WIA 1002 DATA STRUCTURE

SEM 2, SESSION 2024/205

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Recursion

- Concept
- Fibonacci Numbers
- Recursion vs Iteration
- Problem Solving

CONCEPT

Recursion

- Programming technique where **a method calls itself** to fulfil its overall purpose.
- Also known as **Self-Invocation**

Characteristics of Recursion

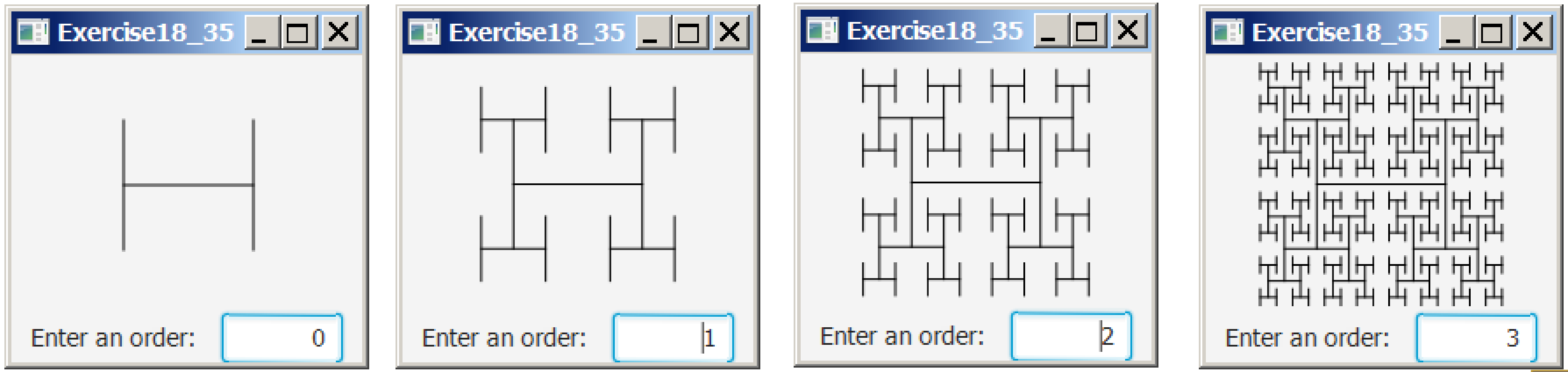
All recursive methods share two essential components:

1. **Base Case(s)**: These are the simplest scenarios where the problem can be solved directly without further recursion. They serve as the stopping condition to prevent infinite recursion.
2. **Recursive Case**: In this part, the method calls itself to solve a smaller or simpler version of the original problem. Each recursive call should move the problem closer to the base case.

Motivations

H-trees - used in a very large-scale integration (VLSI) design as a clock distribution network for routing timing signals to all parts of a chip with equal propagation delays.

How to display H-trees? A good approach is to use recursion.



Computing Factorial

3! = 3 * 2 * 1;
5! = 5 * 4 * 3 * 2 * 1;

The factorial of a number **n** can be recursively defined as follows:

0! = 1; **//Base Case**

n! = n * (n - 1)!; n > 0 **//Recursive Case**

Computing Factorial

How do you find **n!** for a given **n**?

> To find **1!** is easy, because you know that **0!** is **1**, and **1!** is **1** × **0!**. Assuming that you know **(n - 1)!**, you can obtain **n!** immediately by using **n** × **(n - 1)!**. Thus, the problem of computing **n!** is reduced to computing **(n - 1)!**. When computing **(n - 1)!**, you can apply the same idea recursively until **n** is reduced to **0**.

Computing Factorial

Let **factorial(n)** be the method for computing **n!**.

