Project 8

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8.1

Using the crime data set uscrime.txt to build a regression model using: 1. Stepwise regression 2. Lasso 3. Elastic net. For Parts 2 and 3, remember to scale the data first otherwise, the regression coefficients will be on different scales and the constraint won't have the desired effect. For Parts 2 and 3, use the glmnet function in R

```
library(MASS)
library(glmnet)
```

```
## Loading required package: Matrix

## Loaded glmnet 4.1-8
```

Load data:

```
#data <- read.table("Documents/OMSCS/Analytics Modeling/Assignments/Assignment8/uscrime.txt", header =

data <- read.table("uscrime.txt", header = TRUE, stringsAsFactors = FALSE)

data
```

```
##       M So   Ed  Po1  Po2    LF   M.F Pop   NW    U1  U2 Wealth Ineq     Prob
## 1  15.1  1  9.1  5.8  5.6 0.510  95.0  33 30.1 0.108 4.1   3940 26.1 0.084602
## 2  14.3  0 11.3 10.3  9.5 0.583 101.2  13 10.2 0.096 3.6   5570 19.4 0.029599
## 3  14.2  1  8.9  4.5  4.4 0.533  96.9  18 21.9 0.094 3.3   3180 25.0 0.083401
## 4  13.6  0 12.1 14.9 14.1 0.577  99.4 157  8.0 0.102 3.9   6730 16.7 0.015801
## 5  14.1  0 12.1 10.9 10.1 0.591  98.5  18  3.0 0.091 2.0   5780 17.4 0.041399
## 6  12.1  0 11.0 11.8 11.5 0.547  96.4  25  4.4 0.084 2.9   6890 12.6 0.034201
## 7  12.7  1 11.1  8.2  7.9 0.519  98.2   4 13.9 0.097 3.8   6200 16.8 0.042100
## 8  13.1  1 10.9 11.5 10.9 0.542  96.9  50 17.9 0.079 3.5   4720 20.6 0.040099
## 9  15.7  1  9.0  6.5  6.2 0.553  95.5  39 28.6 0.081 2.8   4210 23.9 0.071697
## 10 14.0  0 11.8  7.1  6.8 0.632 102.9   7  1.5 0.100 2.4   5260 17.4 0.044498
## 11 12.4  0 10.5 12.1 11.6 0.580  96.6 101 10.6 0.077 3.5   6570 17.0 0.016201
## 12 13.4  0 10.8  7.5  7.1 0.595  97.2  47  5.9 0.083 3.1   5800 17.2 0.031201
## 13 12.8  0 11.3  6.7  6.0 0.624  97.2  28  1.0 0.077 2.5   5070 20.6 0.045302
## 14 13.5  0 11.7  6.2  6.1 0.595  98.6  22  4.6 0.077 2.7   5290 19.0 0.053200
## 15 15.2  1  8.7  5.7  5.3 0.530  98.6  30  7.2 0.092 4.3   4050 26.4 0.069100
## 16 14.2  1  8.8  8.1  7.7 0.497  95.6  33 32.1 0.116 4.7   4270 24.7 0.052099
## 17 14.3  0 11.0  6.6  6.3 0.537  97.7  10  0.6 0.114 3.5   4870 16.6 0.076299
## 18 13.5  1 10.4 12.3 11.5 0.537  97.8  31 17.0 0.089 3.4   6310 16.5 0.119804
## 19 13.0  0 11.6 12.8 12.8 0.536  93.4  51  2.4 0.078 3.4   6270 13.5 0.019099
```

```
## 20 12.5  0 10.8 11.3 10.5 0.567  98.5  78  9.4 0.130 5.8   6260 16.6 0.034801
## 21 12.6  0 10.8  7.4  6.7 0.602  98.4  34  1.2 0.102 3.3   5570 19.5 0.022800
## 22 15.7  1  8.9  4.7  4.4 0.512  96.2  22 42.3 0.097 3.4   2880 27.6 0.089502
## 23 13.2  0  9.6  8.7  8.3 0.564  95.3  43  9.2 0.083 3.2   5130 22.7 0.030700
## 24 13.1  0 11.6  7.8  7.3 0.574 103.8   7  3.6 0.142 4.2   5400 17.6 0.041598
## 25 13.0  0 11.6  6.3  5.7 0.641  98.4  14  2.6 0.070 2.1   4860 19.6 0.069197
## 26 13.1  0 12.1 16.0 14.3 0.631 107.1   3  7.7 0.102 4.1   6740 15.2 0.041698
## 27 13.5  0 10.9  6.9  7.1 0.540  96.5   6  0.4 0.080 2.2   5640 13.9 0.036099
## 28 15.2  0 11.2  8.2  7.6 0.571 101.8  10  7.9 0.103 2.8   5370 21.5 0.038201
## 29 11.9  0 10.7 16.6 15.7 0.521  93.8 168  8.9 0.092 3.6   6370 15.4 0.023400
## 30 16.6  1  8.9  5.8  5.4 0.521  97.3  46 25.4 0.072 2.6   3960 23.7 0.075298
## 31 14.0  0  9.3  5.5  5.4 0.535 104.5   6  2.0 0.135 4.0   4530 20.0 0.041999
## 32 12.5  0 10.9  9.0  8.1 0.586  96.4  97  8.2 0.105 4.3   6170 16.3 0.042698
## 33 14.7  1 10.4  6.3  6.4 0.560  97.2  23  9.5 0.076 2.4   4620 23.3 0.049499
## 34 12.6  0 11.8  9.7  9.7 0.542  99.0  18  2.1 0.102 3.5   5890 16.6 0.040799
## 35 12.3  0 10.2  9.7  8.7 0.526  94.8 113  7.6 0.124 5.0   5720 15.8 0.020700
## 36 15.0  0 10.0 10.9  9.8 0.531  96.4   9  2.4 0.087 3.8   5590 15.3 0.006900
## 37 17.7  1  8.7  5.8  5.6 0.638  97.4  24 34.9 0.076 2.8   3820 25.4 0.045198
## 38 13.3  0 10.4  5.1  4.7 0.599 102.4   7  4.0 0.099 2.7   4250 22.5 0.053998
## 39 14.9  1  8.8  6.1  5.4 0.515  95.3  36 16.5 0.086 3.5   3950 25.1 0.047099
## 40 14.5  1 10.4  8.2  7.4 0.560  98.1  96 12.6 0.088 3.1   4880 22.8 0.038801
## 41 14.8  0 12.2  7.2  6.6 0.601  99.8   9  1.9 0.084 2.0   5900 14.4 0.025100
## 42 14.1  0 10.9  5.6  5.4 0.523  96.8   4  0.2 0.107 3.7   4890 17.0 0.088904
## 43 16.2  1  9.9  7.5  7.0 0.522  99.6  40 20.8 0.073 2.7   4960 22.4 0.054902
## 44 13.6  0 12.1  9.5  9.6 0.574 101.2  29  3.6 0.111 3.7   6220 16.2 0.028100
## 45 13.9  1  8.8  4.6  4.1 0.480  96.8  19  4.9 0.135 5.3   4570 24.9 0.056202
## 46 12.6  0 10.4 10.6  9.7 0.599  98.9  40  2.4 0.078 2.5   5930 17.1 0.046598
## 47 13.0  0 12.1  9.0  9.1 0.623 104.9   3  2.2 0.113 4.0   5880 16.0 0.052802
##       Time Crime
## 1  26.2011   791
## 2  25.2999  1635
## 3  24.3006   578
## 4  29.9012  1969
## 5  21.2998  1234
## 6  20.9995   682
## 7  20.6993   963
## 8  24.5988  1555
## 9  29.4001   856
## 10 19.5994   705
## 11 41.6000  1674
## 12 34.2984   849
## 13 36.2993   511
## 14 21.5010   664
## 15 22.7008   798
## 16 26.0991   946
## 17 19.1002   539
## 18 18.1996   929
## 19 24.9008   750
## 20 26.4010  1225
## 21 37.5998   742
## 22 37.0994   439
## 23 25.1989  1216
## 24 17.6000   968
## 25 21.9003   523
```

```
## 26 22.1005  1993
## 27 28.4999   342
## 28 25.8006  1216
## 29 36.7009  1043
## 30 28.3011   696
## 31 21.7998   373
## 32 30.9014   754
## 33 25.5005  1072
## 34 21.6997   923
## 35 37.4011   653
## 36 44.0004  1272
## 37 31.6995   831
## 38 16.6999   566
## 39 27.3004   826
## 40 29.3004  1151
## 41 30.0001   880
## 42 12.1996   542
## 43 31.9989   823
## 44 30.0001  1030
## 45 32.5996   455
## 46 16.6999   508
## 47 16.0997   849
```

```
set.seed(1)
```

