Section 8

Inertial Confinement Fusion (ICF)

In this section, we will learn about inertial confinement

- Lawson parameter and ignition criterion for ICF

- The necessary conditions for ICF

- Achieving the conditions

Inertial Confinement Fusion: the concept

In magnetic confinement fusion we can confine the fuel for as long as we like (in principle) – typically the confinement time, τ_E (time-scale of heat loss) is $\sim$ seconds

In inertial confinement fusion, it is only the inertia that keeps the fuel together;

$\Rightarrow$ a finite, very short time

How short?

Consider a pellet of DT fuel of radius r and T=30keV (optimum temperature for fusion)

Material flows at sound speed (see later):

$$c_s = \left(\frac{\gamma k_B T}{m_i}\right)^{1/2}$$

$$m_i = \frac{1}{2}(m_D + m_T) = \frac{5}{2} m_p \quad (m_p = \text{proton mass})$$

$$c_s = \left[\frac{5/3 \times \overbrace{1.602\times10^{-19}}^{\text{conversion } J \to eV} \times \overbrace{3\times10^{4}}^{30\text{keV}}}{5/2 \times \underbrace{1.67\times10^{-27}}_{m_p}}\right]^{1/2}$$

$$\Rightarrow \boxed{c_s = 1.4\times10^{6}\ \text{ms}^{-1}}$$

$\Rightarrow$ inertial confinement time $\tau = \dfrac{r}{c_s}$

e.g. for a 1mm pellet: $\tau = \dfrac{10^{-3}}{1.4\times10^{6}} \sim 10^{-9}\,\text{s}$

Timescales for inertial fusion are ~ns, compared to MCF, where timescales are s (or significant fractions of s)

Lawson-type criterion for ICF

We can derive the ignition criterion for ICF

Recall from Section 1, at optimum temperature of T=30keV, ignition requires $n\tau_E > 1\times10^{20}\text{m}^{-3}\text{s}$

$$\Rightarrow n\tau = \underbrace{\left(\frac{\rho}{m_i}\right)}_{n} \underbrace{\left(\frac{r}{c_s}\right)}_{\tau} > 10^{20}\ \text{m}^{-3}\text{s}$$

ρ = mass density

r = radius of fuel pellet.

For ignition

$$\rho r > 10^{20}\, m_i c_s \quad \text{kg m}^{-2}$$

$m_i = \frac{5}{2} m_p$ = effective ion mass (for DT)

$\Rightarrow$ Ignition criterion is

$$\boxed{\rho r > 0.6\,\text{kg m}^{-2}}$$

To achieve ignition requires a pellet of radius

$$r \gtrsim \frac{0.6}{\rho}\ \text{m}$$

Solid DT has a density ~100kg m^{-3}

$$\boxed{r \gtrsim 6\,\text{mm.}}$$

⇒ ICF works with small pellets of fuel

⇒ But ignition criterion suggests we gain by going to a larger pellet

⇒ Why not work with larger pellets?

⇒ What limits the radius of the pellet?

(1) The energy $\mathcal{E}$ required to heat the pellet to 30keV increases with size

(2) ~~Eng~~ The energy released in the fusion reaction cannot be managed if it is too large (gets dangerous!)

Energy requirements limit pellet size

The problem with going to a larger pellet: consider the energy requirements

Energy to heat a pellet of radius r of density n to temperature T:

$$\mathcal{E} = 3nk_BT \times \frac{4}{3}\pi r^3 = \frac{4\pi}{m_i}\rho k_B T r^3$$

Let ρ_0=100kgm^{-3} (solid density); r_0=1mm

$$\Rightarrow \mathcal{E} = \frac{4\pi}{2.5\times 1.67\times 10^{-27}}\left(\frac{\rho}{\rho_0}\right)\times 100 \times 1.602\times 10^{-19} \times 3\times 10^4 \times \left(\frac{r}{r_0}\right)^3 \times \left(10^{-3}\right)^3 \text{ J}$$

$$\Rightarrow \mathcal{E} = 1.4\left(\frac{\rho}{\rho_0}\right)\left(\frac{r}{r_0}\right)^3 \text{ MJ}$$

$\Rightarrow$ Energy increases with pellet size (naturally!)

