Null-Pair Removal
The Geometric Mechanism of Dimensional Reduction:
From Ledger Kinematics to Lossless Boundary Dynamics

Marek Hubka *

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Abstract

We formalize *Null-Pair Removal* (NPR) as the local, boost-invariant projector that deletes the two normal null directions at a ledger tile, realizing a $D \to D-2$ dimensional reduction (*snap*) in the DCT/QG framework[1]. Starting from the invariant placement rule $\mathcal{I}_{\mathcal{L}} = \mathcal{I}_{\text{crit}}$ (which yields $r_{\mathcal{L}} = \sqrt{\mathcal{T}}\, R_{\text{S}}$ in Schwarzschild), we define the NPR map P, prove idempotence and covariance, and derive its interplay with: (i) lossless, self-adjoint inner boundary dynamics via real–Robin conditions; (ii) the episodic master equation (a dephasing kick at snaps); and (iii) the focused Raychaudhuri impulse with area quantum $\Delta\mathcal{A} = 4\ln 2\, \ell_{\text{P}}^2$ per payload bit W. Worked examples in Schwarzschild, comments on Kerr, and general D-ladder scaling are provided. We conclude with appendices for one solved toy model and research outlook.

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*Independent Researcher, Czech Republic.
Website: tidesofuncertainty.com. Email: marek@tidesofuncertainty.com.

1 Introduction and context

The *ledger* $\mathcal{L}$ is a geometric inner surface placed by the curvature trigger[2]

$$\mathcal{I}_{\mathcal{L}} = \mathcal{I}_{\text{crit}} = \frac{12}{\mathcal{T}},$$

which, for the Schwarzschild family, integrates to the closed-form placement

$$r_{\mathcal{L}} = \sqrt{\mathcal{T}}\, R_{\text{S}}, \qquad \frac{A_{\mathcal{L}}}{A_H} = \mathcal{T}.$$

Between snaps, exterior fields evolve unitarily on the domain with inner boundary $\mathcal{L}$; at a snap the theory prescribes a discrete, reversible kinematic update (Infinity Bits)[1] and a thermodynamic area tick[1] $\Delta\mathcal{A}_{\text{one bit}} = 4\ln 2\,\ell_{\text{P}}^2$ per *written payload* bit W.

This paper addresses the missing *mechanism*: how does the geometry locally enact $D \to D-2$ reduction? The answer is *Null-Pair Removal* (NPR): a boost-invariant map that eliminates the two normal null directions (n_+, n_-), leaving purely tangential data on $T\mathcal{L}$. NPR underwrites the lossless inner boundary (real–Robin), the master equation's dephasing kick[1], and the focused Raychaudhuri impulse[1].

2 Geometric setup on a ledger tile

Let $\mathcal{L}$ be a spacelike $(D-2)$-surface with induced metric h_{ab}, area form ϵ_{ab}, and future-directed null normals ${n_+}^A, {n_-}^A$ obeying

$$n_+ \cdot n_- = -1, \qquad n_+ \cdot n_+ = 0, \qquad n_- \cdot n_- = 0.$$

The *projector* onto the tangent bundle $T\mathcal{L} \subset T\mathcal{M}$ is

$$P^A{}_B \;=\; \delta^A{}_B + {n_+}^A n_{-B} + {n_-}^A n_{+B}, \qquad P^A{}_B\, n_+ = 0, \;\; P^A{}_B\, n_- = 0, \;\; P^A{}_B \circ P^A{}_B = P^A{}_B.$$

Intrinsic optical data:

$$\theta_\pm \equiv h^{ab}\nabla_a n_{\pm b}, \qquad \sigma_{\pm ab} \equiv \tfrac{1}{2}\left(\nabla_a n_{\pm b} + \nabla_b n_{\pm a}\right)^{\text{tf}}, \qquad \omega_a \equiv -\,n_{-B}\,\nabla_a {n_+}^{\;B}.$$

[1] The full, rigorous derivations are reserved for forthcoming companion papers. All concepts are currently in preparation and are introduced in the manifest of DCT/QG[1] framework.