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

ComputeFactorial

Run

LISTING 18.1 ComputeFactorial.java

```java
import java.util.Scanner;

public class ComputeFactorial {
  /** Main method */
  public static void main(String[] args) {
    // Create a Scanner
    Scanner input = new Scanner(System.in);
    System.out.print("Enter a nonnegative integer: ");
    int n = input.nextInt();

    // Display factorial
    System.out.println("Factorial of " + n + " is " + factorial(n));
  }

  /** Return the factorial for the specified number */
  public static long factorial(int n) {
    if (n == 0) // Base case
      return 1;
    else
      return n * factorial(n - 1); // Recursive call
  }
}
```

base case

recursion

```
Enter a nonnegative integer: 4 ↵Enter
Factorial of 4 is 24
```

animation

Computing Factorial

factorial(0) = 1;
factorial(n) = n*factorial(n-1);

factorial(4)

animation

Computing Factorial

factorial(0) = 1;
factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)

animation

Computing Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)

= 4 * (3 * factorial(2))

animation

Computing Factorial

factorial(0) = 1;
factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)
= 4 * (3 * factorial(2))
= 4 * (3 * (2 * factorial(1)))

animation

Computing Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)

= 4 * (3 * factorial(2))

= 4 * (3 * (2 * factorial(1)))

= 4 * (3 * (2 * (1 * factorial(0))))

1. temporarily suspended until invocation complete

2. Invocation complete when it reaches base case (n==0)

animation

Computing Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)

= 4 * (3 * factorial(2))

= 4 * (3 * (2 * factorial(1)))

= 4 * (3 * (2 * (1 * factorial(0))))

= 4 * (3 * (2 * (1 * 1)))

animation

Computing Factorial

factorial(0) = 1;
factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)
= 4 * (3 * factorial(2))
= 4 * (3 * (2 * factorial(1)))
= 4 * (3 * (2 * (1 * factorial(0))))
= 4 * (3 * (2 * (1 * 1)))
= 4 * (3 * (2 * 1))

animation

Computing Factorial

factorial(0) = 1;
factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)
= 4 * (3 * factorial(2))
= 4 * (3 * (2 * factorial(1)))
= 4 * (3 * (2 * (1 * factorial(0))))
= 4 * (3 * (2 * (1 * 1)))
= 4 * (3 * (2 * 1))
= 4 * (3 * 2)

animation

Computing Factorial

factorial(0) = 1;
factorial(n) = n*factorial(n-1);

```
factorial(4) = 4 * factorial(3)
             = 4 * (3 * factorial(2))
             = 4 * (3 * (2 * factorial(1)))
             = 4 * (3 * ( 2 * (1 * factorial(0))))
             = 4 * (3 * ( 2 * ( 1 * 1)))
             = 4 * (3 * ( 2 * 1))
             = 4 * (3 * 2)
             = 4 * 6
```