###1. Stepwise Regression model

Analyze variable importance:

```
linear <- lm(Crime~., data)

stepwise <- stepAIC(linear, direction = "both")
```

```
## Start:  AIC=514.65
## Crime ~ M + So + Ed + Po1 + Po2 + LF + M.F + Pop + NW + U1 +
##     U2 + Wealth + Ineq + Prob + Time
##
##          Df Sum of Sq     RSS    AIC
## - So      1        29 1354974 512.65
## - LF      1      8917 1363862 512.96
## - Time    1     10304 1365250 513.00
## - Pop     1     14122 1369068 513.14
## - NW      1     18395 1373341 513.28
## - M.F     1     31967 1386913 513.74
## - Wealth  1     37613 1392558 513.94
## - Po2     1     37919 1392865 513.95
## <none>                1354946 514.65
## - U1      1     83722 1438668 515.47
## - Po1     1    144306 1499252 517.41
## - U2      1    181536 1536482 518.56
## - M       1    193770 1548716 518.93
## - Prob    1    199538 1554484 519.11
## - Ed      1    402117 1757063 524.86
## - Ineq    1    423031 1777977 525.42
```

```
##
## Step:  AIC=512.65
## Crime ~ M + Ed + Po1 + Po2 + LF + M.F + Pop + NW + U1 + U2 +
##     Wealth + Ineq + Prob + Time
##
##          Df Sum of Sq     RSS    AIC
## - Time    1     10341 1365315 511.01
## - LF      1     10878 1365852 511.03
## - Pop     1     14127 1369101 511.14
## - NW      1     21626 1376600 511.39
## - M.F     1     32449 1387423 511.76
## - Po2     1     37954 1392929 511.95
## - Wealth  1     39223 1394197 511.99
## <none>                1354974 512.65
## - U1      1     96420 1451395 513.88
## + So      1        29 1354946 514.65
## - Po1     1    144302 1499277 515.41
## - U2      1    189859 1544834 516.81
## - M       1    195084 1550059 516.97
## - Prob    1    204463 1559437 517.26
## - Ed      1    403140 1758114 522.89
## - Ineq    1    488834 1843808 525.13
##
## Step:  AIC=511.01
## Crime ~ M + Ed + Po1 + Po2 + LF + M.F + Pop + NW + U1 + U2 +
##     Wealth + Ineq + Prob
##
##          Df Sum of Sq     RSS    AIC
## - LF      1     10533 1375848 509.37
## - NW      1     15482 1380797 509.54
## - Pop     1     21846 1387161 509.75
## - Po2     1     28932 1394247 509.99
## - Wealth  1     36070 1401385 510.23
## - M.F     1     41784 1407099 510.42
## <none>                1365315 511.01
## - U1      1     91420 1456735 512.05
## + Time    1     10341 1354974 512.65
## + So      1        65 1365250 513.00
## - Po1     1    134137 1499452 513.41
## - U2      1    184143 1549458 514.95
## - M       1    186110 1551425 515.01
## - Prob    1    237493 1602808 516.54
## - Ed      1    409448 1774763 521.33
## - Ineq    1    502909 1868224 523.75
##
## Step:  AIC=509.37
## Crime ~ M + Ed + Po1 + Po2 + M.F + Pop + NW + U1 + U2 + Wealth +
##     Ineq + Prob
##
##          Df Sum of Sq     RSS    AIC
## - NW      1     11675 1387523 507.77
## - Po2     1     21418 1397266 508.09
## - Pop     1     27803 1403651 508.31
## - M.F     1     31252 1407100 508.42
```

```
## - Wealth  1     35035 1410883 508.55
## <none>                 1375848 509.37
## - U1      1     80954 1456802 510.06
## + LF      1     10533 1365315 511.01
## + Time    1      9996 1365852 511.03
## + So      1      3046 1372802 511.26
## - Po1     1    123896 1499744 511.42
## - U2      1    190746 1566594 513.47
## - M       1    217716 1593564 514.27
## - Prob    1    226971 1602819 514.54
## - Ed      1    413254 1789103 519.71
## - Ineq    1    500944 1876792 521.96
##
## Step:  AIC=507.77
## Crime ~ M + Ed + Po1 + Po2 + M.F + Pop + U1 + U2 + Wealth + Ineq +
##     Prob
##
##          Df Sum of Sq     RSS    AIC
## - Po2     1     16706 1404229 506.33
## - Pop     1     25793 1413315 506.63
## - M.F     1     26785 1414308 506.66
## - Wealth  1     31551 1419073 506.82
## <none>                 1387523 507.77
