

**0.1 Tutorial 4 - Randomized algorithm**

1. (a) probability that you hire exactly one person

$$\begin{aligned} P(\text{one person}) &= \sum_{i=1}^n \frac{1}{i} \\ &= O(n \log n) \end{aligned}$$

correct answer:  $\frac{1}{n}$

- (b) probability that you hire exactly n person

$$\begin{aligned} P(\text{n person}) &= \prod_{i=1}^n \frac{1}{i} \\ &= \frac{1}{n!} \end{aligned}$$

2. Let  $x_i = 1$  when the hat is correct

$$\begin{aligned} E[x_i] &= P[x_i] \\ &= \frac{n-i}{n} \end{aligned} \quad \text{correct answer: } \frac{1}{n}$$

$$\begin{aligned} E[x] &= \sum_{i=1}^n E[x_i] \\ &= O(n \log n) \end{aligned} \quad \text{correct answer: } 1$$

3. Let  $x_{i,j} = 1$  when  $A[i] > A[j]$

$$\begin{aligned} E[x_{i,j}] &= \frac{1}{2} \\ E[x] &= \frac{\binom{n}{2}}{2} \\ &= \frac{n(n-1)}{4} \end{aligned}$$