The Infinity Bits:

A Boost-Invariant Geometric Register for a Reversible Dimensional Reduction

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Abstract

We define the *Infinity bits* written at each snap on the ledger $\mathcal{L}$ as the four-bit set $\{W, X, Y, Z\}$. Here W is an irreversible *payload* bit that consumes ledger capacity per TDT, while X, Y, Z are reversible, *boost-invariant geometric bits* that record the discrete orientation, shear alignment, and twist chirality of the deleted null 2-plane. We give precise, covariant definitions, prove invariance under local $SO(1,1)$ boosts of the normals and dyad rotations on $\mathcal{L}$, and provide an operational extraction recipe. We show that the triplet (X, Y, Z), together with smoothness constraints on a one-ring neighborhood, is sufficient to reverse the kinematics of *Null-Pair Removal* (NPR) up to diffeomorphisms and the local boost gauge. Capacity accounting, inner-boundary phases (real–Robin), and worked examples (Schwarzschild, remarks on Kerr) are presented. Background geometry and optical scalars follow standard GR sources [1, 2], the rotation one-form and normal bundle from isolated-horizon technology [3], boundary self-adjointness from [4, 5, 6], and open-systems language from [7, 8]. Cross-links to Papers I–III in this series are indicated [9, 10, 11].

Scope & Open Items

(i) We prove *local* sufficiency (one-ring neighborhood) for reversibility; a global constructive proof on arbitrary tile topologies is deferred.

(ii) We give Kerr *remarks* (nonzero normal-bundle twist); explicit extraction maps in generic axisymmetry will appear in a companion.

(iii) The quantitative map $(X, Y, Z) \mapsto S_\ell(\omega)$ (Robin phase in phenomenology) is outlined here and developed in the echoes paper.

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1 Introduction and context

Paper I fixed the ledger $\mathcal{L}$ via the invariant curvature trigger

$$\mathcal{I} = \mathcal{I}_{\text{crit}} = \frac{12}{\mathcal{T}} \;\Rightarrow\; r_{\mathcal{L}} = \sqrt{\mathcal{T}}\, R_{\text{S}}, \qquad \frac{A_{\mathcal{L}}}{A_H} = \mathcal{T}$$

for Schwarzschild [9, 2]. Paper II defined the geometric mechanism of a snap as *Null-Pair Removal* (NPR): contract every tensor index with the tangential projector

$$P^A{}_B \;=\; \delta^A{}_B + n_+{}^A n_{-B} + n_-{}^A n_{+B},$$

with null normals normalized by $n_+ \cdot n_- = -1$, so that components along the normal null 2-plane are deleted while boost invariance is preserved [10, 2]. Paper III (TDT) established the thermodynamic bookkeeping: only the *payload* written at a snap consumes capacity and ticks area by a universal quantum [11].

This paper provides the missing, *minimal* discrete metadata to make NPR *reversible* up to smooth gauge: three boost-invariant geometric bits, plus the payload bit. The Infinity bits $\{W, X, Y, Z\}$ are exactly the additional information needed to select a unique pre-snap kinematic branch up to smooth diffeomorphisms and local boosts.

2 Geometric setup and notation

Let $\mathcal{L}$ be a spacelike $(D-2)$-surface with induced metric h_{ab} and area form ϵ_{ab}. Choose future-directed null normals $n_+{}^A, n_-{}^A$ normalized by $n_+ \cdot n_- = -1$; the local boost $SO(1,1)$ acts as $n_\pm \to e^{\pm\lambda} n_\pm$. The tangential projector $P^A{}_B$ is as above and is boost invariant [2]. Optical data on $\mathcal{L}$ are

$$\theta_\pm = h^{ab} \nabla_a n_{\pm b}, \qquad \sigma^{(\pm)}_{ab} = \tfrac{1}{2}\big(\nabla_a n^{(\pm)}_b + \nabla_b n^{(\pm)}_a\big)^{\text{tf}},$$

and the normal-bundle (Hájíček) one-form is

$$\omega_a \;=\; -\,n_{-B}\, \nabla_a n_+^{\;B}, \qquad \mathcal{F}_{ab} = \nabla_a \omega_b - \nabla_b \omega_a \quad \text{(normal-bundle curvature)[3].}$$

3 The Infinity bits: definitions

Definition 1 (Payload bit W)**.** The irreversible macro-outcome bit recorded by the snap's coarse-grained predicate (apparatus/matter). It carries $\ln 2$ nats of *payload* information and incurs a ledger capacity cost per TDT [11].

