WIA 1002 DATA STRUCTURE

SEM 2, SESSION 2024/205

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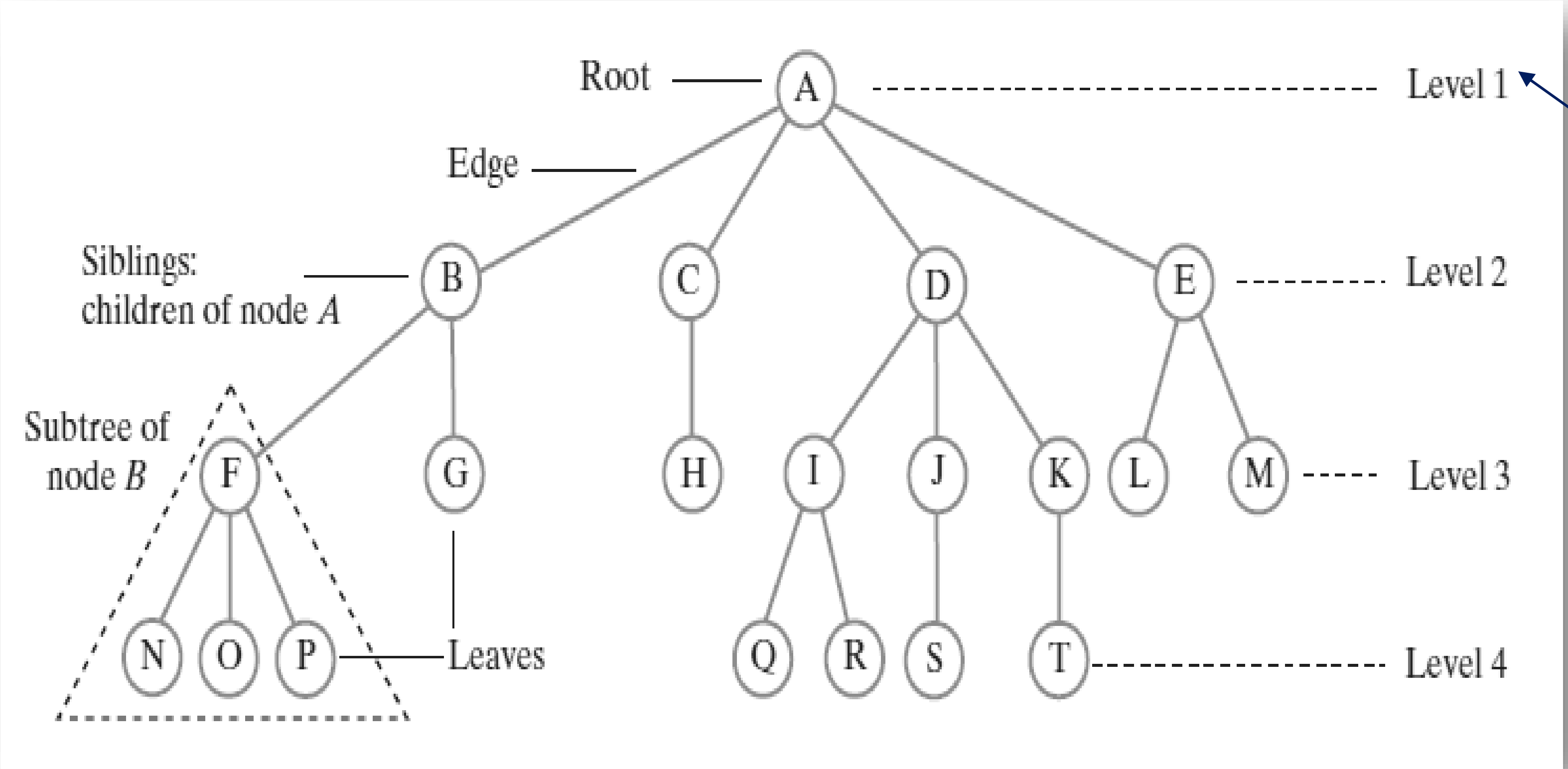
BINARY SEARCH TREE

Objectives

- To **design** and **implement** a binary search tree.
- To **represent** binary trees using linked data structures.
- To **search** an element in binary search tree.
- To **insert** an element into a binary search tree.
- To **traverse** elements in a binary tree.
- To **delete** elements from a binary search tree.

Tree – hierarchical structure

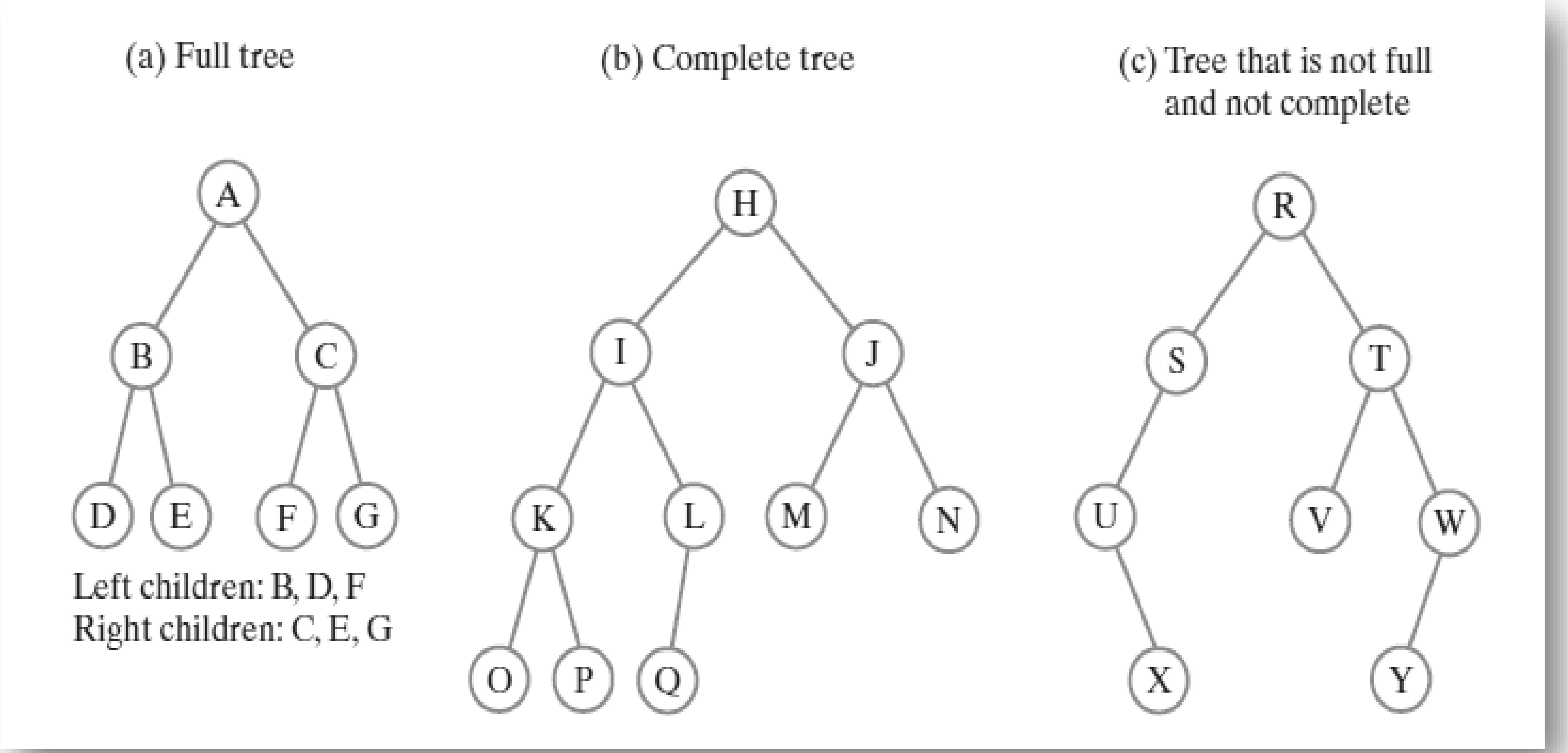


Or level 0 in other books

Tree

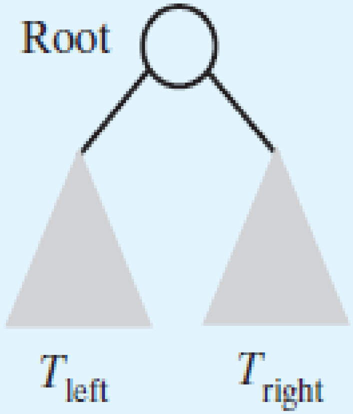
- In general, each node in a tree can have an arbitrary number of children. We sometimes call such a tree a **general tree**. If each node has no more than n children, the tree is called an ***n*-ary tree**.
- Not every general tree is an n-ary tree. If each node has at most two children, the tree is called a **binary tree**.

Binary Trees



Binary Trees

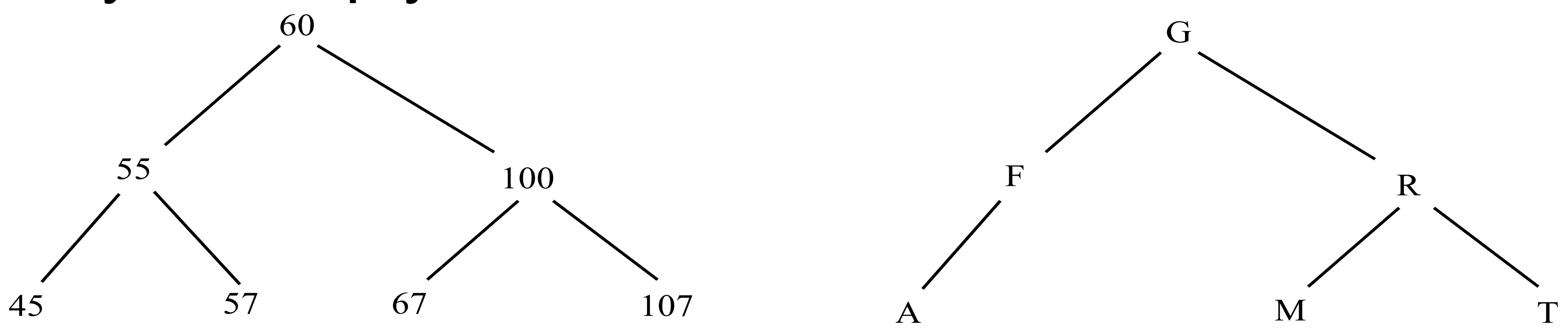
Note: A binary tree is either empty or has the following form:



where T_{left} and T_{right} are binary trees.

Binary Trees

A **binary tree** is a hierarchical structure. It is either empty or consists of an element, called the *root*, and two distinct binary trees, called the *left subtree* and *right subtree*, either or both of which may be empty



(A) (B)

Each node in a binary tree has zero, one, or two subtrees.