animation

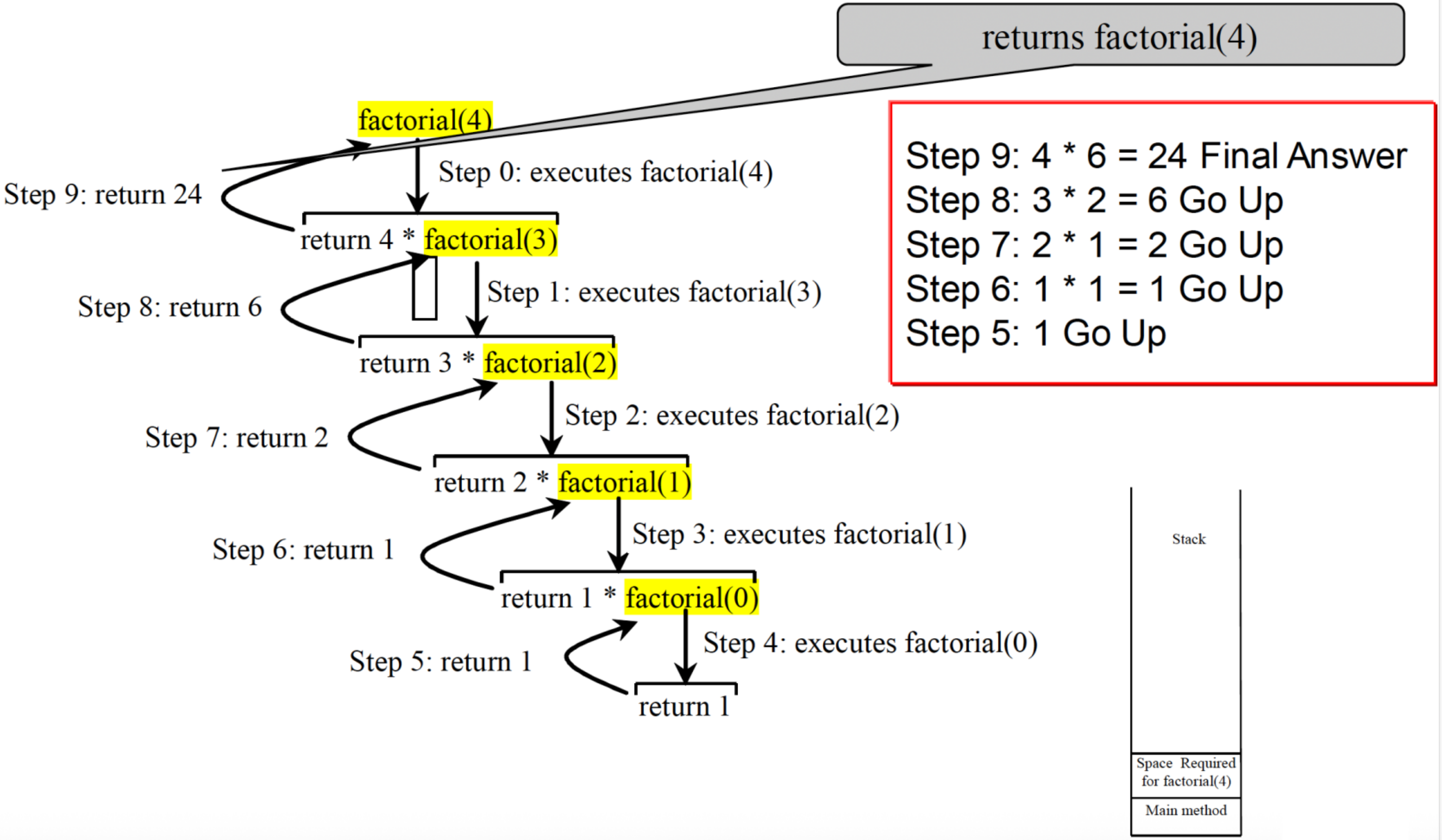
Computing Factorial

factorial(0) = 1;