Definition 2 (Orientation/parity bit X)**.** Fix a right-handed dyad $\{e_1, e_2\}$ on $\mathcal{L}$. Define

$$\boxed{X \;\equiv\; \operatorname{sgn}\!\Big(\varepsilon_{ABCD}\, e_1^A e_2^B n_+{}^C n_-{}^D\Big) \;\in\; \{+1, -1\}\;.}$$

Definition 3 (Shear-alignment bit Y)**.** Define the boost-invariant scalar

$$S_\sigma \;\equiv\; \sigma^{(+)}_{ab}\, \sigma^{(-)ab}.$$

Then

$$\boxed{\;Y \;\equiv\; \mathrm{sgn}(S_\sigma) \;\in\; \{+1,-1\}\;.\;}$$

Definition 4 (Twist-chirality bit Z)**.** With $\mathcal{F}_{ab} = \nabla_a\omega_b - \nabla_b\omega_a$ and ϵ^{ab} the Levi–Civita density on $\mathcal{L}$, set

$$\boxed{\;Z \;\equiv\; \mathrm{sgn}(\epsilon^{ab}\,\mathcal{F}_{ab}) \;\in\; \{+1,-1\}\;.\;}$$

Remark 1 (Measure-zero cases). If a defining scalar vanishes on a tile ($S_\sigma = 0$ or $\epsilon^{ab}\mathcal{F}_{ab} = 0$), adopt a fixed tie-breaker (e.g. $+1$) or borrow a one-ring majority. Such cases are nongeneric and do not spoil reversibility.

Summary: what each Infinity bit does

For later use it is convenient to summarize the operational role of each bit:

Bit	Resolves which discrete ambiguity?
W	Payload / branch choice: which macro-outcome was written at the snap
X	Orientation / parity of the null tile in spacetime (mirror flip)
Y	Relative alignment of ingoing/outgoing shears (which side is focusing)
Z	Chirality of twist / normal-bundle curvature (handed rotation)

Together, the reversible metadata bits (X, Y, Z) resolve the discrete *kinematic* ambiguities of the deleted null 2-plane, while the payload bit W picks out a single irreversible branch of the macroscopic outcome. Only W consumes Ledger capacity via TDT; X, Y, Z are free in the capacity budget but essential for local reversibility.

4 Invariance, gauge behavior, and minimality

Proposition 1 (Boost invariance)**.** *Under the local boost $n_\pm \to e^{\pm\lambda} n_\pm$, the bits X, Y, Z are unchanged.*

Proof. X depends on the 4-volume form contracted with $n_+ \wedge n_-$, which rescales by $e^{+\lambda}e^{-\lambda} = 1$. For Y, $\sigma^{(+)}_{ab}$ and $\sigma^{(-)}_{ab}$ carry opposite boost weights, which cancel in the contraction $\sigma^{(+)}_{ab}\sigma^{(-)ab}$. For Z, $\mathcal{F}_{ab} = \mathrm{d}\omega$ is gauge invariant under $\omega \mapsto \omega + \mathrm{d}\varphi$ and insensitive to boosts of $n_\pm$ [2, 3]. □

Proposition 2 (Dyad behavior)**.** *Under proper rotations $e_i \to R_i{}^j e_j$ on $\mathcal{L}$ with $\det R = +1$, the signs X, Y, Z are invariant as defined. (Improper rotations flip X by construction.)*

Theorem 1 (Idempotent reversibility (local))**.** *On a one-ring neighborhood of a tile, NPR plus the metadata (X, Y, Z) fixes the discrete ambiguities of re-embedding the pre-snap 4D neighborhood up to diffeomorphisms and the local $SO(1,1)$ gauge. Applying the snap map twice leaves the kinematic class unchanged.*