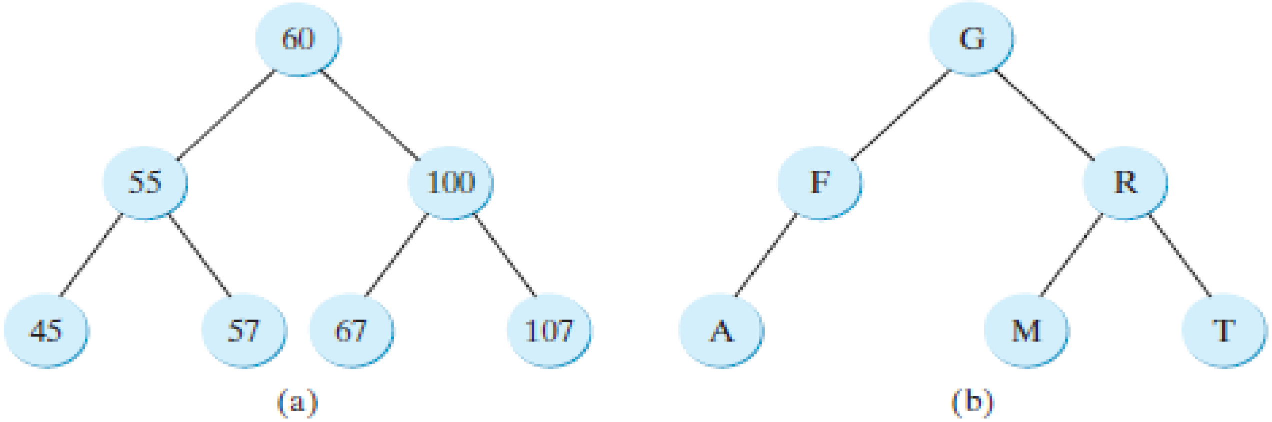


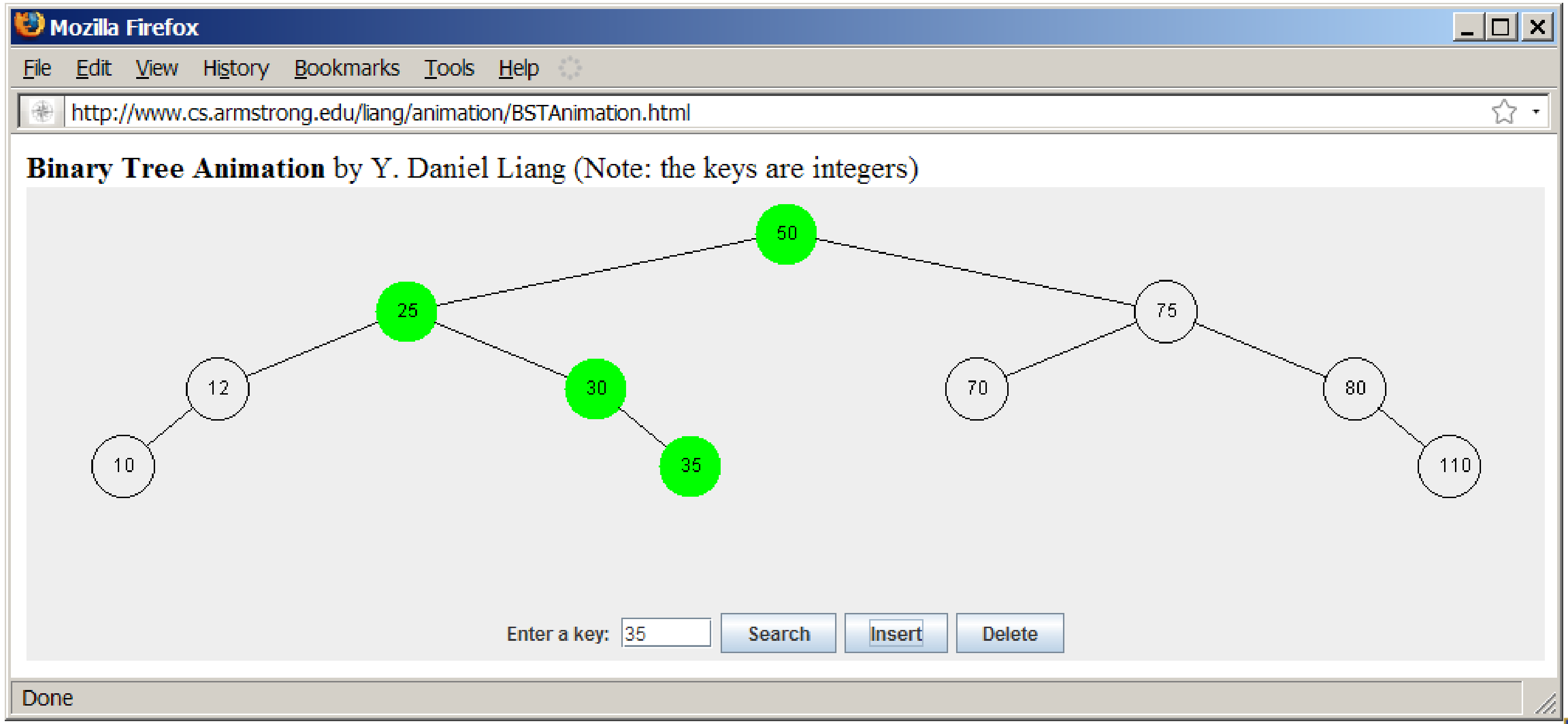
FIGURE 25.1 Each node in a binary tree has zero, one, or two subtrees.

Terms:

- Length of path – num of edges in the path (eg:60-45 → 2)
- Depth – length of path from root to node (eg:60 → 0; 55→1)
- Level – set of all nodes at a given depth (e.g:55,100)
- Siblings – nodes share same parent node (eg: 45-57)
- Left child- root of the left subtree of a node
- Leaf – node without children (eg: A)
- Height – length of path fr root node to its furthest leaf (e.g 2)

See How a Binary Tree Works

http://www.cs.armstrong.edu/liang/animation/web/BST.html



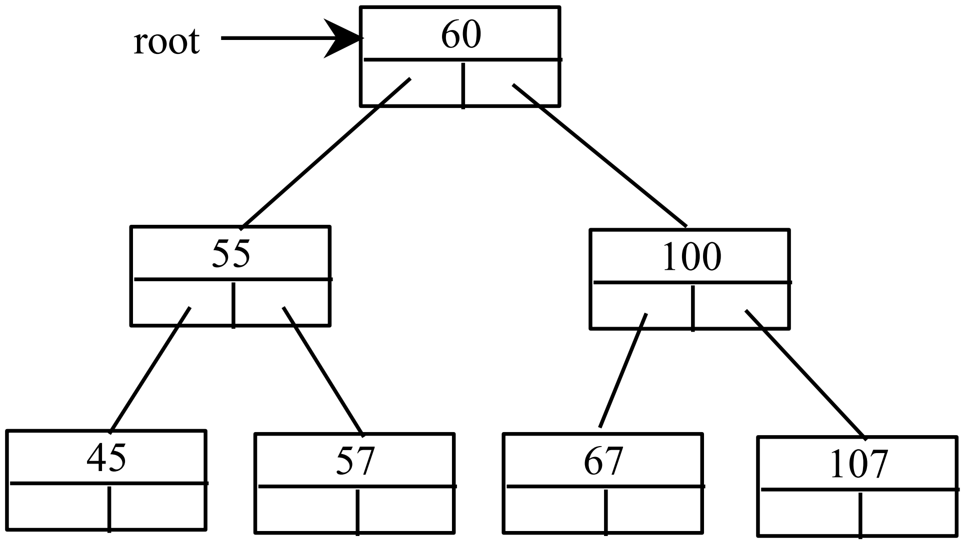
Binary Tree Terms

- A special type of binary tree called a *binary search tree* is often useful.

- A binary search tree (with **no duplicate elements**) has the property that for every node in the tree the value of any node in its *left subtree* is **less than** the value of the node and the value of any node in its *right subtree* is greater than the value of the node.

Representing Binary Trees

A binary tree can be represented using a set of linked nodes. Each node contains a value and two links named *left* and *right* that reference the left child and right child, respectively as shown below.




```
class TreeNode<E> {
  E element;
  TreeNode<E> left;
  TreeNode<E> right;

  public TreeNode(E o) {
    element = o;
  }
}
```

Create node

- Variable `root` refers to the root node of the tree.
- If tree is empty, `root` is null.
- Create root node:

```
TreeNode<Integer> root = new TreeNode<Integer>(new Integer(60));
```

- Create left child node:

```
root.left = new TreeNode<Integer>(new Integer(55));
```

- Create right child node:

```
root.right = new TreeNode<Integer>(new Integer(100));
```

Searching an Element in a BST

```
public boolean search(E element) {
  TreeNode<E> current = root; // Start from the root
  while (current != null)
    if (element < current.element) {
      current = current.left; // Go left
    }
    else if (element > current.element) {
      current = current.right; // Go right
    }
    else // Element matches current.element
      return true; // Element is found
  return false; // Element is not in the tree
}
```

Inserting an Element to a BST

If a BST is empty, create a root node with the new element.

Otherwise, **locate** the **parent node** for the new element node.

If the new element is less than the parent element, the node for the new element becomes the left child of the parent.

If the new element is greater than the parent element, the node for the new element becomes the right child of the parent. Here is the algorithm/code:

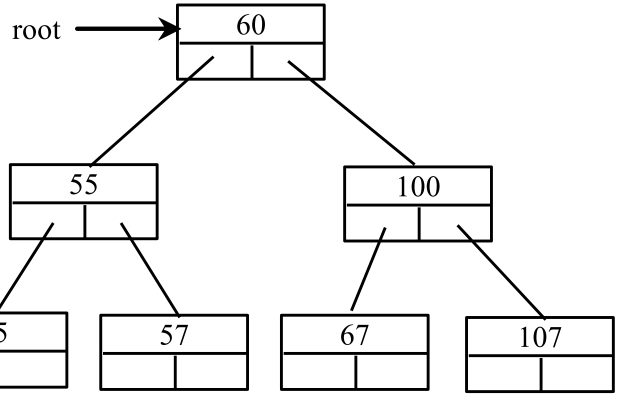
Inserting an Element to a BST

```
if (root == null)
  root = new TreeNode(element);
else {
  // Locate the parent node
  current = root;
  while (current != null)
    if (element value < the value in current.element) {
      parent = current;
      current = current.left;
    }
    else if (element value > the value in current.element) {
      parent = current;
      current = current.right;
    }
    else
      return false; // Duplicate node not inserted

  // Create the new node and attach it to the parent node
  if (element < parent.element)
    parent.left = new TreeNode(elemenet);
  else
    parent.right = new TreeNode(elemenet);

  return true; // Element inserted
}
```

Insert 101 into the following tree.



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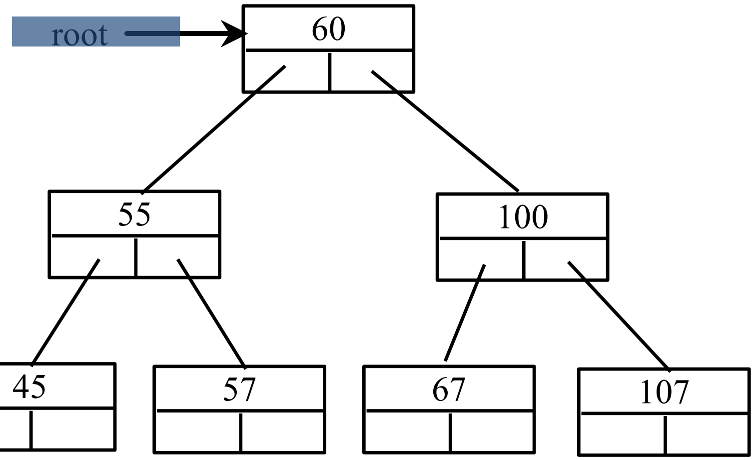

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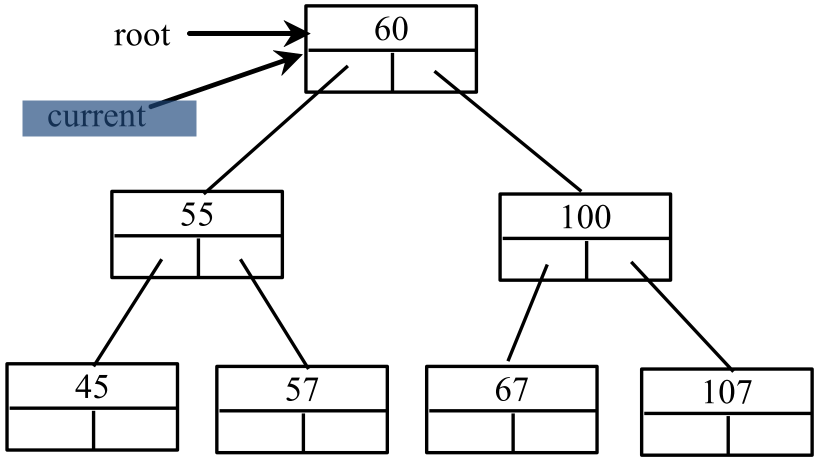

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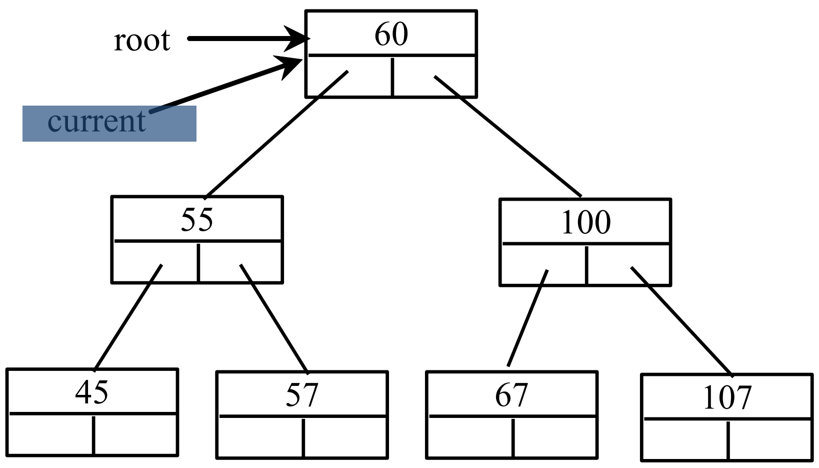

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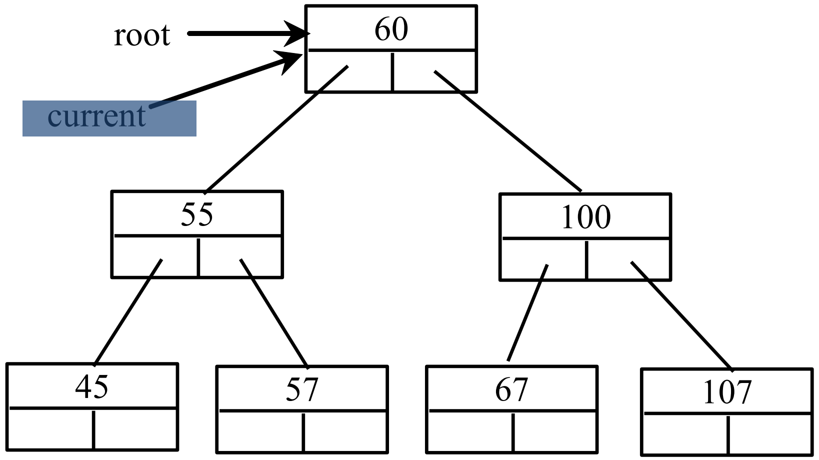
Insert 101 into the following tree.