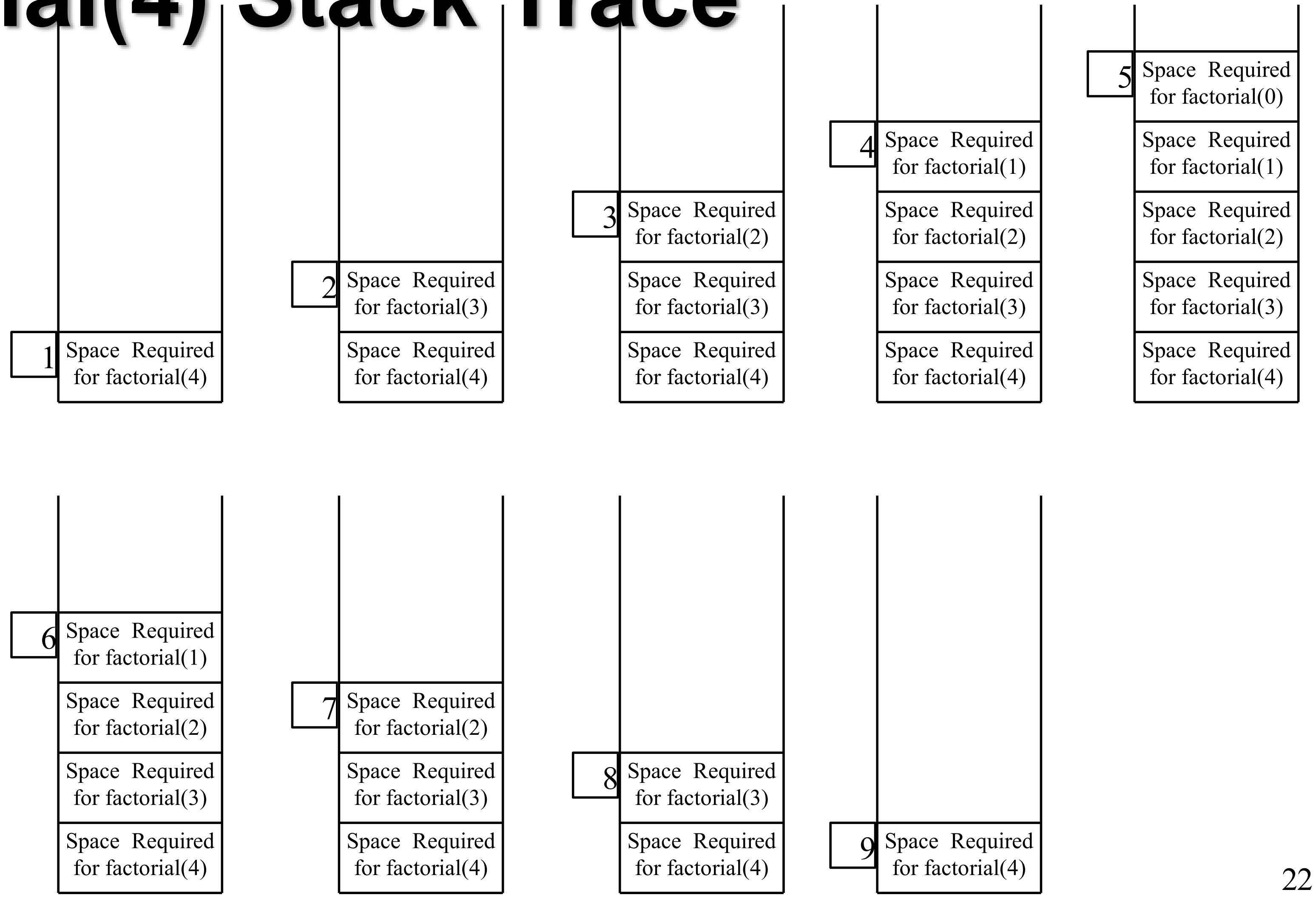
factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)
= 4 * (3 * factorial(2))
= 4 * (3 * (2 * factorial(1)))
= 4 * (3 * (2 * (1 * factorial(0))))
= 4 * (3 * (2 * (1 * 1)))
= 4 * (3 * (2 * 1))
= 4 * (3 * 2)
= 4 * 6
= 24

Trace Recursive factorial



factorial(4) Stack Trace



FIBONACCI NUMBERS

Fibonacci Numbers

```
Fibonacci series: 0 1 1 2 3 5 8 13 21 34 55 89…
          indices: 0 1 2 3 4 5 6 7  8  9  10 11
```

The Fibonacci series begins with **0** and **1**, and each subsequent number is the sum of the preceding two. The series can be recursively defined as:

fib(0) = 0; **//Base case**

fib(1) = 1; **//Base case**

fib(index) = fib(index -1) + fib(index -2); index >=2 **//Recursive Case**

Fibonacci Numbers

```
Fibonacci series: 0 1 1 2 3 5 8 13 21 34 55 89…
         indices: 0 1 2 3 4 5 6 7  8  9  10 11
```

fib(3) = fib(2) + fib(1)
= (fib(1) + fib(0)) + fib(1)
= (1 + 0) +fib(1)
= 1 + fib(1)
= 1 + 1
= 2

Fibonacci Numbers

How do you find **fib(index)** for a given **index**?

It is easy to find **fib(2)**, because you know **fib(0)** and **fib(1)**. Assuming that you know **fib(index - 2)** and **fib(index - 1)**, you can obtain **fib(index)** immediately. Thus, the problem of computing **fib(index)** is reduced to computing **fib(index - 2)** and **fib(index - 1)**. When doing so, apply the idea recursively until **index** is reduced to **0** or **1**.

The base case is **index = 0** or **index = 1**. If you call the method with **index = 0** or **index = 1**, it immediately returns the result. If you call the method with **index >= 2**, it divides the problem into two subproblems for computing **fib(index - 1)** and **fib(index - 2)** using recursive calls.