$\Rightarrow$ For $\rho=\rho_0$ (solid DT density) and r=6mm (required by the ignition criterion)

$$\mathcal{E} \sim 1.4 \times 1 \times 6^3 \sim 300\text{MJ !!}$$

This is way above what can be achieved by the most powerful lasers (which are ~1MJ)

Solution: combine the energy required

$$\mathcal{E} = 1.4\left(\frac{\rho}{\rho_0}\right)\left(\frac{r}{r_0}\right)^3 \text{ MJ}$$

With ignition criterion, $\rho r = 0.6 \text{ kgm}^{-2}$ to eliminate r:

$$\Rightarrow \frac{\rho}{\rho_0} \simeq \frac{18}{\sqrt{\mathcal{E}(\text{MJ})}}$$

$\Rightarrow$ to get energy below 1MJ requires significant compression of fuel, i.e. ρ must be much greater than solid density, ρ_0 – 18 times solid density to limit heating energy to 1MJ

$\Rightarrow$ Compression is key to ICF $\Rightarrow \rho \gg \rho_0$

The system to achieve the compression is called the "driver" – can be lasers or x-rays (& others).

Burn fraction and burn parameter

There is no guarantee that all the DT fuel in the pellet is burnt during the confinement time
– if not, this reduces the efficiency

Consider an equal D-T mix, initial density n, with a fraction f burnt at time t: $\Rightarrow$ at time t $n_D = n_T = (1-f)n$

$\Rightarrow$ reaction rate (rate at which D or T are burned)

$$-\frac{dn_D}{dt} = n\frac{df}{dt} = \underbrace{n(1-f)}_{n_D}\,\underbrace{n(1-f)}_{n_T}\,\underbrace{\langle\sigma v\rangle}_{\text{reactivity}}$$

$$\Rightarrow \boxed{\frac{df}{dt} = n(1-f)^2\langle\sigma v\rangle}$$

To find f after one confinement time we integrate from $t=0$ $(f=0)$ to τ [assume $\langle\sigma v\rangle$ = constant]

$$\Rightarrow \int_0^f \frac{df'}{(1-f')^2} = n\langle\sigma v\rangle\int_0^\tau dt$$

$$\Rightarrow \frac{1}{(1-f)} - 1 = n\langle\sigma v\rangle\tau = \underbrace{\left(\frac{\rho}{m_i}\right)}_{n}\langle\sigma v\rangle\underbrace{\left(\frac{r}{c_s}\right)}_{\tau}$$

Define the burn parameter, $H_B = m_i c_s / \langle\sigma v\rangle$

$$\Rightarrow \frac{1}{(1-f)} = 1 + \frac{\rho r}{H_B} \quad \Rightarrow \quad \boxed{f = \frac{\rho r}{\rho r + H_B}}$$

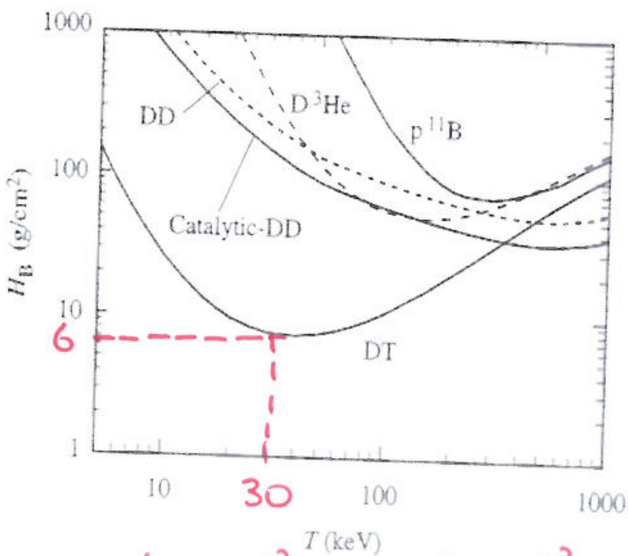
Low H_B $\Rightarrow$ high burn fraction

Take H_B=60kgm^{-2} and the ignition criterion ρr=~~60~~ 0.6kgm^{-2}
$\Rightarrow$ Fraction of fuel burnt is only $f \sim \frac{0.6}{60} \sim 1\%$.