101 < 60?



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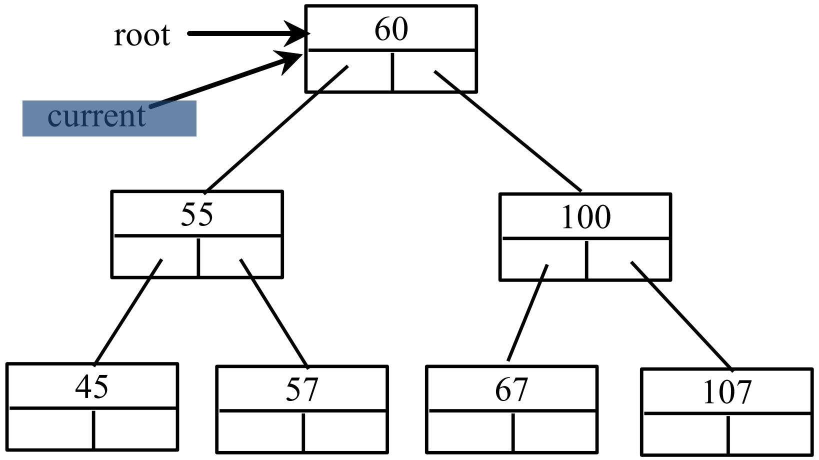

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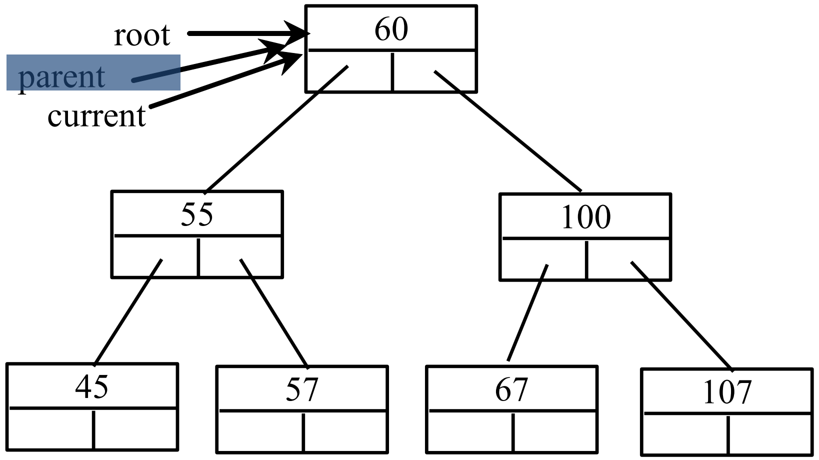

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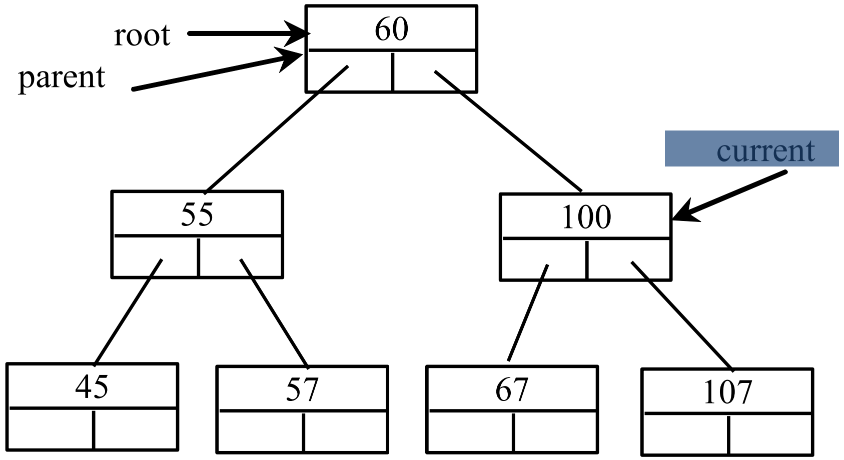

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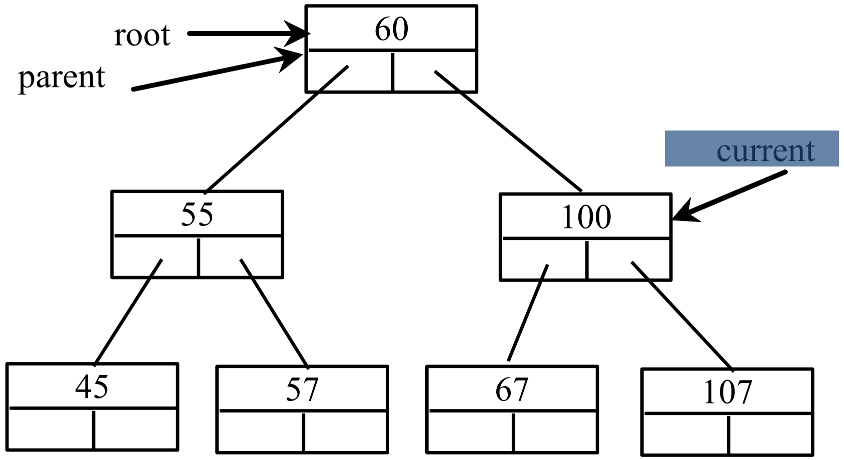

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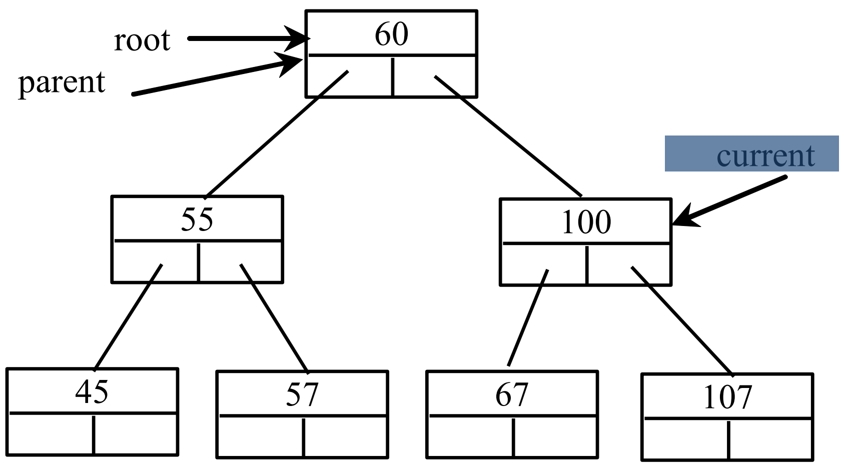

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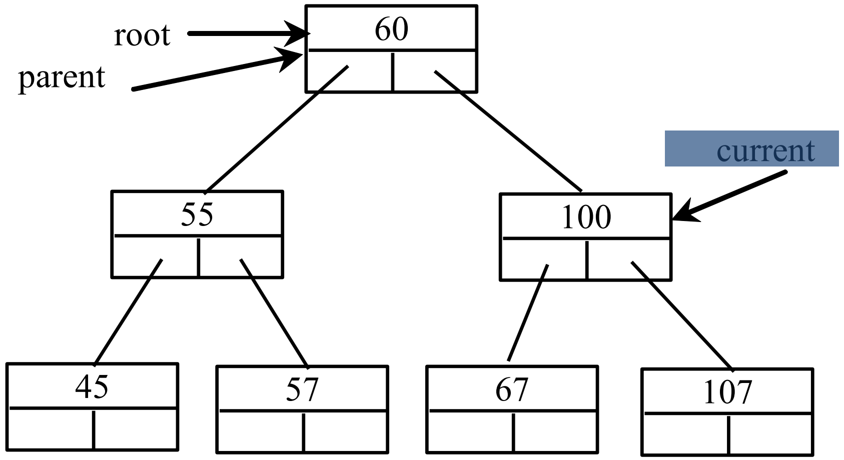

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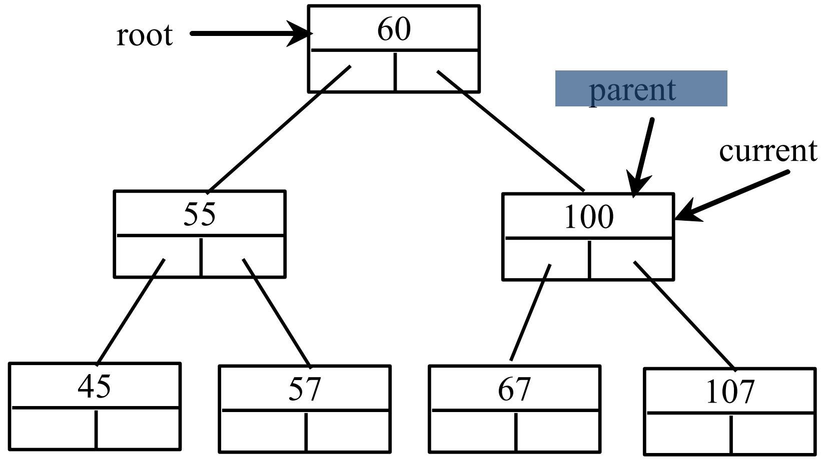

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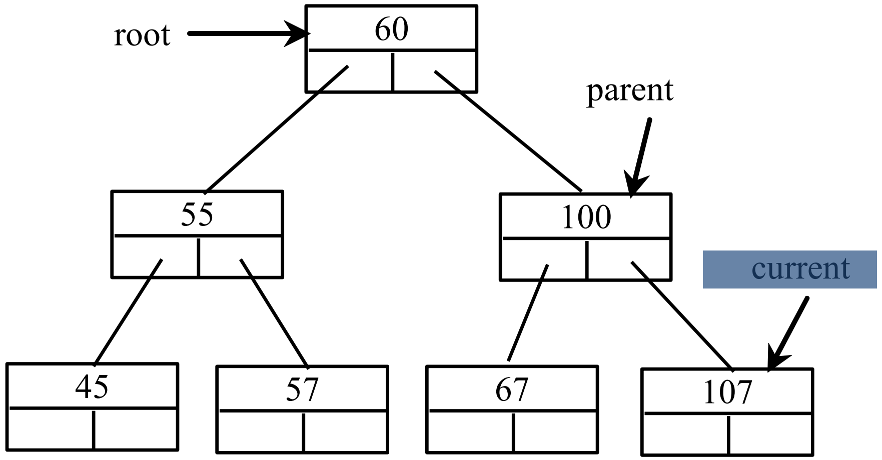

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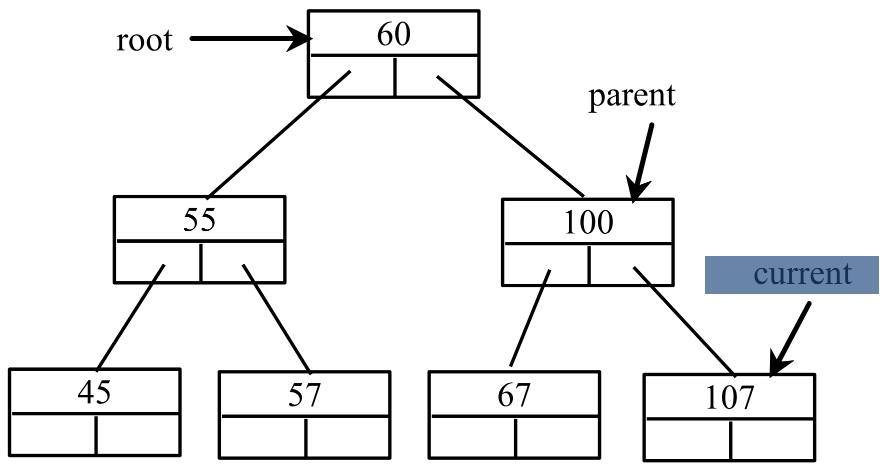

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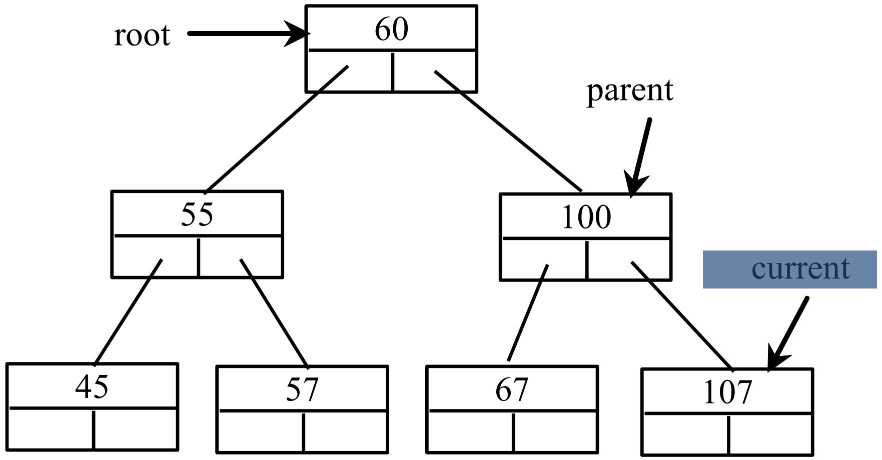
Insert 101 into the following tree.

