problem 1:

a) approximate $u''(x)$ using centered (2,2) and off-centered (4,0) FDM.

1.

$$\frac{d^2u(\bar{x})}{d\bar{x}^2} = a_0 u(\bar{x}-4\cdot h) + a_1 u(\bar{x}-3\cdot h) + a_2 u(\bar{x}-2\cdot h) + a_3 u(\bar{x}-h) + a_4 u(\bar{x})$$

taylor expansion:

$$h(\bar{x}+ih) = h(\bar{x}) + (ih)\,h'(x) + \dots + \frac{(ih)^k}{k!} h''(x)$$

so,

$$u(\bar{x}-n\cdot h) = h(\bar{x}) - n\cdot h\, u'(\bar{x}) - \frac{(n\cdot h)^2}{2!} u''(\bar{x}) - \frac{(nh)^3}{3!} u^{(3)}(\bar{x}) - \frac{(nh)^4}{4!} u^{(4)}(\bar{x})$$

$$\begin{aligned}\frac{d^2u(\bar{x})}{d\bar{x}^2} = {} & (a_0+a_1+a_2+a_3+a_4)\cdot u(\bar{x}) - a_0\Big(4\cdot h\cdot u'(\bar{x}) + \frac{(4\cdot h)^2}{2}\cdot u''(\bar{x}) + \frac{(4\cdot h)^3}{6} u^{(3)}(\bar{x}) + \frac{(4\cdot h)^4}{24} u^{(4)}(\bar{x})\Big) \\ & - a_1\Big(3\cdot h\cdot u'(\bar{x}) + \frac{(3\cdot h)^2}{2}\cdot u''(\bar{x}) + \frac{(3\cdot h)^3}{6} u^{(3)}(\bar{x}) + \frac{(3\cdot h)^4}{24} u^{(4)}(\bar{x})\Big) \\ & - a_2\Big(2\cdot h\cdot u'(\bar{x}) + \frac{(2\cdot h)^2}{2}\cdot u''(\bar{x}) + \frac{(2\cdot h)^3}{6} u^{(3)}(\bar{x}) + \frac{(2\cdot h)^4}{24} u^{(4)}(\bar{x})\Big) \\ & - a_3\Big(h\cdot u'(\bar{x}) + \frac{(h)^2}{2}\cdot u''(\bar{x}) + \frac{(h)^3}{6} u^{(3)}(\bar{x}) + \frac{(h)^4}{24} u^{(4)}(\bar{x})\Big)\end{aligned}$$

$$\begin{aligned}= {} & (a_0+a_1+a_2+a_3+a_4)\cdot u(\bar{x}) - \\ & (a_0\cdot 4 + a_1\cdot 3 + a_2\cdot 2 + a_3)\cdot h\cdot u'(\bar{x}) - \\ & \Big(a_0\frac{4^2}{2} + a_1\frac{3^2}{2} + a_2\frac{2^2}{2} + a_3\frac{1}{2}\Big)\cdot h^2\cdot u''(x) - \\ & \Big(a_0\frac{4^3}{3} + a_1\frac{3^3}{3} + a_2\frac{2^3}{3} + a_3\frac{1}{3}\Big) h^3\cdot u^{(3)}(x) - \\ & \Big(a_0\frac{4^4}{4} + a_1\frac{3^4}{4} + a_2\frac{2^4}{4} + a_3\frac{1}{4}\Big)\cdot h^4\cdot u^{(4)}(x)\end{aligned}$$

så alt lig 0 men

$$\Big(a_0\frac{4^2}{2} + a_1\frac{3^2}{2} + a_2\frac{2^2}{2} + a_3\frac{1}{2}\Big)\cdot h^2 = 1 \quad \text{så} \Rightarrow \frac{1}{h^2}$$

forklaring på

$$\begin{aligned} r_j &= \partial_{xx} u(x)\big|_{x=x_j} \\ &= \partial_{xx} u(x)\big|_{x=x_j} - \\ &= \sum_{i=0}^{\infty} \frac{u^{(i)} h^i}{i!} - \\ &= \end{aligned}$$

graph.

$- D^2 u_j$

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