DM549/DS(K)820/MM537/DM547

Lecture 1: Propositional Logic

Kevin Schewior Email: kevs@sdu.dk

University of Southern Denmark

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Logical Propositions

Definition (Definition 1.1.1)

A *proposition* (et udsagn) is a declarative statement (that is, a statement that declares a fact) that is true (sand) or false (falsk) but not both.

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- We denote a true proposition as T and a false one as F.
- Alternatively, one can also think of bits, where 1 corresponds to T and 0 corresponds to F.

Logical Operators

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Today, we will get to know the following operators:

- the negation ¬,
- \blacksquare the conjunction \land ,
- the disjunction ∨.
- lacksquare the implication \Rightarrow ,
- the bi-implication ⇔,
- \blacksquare the exclusive or \oplus .

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p & \neg p \\
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T & F \\
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- Alternative notation: \bar{p} , !p.

We can also define binary (as opposed to unary) operators through truth tables:

р	q	$p \wedge q$	p	q	$p \lor q$
Т	Т	Т	Т	Т	Т
Т	F	F	Т	F	Т
F	Т	F	F	Т	Т
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F

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- In words:
 - For $p \land q$ to be T, both p and q must be T.
 - For $p \lor q$ to be T, at least one of p and q must be T.

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- How to memorize? Perhaps "a\d" helps.

A Quiz

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 - Alternative notation: → (book!)

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 - "p iff q" ("p hviss q")

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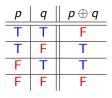
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- Read:
 - "either p or q" ("enten p eller q")
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- In words: for $p \oplus q$ to be T, p and q must have different truth values.
- Caution: The book uses
 - "either p or q" to say $p \lor q$ and
 - "either p or q but not both" to say $p \oplus q$.

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Precedence order ("order of evaluation") of operators:

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 \neg , \land , \lor , \Rightarrow , \Leftrightarrow

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- Similarly, if you are not sure about the precedence, just put parentheses.