Pr[X_{ij} = 1] = \frac{2}{k-i+1}$

(c) i and j are on the right side of the decision tree, the number of elements $= k - j + 1$. So the probability that z_i or z_j is selected is $Pr[X_{ij} = 1] = \frac{2}{j-k+1}$

`better answer: analyze case by case (i.e q1b, split to` $t = i$ `or` $t = j, i < t < j, k \leq t \leq i$`)`

2. (a) $\Theta(n^2)$

Algorithm 1 partition'

(b) **Input:** *input*
Output: *output*

```
1: i ← randomp, r
2: x ← A[r]
3: i ← p − 1; k ← p − 1;
4: for j ← p to r − 1 do
5:     if A[j] = x then
6:         k ← k + 1
7:         SWAP(A[k], A[j])
8:     else if A[j] < x then
9:         i ← i + 1
10:        k ← k + 1
11:        SWAP(A[k], A[j])
12:        SWAP(A[i], A[k])
13:    end if
14:    SWAP(A[k + 1], A[r])
15:    return state
```

Algorithm 2 quicksort'

(c) **Input:** A, p, r
Output: A

```
1: if p ≥ r then
2:     return A
3: end if
4: q, t = PARTITION'(A, p, r)
5: l = QUICKSORT'(A, q − 1)
6: r = QUICKSORT'(A, t + 1)
7: return A =0
```

3. key: $Pr[x_{i'j'}] \leq Pr[x_{ij}] \implies E[X'] \leq O(n \log n)$