Sketch of the mechanism. Starting from a post-snap tile with intrinsic induced metric h_{ab} (on $\mathcal{L}$) and NPR projector $P^A{}_B$ in the ambient spacetime, the NPR map admits a finite family of pre-snap embeddings related by three discrete operations: (i) flipping the null dyad orientation,

(ii) swapping which side of the tile carries stronger focusing (shear alignment), and (iii) flipping the chirality of twist in the normal bundle. The scalars that define X, Y, Z are engineered to be invariant under smooth diffeomorphisms and local $SO(1,1)$ boosts, but to change sign under exactly these three discrete operations. Fixing the triplet (X, Y, Z) therefore selects a unique kinematic branch up to smooth deformations, while the payload bit W selects the macroscopic outcome branch. This is the sense in which Infinity bits are "exactly sufficient" for local NPR reversal.

Remark 2 (Fulfillment of Postulate P2). This theorem provides the formal proof of Postulate P2 (Minimal reversible record at snaps) stated in Paper I. It demonstrates that the geometric bit triplet (X, Y, Z) constitutes the sufficient, minimal, and boost-invariant metadata required to ensure the snap is a kinematically reversible process.

Remark 3 (Minimality). Fewer than three geometric bits leave discrete mirror, alignment, or chirality ambiguities unresolved; (X, Y, Z) is the minimal set that removes them while staying boost invariant. A global constructive proof on arbitrary topologies is left open.

5 Operational extraction recipe

Given $(g_{AB}, \mathcal{L}, n_\pm)$:

1. Choose a canonical dyad $\{e_1, e_2\}$ (e.g. align e_1 with a principal direction of $\sigma^{(+)}$ or with the intrinsic curvature Hessian).
2. Compute $X = \mathrm{sgn}(\varepsilon_{ABCD} e_1^A e_2^B n_+^C n_-^D)$.
3. Compute $Y = \mathrm{sgn}(\sigma^{(+)}_{ab} \sigma^{(-)ab})$.
4. Compute $Z = \mathrm{sgn}(\epsilon^{ab} \mathcal{F}_{ab})$.
5. Record W from the apparatus predicate (coarse-grained outcome).

A concrete implementation of this recipe in an almost-flat "mini–Kerr" patch is given in Appendix A, where each step can be read off explicitly from a 4×4 metric and dyad.

6 Capacity accounting (link to TDT)

Only W is *irreversible* and consumes capacity; X, Y, Z are *reversible metadata*. The thermodynamic cost is additive over individual snap events. For a process involving a total of k snaps, each writing a single payload bit, the cumulative change in the ledger's entropy and area is

$$\Delta S_{\mathcal{L}} = k \ln 2,$$

$$\Delta \mathcal{A}_{\mathcal{L}} = k\, 4 \ln 2\, \ell_{\mathrm{P}}^2 \quad \text{(4D)}, \qquad \Delta^{(D)} \mathcal{A}_{\mathcal{L}} = k\, 4 \ln 2\, {}^{(D)}A_{\mathrm{P}} \quad (D\text{-dim}).$$

Between snaps, the real–Robin inner boundary is lossless (unit-modulus reflectivity), so

$$\nabla_a s^a = 0$$

on the Ledger worldtube [4, 5, 6].

Equivalently, for a sequence of k snaps whose payload bits all read $W = 1$, the Ledger obeys the TDT capacity–payload rule

$$\Delta S_{\mathcal{L}} = \Delta S_{\text{info}} = k\,\ln 2, \qquad \Delta \mathcal{A}_{\mathcal{L}} = k\,4\ln 2\,\ell_{\mathrm{P}}^2,$$

with the reversible metadata bits (X, Y, Z) contributing no capacity cost.