101 < 107 true



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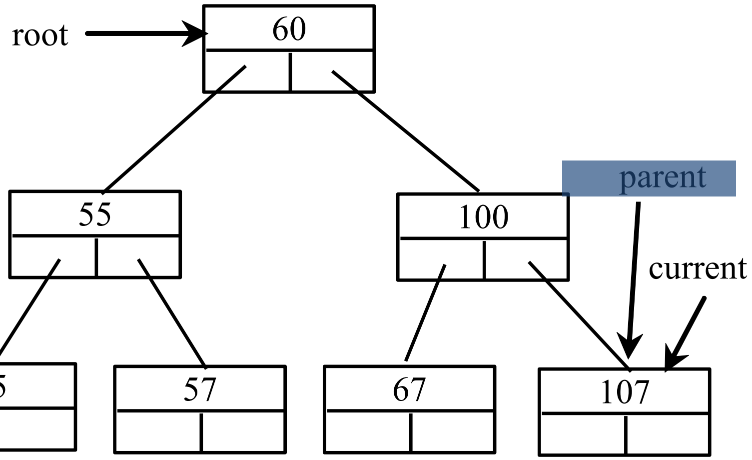

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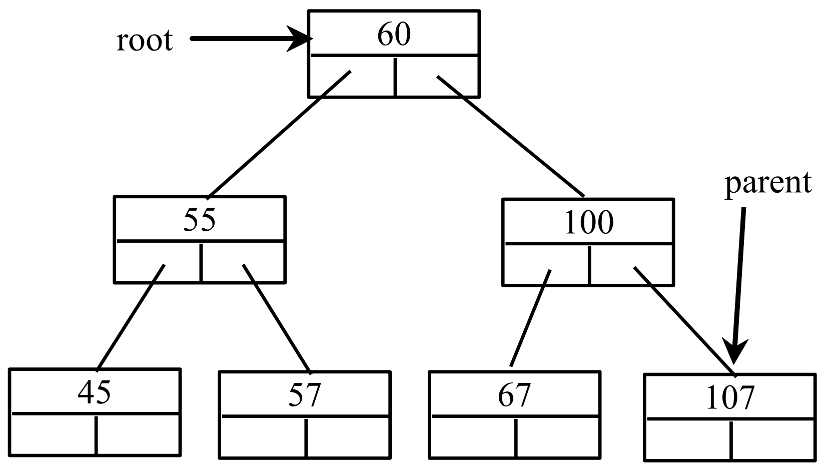

Since current.left is null,current becomes null

Trace Inserting 101 into the following tree, cont.

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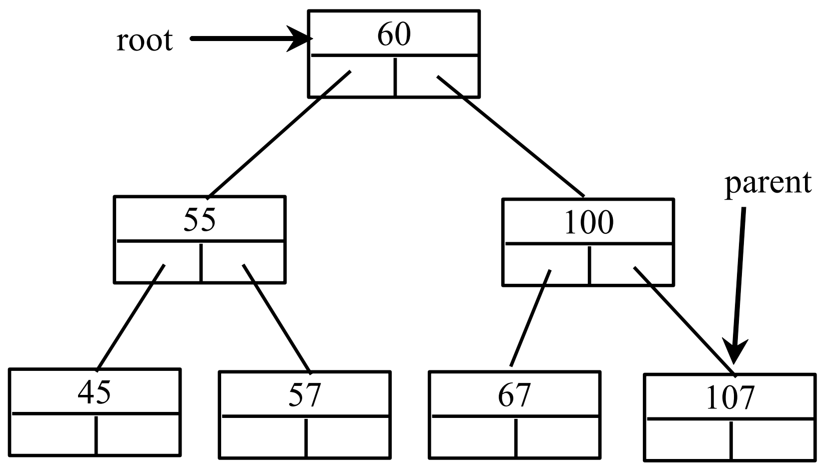
current is null now

Insert 101 into the following tree.



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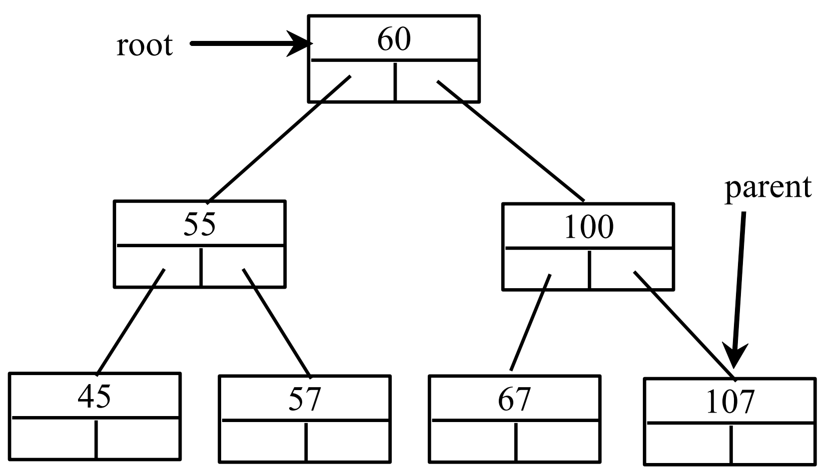
Trace Inserting 101 into the following tree, cont.

```
if (root == null)
  root = new TreeNode(element);
else {
  // Locate the parent node
  current = root;
  while (current != null)
    if (element value < the value in current.element) {
      parent = current;
      current = current.left;
    }
    else if (element value > the value in current.element) {
      parent = current;
      current = current.right;
    }
    else
      return false; // Duplicate node not inserted

  // Create the new node and attach it to the parent node
  if (element < parent.element)
    parent.left = new TreeNode(elemenet);
  else
    parent.right = new TreeNode(elemenet);

  return true; // Element inserted
}
```

101 < 107 true

Insert 101 into the following tree.



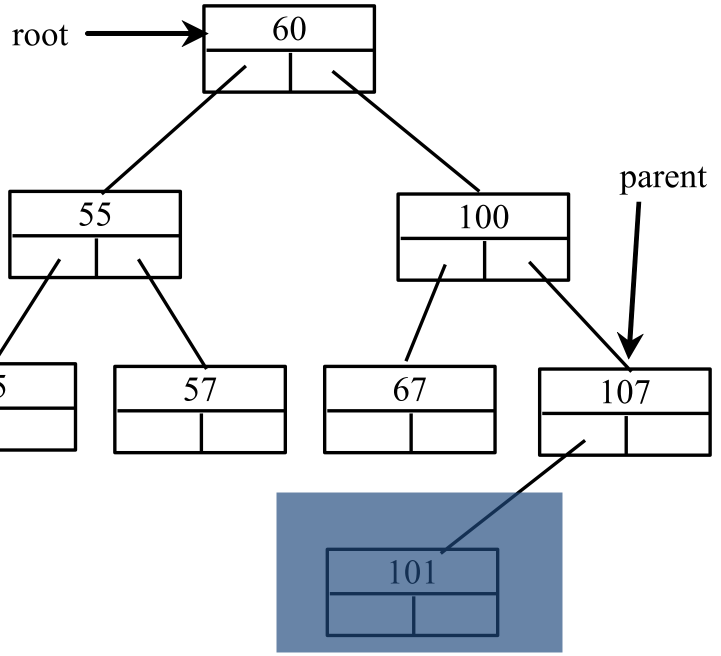
Since current.left is null,current becomes null

Trace Inserting 101 into the following tree, cont.

```
if (root == null)
  root = new TreeNode(element);
else {
  // Locate the parent node
  current = root;
  while (current != null)
    if (element value < the value in current.element) {
      parent = current;
      current = current.left;
    }
    else if (element value > the value in current.element) {
      parent = current;
      current = current.right;
    }
    else
      return false; // Duplicate node not inserted

  // Create the new node and attach it to the parent node
  if (element < parent.element)
    parent.left = new TreeNode(elemenet);
  else
    parent.right = new TreeNode(elemenet);

  return true; // Element inserted
}
```

101 < 107 true

Insert 101 into the following tree.

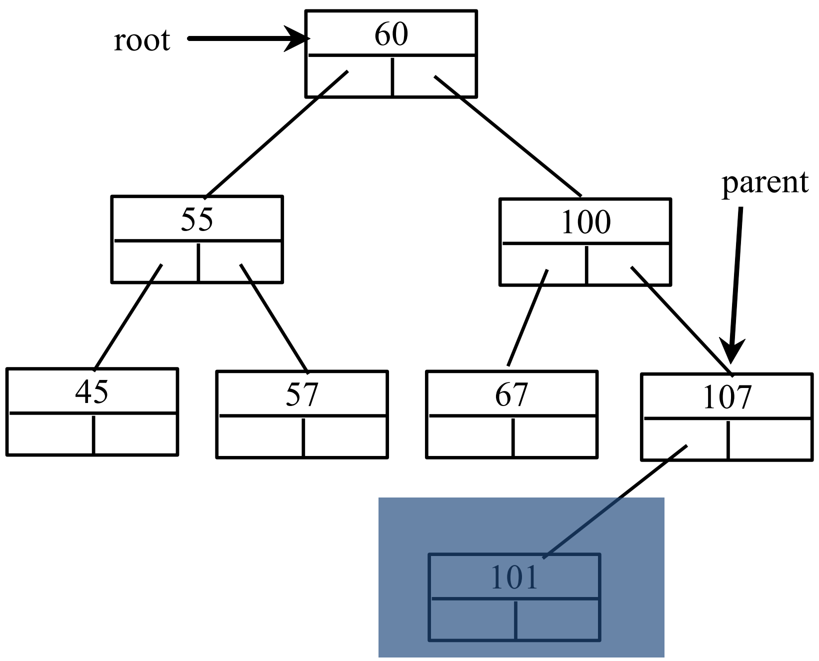


Trace Inserting 101 into the following tree, cont.

```
if (root == null)
  root = new TreeNode(element);
else {
  // Locate the parent node
  current = root;
  while (current != null)
    if (element value < the value in current.element) {
      parent = current;
      current = current.left;
    }
    else if (element value > the value in current.element) {
      parent = current;
      current = current.right;
    }
    else
      return false; // Duplicate node not inserted

  // Create the new node and attach it to the parent node
  if (element < parent.element)
    parent.left = new TreeNode(elemenet);
  else
    parent.right = new TreeNode(elemenet);

  return true; // Element inserted
}
```

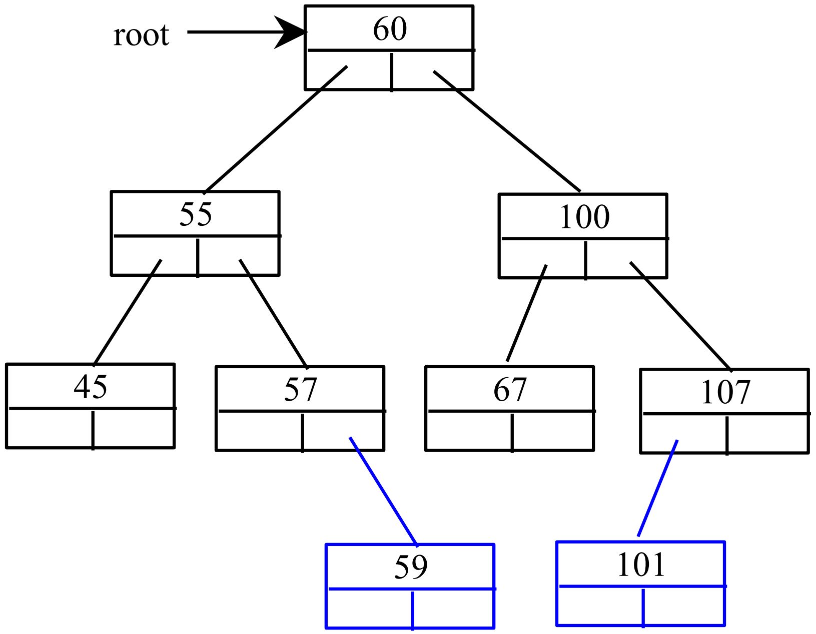
Insert 101 into the following tree.

