DM549/DS(K)820/MM537/DM547

Lecture 1: Propositional Logic

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Logical Propositions

Definition (Definition 1.1.1)

A *proposition* (et udsagn) is a declarative statement (that is, a statement that declares a fact) that is true (sand) or false (falsk) but not both.

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Remarks:

- We denote a true proposition as T and a false one as F.
- Alternatively, one can also think of bits, where 1 corresponds to T and 0 corresponds to F.

Logical Operators

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Today, we will get to know the following operators:

- the negation $\neg$,
- the conjunction $\wedge$,
- the disjunction $\vee$,
- the implication $\Rightarrow$,
- the bi-implication $\Leftrightarrow$,
- the exclusive or $\oplus$.

The Logical Negation

We can define an operator through a so-called *truth table* (sandhedstabel):

p	$\neg p$
T	F
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For every value of the operand p, the value of the compound proposition is given.

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- Alternative notation: $\bar{p}$, $!p$.

The Logical And and the Logical Or

We can also define binary (as opposed to unary) operators through truth tables:

p	q	$p \wedge q$
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- How to memorize? Perhaps "a∧d" helps.

A Quiz

Go to `pollev.com/kevs`

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Another operator:

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- Alternative notation: $\rightarrow$ (book!)

The Logical Bi-implication

A similarly looking operator:

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The last operator:

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- In words: for $p \oplus q$ to be T, p and q must have different truth values.
- Caution: The book uses
  - "either p or q" to say $p \vee q$ and
  - "either p or q but not both" to say $p \oplus q$.

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Precedence order ("order of evaluation") **of operators:**

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Note:

- I (and I believe I speak for a larger group of Mathematicians) would not assume that the reader knows the precedence order among $\wedge$ and $\vee$; neither among $\Rightarrow$ and $\Leftrightarrow$. In these cases, I am putting parentheses.

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- Similarly, if you are not sure about the precedence, just put parentheses.