Department of Mathematics and Computer Science
University of Southern Denmark, Odense

September 16, 2024
KS

DM549, DS(K)820, MM537, and DM547

Exercise Sheet 5 (Week 39-1)

Before the session

Solve the following exercises (you are welcome to do so in your study group):

1. Section 1.8:
   - Exercise 32
2. For $n \in \mathbb{N}$ let $P(n)$ be the proposition

$$\sum_{i=0}^{n} 3^i = \frac{3^{n+1} - 1}{2}.$$

   Show by induction that $P(n)$ is true for all $n \in \mathbb{N}$, using the following steps.

   (a) What is the proposition $P(0)$?
   (b) Show $P(0)$, i.e., carry out the basis step.
   (c) Write down the inductive hypothesis.
   (d) What needs to be shown in the inductive step?
   (e) Carry out the inductive step. Specify where you use the inductive hypothesis.
   (f) Explain why these steps establish that $P(n)$ holds for all $n \in \mathbb{N}$.

During the session

Solve the following exercises in your study group:

1. Section 5.1:
   - Exercise 18
   - Exercise 11
   - Exercise 78
   - Exercise 79
   - Generalize your solution for Exercises 78 and 79 to chess boards of size $2^n \times 2^n$ for any $n \in \mathbb{Z}^+$, again with the upper left square removed.
2. Go through Test 1 as needed.