## - U1      1     83881 1471404 508.52
## + NW      1     11675 1375848 509.37
## + So      1      7207 1380316 509.52
## + LF      1      6726 1380797 509.54
## + Time    1      4534 1382989 509.61
## - Po1     1    118348 1505871 509.61
## - U2      1    201453 1588976 512.14
## - Prob    1    216760 1604282 512.59
## - M       1    309214 1696737 515.22
## - Ed      1    402754 1790276 517.74
## - Ineq    1    589736 1977259 522.41
##
## Step:  AIC=506.33
## Crime ~ M + Ed + Po1 + M.F + Pop + U1 + U2 + Wealth + Ineq +
##     Prob
##
##          Df Sum of Sq     RSS    AIC
## - Pop     1     22345 1426575 505.07
## - Wealth  1     32142 1436371 505.39
## - M.F     1     36808 1441037 505.54
## <none>                 1404229 506.33
## - U1      1     86373 1490602 507.13
## + Po2     1     16706 1387523 507.77
## + NW      1      6963 1397266 508.09
## + So      1      3807 1400422 508.20
## + LF      1      1986 1402243 508.26
## + Time    1       575 1403654 508.31
## - U2      1    205814 1610043 510.76
## - Prob    1    218607 1622836 511.13
## - M       1    307001 1711230 513.62
## - Ed      1    389502 1793731 515.83
```

```
## - Ineq    1    608627 2012856 521.25
## - Po1     1   1050202 2454432 530.57
##
## Step:  AIC=505.07
## Crime ~ M + Ed + Po1 + M.F + U1 + U2 + Wealth + Ineq + Prob
##
##          Df Sum of Sq     RSS    AIC
## - Wealth  1     26493 1453068 503.93
## <none>                1426575 505.07
## - M.F     1     84491 1511065 505.77
## - U1      1     99463 1526037 506.24
## + Pop     1     22345 1404229 506.33
## + Po2     1     13259 1413315 506.63
## + NW      1      5927 1420648 506.87
## + So      1      5724 1420851 506.88
## + LF      1      5176 1421398 506.90
## + Time    1      3913 1422661 506.94
## - Prob    1    198571 1625145 509.20
## - U2      1    208880 1635455 509.49
## - M       1    320926 1747501 512.61
## - Ed      1    386773 1813348 514.35
## - Ineq    1    594779 2021354 519.45
## - Po1     1   1127277 2553852 530.44
##
## Step:  AIC=503.93
## Crime ~ M + Ed + Po1 + M.F + U1 + U2 + Ineq + Prob
##
##          Df Sum of Sq     RSS    AIC
## <none>                1453068 503.93
## + Wealth  1     26493 1426575 505.07
## - M.F     1    103159 1556227 505.16
## + Pop     1     16697 1436371 505.39
## + Po2     1     14148 1438919 505.47
## + So      1      9329 1443739 505.63
## + LF      1      4374 1448694 505.79
## + NW      1      3799 1449269 505.81
## + Time    1      2293 1450775 505.86
## - U1      1    127044 1580112 505.87
## - Prob    1    247978 1701046 509.34
## - U2      1    255443 1708511 509.55
## - M       1    296790 1749858 510.67
## - Ed      1    445788 1898855 514.51
## - Ineq    1    738244 2191312 521.24
## - Po1     1   1672038 3125105 537.93
```

```
summary(stepwise)
```

```
##
## Call:
## lm(formula = Crime ~ M + Ed + Po1 + M.F + U1 + U2 + Ineq + Prob,
##     data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
```

```
## -444.70 -111.07    3.03  122.15  483.30
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -6426.10    1194.61  -5.379 4.04e-06 ***
## M              93.32      33.50   2.786  0.00828 **
## Ed            180.12      52.75   3.414  0.00153 **
## Po1           102.65      15.52   6.613 8.26e-08 ***
## M.F            22.34      13.60   1.642  0.10874
## U1          -6086.63    3339.27  -1.823  0.07622 .
## U2            187.35      72.48   2.585  0.01371 *
## Ineq           61.33      13.96   4.394 8.63e-05 ***
## Prob        -3796.03    1490.65  -2.547  0.01505 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 195.5 on 38 degrees of freedom
## Multiple R-squared:  0.7888, Adjusted R-squared:  0.7444
## F-statistic: 17.74 on 8 and 38 DF,  p-value: 1.159e-10
```