The formalism for these optical scalars and normal-bundle connections follows standard treatments [3].

Lemma 1 (Boost invariance)**.** *Under a local* $\mathrm{SO}(1,1)$ *boost* $n_\pm \to e^{\pm\lambda} n_\pm$*, the projector* $P^A{}_B$ *is invariant.*

Proof. A direct substitution gives $\delta^A{}_B + \mathrm{e}^{+\lambda} n_+{}^A\, \mathrm{e}^{-\lambda} n_{-B} + \mathrm{e}^{-\lambda} n_-{}^A\, \mathrm{e}^{+\lambda} n_{+B} = P^A{}_B$. □

3 NPR: definition and core properties

Definition 1 (NPR on tensors)**.** For any rank-(p,q) tensor T, define $\mathsf{P}[T]$ by contracting *each* index with $P^A{}_B$:

$$\mathsf{P}[T]^{A_1\cdots A_p}{}_{B_1\cdots B_q} = P^{AA_1}_{\;\;B}{}_{C_1} \cdots P^{AA_p}_{\;\;B}{}_{C_p}\, P^{AD_1}_{\;\;B}{}_{B_1} \cdots P^{AD_q}_{\;\;B}{}_{B_q}\, T^{C_1\cdots C_p}{}_{D_1\cdots D_q}.$$

Proposition 1 (Idempotence and covariance)**.** $\mathsf{P} \circ \mathsf{P} = \mathsf{P}$*. Moreover* P *is tensorial and invariant under local boosts* $\mathrm{SO}(1,1)$ *in the normal 2-plane.*

Proof. Idempotence follows from $P^A{}_B \circ P^A{}_B = P^A{}_B$ applied to each slot. Covariance is immediate since $P^A{}_B$ is a tensor built algebraically from $n_\pm$ and the metric, and Lemma 1 shows boost invariance. □

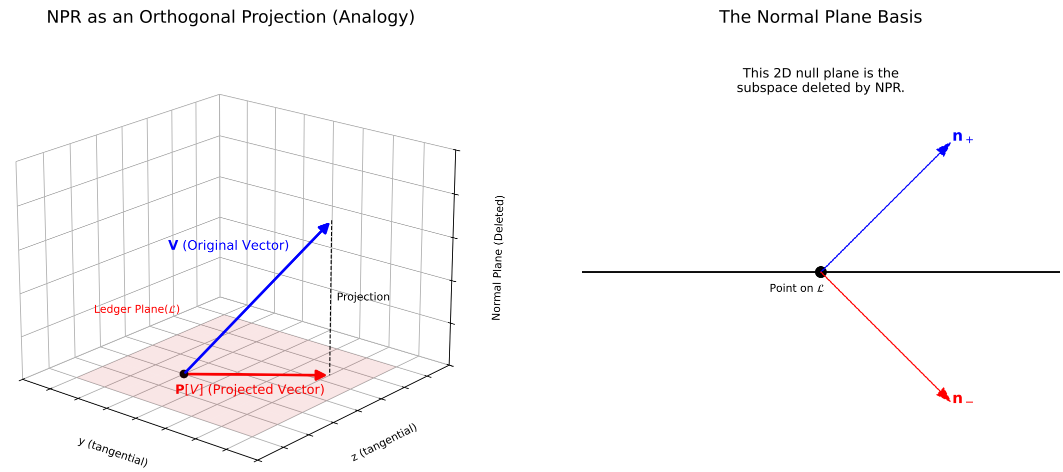

Figure 1: **A Visual Analogy for Null-Pair Removal (NPR) as an Orthogonal Projection.** (*Left*) An arbitrary vector **V** (blue) in the ambient spacetime has components both along the Ledger plane $\mathcal{L}$ (light red) and in the direction normal to it. The NPR projector $P^A{}_B$ acts like a light shining from directly above, removing all components in the normal direction. The result is the projected vector $\mathbf{P}[V]$ (red), a purely tangential object that lies entirely on the Ledger. (*Right*) The inset shows the physical basis of the deleted normal plane. It is not a single spatial dimension, but a 2D plane spanned by a pair of null vectors, $\mathbf{n}_+$ and $\mathbf{n}_-$.