LISTING 18.2 ComputeFibonacci.java

```java
import java.util.Scanner;

public class ComputeFibonacci {
  /** Main method */
  public static void main(String[] args) {
    // Create a Scanner
    Scanner input = new Scanner(System.in);
    System.out.print("Enter an index for a Fibonacci number: ");
    int index = input.nextInt();

    // Find and display the Fibonacci number
    System.out.println("The Fibonacci number at index "
      + index + " is " + fib(index));
  }

  /** The method for finding the Fibonacci number */
  public static long fib(long index) {
    if (index == 0) // Base case
      return 0;
    else if (index == 1) // Base case
      return 1;
    else  // Reduction and recursive calls
      return fib(index - 1) + fib(index - 2);
  }
}
```

base case

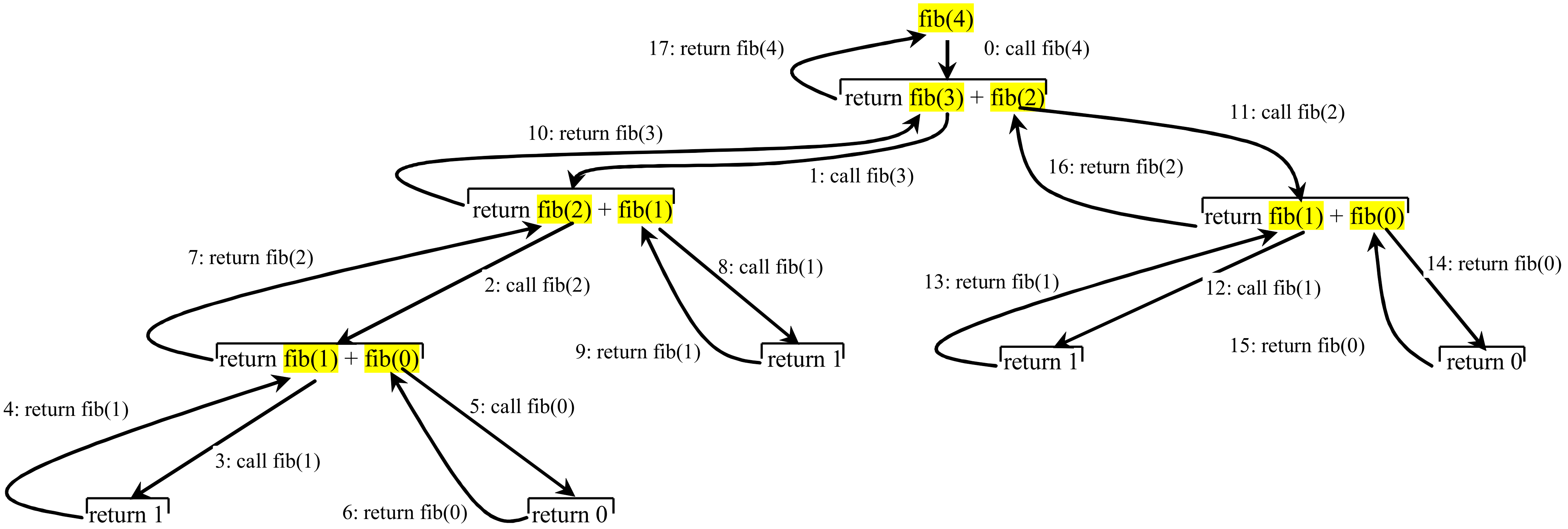
recursion

```
Enter an index for a Fibonacci number: 1 Enter
The Fibonacci number at index 1 is 1
```

```
Enter an index for a Fibonacci number: 6 Enter
The Fibonacci number at index 6 is 8
```

```
Enter an index for a Fibonacci number: 7 Enter
The Fibonacci number at index 7 is 13
```

Fibonnaci Numbers, cont.



RECAP

Characteristics of Recursion

All recursive methods share two essential components:

1. **Base Case(s)**: These are the simplest scenarios where the problem can be solved directly without further recursion. They serve as the stopping condition to prevent infinite recursion.
2. **Recursive Case**: In this part, the method calls itself to solve a smaller or simpler version of the original problem. Each recursive call should move the problem closer to the base case.

What happens if a recursive method never reaches a base case?