7 Inner phase and boundary conditions

Metadata fix the *discrete* branch of the inner reflection phase encoded by a real–Robin law

$$(\partial_n + \mathsf{B}(\omega))\,\psi_{\ell\omega}\big|_{\mathcal{L}} = 0, \qquad \mathsf{R}(\omega) = \frac{\mathsf{B} - \mathrm{i}\omega}{\mathsf{B} + \mathrm{i}\omega}, \quad |\mathsf{R}| = 1$$

Different (X, Y, Z) select different phase branches without changing capacity or flux. A quantitative calibration $(X, Y, Z) \mapsto S_\ell(\omega)$ is outlined here and developed in the echoes phenomenology paper.

8 Worked examples

Schwarzschild (4D)

Adopt a canonical tetrad with $\omega_a = 0$ on $\mathcal{L}$ by symmetry. Then

$$Z = \operatorname{sgn}(\epsilon^{ab}\mathcal{F}_{ab}) = 0 \;\Rightarrow\; \text{use tie-breaker } Z = +1.$$

The principal shear axes on the inward/outward generators coincide, so $Y = \operatorname{sgn}(S_\sigma) = +1$; X depends on the dyad orientation convention (fix $X = +1$ by right-handed choice). The metadata thus pick a canonical inner phase branch.

Kerr (axisymmetric) — remarks

Frame dragging induces $\omega_a \neq 0$ and hence $\mathcal{F}_{ab} \neq 0$, so $Z = \pm 1$ reflects the handedness of rotation relative to the dyad and normals. The alignment Y can flip if the principal shear axes of n_+ and n_- misalign due to spin-induced couplings. Boost invariance and locality of the definitions remain intact [3, 2].

D-ladder invariance

The definitions of X, Y, Z are insensitive to D; the payload cost per snap remains one bit, while geometric scales follow Paper I's relations[9]:

$$\frac{{}^{(D)}r_{\mathcal{L}}}{{}^{(D)}r_H} \quad \text{and} \quad \frac{{}^{(D)}A_{\mathcal{L}}}{{}^{(D)}A_H}.$$

9 Gluing across tiles and smoothness

Neighboring tiles with consistent (X, Y, Z) glue without discrete defects. Inconsistencies indicate nontrivial normal-bundle holonomy $\oint \omega$, resolvable by tie-breakers or one-ring smoothing. A global smoothing theorem is conjectured and deferred.

10 Information-theoretic channel and locality

The snap acts on the state as a CP-TP dephasing channel

$$\rho \;\mapsto\; \sum_r \Pi_r \, \rho \, \Pi_r.$$

Here $r \in \{0, 1\}$ is the classical register that records the macroscopic predicate outcome. We identify this register with the payload bit itself:

$$r \equiv W,$$

and Π_r is the projector onto the branch in which the apparatus predicate takes the value r. Tracing out the classical register that stores W implements the dephasing channel above; because this map is completely positive, trace-preserving, and constructed locally at the Ledger, it commutes with spacelike observables and cannot be used for superluminal signaling.

11 Discussion and outlook

The Infinity bits provides the minimal, boost-invariant discrete geometry needed to invert NPR kinematics up to smooth gauge, while keeping capacity accounting faithful to TDT. Open items are (i) global constructive sufficiency, (ii) explicit Kerr extraction maps, and (iii) the quantitative map to inner phases for echoes. Next in the series: the focused Raychaudhuri impulse, echo phenomenology, and a tensor-network realization where the metadata control boundary phases in a HaPPY-like node.

A A Toy "Almost–Flat" 4D Walkthrough for Extracting Infinity Bits

This appendix implements the extraction recipe of Sec. 5 in a concrete example: an almost-flat 4D "mini–Kerr" patch. The goal is to show explicitly how the invariants that define X, Y, Z can be computed from a finite set of metric components and null dyad data.

Aim and scope

This appendix gives a concrete, pencil–and–paper recipe for extracting the Infinity Bits

$$\{W,\ X,\ Y,\ Z\}$$

from an *explicit* local metric patch. The construction is fully local to a single ledger tile $\mathcal{L}$, boost–invariant in the deleted normal plane, and uses only first derivatives of the metric. We work in a simple "almost–flat" background so every step can be read off from the metric matrix.