Energy–momentum after NPR. For T_{AB},

$$\widetilde{T}_{ab} \equiv q_a{}^C q_b{}^D T_{CD}, \qquad \widetilde{T}_{ab}\, n^b_{\pm} = 0.$$

Components $T_{++}, T_{--}, T_{+-}, T_{\pm a}$ are removed; only tangential T_{ab} remains.

4 Transport across the ledger between snaps

Notation. We write B for the (real, Hermitian) boundary operator on $\mathcal{L}$; in a $Y_{\ell m}$ basis this reduces to real scalars $\mathsf{B}_{\ell m}(\omega)$, and in spherically symmetric cases $\mathsf{B}_{\ell}(\omega)$.

Between snaps, the exterior fields solve the projected equations on the domain with inner boundary $\mathcal{L}$, endowed with a *real–Robin* law:

$$(\partial_n + \mathsf{B})\, \psi\big|_{\mathcal{L}} = 0, \qquad \mathsf{B} \in \mathbb{R}.$$

Self-adjointness and frequency dependence. In practice B may depend on frequency ω and angular channel (ℓ, m) via short-distance physics, $\mathsf{B} = \mathsf{B}_{\ell m}(\omega)$, but it must remain *real* to define a self-adjoint extension on the half-line. Consequently the reflectivity

$$\mathsf{R}_\ell(\omega) = \frac{\mathsf{B}_\ell(\omega) - \mathrm{i}\, k_\ell(\omega)}{\mathsf{B}_\ell(\omega) + \mathrm{i}\, k_\ell(\omega)}\, \mathrm{e}^{2\mathrm{i}\varphi_\ell(\omega)}, \quad \varphi_\ell(\omega) \in \mathbb{R}$$

has unit modulus, $|\mathsf{R}_\ell(\omega)| = 1$, implying vanishing normal energy flux at $\mathcal{L}$. See also the self-adjoint extension literature for radial Hamiltonians[4, 5] and unitarity[6, 7, 8].
Local wavenumber. Near the inner boundary, each separated mode obeys

$$\big[-\partial^2_{r_*} + V_\ell(r)\big]\, \psi_{\ell\omega} = \omega^2 \psi_{\ell\omega},$$

so the normal wavenumber at the ledger is

$$k_\ell(\omega) \equiv \sqrt{\omega^2 - V_\ell(r_*^{\mathcal{L}})}\,.$$

In high-frequency or locally flat limits where $V_\ell(r_*^{\mathcal{L}}) \approx 0$, one has $k_\ell(\omega) \approx \omega$.

Proposition 2 (No flux through $\mathcal{L}$)**.** *For any field obeying the real–Robin boundary condition, the conserved flux normal to the ledger is identically zero.*

Sketch. The conserved flux (Noether current) for a complex scalar field ψ is $J^\mu \propto \mathrm{Im}[\psi^* \partial^\mu \psi]$. The component normal to the boundary $\mathcal{L}$ is therefore proportional to $J^n \propto \mathrm{Im}[\psi^* \partial_n \psi]$. We apply the real–Robin condition, $\partial_n \psi = -\mathsf{B}\psi$. Since B is a real operator, this also means $\partial_n \psi^* = -\mathsf{B}\psi^*$. Substituting this into the flux expression gives:

$$J^n \propto \mathrm{Im}[\psi^*(-\mathsf{B}\psi)] = \mathrm{Im}[-\mathsf{B}(\psi^*\psi)] = \mathrm{Im}[-\mathsf{B}|\psi|^2].$$