To get a reasonable burn fraction, eg f=30%, requires:

$$\boxed{\rho r \sim 30\,\mathrm{kgm^{-2}}}$$

$\gg$ ignition criterion.



$6\,\mathrm{gcm^{-2}} = 60\,\mathrm{kgm^{-2}}$

Compression is key

We must limit the size of the fuel pellet that we burn to avoid excess energy production, and damage!

Consider the energy produced by burning *m*kg of DT fuel:

$$E = \underbrace{N_D}_{\text{number D nuclei = number of reactions.}} \times \underbrace{\mathcal{E}_{DT}}_{\text{energy per reaction}} \qquad N_D = \frac{m}{m_D + m_T} = \frac{m}{5m_p} \; ; \; \mathcal{E}_{DT} = 17.6\text{MeV}$$

$$\Rightarrow \boxed{E = 3.38m \times 10^{14}\ \text{J}}$$

$\Rightarrow$ energy produced $= 3.38 \times 10^{14}$ J per kg of DT fuel.

Just 1mg of fuel would produce 338MJ of energy (about the limit that can be handled). This corresponds to a fuel pellet of radius *r*~1mm if uncompressed

$\Rightarrow$ ρ*r* is well below ignition if fuel is not compressed

[ρ if uncompressed $\rho r = \rho_0 r_0 = 0.1\ \text{kgm}^{-2} \ll 0.6\ \text{kgm}^{-2}$]

To calculate the required compression:

Assume a burn fraction of 30%, so $\rho r = 30\text{kgm}^{-2}$

$\Rightarrow$ Maximum mass of DT fuel ~3mg (ie. 1mg burned)

$\rho r = 30\text{kgm}^{-2}$ ~~sets the~~ ensures a reasonable burn fraction [ignition criterion easily satisfied]

Combine this with the laser energy required to heat the fuel to fusion conditions:

$$\mathcal{E} = 1.4\left(\frac{\rho}{\rho_0}\right)\left(\frac{r}{r_0}\right)^3 = 1.4\left(\frac{\rho_0}{\rho}\right)^2\left(\frac{\rho r}{\rho_0 r_0}\right)^3 \qquad \rho_0 = 100\text{kgm}^{-3}, \; r_0 = 1\text{mm}.$$

$$\Rightarrow \left(\frac{\rho}{\rho_0}\right)^2 = \frac{1.4}{\mathcal{E}(\text{MJ})}\left[\frac{30}{100 \times 10^{-3}}\right]^3$$

This yields the required compression factor:

$$\boxed{\frac{\rho}{\rho_0} \sim \frac{6000}{\sqrt{\mathcal{E}(\text{MJ})}}}$$

Must achieve compression factors in excess of 1000!

NB. – much more difficult than just ignition.