- Infinite recursion - occurs if recursion does not reduce the problem in a manner that allows it to eventually converge into the base case
- The stack will never stop growing.
- But OS limits the stack to a particular height, so that no program eats up too much memory.
- If a program's stack exceeds this size, the computer initiates an exception (**StackOverflowError)**, which typically would crash the program.

What is the output? What is the base case?

```
public static void main(String[] args) {
    recursion(735);
     // System.out.println(result);
}
public static void recursion(int n) {
        if (n > 0) {
  System.out.print(n % 10);
  recursion(n / 10);
}
  }
```

What is the output? What is the base case?

```
public static void main(String[] args) {
    recursion(735);
     // System.out.println(result);
}
public static void recursion(int n) {
        if (n > 0) {
   System.out.print(n % 10);
   recursion(n / 10);
}
   }
```

Output : 537
Base case : n <= 0

What is the ouput?

```
public static long factorial(int n) {
    return n * factorial(n - 1);
}
```

What is the ouput?

```
public static long factorial(int n) {
    return n * factorial(n - 1);
}
```

Output : The method runs infinitely and causes a StackOverflowError.

RECURSION VS ITERATION

Recursion vs Iteration

- Recursion and loop are related concepts.
- Anything you can do with a loop, you can do with recursion, and vice versa.
- Sometimes recursion is simpler to write, and sometimes loop is, but in principle they are interchangeable.

Recursion vs Iteration

Implementing factorial using a loop:

```
public static long factorialLoop(int n) {
    long result = 1;
    while (n>0) {
        result *= n;
        n--;
    }
    return result;
  }
```

Recursion vs Iteration

Recursion

- Terminate when a base case is reached
- Each recursive call requires extra space on the stack frame (memory)
- If we get infinite recursion, it may result in stack overflow

Iteration

- Terminates when a condition is proven to be false
- Each iteration does not require any extra space
- An infinite loop could loop forever since there is no extra memory being created

Recursion

- An alternative form of program control.
- Repetition without a loop.
- Substantial overhead –
  - The system must assign space for all of the method's local variables and parameters each time a method is called.
  - Consume considerable memory and requires extra time to manage the additional space.
- However, it is good for solving the problems that are inherently recursive.

PROBLEM SOLVING

Problem Solving Using Recursion - Think Recursively

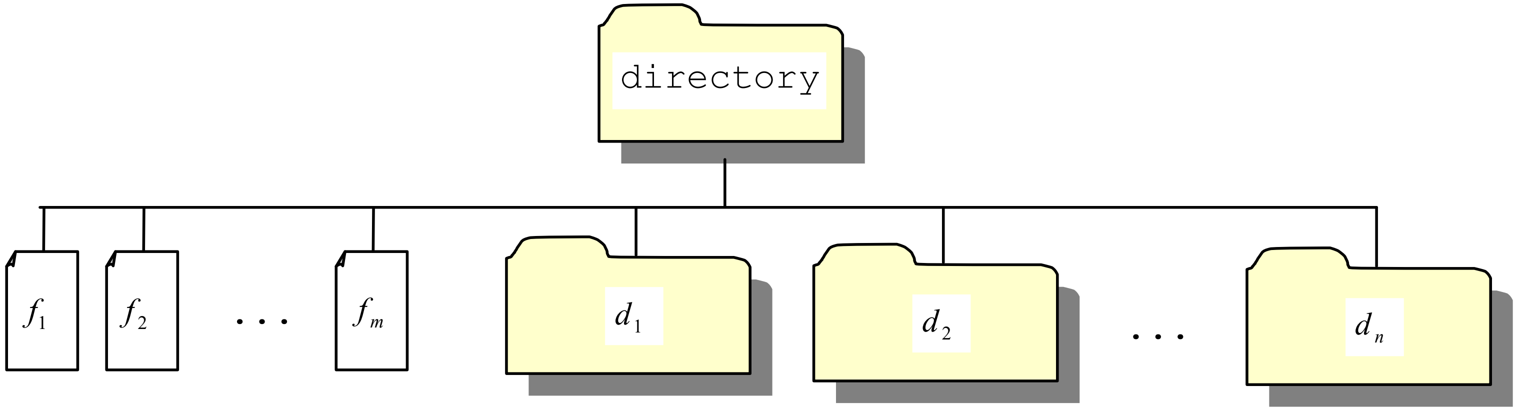
- Example:
  - a simple problem of printing a message for n times.
  - break the problem into two subproblems:
    - one problem is to print the message one time
    - the other problem is to print the message for n-1 times. The 2nd problem is the same as the original problem with a smaller size.
  - the base case for the problem is n==0.

```
public static void nPrintln(String message, int times) {
   if (times >= 1) {
         System.out.println(message);
         nPrintln(message, times - 1);
   } // The base case is times == 0
}
```

nPrintln("Welcome", 5);