Because B is real and $|\psi|^2$ is real, the quantity inside the imaginary part is purely real. Therefore, its imaginary part is zero:

$$J^n \propto \mathrm{Im}[\text{real value}] = 0.$$

There is no energy or probability current flowing across the boundary. $\square$

5 Snap events: state map, NPR delete, and focusing impulse

A snap combines: (i) a *state* dephasing onto a coarse-grained predicate $\{\Pi_r\}$,

$$\rho \;\longmapsto\; \sum_r \Pi_r\,\rho\,\Pi_r,$$

which is a standard decoherence or measurement channel [9], with (ii) a *geometric* NPR deletion of normal components and (iii) a localized Raychaudhuri impulse that increments the tile area by

$$\Delta\mathcal{A} \;=\; 4\ln 2\,\ell_{\mathrm{P}}^2 \times (\#\text{ payload bits } W).$$

The congruence expansion integrates to

$$\int_{\lambda^-}^{\lambda^+} \theta\,\mathrm{d}\lambda \;=\; \ln\Big(1+\frac{\Delta\mathcal{A}}{A}\Big) \;\approx\; \frac{\Delta\mathcal{A}}{A},$$

ensuring finite focusing and preventing singular blow-up.

6 Conservation and locality

Let $\widetilde{T}_{ab}$ be the tangential stress after NPR. The intrinsic divergence on $\mathcal{L}$ satisfies a Gauss–Codazzi relation schematically

$$D^a\,\widetilde{T}_{ab} \;=\; -\,\mathcal{K}_b{}^c{}_a\,T^a{}_c,$$

where $\mathcal{K}$ contains extrinsic curvature data. With real–Robin, normal fluxes vanish, so there is no energy leakage; the only nonzero *source* is the localized impulse already accounted for by $\Delta\mathcal{A}$.

Locality and no-FTL. The dephasing channel $\rho \mapsto \sum_r \Pi_r\rho\Pi_r$ is completely positive and trace-preserving. For observables spacelike to the tile, commutators vanish; hence NPR snaps do not transmit superluminal signals.

7 Curvature data that survive NPR

Applying P to the Riemann tensor removes mixed/normal components; the *intrinsic* curvature of $\mathcal{L}$ obeys the Gauss equation. By the Gauss–Codazzi relation for a codimension-2 surface with null normals $n_\pm^A$ and induced metric h_{AB},

$${}^{(d)}\!R = h^{AC}h^{BD}R_{ABCD} + \theta_+\theta_- - \sigma_{+ab}\,\sigma_-{}^{ab},$$

up to total divergences and choice of normalization for $n_\pm$[4, 5]. Thus ${}^{(d)}\!R$ and the dyad-invariant contractions built from $\sigma_{\pm ab}$ survive NPR and furnish the reversible metadata.

NPR deletes all curvature components carrying legs in the normal null plane, e.g.

$$R_{ACBD}\, n_+{}^C n_+{}^D, \qquad R_{ACBD}\, n_-{}^C n_-{}^D, \qquad R_{ACBD}\, n_+{}^C n_-{}^D, \qquad R_{ACBD}\, n_-{}^C n_+{}^D$$

leaving the intrinsic ${}^{(d)}R_{abcd}$ (all indices tangent to $\mathcal{L}$) and the normal-bundle connection ω_a (through $\mathcal{F}_{ab} = \nabla_a \omega_b - \nabla_b \omega_a$) as the relevant geometric remnants.