From the summary above it is evident that the variables M, Ed, M.F, Po1, U1, U2, Ineq and Prob are of greater importance than the rest. As a result these are the variables that will be further analyzed:

Create stepwise model:

```r
linear2 <- lm(Crime~M+Ed+M.F+Po1+U1+U2+Ineq+Prob, data)

stepwise2 <- stepAIC(linear, direction = "both")
```

```
## Start:  AIC=514.65
## Crime ~ M + So + Ed + Po1 + Po2 + LF + M.F + Pop + NW + U1 +
##     U2 + Wealth + Ineq + Prob + Time
##
##          Df Sum of Sq     RSS    AIC
## - So      1        29 1354974 512.65
## - LF      1      8917 1363862 512.96
## - Time    1     10304 1365250 513.00
## - Pop     1     14122 1369068 513.14
## - NW      1     18395 1373341 513.28
## - M.F     1     31967 1386913 513.74
## - Wealth  1     37613 1392558 513.94
## - Po2     1     37919 1392865 513.95
## <none>                1354946 514.65
## - U1      1     83722 1438668 515.47
## - Po1     1    144306 1499252 517.41
## - U2      1    181536 1536482 518.56
## - M       1    193770 1548716 518.93
## - Prob    1    199538 1554484 519.11
## - Ed      1    402117 1757063 524.86
## - Ineq    1    423031 1777977 525.42
##
## Step:  AIC=512.65
## Crime ~ M + Ed + Po1 + Po2 + LF + M.F + Pop + NW + U1 + U2 +
##     Wealth + Ineq + Prob + Time
```

```
##
##          Df Sum of Sq     RSS    AIC
## - Time    1     10341 1365315 511.01
## - LF      1     10878 1365852 511.03
## - Pop     1     14127 1369101 511.14
## - NW      1     21626 1376600 511.39
## - M.F     1     32449 1387423 511.76
## - Po2     1     37954 1392929 511.95
## - Wealth  1     39223 1394197 511.99
## <none>                1354974 512.65
## - U1      1     96420 1451395 513.88
## + So      1        29 1354946 514.65
## - Po1     1    144302 1499277 515.41
## - U2      1    189859 1544834 516.81
## - M       1    195084 1550059 516.97
## - Prob    1    204463 1559437 517.26
## - Ed      1    403140 1758114 522.89
## - Ineq    1    488834 1843808 525.13
##
## Step:  AIC=511.01
## Crime ~ M + Ed + Po1 + Po2 + LF + M.F + Pop + NW + U1 + U2 +
##     Wealth + Ineq + Prob
##
##          Df Sum of Sq     RSS    AIC
## - LF      1     10533 1375848 509.37
## - NW      1     15482 1380797 509.54
## - Pop     1     21846 1387161 509.75
## - Po2     1     28932 1394247 509.99
## - Wealth  1     36070 1401385 510.23
## - M.F     1     41784 1407099 510.42
## <none>                1365315 511.01
## - U1      1     91420 1456735 512.05
## + Time    1     10341 1354974 512.65
## + So      1        65 1365250 513.00
## - Po1     1    134137 1499452 513.41
## - U2      1    184143 1549458 514.95
## - M       1    186110 1551425 515.01
## - Prob    1    237493 1602808 516.54
## - Ed      1    409448 1774763 521.33
## - Ineq    1    502909 1868224 523.75
##
## Step:  AIC=509.37
## Crime ~ M + Ed + Po1 + Po2 + M.F + Pop + NW + U1 + U2 + Wealth +
##     Ineq + Prob
##
##          Df Sum of Sq     RSS    AIC
## - NW      1     11675 1387523 507.77
## - Po2     1     21418 1397266 508.09
## - Pop     1     27803 1403651 508.31
## - M.F     1     31252 1407100 508.42
## - Wealth  1     35035 1410883 508.55
## <none>                1375848 509.37
## - U1      1     80954 1456802 510.06
## + LF      1     10533 1365315 511.01
```

```