101 < 107 true



Inserting 59 into the Tree

```
if (root == null)
  root = new TreeNode(element);
else {
  // Locate the parent node
  current = root;
  while (current != null)
    if (element value < the value in current.element) {
      parent = current;
      current = current.left;
    }
    else if (element value > the value in current.element) {
      parent = current;
      current = current.right;
    }
    else
      return false; // Duplicate node not inserted

  // Create the new node and attach it to the parent node
  if (element < parent.element)
    parent.left = new TreeNode(elemenet);
  else
    parent.right = new TreeNode(elemenet);

  return true; // Element inserted
}
```

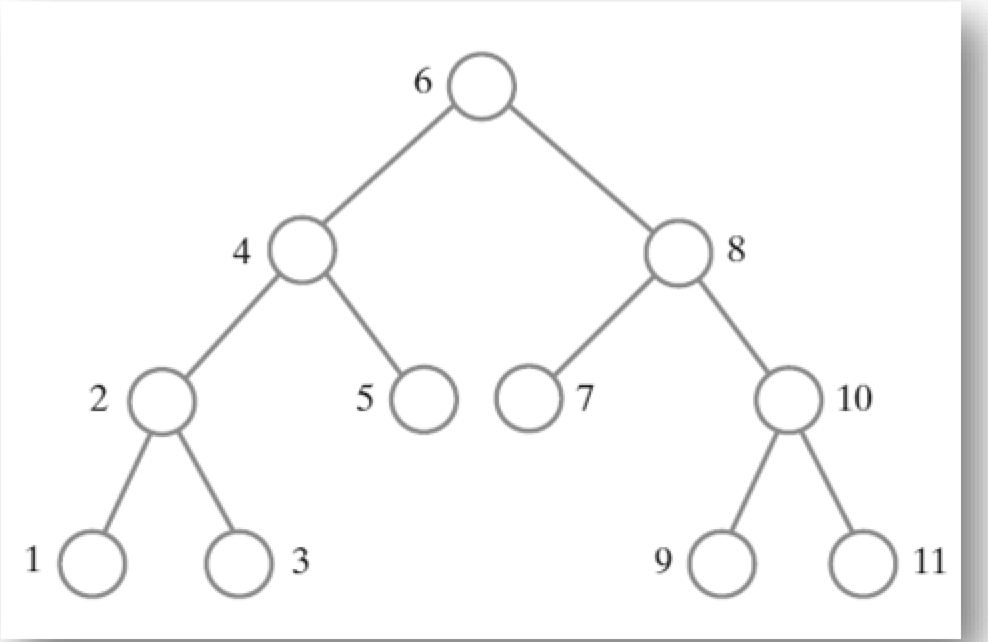


Tree Traversal

- **Tree traversal** is the *process of visiting each node in the tree exactly once*.
- We will say that traversal can pass through a node without visiting it at that moment.
- Traversals of a binary tree are somewhat easier to understand
- There are several ways to traverse a tree:
  - *Inorder*
  - *Postorder*
  - *Preorder (depth-first), and*
  - *Breadth-first (level-order)*

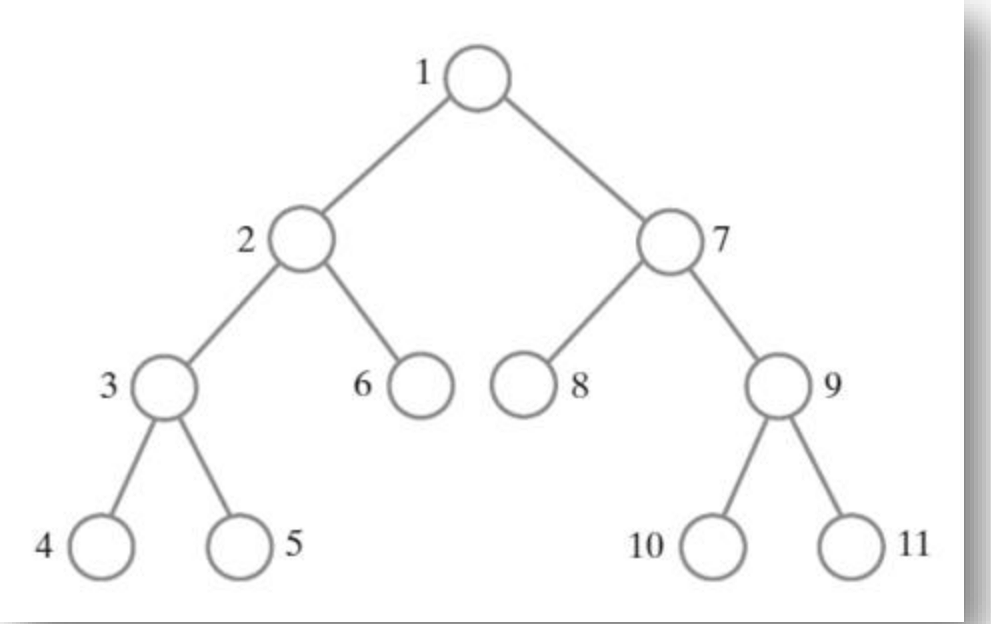
Tree Traversal, cont.

The inorder traversal is to **visit the left subtree of the current node first recursively**, **then** the **current node** itself, and **finally** the **right subtree of the current node recursively**.



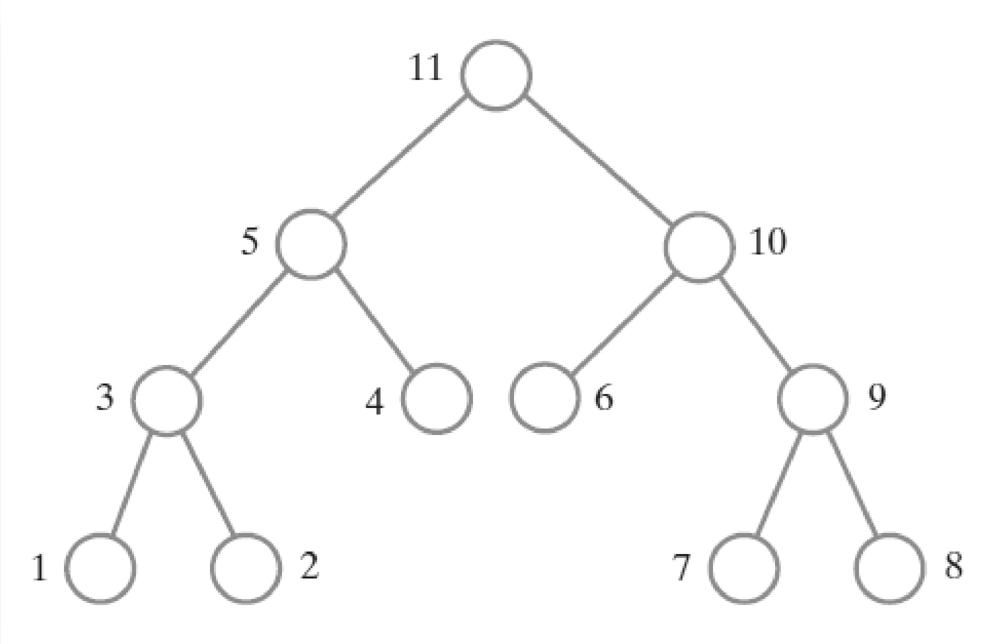
Tree Traversal, cont.

The preorder/depth-first search traversal is to **visit the current node first, then the left subtree of the current node recursively, and finally the right subtree of the current node recursively.**



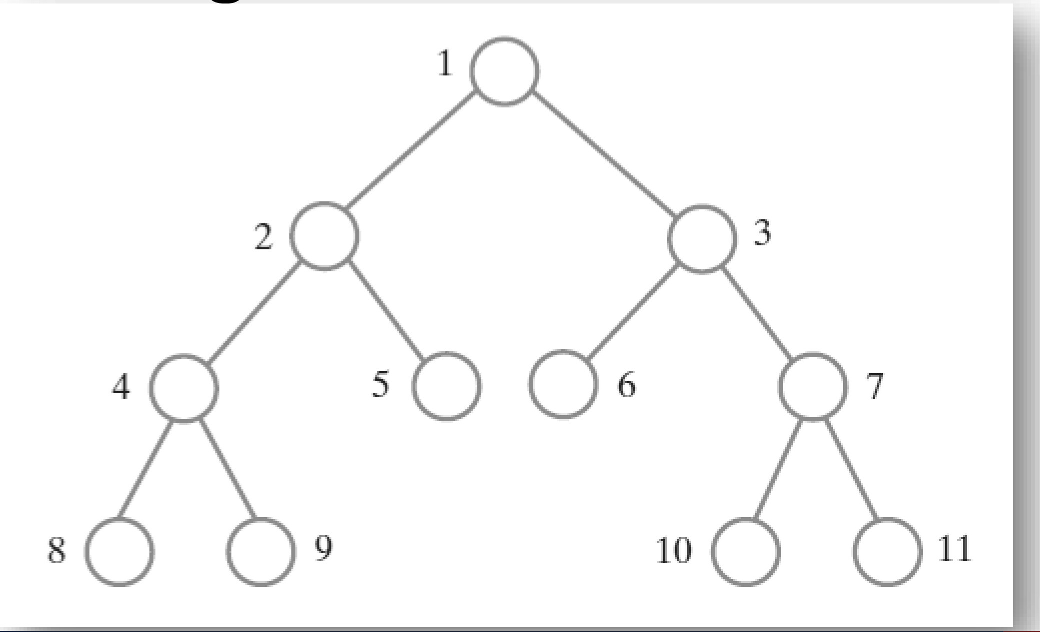
Tree Traversal, cont.

The postorder traversal is to **visit** the **left subtree of the current node first**, **then** the **right subtree of** the **current node**, and **finally** the **current node** itself.

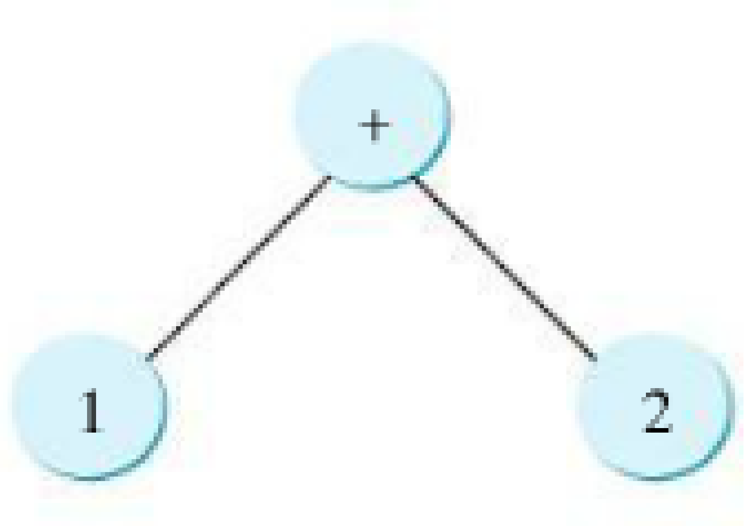


Tree Traversal, cont.

The breadth-first (level-order) traversal is to **visit the nodes level by level. First visit the root, then all children of the root from left to right, then grandchildren of the root from left to right**, and so on.