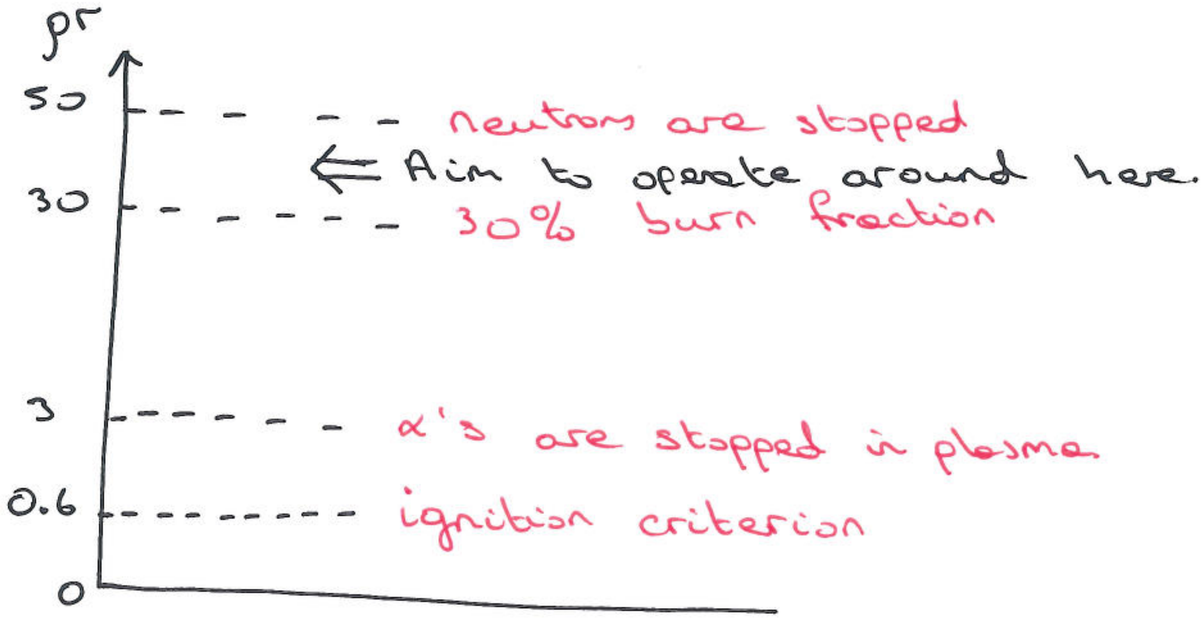
The importance of ρr

We have already seen that the parameter ρr is important for:

- ignition criterion
- burn fraction

It is also important for stopping the fusion products (α's and neutrons)

- α's are stopped if $\rho r > 3$ kg m^{-3}
- neutrons are stopped if $\rho r > 50$ kg m^{-3}



For inertial fusion to be effective, one must exceed the ignition criterion by a significant margin.

Operating at around $\rho r \sim 30$kgm^{-2} provides a good burn fraction (~30%)

⇒ also, alpha particle energy is given up to pellet, providing additional heating ⇒ we can use this to reduce the requirements of the "driver" (eg lasers) and get increased gain

⇒ in addition the neutrons escape to interact with the surrounding Li-blanket and create T

The route to high gain

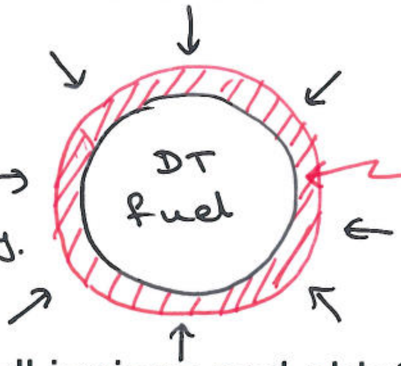
To get high gain (ie more electrical energy out than we put in) for a given fusion power we must minimise the input power

– Low input power $\Rightarrow$ only a small central volume is ignited by the driver – the fusion products do the rest

The process:

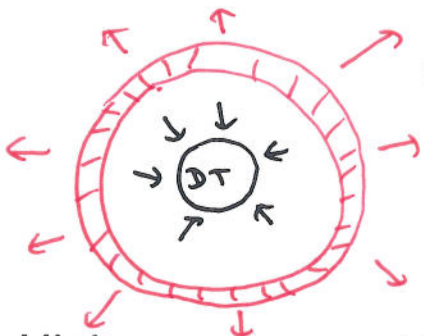
1. Take a sphere of DT encased in a spherical shell
2. Use a "driver" to heat the shell
   – Could be lasers or X-rays

Driver heats shell uniformly.



shell surrounding fuel.

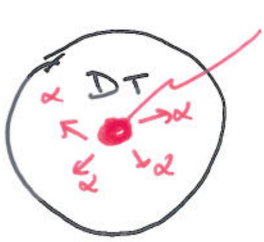
3. The shell ionises and ablates; the reaction drives a pressure shock wave into the centre



Shell ablates.

Rocket effect compresses DT fuel.

4. High pressure and temperature are reached, just at the very centre, satisfying the ignition condition
5. The resulting fusion processes produce α's which heat up larger radii, causing that plasma to fuse: this "burn wave" propagates out towards the edge



Hot spot at centre reaches fusion conditions.