## + Time    1      9996 1365852 511.03
## + So      1      3046 1372802 511.26
## - Po1     1    123896 1499744 511.42
## - U2      1    190746 1566594 513.47
## - M       1    217716 1593564 514.27
## - Prob    1    226971 1602819 514.54
## - Ed      1    413254 1789103 519.71
## - Ineq    1    500944 1876792 521.96
##
## Step:  AIC=507.77
## Crime ~ M + Ed + Po1 + Po2 + M.F + Pop + U1 + U2 + Wealth + Ineq +
##     Prob
##
##          Df Sum of Sq     RSS    AIC
## - Po2     1     16706 1404229 506.33
## - Pop     1     25793 1413315 506.63
## - M.F     1     26785 1414308 506.66
## - Wealth  1     31551 1419073 506.82
## <none>                1387523 507.77
## - U1      1     83881 1471404 508.52
## + NW      1     11675 1375848 509.37
## + So      1      7207 1380316 509.52
## + LF      1      6726 1380797 509.54
## + Time    1      4534 1382989 509.61
## - Po1     1    118348 1505871 509.61
## - U2      1    201453 1588976 512.14
## - Prob    1    216760 1604282 512.59
## - M       1    309214 1696737 515.22
## - Ed      1    402754 1790276 517.74
## - Ineq    1    589736 1977259 522.41
##
## Step:  AIC=506.33
## Crime ~ M + Ed + Po1 + M.F + Pop + U1 + U2 + Wealth + Ineq +
##     Prob
##
##          Df Sum of Sq     RSS    AIC
## - Pop     1     22345 1426575 505.07
## - Wealth  1     32142 1436371 505.39
## - M.F     1     36808 1441037 505.54
## <none>                1404229 506.33
## - U1      1     86373 1490602 507.13
## + Po2     1     16706 1387523 507.77
## + NW      1      6963 1397266 508.09
## + So      1      3807 1400422 508.20
## + LF      1      1986 1402243 508.26
## + Time    1       575 1403654 508.31
## - U2      1    205814 1610043 510.76
## - Prob    1    218607 1622836 511.13
## - M       1    307001 1711230 513.62
## - Ed      1    389502 1793731 515.83
## - Ineq    1    608627 2012856 521.25
## - Po1     1   1050202 2454432 530.57
##
## Step:  AIC=505.07
```

```
## Crime ~ M + Ed + Po1 + M.F + U1 + U2 + Wealth + Ineq + Prob
##
##          Df Sum of Sq     RSS    AIC
## - Wealth  1     26493 1453068 503.93
## <none>                1426575 505.07
## - M.F     1     84491 1511065 505.77
## - U1      1     99463 1526037 506.24
## + Pop     1     22345 1404229 506.33
## + Po2     1     13259 1413315 506.63
## + NW      1      5927 1420648 506.87
## + So      1      5724 1420851 506.88
## + LF      1      5176 1421398 506.90
## + Time    1      3913 1422661 506.94
## - Prob    1    198571 1625145 509.20
## - U2      1    208880 1635455 509.49
## - M       1    320926 1747501 512.61
## - Ed      1    386773 1813348 514.35
## - Ineq    1    594779 2021354 519.45
## - Po1     1   1127277 2553852 530.44
##
## Step:  AIC=503.93
## Crime ~ M + Ed + Po1 + M.F + U1 + U2 + Ineq + Prob
##
##          Df Sum of Sq     RSS    AIC
## <none>                1453068 503.93
## + Wealth  1     26493 1426575 505.07
## - M.F     1    103159 1556227 505.16
## + Pop     1     16697 1436371 505.39
## + Po2     1     14148 1438919 505.47
## + So      1      9329 1443739 505.63
## + LF      1      4374 1448694 505.79
## + NW      1      3799 1449269 505.81
## + Time    1      2293 1450775 505.86
## - U1      1    127044 1580112 505.87
## - Prob    1    247978 1701046 509.34
## - U2      1    255443 1708511 509.55
## - M       1    296790 1749858 510.67
## - Ed      1    445788 1898855 514.51
## - Ineq    1    738244 2191312 521.24
## - Po1     1   1672038 3125105 537.93
```

```r
stepwise$anova
```

```
## Stepwise Model Path
## Analysis of Deviance Table
##
## Initial Model:
## Crime ~ M + So + Ed + Po1 + Po2 + LF + M.F + Pop + NW + U1 +
##     U2 + Wealth + Ineq + Prob + Time
##
## Final Model:
## Crime ~ M + Ed + Po1 + M.F + U1 + U2 + Ineq + Prob
##
##
```

```
##       Step Df    Deviance Resid. Df Resid. Dev      AIC
## 1                                31    1354946 514.6488
## 2     - So  1    28.57405        32    1354974 512.6498
## 3   - Time  1 10340.66984        33    1365315 511.0072
## 4     - LF  1 10533.15902        34    1375848 509.3684
## 5     - NW  1 11674.63991        35    1387523 507.7655
## 6    - Po2  1 16706.34095        36    1404229 506.3280
## 7    - Pop  1 22345.36638        37    1426575 505.0700
## 8 - Wealth  1 26493.24677        38    1453068 503.9349
```