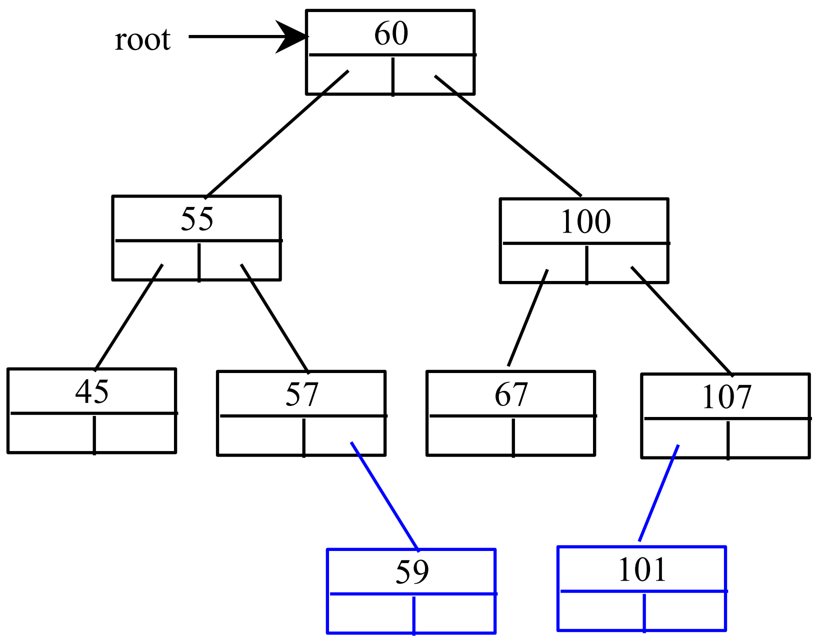


Remembering traversal order



- Inorder is 1 + 2
- Postorder is 1 2 +
- Preorder + 1 2

Tree Traversal, cont.



For example, in the tree above,

the inorder is 45 55 57 59 60 67 100 101 107.

the postorder is 45 59 57 55 67 101 107 100 60.

the preorder is 60 55 45 57 59 100 67 107 101.

the breadth-first traversal is 60 55 100 45 57 67 107 59 101.

The Tree Interface

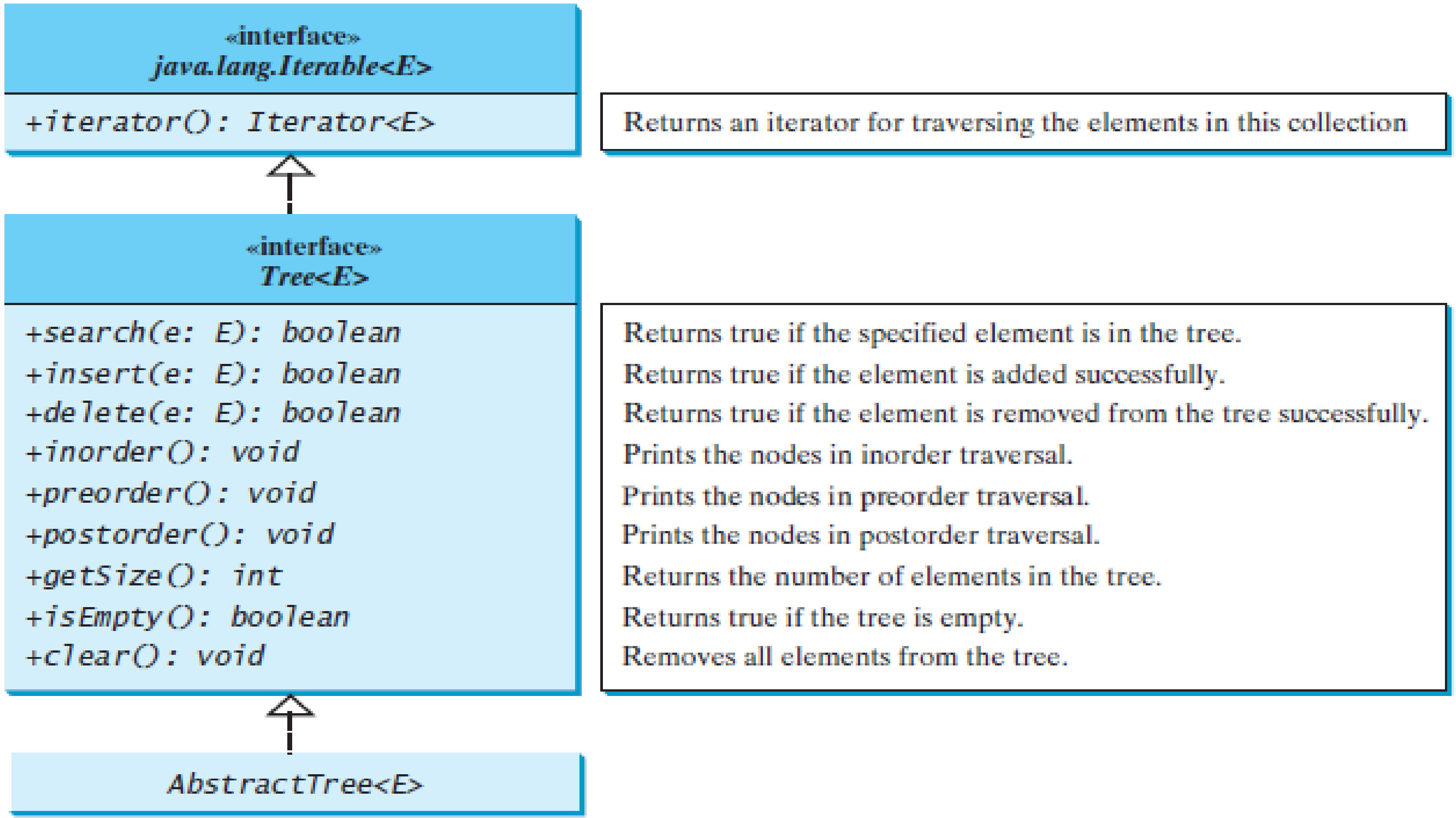


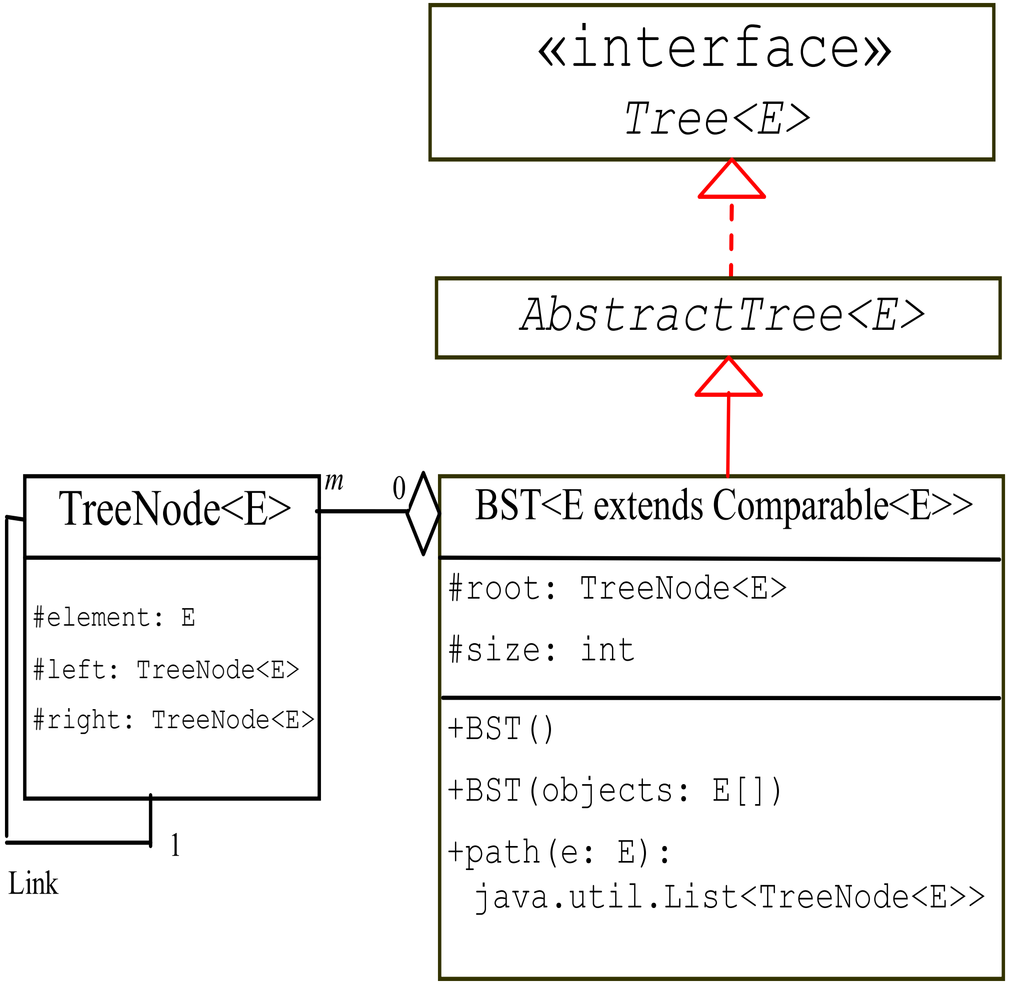
FIGURE 25.7 The `Tree` interface defines common operations for trees, and the `AbstractTree` class partially implements `Tree`.

http://www.cs.armstrong.edu/liang/intro10e/html/Tree.html

http://www.cs.armstrong.edu/liang/intro10e/html/AbstractTree.html

The BST Class

A concrete BST class can be defined to extend AbstractTree.



The root of the tree.

The number of nodes in the tree.

Creates a default BST.

Creates a BST from an array of elements.

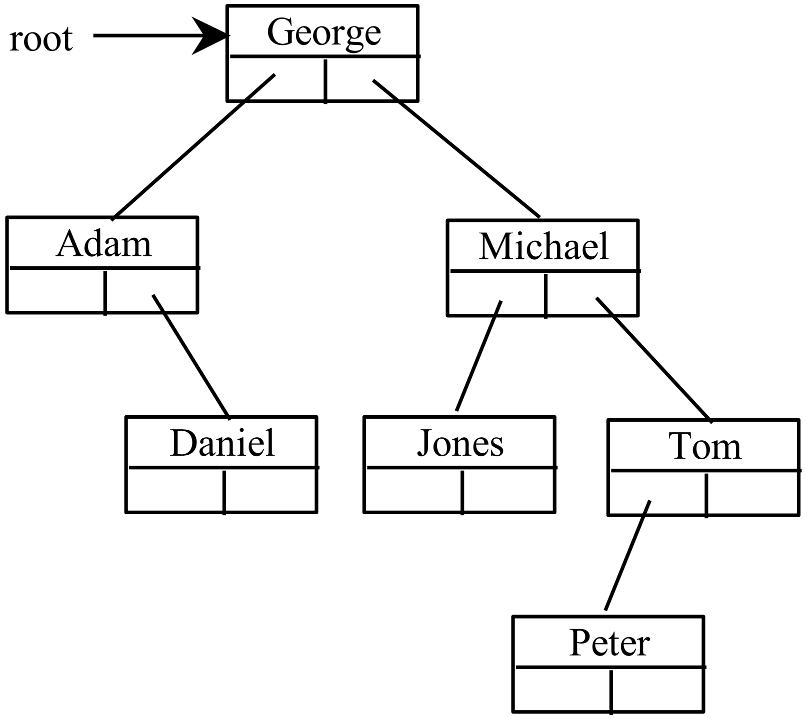
Returns the path of nodes from the root leading to the node for the specified element. The element may not be in the tree.