8 Boost-invariant metadata and reversibility

The *Infinity Bits* $\{W, X, Y, Z\}$tore: one irreversibly written *payload* bit W and three reversible, boost-invariant geometric bits:

$$X = \mathrm{sgn}\big(\varepsilon_{ABCD} e_1^A e_2^B n_+{}^C n_-{}^D\big), \quad Y = \mathrm{sgn}\big(\sigma_{+ab}\, \sigma_-{}^{ab}\big), \quad Z = \mathrm{sgn}\big(\epsilon^{ab} \mathcal{F}_{ab}\big)$$

$$X, Y, Z \in \{+1, -1\}$$

Given $\{X, Y, Z\}$ on a one-ring neighborhood and smoothness, NPR is kinematically reversible up to diffeomorphisms and local $\mathrm{SO}(1,1)$ gauge. Only W consumes ledger capacity.

9 Worked examples

Schwarzschild (4D)

With $K = 48M^2/r^6$, the invariant $\mathcal{I}(r) \equiv K r^4 = 12(R_\mathrm{S}/r)^2$ places the ledger at

$$r_\mathcal{L} = \sqrt{\mathcal{T}}\, R_\mathrm{S}, \qquad \frac{A_\mathcal{L}}{A_\mathrm{H}} = \mathcal{T}.$$

After spherical-harmonic reduction to a master field $u(r_*, t)$, impose at $r_*^\mathcal{L}$ a real–Robin law

$$\Big(\frac{\mathrm{d}}{\mathrm{d}r_*} + \mathsf{B}_\ell(\omega)\Big) u_{\ell\omega}\Big|_{r_*^\mathcal{L}} = 0, \qquad \mathsf{R}_\ell(\omega) = \frac{\mathsf{B}_\ell(\omega) - \mathrm{i}\, k_\ell(\omega)}{\mathsf{B}_\ell(\omega) + \mathrm{i}\, k_\ell(\omega)}\, \mathrm{e}^{2\mathrm{i}\varphi_\ell(\omega)}, \quad \varphi_\ell(\omega) \in \mathbb{R}$$

$$k_\ell(\omega) \equiv \sqrt{\omega^2 - V_\ell(r_*^\mathcal{L})}\,,$$

with $k_\ell \approx \omega$ when $V_\ell(r_*^\mathcal{L}) \approx 0$. NPR removes normal components; only tangential stress couples to the inner boundary.

Kerr (axisymmetric) — remarks

Adopt a tetrad aligned with principal null directions; choose n_+, n_- compatible with horizon-penetrating slices. Frame-dragging enters through ω_a, affecting the sign of Z. Boost invariance of $P^A{}_B$ and locality of NPR remain intact.

D-ladder step $D \to D-2$

Let ${}^{(D)}r_{\mathrm{H}}$ be the horizon scale and ${}^{(D)}r_{\mathcal{L}}$ the ledger scale in D dimensions. The universal placement and area ratios read:

$$\frac{{}^{(D)}r_{\mathcal{L}}}{{}^{(D)}r_{\mathrm{H}}} = \mathcal{T}^{\frac{1}{2(D-3)}}, \qquad \frac{{}^{(D)}A_{\mathcal{L}}}{{}^{(D)}A_{\mathrm{H}}} = \mathcal{T}^{\frac{D-2}{2(D-3)}}.$$

Composing NPR with fresh normals at the next ledger reproduces the ladder, consistent with the scaling laws derived in [2].

10 Interfaces

The NPR mechanism established here is self-contained: a local, boost-invariant projector

$$P^A{}_B = \delta^A{}_B + n_+{}^A n_{-B} + n_-{}^A n_{+B}, \qquad n_+ \cdot n_- = -1,$$

that deletes the normal null pair and preserves the Ledger tangent, plus a lossless (real–Robin) inner boundary between snaps. Its *interfaces* with other pillars of the framework are as follows (details in subsequent papers):

- **Capacity bookkeeping (TDT).** At a snap, the irreversible write W increases the Ledger entropy by $\Delta S_{\mathcal{L}} = \ln 2$, costing area $\Delta\mathcal{A} = 4\ln 2\,\ell_P^2$ Between snaps, NPR imposes no flux and no growth.
- **Reversible metadata (Infinity Bits).** Three boost-invariant parities $X, Y, Z \in \{\pm 1\}$ tored on $\mathcal{L}$ make the NPR map invertible up to local gauge, while carrying no area cost.
- **Dynamics between snaps (Master Equation / EFT).** Exterior evolution on the half-line with a real–Robin boundary is unitary[6] self-adjoint radial Hamiltonian); snaps appear as localized CP–TP instruments supported on $\mathcal{L}$.