Calculate r-squared for stepwise model:

```
tot = sum((data$Crime - mean(data$Crime))^2)

sum_step = sum(stepwise2$residuals^2)

r2 = 1 - sum_step/tot

r2
```

```
## [1] 0.7888268
```

The r-squared result for the stepwise model is around .788, which is fairly good for the model. Next we will compare this result to the lasso and elastic net models.

###2. Lasso Model

Organize data:

```
scale <- scale(data)

x <- as.matrix(scale[,1:15])
y <- scale[,16]

training <- sample(1:47, .66*47)

training
```

```
##  [1]  4 39  1 34 23 14 18 33 21 46 10  7  9 15 38  5 35 25 42 32 28  2 37 12 44
## [26] 45 20  3  6 30 41
```

Create x test and train data:

```
x.train <- x[training,]
x.test <- x[-training,]
```

Create y test and train data:

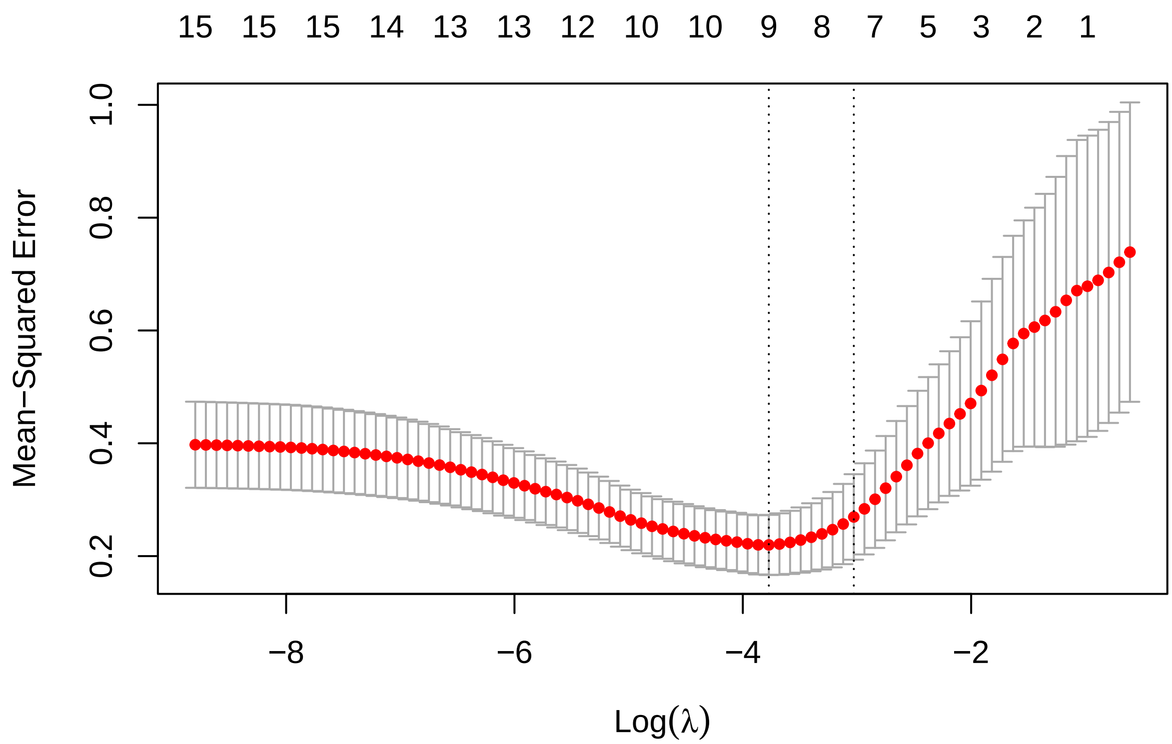
```
y.train <- y[training]
y.test <- y[-training]
```

Create lasso model:

```r
lasso <- glmnet(x.train, y.train, alpha = 1, family = "mgaussian")

cv.lasso <- cv.glmnet(x.train, y.train, alpha = 1)

plot(cv.lasso)
```



From the plot aboive we can see that the mean square error for the lasso model dips then begins to rise around the log of lambda -3 for about 8 variables.

Analyze lasso model variables:

```r
best.lambda <- cv.lasso$lamda.min

coef(cv.lasso, s = "lambda.min")
```

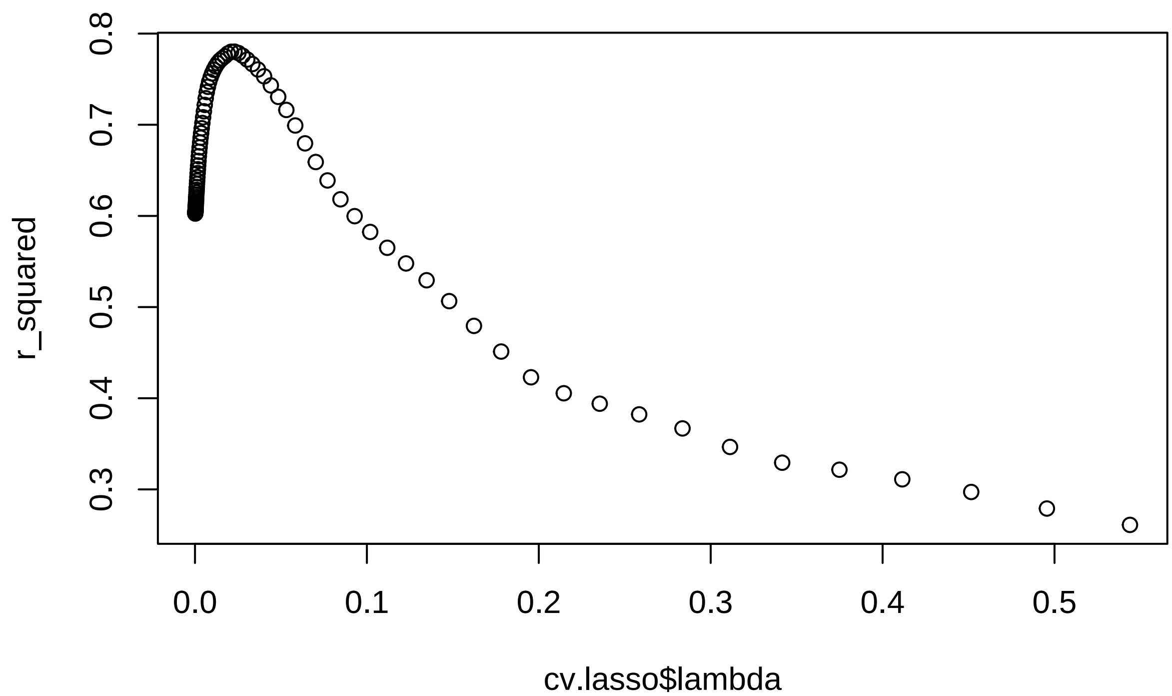
```
## 16 x 1 sparse Matrix of class "dgCMatrix"
##                      s1
## (Intercept)  0.05389714
## M            0.25752120
## So           .
## Ed           0.47742812
## Po1          0.42891382
## Po2          0.40733772
## LF          -0.05318421
## M.F          .
## Pop          .
```

```
## NW           0.05585174
## U1           .
## U2           0.09120898
## Wealth       .
## Ineq         0.59956949
## Prob        -0.18654819
## Time         .
```

From this summary we can see that the important variables to for this data are similar to the previous stepwise model.