Energetic α's ~~heat~~ (from core fusion reaction) heats surroundi plasma to fusion conditions $\Rightarrow$ more α's $\Rightarrow$ a "burn" wave".

Challenges, and a possible solution:

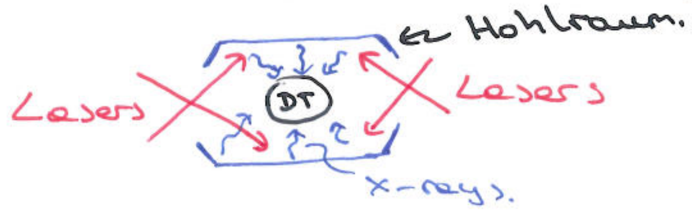
The conventional ICF process is challenging because of the Rayleigh-Taylor instability

- Exists when you place a dense liquid over a light one, for example
- The instability enhances non-uniformities
- Makes achieving the desired compression difficult

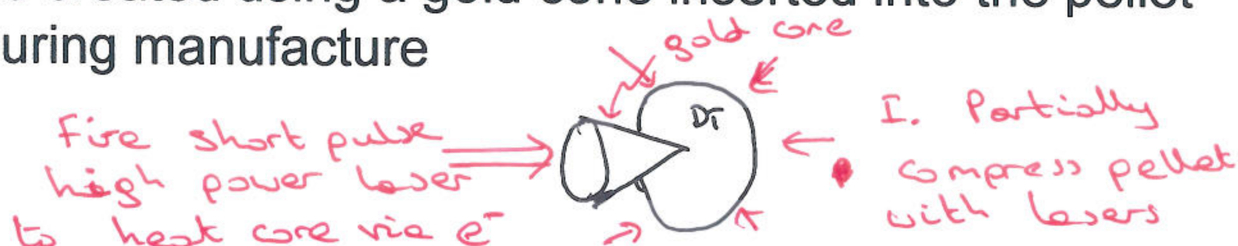
Possible ways out

1. Carefully design pellet, eg make the exploding shell as smooth and uniform as possible

2. Make the driver radiation as uniform as possible
   - One way is to place the pellet of DT in a cell of heavy metal (eg gold), called a hohlraum, and irradiate the inside of the hohlraum with lasers
     $\Rightarrow$ X-rays, which then impinge on the target (shorter wavelength minimises Rayleigh-Taylor)



3. Fast ignition:
   - Use conventional process to partially compress
   - Use a second high energy laser to bore a hole towards the centre
   - Fire a short, high power third laser through this hole towards the centre: generates electrons which heat the core to provide the spark for ignition
   - Instead of the second laser, the path to the core can be created using a gold cone inserted into the pellet during manufacture



What you need to know

You need to understand the process that governs the confinement time in ICF: ie, the inertia. You need to know that this means flow of plasma out of the confinement region (the pellet) at the sound speed.

You should be able to show that the ignition criterion for ICF corresponds to a critical value of ρr, and know how to derive this. You should also be able to argue what processes limit the size of the pellet (that is the power required to heat it to fusion conditions becomes excessive for large pellets, and the energy generated by the fusion of a large pellet cannot be contained with existing materials.

You should be able to give an argument for the compression requirement, both in words and quantify this with a calculation.

You should understand what is meant by the burn fraction. You do not need to learn the expression for the burn parameter, H_B, but if you were given the expression for it, you should be able to derive the relationship between the burn fraction and H_B. You should be able to demonstrate that a large value of ρr is required for a large burn fraction.

You should understand that a sufficiently large ρr is required for a reasonable burn fraction and that to limit the fusion power produced by a single pellet to a level that materials can handle requires a high compression of the fuel. You should be able to calculate this compression.

You should be able to give an overview of the difference mechanisms that influence the choice of ρr.

You should be able to describe the compression process and the different stages to generate the fusion conditions, from the initial fusion in the very centre of the compressed pellet, through to the burn wave that propagates out to progressively create the conditions for fusion throughout the whole pellet. You should understand why uniformity in the pellet surface is important, and why a uniform radiation (ideally with short wavelength) is important. It would be helpful to know what is meant by “fast ignition”.