These pointers clarify *interfaces* only; full derivations are deferred to the upcoming dedicated TDT, Infinity Bits, and EFT papers.

11 Discussion and outlook

We have isolated the purely geometric core of the snap (*NPR*) and tied it to thermodynamic bookkeeping and boundary dynamics. Immediate next steps are: (i) explicit Kerr calculations (RW–Zerilli–Teukolsky sectors) with inner Robin phases; (ii) numerical relativity implementation of NPR+impulse layers; and (iii) phenomenology (echo templates), where the inner phase $\mathsf{B}_\ell(\omega)$ enters observables.

A The Projector in a Simple Basis (4D to 2D)

Conventions. Metric signature $(-,+,+,+)$ in 4D; generalizations keep one timelike, $D-1$ spacelike. Levi–Civita symbols follow $\epsilon_{0123} = +\sqrt{|g|}$.

The covariant formula for the NPR projector, $P^A{}_B = \delta^A{}_B + n_+{}^A n_{-B} + n_-{}^A n_{+B}$, is constructed to be manifestly independent of any specific coordinate system. It is illuminating to see its action in a simple, concrete basis to understand that it is equivalent to the intuitive act of **zeroing out** the two normal dimensions.

Setup: A 4D Toy Model of the Black Hole Interior

Let's consider a local region of 4-dimensional spacetime inside a black hole, with coordinates $X^A = (t, r, \theta, \phi)$. In the DCT/QG framework, the final '4D -> 2D' snap resolves the singularity by recording information on a 2-dimensional Ledger.

- **The Ledger (Tangent Space):** The information is recorded on the 2-sphere. The Ledger is therefore the 2D surface spanned by the angular basis vectors ∂_θ and ∂_ϕ.
- **The Normal Space:** The 2D plane orthogonal to this spherical Ledger is the one spanned by the time and radial coordinates, ∂_t and ∂_r. This is the plane that will be deleted.

Inside the event horizon, the roles of t and r have swapped. r is the time-like coordinate, and t is a space-like one. The metric for the normal plane is therefore Lorentzian, with a signature we can write as $(-,+)$ for the (r,t) subspace in the appropriate frame. We will use a simplified local Minkowski metric for this plane: $g_{\text{normal}} = \text{diag}(-1, 1)$ for the (r,t) coordinates.

Basis order: In what follows we use the component order (r, t, θ, ϕ) for matrix display. This differs from the listing $X^A = (t, r, \theta, \phi)$ above only by a cosmetic swap of the first two entries.

From the Standard Basis to the Null Basis

The NPR formalism is built on two null vectors, n_+ and n_-, that span this '(r,t)' normal plane. We construct them as linear combinations of our normal basis vectors, ∂_r and ∂_t.

$$n_+{}^A = \frac{1}{\sqrt{2}}(\partial_r^A + \partial_t^A) = \frac{1}{\sqrt{2}}(1,1,0,0)^T \quad \text{(in (r,t,theta,phi) order)}$$
$$n_-{}^A = \frac{1}{\sqrt{2}}(\partial_r^A - \partial_t^A) = \frac{1}{\sqrt{2}}(1,-1,0,0)^T$$

These vectors are null with respect to the $(-,+)$ signature of the normal plane, and we normalize them such that they form a valid null basis.

Constructing the Projector Matrix

The projector onto the normal space is $Q^A{}_B$. The projector onto the Ledger's tangent space (the part we want to keep) is then $P = I - Q$. Let's construct the matrix for Q.