TestBST class

- Write a test program that creates a binary search tree using BST. Add strings into the tree and traverse the tree in inorder, postorder, and preorder.

http://www.cs.armstrong.edu/liang/intro10e/html/TestBST.html

Tree After Insertions



Inorder: Adam, Daniel George, Jones, Michael, Peter, Tom

Postorder: Daniel Adam, Jones, Peter, Tom, Michael, George

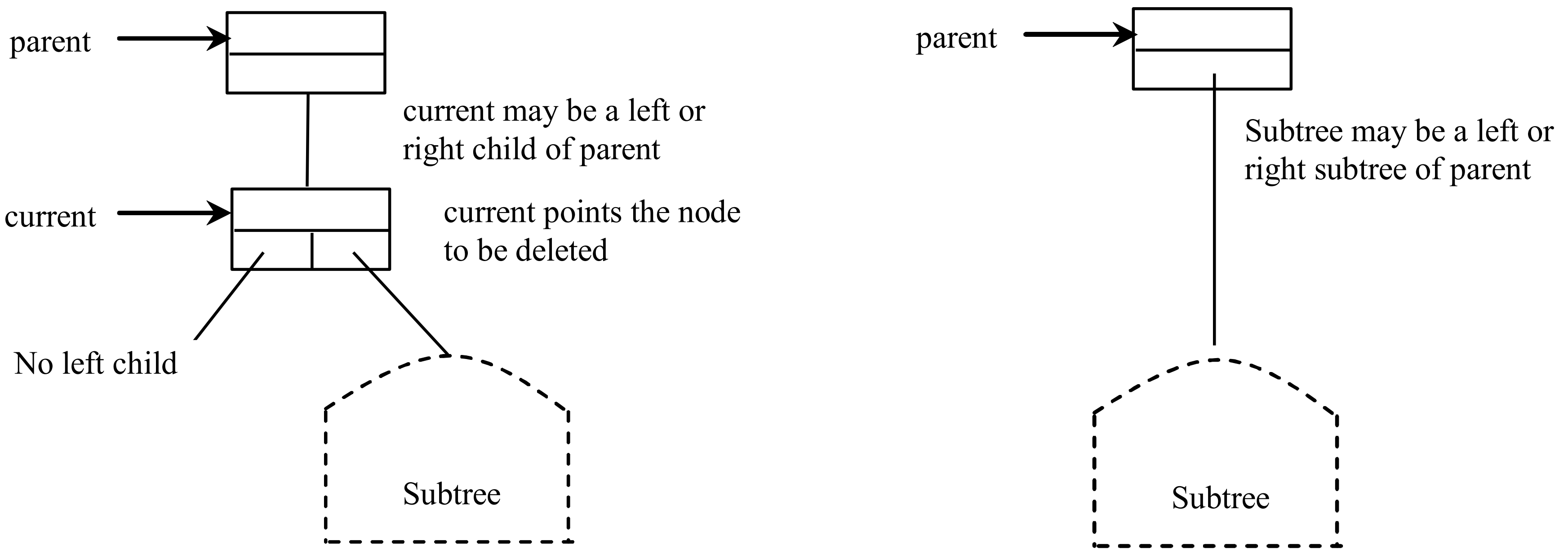
Preorder: George, Adam, Daniel, Michael, Jones, Tom, Peter

Deleting Elements in a Binary Search Tree

- Need to **first locate** the **node** that **contains** the **element and** also **its parent** node.
- Let **current point to** the **node** that **contains the element** in the binary tree and **parent point to** the **parent of** the **current node**.
- The **current** node **may be** a **left** child **or** a **right** child **of** the **parent** node. There are two cases to consider:

Deleting Elements in a Binary Search Tree

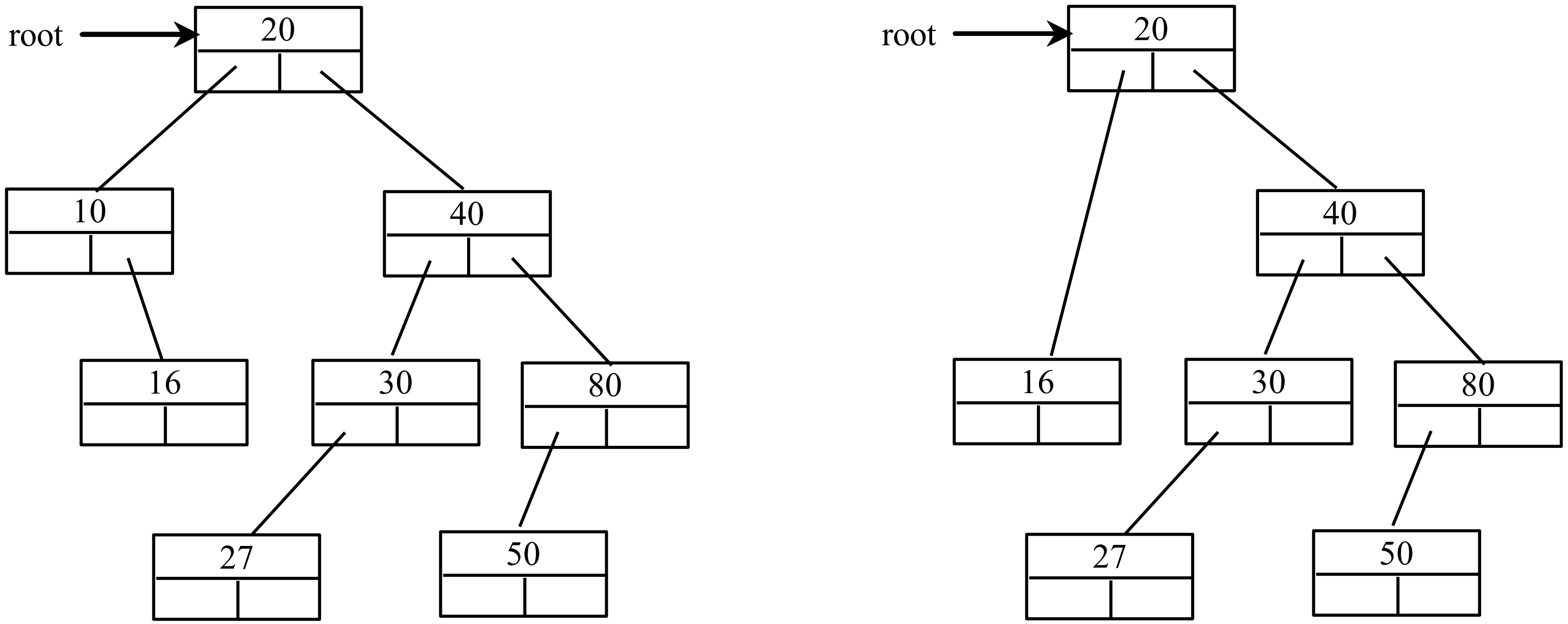
Case 1: The **current node does not have a left child**, as shown in this figure (a). Simply **connect the parent with the right child of the current node**, as shown in this figure (b).



(a) (b)

Deleting Elements in a Binary Search Tree

For example, to delete node <u>10</u> in figure below. Connect the parent of node <u>10</u> with the right child of node <u>10</u> as shown in part (b).



(a) (b)

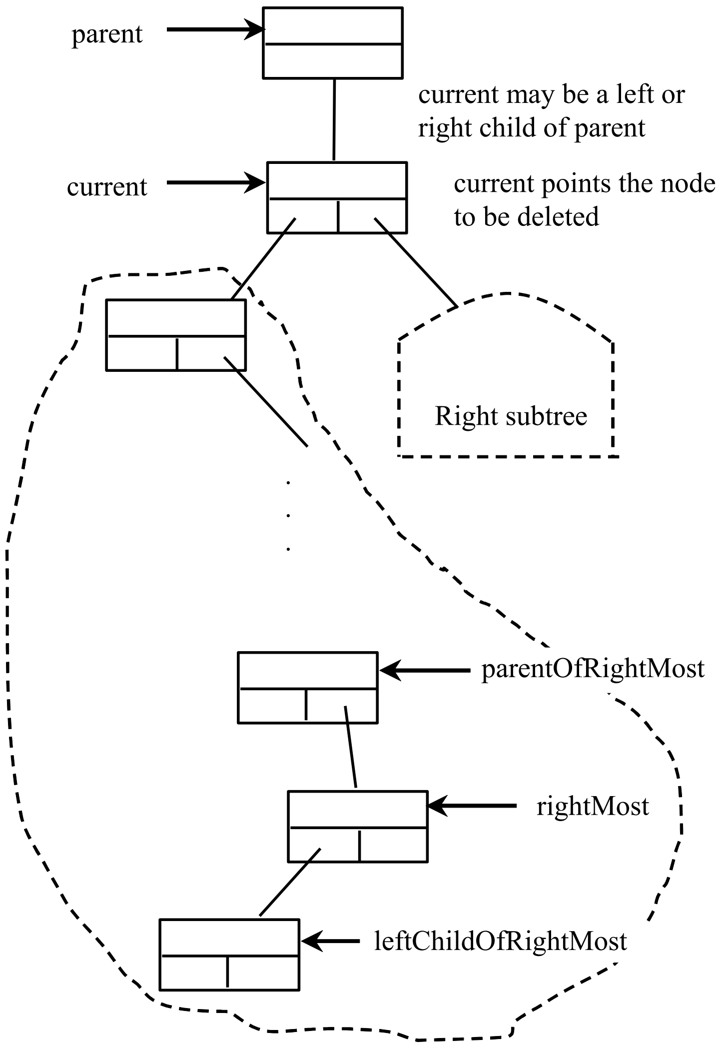
Deleting Elements in a Binary Search Tree

Case 2: The **current node has a left child**.

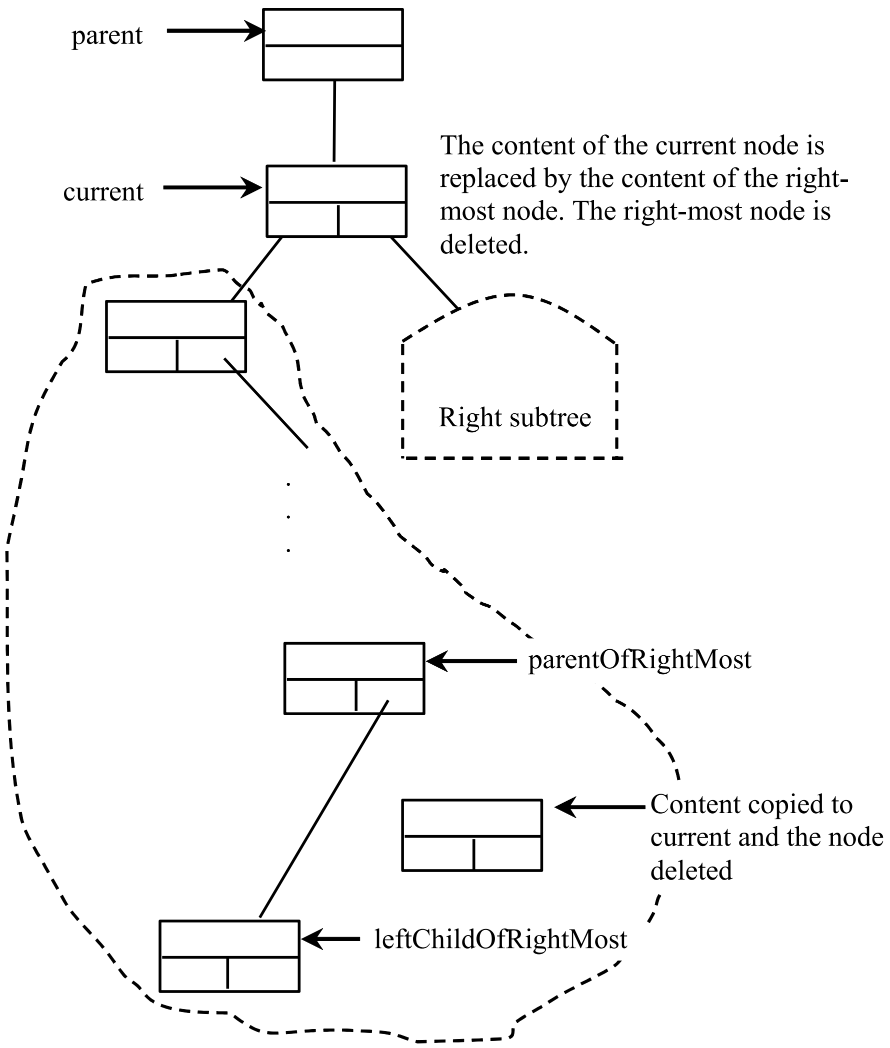
- Let **rightMost point** to the **node** that contains the **largest element in** the **left subtree of** the **current node** and **parentOfRightMost point to** the **parent** node **of** the **rightMost node**, as shown in next figure part (a).
- Note that the **rightMost node cannot have a right child**, but may have a left child.
- **Replace** the **element** value **in** the **current** node **with** the one in the **rightMost node**, **connect** the **parentOfRightMost** node **with** the **left** child **of** the **rightMost node**, and **delete** the **rightMost node**, as shown in next figure part (b).

