COMP2711H - Honours discrete mathematics Spring 2023

Notes by Marcus Chan

May 28, 2023

Contents

1 Lecture 1 - proposition and logic

1.1 Introduction to Propositions

Definition 1.1. proposition is either true or false

Remark 1.2 — contradiction is not a proposition

Definition 1.3 (Conditional statement)**.** $p \to q$, p is the **hypothesis**, and q is the **conclusion**. If the statement is true, then p is the **necessary condition** and q is the **sufficient condition**

1.2 Compound propositions

1. converse: $p \to \neg q$
2. contrapositive: $\neg q \to \neg p$
3. inverse: $\neg p \to \neg q$

Definition 1.4 (Biconditional statement)**.** $p \iff q$ means p if and only if q, also means p $\equiv$ q

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\to$	4
$\leftrightarrow$	5

Figure 1: precedence table

1.3 Propositional Equivalence

Definition 1.5 (tautology)**.** A compound proposition that is always **true**, no matter what the truth values of the propositional variables that occur in it.

Definition 1.6 (contradiction)**.** A compound proposition that is always **false**

Definition 1.7 (contingency)**.** A compound proposition that is neither a tautology nor a contradiction

Equivalence	Name
$p \wedge \mathrm{T} \equiv p$ $p \vee \mathrm{F} \equiv p$	Identity laws
$p \vee \mathrm{T} \equiv \mathrm{T}$ $p \wedge \mathrm{F} \equiv \mathrm{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws

Figure 2: propositional equivalences

Equivalence	Name
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathrm{T}$ $p \wedge \neg p \equiv \mathrm{F}$	Negation laws

Figure 3: propositional equivalences

Equivalence	Name
$p \to q \equiv \neg p \lor q$ $p \to q \equiv \neg q \to \neg p$ $(p \to q) \land (p \to r) \equiv p \to (q \land r)$ $(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$ $(p \to r) \land (q \to r) \equiv (p \lor q) \to r$ $(p \to r) \lor (q \to r) \equiv (p \land q) \to r$	Involving conditional statements
$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$ $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$ $p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$	Involving biconditional statements

Figure 4: propositional equivalences

2 Lecture 2 - Predicate Logic (First-order logic)

2.1 Predicates

Definition 2.1 (Predicate)**.** A predicate is a statement that may be true or false depending on its variables

2.2 Quantifiers

Definition 2.2 (Universal qunatification)**.** $\forall x P(x)$

Definition 2.3 (Domain)**.** The set of all possible values of a variable. **Must be defined** when using quantification.

Definition 2.4 (Existential quantification)**.** $\exists x P(x)$ (at least one x in domain)

2.3 Logical equivaleces

Fact 2.5. $\forall x P(x) \lor Q(x) \not\equiv \forall x P(x) \lor \forall x Q(x)$

Fact 2.6. $\exists x P(x) \land Q(x) \not\equiv \exists x P(x) \lor \exists x Q(x)$

2.4 Negating Quantified Expressions

$$\neg \forall x P(x) \equiv \exists x \neg P(x).$$

$$\neg \exists x Q(x) \equiv \forall x \neg Q(x).$$

2.5 Nested quantifiers