Analyze R-squared for lasso model:

```
r_squared <- 1 - cv.lasso$cvm/var(y)
plot(cv.lasso$lambda, r_squared)
```



From the plot of the r-squared values for the cross-validation for the lasso model the r-squared values peak around .7. This is very close to the r-squared value calculated previously by the stepwise model. Now we can compare these values to the elastic model.

###3. Elastic Net

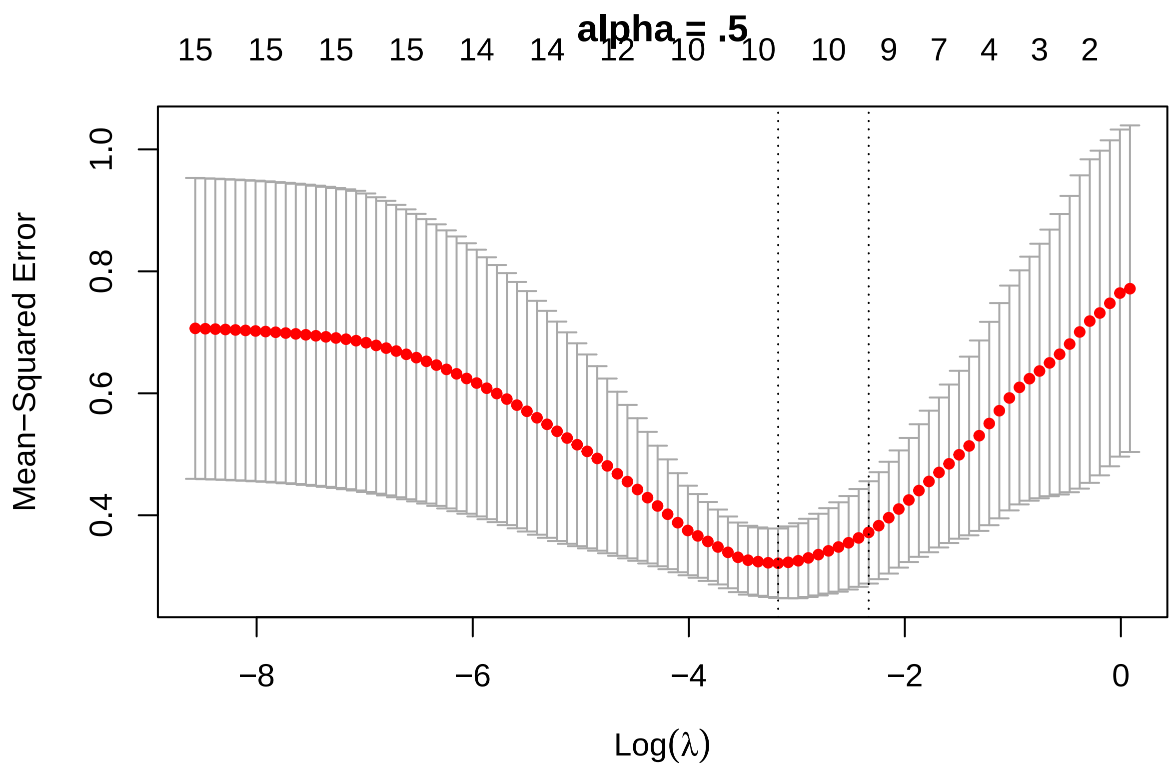
Create elastic net model:

```
elastic <- glmnet(x.train, y.train, alpha =- .5, family = "mgaussian")
```

```
## Warning in glmnet(x.train, y.train, alpha = -0.5, family = "mgaussian"):
## alpha<0; set to 0
```

```r
cv.elastic <- cv.glmnet(x.train, y.train, alpha = .5)

plot(cv.elastic, main = "alpha = .5")
```



From the plot aboive we can see that the mean square error for the lasso model dips then starts to rise around the log of lambda -3 for about 9 variables.

Analyze elastic model variables:

```r
best.lambda <- cv.elastic$lamda.min

coef(cv.elastic, s = "lambda.min")
```

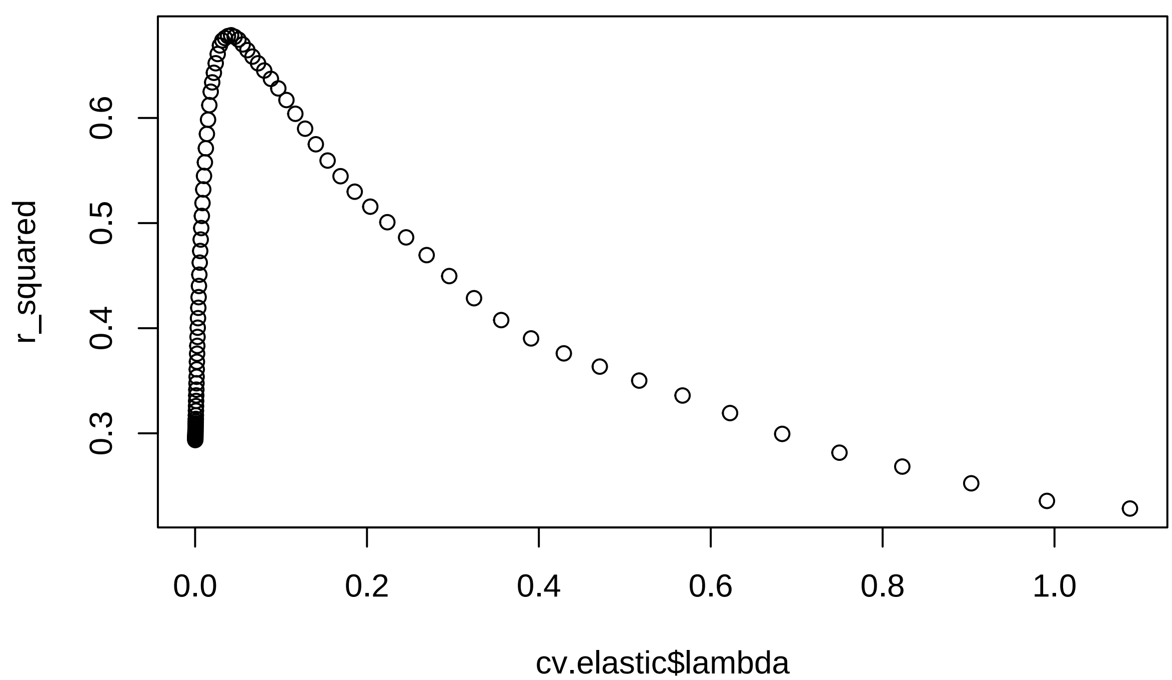
```
## 16 x 1 sparse Matrix of class "dgCMatrix"
##                          s1
## (Intercept)  0.0535071182
## M            0.2625285561
## So           .
## Ed           0.4156941351
## Po1          0.3926975206
## Po2          0.4147124816
## LF          -0.0445084909
## M.F          .
## Pop          0.0007296041
## NW           0.0519799897
## U1           .
```

```
## U2           0.0862082350
## Wealth       .
## Ineq         0.5210074904
## Prob        -0.1892576983
## Time         .
```