Directory Size

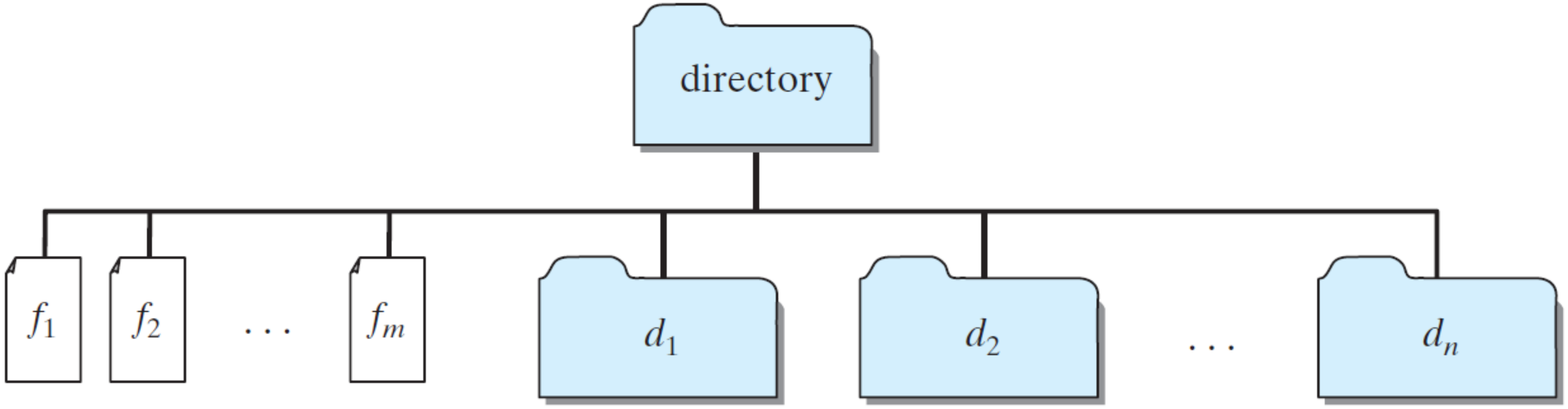
- A problem that is difficult to solve without using recursion.
- The size of a directory is the sum of the sizes of all files in the directory.
- A directory may contain subdirectories.



Directory Size

The size of the directory can be defined recursively as follows:

$$size(d) = size(f_1) + size(f_2) + \ldots + size(f_m) + size(d_1) + size(d_2) + \ldots + size(d_n)$$



LISTING 18.7 DirectorySize.java

```java
import java.io.File;
import java.util.Scanner;

public class DirectorySize {
  public static void main(String[] args) {
    // Prompt the user to enter a directory or a file
    System.out.print("Enter a directory or a file: ");
    Scanner input = new Scanner(System.in);
    String directory = input.nextLine();

    // Display the size
    System.out.println(getSize(new File(directory)) + " bytes");
  }

  public static long getSize(File file) {
    long size = 0; // Store the total size of all files

    if (file.isDirectory()) {
      File[] files = file.listFiles(); // All files and subdirectories
      for (int i = 0; files != null && i < files.length; i++) {
        size += getSize(files[i]); // Recursive call
      }
    }
    else { // Base case
      size += file.length();
    }

    return size;
  }
}
```

```
Enter a directory or a file: c:\book ↵Enter
48619631 bytes
```

```
Enter a directory or a file: c:\book\Welcome.java ↵Enter
172 bytes
```

References

Chapter 18 Recursion, Liang, Introduction to Java Programming, 10th Edition, Global Edition, Pearson, 2015

Q&A