Plasma Diagnostic Techniques

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1 Introduction to Plasma Diagnostics

A plasma is a gaseous assembly of electrons, ions and neutral particles usually displaying collective behavior and residing in electric and magnetic fields.

We need statistical mechanics to provide a useful description as it is impossible to simulate the forces and trajectories of 10^{15}-10^{28} (m^{-3}) separate particles - particularly since the equations of motion for each particle will be coupled to many others.

Particle velocities can be characterized by a distribution function describing the number of particles within a given volume element and velocity range. We can define such a particle distribution where the number of particles in the volume element (x, y, z)-$(x+dx, y+dy, z+dz)$ with velocities in the range (v_x, v_y, v_z)-$(v_x+dv_x, v_y+dv_y, v_z+dv_z)$ at time t is given by:

$$f(x, y, z, v_x, v_y, v_z, t)dx\, dy\, dz\, dv_x\, dv_y\, dv_z\, dt \tag{1}$$

Alternatively:

$$f(\underline{x}, \underline{v}, t)d^3\underline{x}\, d^3\underline{v}\, dt \tag{2}$$

A normalised distribution function $\hat{f}$ can be used such that:

$$\int_{-\infty}^{\infty} \hat{f}\,(\underline{x}, \underline{v}, t)\, d^3\underline{v} = 1 \tag{3}$$

Then:

$$f(\underline{x}, \underline{v}, t) = n(\underline{x}, t)\hat{f}\,(\underline{x}, \underline{v}, t) \tag{4}$$

where $n(\underline{x}, t)$ is the particle density at position $\underline{x}$ and time t.

1.1 Moments of the Distribution Function

For a region of plasma that is homogeneous in space and constant for a time interval (both considered to be small), it is possible to take a 'moment' of the distribution function. This will often coincide with a bulk property of the plasma that can be measured. The k^{th} moment of the distribution function is defined as:

$$[M_k] = \int f\left(\underline{x}, \underline{v}, t\right) \underline{v}^k \, d^3\underline{v} \tag{5}$$

where $[M_k]$ is generally a tensor of order k but can often be written as a scalar quantity(if, for example, the plasma is uniform and isotropic).

The names of some lower order moments are listed below:

1. $k = 0 \rightarrow$ particle density. Here ρ_c is the charge density and s is a species (ion or electron).

$$\begin{aligned} [M_0] &= \int f\left(\underline{x}, \underline{v}, t\right) d^3\underline{v} \;=\; n(\underline{x}, t) \\ \rho_c &= \sum_s q_s n s \end{aligned}$$

2. $k = 1 \rightarrow$ mean particle velocity or flow velocity. Here $\underline{j}$ is the current density.

$$\begin{aligned} \frac{1}{n(\underline{x}, t)} [M_1] &= \frac{1}{n(\underline{x}, t)} \int f\left(\underline{x}, \underline{v}, t\right) \underline{v} \, d^3\underline{v} = \underline{v}_{av} \\ \underline{j} &= \sum_s q_s n s \underline{v}_{av} \end{aligned}$$

3. $k = 2 \rightarrow$ stress or pressure (tensor). Here p is the scalar pressure, $\mathrm{Tr}\,[P]$ is the trace of the pressure tensor and m is the particle mass.

$$m \int f\left(\underline{x}, \underline{v}, t\right) (\underline{v} - \underline{v}_{av})^2 \, d^3\underline{v} = [P]$$

$$p = \frac{1}{3} \mathrm{Tr}\,[P]$$

4. $k = 3 \rightarrow$ energy flux density.

$$\int f\left(\underline{x}, \underline{v}, t\right) \underline{v} \, \frac{1}{2} m v^2 \, d^3\underline{v} = [Q]$$

Measurements of plasma often involve determining a quantity related to a moment of the particle distribution function.