Using the formula for the projector onto the normal space, $Q^A{}_B = -(n_+{}^A n_{-B} + n_-{}^A n_{+B})$ (with the $(-,+)$ signature and $n_+ \cdot n_- = -1$ normalization), we can calculate its components. This construction results in the identity matrix for the subspace it projects onto. For the (r,t) block, we get:

$$(Q^A{}_B)_{\text{normal}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

This is the identity matrix in the normal subspace, as expected. The full 4D matrix for Q is therefore $\text{diag}(1,1,0,0)$ in our (r,t,θ,ϕ) coordinate order.

The projector onto the Ledger's tangent space, $P^A{}_B$, is $I - Q$. Therefore, the full 4D matrix for the NPR projector is:

$$P^A{}_B = \text{diag}(1,1,1,1) - \text{diag}(1,1,0,0) = \text{diag}(0,0,1,1).$$

Conclusion: The Physical Meaning

This result makes the physical action of the final snap transparent. When we apply this projector to any 4D object (a vector, a tensor, the metric) in the black hole interior, it performs a very simple operation:

- It multiplies all components in the time (t) and radial (r) directions by zero, effectively **deleting them**.
- It leaves the components in the angular (θ,ϕ) directions completely **unchanged**.

For example, applying this to the 4D metric results in a projected metric g'_{AB} where only the angular components are non-zero. This is the metric of the 2D spherical Ledger. The time and radial dimensions have been **frozen out**, leaving a static, timeless record.

The complex-looking covariant formula is the necessary and elegant machinery to perform this simple **zeroing out** operation in a way that is independent of our choice of coordinates, ensuring that NPR is a robust physical principle.

B Technical Appendix and Research Outlook

This appendix provides technical sketches for key results and outlines the primary open problems and future research directions related to the Null-Pair Removal (NPR) mechanism.

Core Algebraic Properties of the NPR Projector

Proof of Boost Invariance. Under a local $\text{SO}(1,1)$ boost in the normal plane, the null directors transform as $n_+ \to e^{+\lambda} n_+$ and $n_- \to e^{-\lambda} n_-$. The projector P' in the new frame is:

$$P'^A{}_B = \delta^A{}_B + (\mathrm{e}^{+\lambda} n_+{}^A)(\mathrm{e}^{-\lambda} n_{-B}) + (\mathrm{e}^{-\lambda} n_-{}^A)(\mathrm{e}^{+\lambda} n_{+B}) = \delta^A{}_B + n_+{}^A n_{-B} + n_-{}^A n_{+B} = P^A{}_B.$$

The projector is manifestly invariant, making NPR a physically robust, gauge-independent operation.

Proof of Idempotence. The operator $P^A{}_B$ projects any vector into the subspace tangential to $\mathcal{L}$. For any vector V^A already in this tangential subspace, the identity $P^A{}_B V^B = V^A$ holds. The column vectors of $P^A{}_B$ are themselves, by definition, in this subspace. Therefore, applying the projector a second time has no further effect: $P^A{}_C P^C{}_B = P^A{}_B$.

Lossless Boundary Dynamics

Technical Sketch of Self-Adjointness and Losslessness. The lossless nature of the ledger between snaps is guaranteed by the real–Robin boundary condition. For a self-adjoint spatial operator, Green's identity requires the boundary form to vanish. For an operator like $H = -\partial_x^2$ on the half-line $x \geq 0$, this requires that $[\psi_1^* \partial_x \psi_2 - (\partial_x \psi_1^*)\psi_2]_0^\infty = 0$. The real–Robin condition, $(\partial_x + \mathsf{B})\psi|_{x=0} = 0$ with real B, enforces this at the boundary $x = 0$. The corresponding reflection coefficient is:

$$\mathsf{R}(\omega) = \frac{\mathsf{B} - \mathrm{i}\omega}{\mathsf{B} + \mathrm{i}\omega}.$$

Since B and ω are both real, this is the ratio of a complex number to its conjugate, which has a modulus of exactly 1. Thus, $|\mathsf{R}| = 1$, guaranteeing perfect reflection and no energy loss at the boundary[7, 10].