Deleting Elements in a Binary Search Tree

Case 2:



(a)



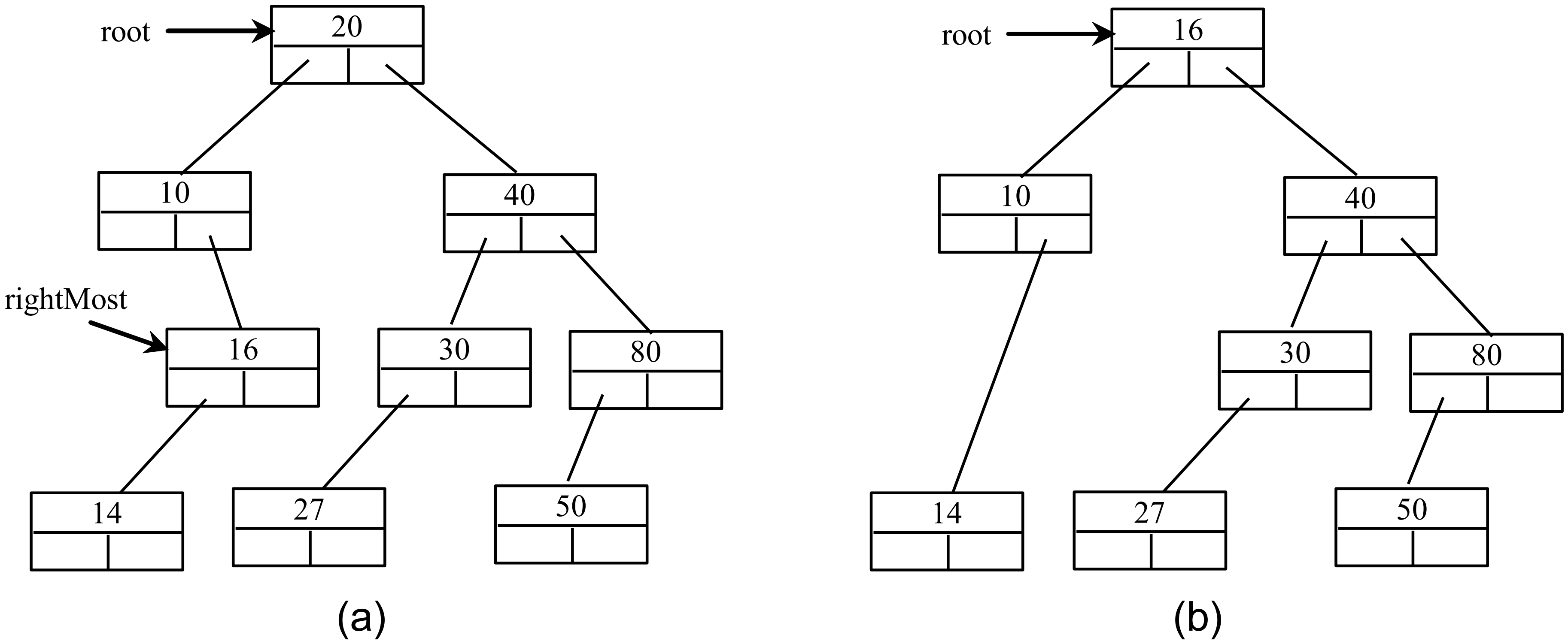
(b)

Algorithm for deleting an element

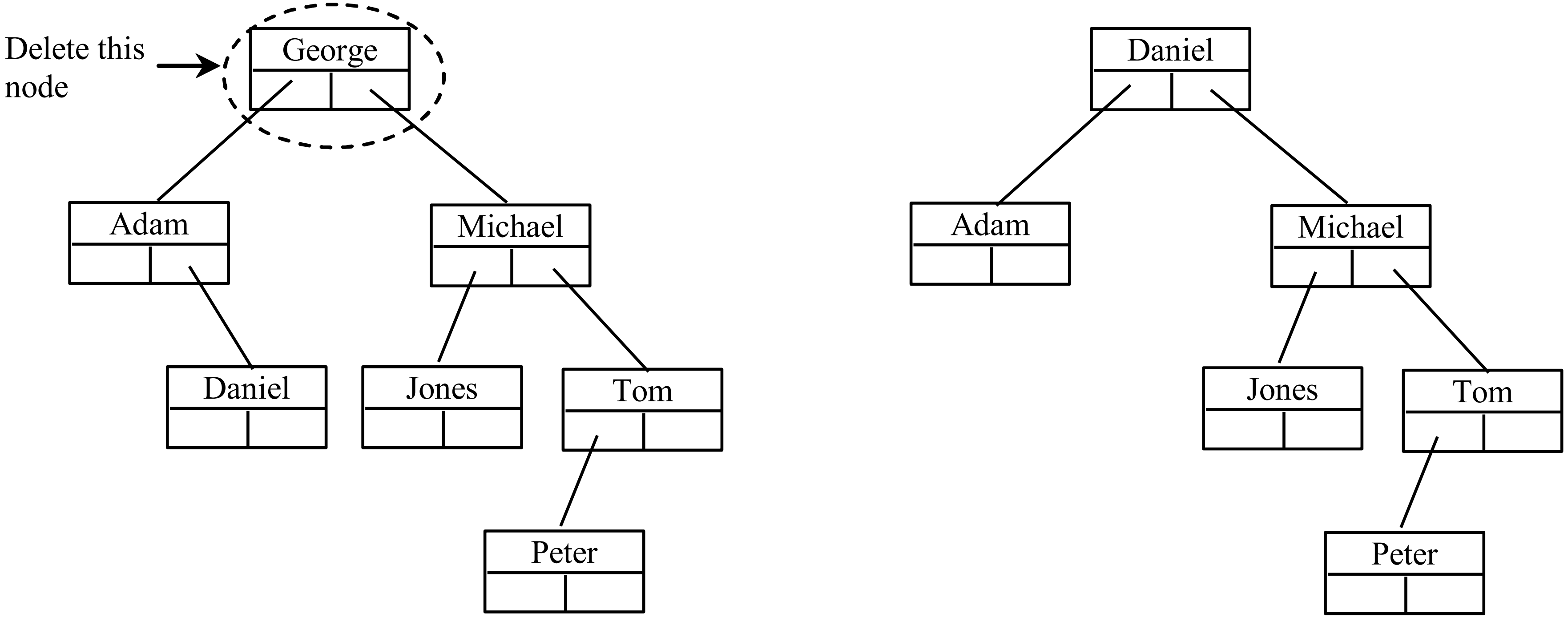
```
boolean delete(E e) {
  Locate element e in the tree;
  if element e is not found
    return true;

  Let current be the node that contains e and parent be
    the parent of current;

  if (current has no left child) // Case 1
    Connect the right child of
      current with parent; now current is not referenced, so
    it is eliminated;
  else // Case 2
    Locate the rightmost node in the left subtree of current.
    Copy the element value in the rightmost node to current.
    Connect the parent of the rightmost node to the left child
      of rightmost node;

  return true; // Element deleted
}
```

Deleting Elements in a Binary Search Tree

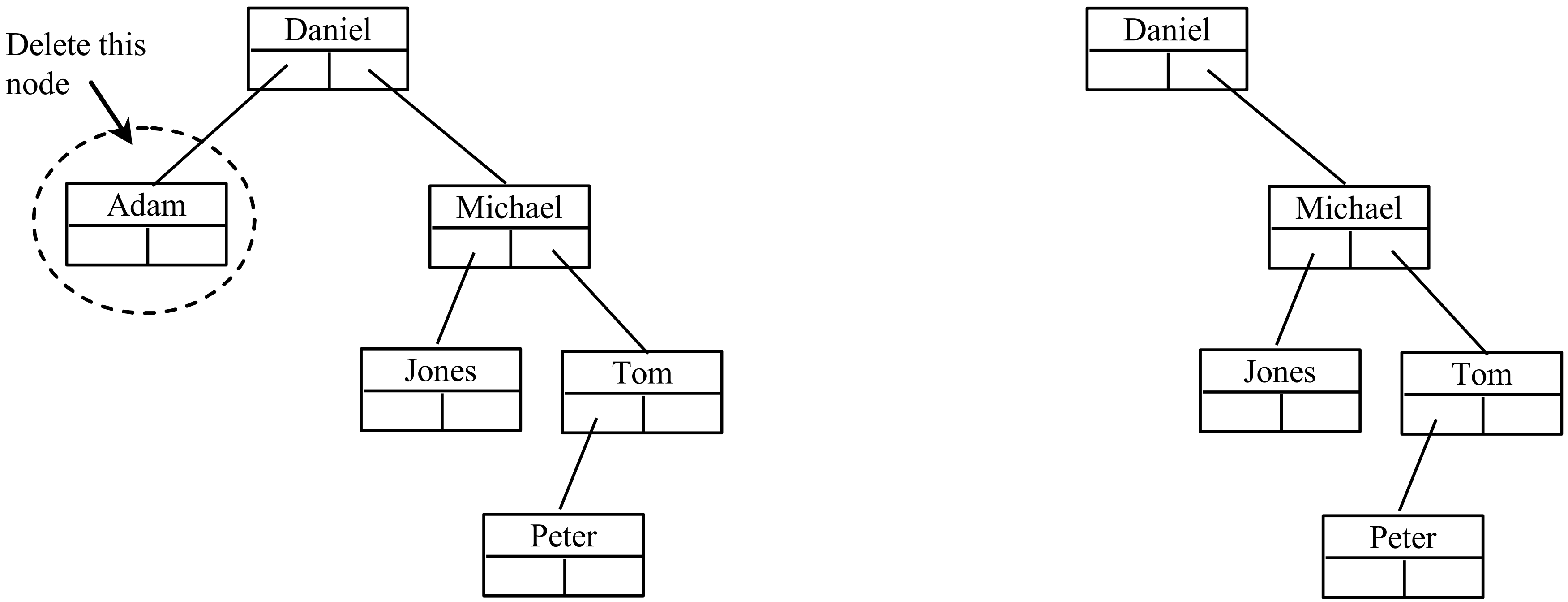
Case 2 example: delete 20



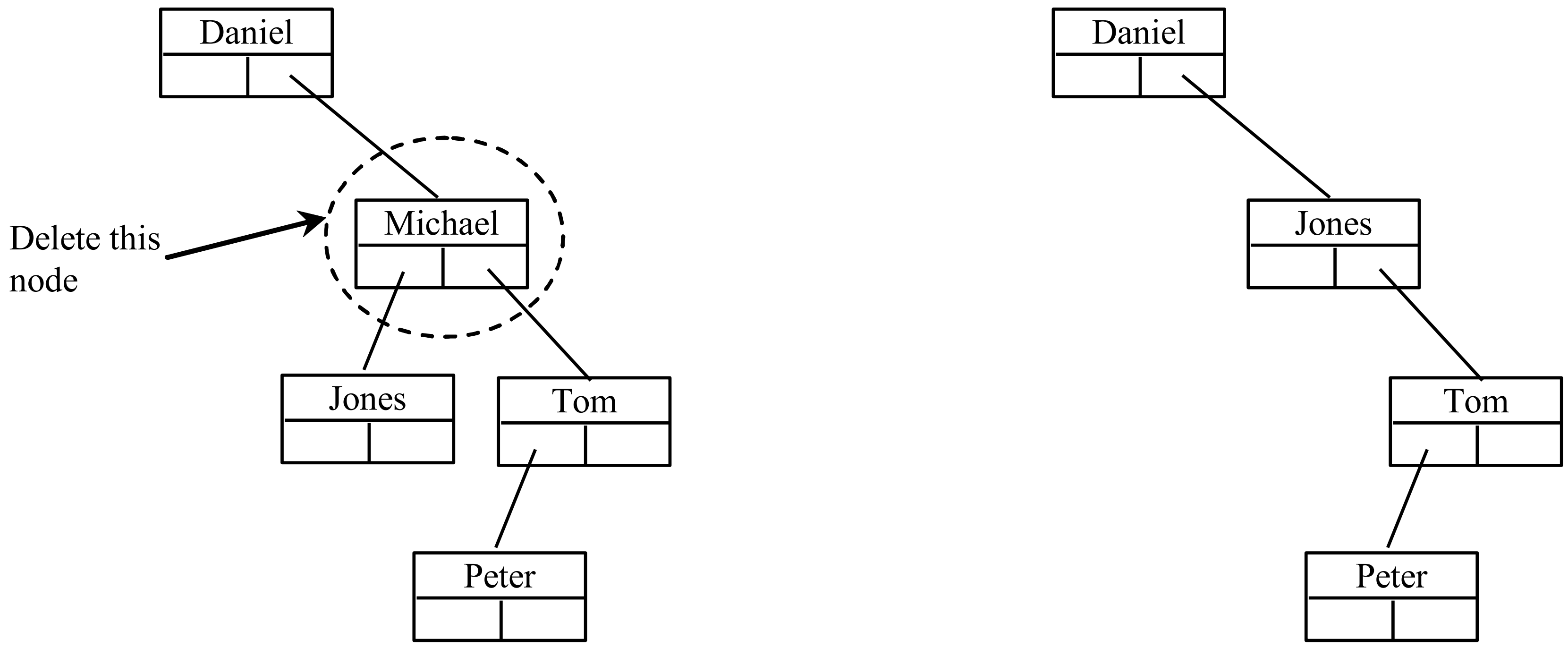
Examples


Delete this node
George
Adam
Michael
Daniel
Jones
Tom
Peter
Daniel
Adam
Michael
Jones
Tom
Peter

Examples


Delete this node
Daniel
Adam
Michael
Jones
Tom
Peter
Daniel
Michael
Jones
Tom
Peter

Examples



http://www.cs.armstrong.edu/liang/intro10e/html/TestBSTDelete.html

[TestBSTDelete]

Run

Reference

- Chapter 25, Liang, Introduction to Java Programming, 10^{th} Edition, Global Edition, Pearson, 2015
- Chapter 23, Frank M. Carrano and Timothy Henry. 2015. Data Structures and Abstractions with Java, 4th Edition. Pearson.

THANK YOU