As we can see from the summary of the elastic net model the variables that are important to analyze are similar to the previous models but now include slightly more variables

Analyze R-squared for elastic model:

```r
r_squared <- 1 - cv.elastic$cvm/var(y)
plot(cv.elastic$lambda, r_squared)
```




```r
#install.packages('tinytex')
#tinytex::install_tinytex()
```

From the plot of the r-squared values for the cross-validation for the elastic model the r-squared values peak around .7. This is very close to the r-squared value calculated previously by the lasso model.

8.2

In this problem you, can simulate a simplified airport security system at a busy airport. Passengers arrive according to a Poisson distribution with lambda1 = 5 per minute (i.e., mean

interarrival rate 1 = 0.2 minutes) to the ID/boarding-pass check queue, where there are several servers who each have exponential service time with mean rate 2 = 0.75 minutes. [Hint: model them as one block that has more than one resource.] After that, the passengers are assigned to the shortest of the several personal-check queues, where they go through the personal scanner (time is uniformly distributed between 0.5 minutes and 1 minute).

Python Code -Simpy

```
# import simpy
# import random
# import numpy as np
#
#
#
# check_delay = 0
# scan_delay = 0
# system_time = 0
# time_delay = 0
# time_in = 0
# time_out = 0
# time_scan = 0
# time_scanned = 0
#
# mean_time_delay = []
# mean_check_delay = []
# mean_scan_delay = []
# mean_system_time = []
#
# class Setting(object):
#
#     def __init__(self, env):
#
#         self.env = env
#
#         self.c = simpy.Resource(env, 10)
#
#         self.s = []
#
#         for i in range(0,25):
#             r = simpy.Resource(env, capacity = 1)
#             self.s.append(r)
#
#
#     def generate_check(self, customer):
#         yield self.env.timeout(random.expovariate(1.0/.75))
#
#     def generate_scan(self, customer):
#         yield self.env.timeout(random.uniform(.5, 1.0))
#
# def airport_run(env):
#
#     airport = Setting(env)
```

```
#     i = 0
#     while True:
#         i+= 1
#         yield env.timeout(random.expovariate(1.0 / .75))
#         i += 1
#         env.process(customer(env, 'Customer %d' % i, airport))
#
# def customer(env, customer, servers):
#     timeInit = env.now
#     print('arrives at', timeInit)
#
#     with servers.c.request() as request:
#
#         print(env.now, 'customer {} arrives'.format(customer))
#         yield request
#         t_in = env.now
#         print(env.now, 'customer {} is being checked'.format(customer))
#         yield env.process(servers.generate_check(customer))
#         t_out = env.now
#         print(env.now, 'customer {} scanned'.format(customer))
#
#     m_queue = 0
#
#     with servers.s[m_queue].request() as request:
#         yield request
#         print('queue length: ', len(servers.s))
#
#         for j in range(1,25):
#             if (len(servers.s[j].queue) < len(servers.s[m_queue].queue)):
#                 m_queue = j
#
#         t_scan = env.now
#         yield env.process(servers.generate_scan(customer))
#         t_scanned = env.now
#
#     time_end = env.now
#
#     global system_time
#     system_time += time_end - timeInit
#
#     global check_delay
#     check_delay += t_in - timeInit
#
#     global scan_delay
#     scan_delay += t_scanned - t_out
#
#     global time_delay
#     time_delay = check_delay + scan_delay
#
# for i in range(0, 100):
#
#     env = simpy.Environment()
#     env.process(airport_run(env))
```

```
#     env.run(until = 100)
#
#     mean_time_delay.append(time_delay/ 125)
#     mean_check_delay.append(check_delay / 125)
# sim_wait = sum(mean_time_delay) / 100
# sim_check = sum(mean_check_delay) / 100
# sim_scan = sum(mean_scan_delay) / 100
# sim_sys_time = sum(mean_system_time) / 100
#
# print("")
# print('Average wait time: ' + str(sim_wait) + '\n' + 'Average check: ' + str(sim_check) + '\n' + 'Ave
# print("")
```

Results

```
Average wait time: 4.838188608088583
```

The results shown above that we were able to measure that the wait time for the airport passengers was less than 15 minutes and around approximately 5 minutes. The number of checkers and queues in this example was 10 and 25 respectively. Increasing these amounts assisted in reducing the wait times and delays by spreading out the amount of passengers at each queue. In addition, the checking rate was .75 minutes per passenger and rate of 5 passengers per minute, for this example. Increasing these amounts, also significantly reduces the amount of wait times.