Scaling Relations and the Dimensional Ladder

Formulas and Derivation Sketch. For the Tangherlini black hole family in D dimensions, the dimensionless curvature invariant scales as ${}^{(D)}\mathcal{I} \propto (r_\mathrm{H}/r)^{2(D-3)}$. The trigger condition ${}^{(D)}\mathcal{I}(r_\mathcal{L})/{}^{(D)}\mathcal{I}(r_\mathrm{H}) = 1/\mathcal{T}$ then uniquely fixes the radius and area ratios:

$$\frac{{}^{(D)}r_\mathcal{L}}{{}^{(D)}r_\mathrm{H}} = \mathcal{T}^{\frac{1}{2(D-3)}}, \qquad \frac{{}^{(D)}A(\mathcal{L})}{{}^{(D)}A_\mathrm{H}} = \left(\frac{{}^{(D)}r_\mathcal{L}}{{}^{(D)}r_\mathrm{H}}\right)^{D-2} = \mathcal{T}^{\frac{D-2}{2(D-3)}}.$$

These universal scaling laws ensure that the geometric structure of the theory is consistent as one moves up or down the dimensional ladder.

The Snap Impulse and Finite Focusing

Distributional Model. The Raychaudhuri impulse at a snap is modeled as a distributional source. This can be formalized by considering the snap to occur in an infinitesimally thin layer in the affine parameter λ. The expansion θ satisfies:

$$\frac{\mathrm{d}}{\mathrm{d}\lambda} \ln \mathcal{A}(\lambda) = \theta(\lambda) = \bar{\theta}(\lambda) + \Delta\theta\, \delta(\lambda - \lambda_0),$$

where $\bar{\theta}$ is the smooth part and the integrated impulse is given by the TDT area tick from Paper III:

$$\int \Delta\theta\, \delta(\lambda - \lambda_0)\, \mathrm{d}\lambda = \ln\left(1 + \frac{\Delta\mathcal{A}}{\mathcal{A}}\right).$$

This provides a well-defined mathematical structure for the "kick" that prevents the formation of a caustic.

Open Problems and Future Research Directions

The formalization of NPR opens several clear and important avenues for future research.

- **Numerical Implementation of the Snap Impulse:** A key open problem is the implementation of the distributional impulse model described above in a numerical relativity code. This likely involves creating a new type of "puncturing" or dynamic inner boundary condition that is activated when the $\mathcal{I}_{\text{crit}}$ threshold is met, and which smoothly matches the metric data across the impulse layer.

- **Explicit Calculations in Kerr Geometry:** While this paper outlines the principles in axisymmetry, a full analysis requires applying the NPR projector within the Teukolsky formalism for gravitational perturbations and the Dirac equation for fermionic fields in a Kerr background. This is essential for building realistic echo templates for spinning black holes and is a major task for future phenomenological work.

- **Global Sufficiency for Reversibility:** This paper and forthcoming paper for part IV of DCT/QG argue for the *local* sufficiency of the Infinity Bits to reverse an NPR snap on a single tile or a small neighborhood. A full, formal proof of *global* sufficiency on an arbitrarily tiled ledger surface with complex topology (e.g., handles or defects) remains an open mathematical problem.

- **The Gauss–Codazzi Formalism:** A complete description requires expressing the intrinsic curvature of the ledger, ${}^{(D-2)}R_{abcd}$, in terms of the ambient curvature and extrinsic data via the Gauss–Codazzi equations. The open task is to provide a full, explicit decomposition showing precisely which curvature components survive the NPR projection and how they relate to the normal-bundle curvature $\mathcal{F}_{ab}$ that defines the Z bit of the Infinity Bits register.

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