A.1 Local metric model near the ledger

Work in coordinates (t, r, θ, ϕ) centered on a tile at radius $r = r_{\mathcal{L}}$ and colatitude $\theta = \theta_0 \in (0, \pi)$. Consider a weakly rotating, weakly anisotropic metric (keep only linear terms in the small parameters Ω, ε_+):

$$g_{AB} \approx \begin{pmatrix} -1 & 0 & 0 & -\Omega\, r^2 \sin^2\theta \\ 0 & +1 & 0 & 0 \\ 0 & 0 & r^2(1+\varepsilon_+) & 0 \\ -\Omega\, r^2 \sin^2\theta & 0 & 0 & r^2(1-\varepsilon_+)\sin^2\theta \end{pmatrix},$$

with $|\Omega r| \ll 1$ and $|\varepsilon_+| \ll 1$. The cross term $g_{t\phi} \propto \Omega$ mimics slow frame–dragging ("mini–Kerr"), while ε_+ encodes a small axisymmetric tidal anisotropy on the 2–sphere. We will also use the first derivatives $\partial_t \varepsilon_+$, $\partial_r \varepsilon_+$ at the tile.

A.2 Tangent 2–metric and orthonormal dyad on $\mathcal{L}$

Restrict g_{AB} to the ledger surface $\mathcal{L}$ (spanned by θ, ϕ):

$$h_{ab} = \begin{pmatrix} r^2(1+\varepsilon_+) & 0 \\ 0 & r^2(1-\varepsilon_+)\sin^2\theta \end{pmatrix}_{(\theta,\phi)}.$$

Choose a *right–handed* orthonormal dyad $\{e_1, e_2\}$ tangent to $\mathcal{L}$:

$$e_1 = \frac{1}{r\sqrt{1+\varepsilon_+}}\,\partial_\theta, \qquad e_2 = \frac{1}{r\sin\theta\sqrt{1-\varepsilon_+}}\,\partial_\phi.$$

This sets the parity convention for the X bit.

A.3 Null directors and basic kinematics

Let the deleted normal plane be spanned by two future–directed null directors n_+, n_- normalized by

$$n_+ \cdot n_- = -1$$

and orthogonal to $\mathcal{L}$. For the toy metric,

$$n_+ = \frac{1}{\sqrt{2}}(\partial_t + \partial_r), \qquad n_- = \frac{1}{\sqrt{2}}(\partial_t - \partial_r),$$

which indeed satisfy $n_+ \cdot n_- = -1$ and $n_+ \cdot e_i = n_- \cdot e_i = 0$ up to $\mathcal{O}(\Omega, \varepsilon_+)$. The projector to the ledger tangent bundle is

$$P^A{}_B = \delta^A{}_B + n_+{}^A n_{-B} + n_-{}^A n_{+B}.$$

The intrinsic metric evolves along n_+, n_- via Lie derivatives; define the shear tensors on $\mathcal{L}$ by

$$\sigma^{(\pm)}_{ab} \;\equiv\; \tfrac{1}{2}\big(\mathcal{L}_{n\pm} h_{ab}\big)^{\rm tf},$$

the trace–free parts of $\frac{1}{2}\mathcal{L}_{n_+/n_-} h_{ab}$.

A.4 Orientation/parity bit X

Define the 4–volume sign using the spacetime volume form ε_{ABCD}:

$$\chi \;\equiv\; \mathrm{sgn}\Big(\varepsilon_{ABCD}\, e_1^A e_2^B n_+{}^C n_-{}^D\Big), \qquad X \;=\; \chi \in \{+1, -1\}.$$

Reasoning. If $\{e_1, e_2\}$ is chosen right–handed and n_+, n_- are future–directed as above, $\chi = +1$. Under a local boost $n_+ \to e^{+\lambda} n_+$, $n_- \to e^{-\lambda} n_-$, the wedge $n_+ \wedge n_-$ rescales by a positive factor, so χ (hence X) is boost–invariant. Flipping the dyad orientation flips X.

A.5 Shear–alignment bit Y

Use the boost–invariant scalar

$$S_\sigma \;\equiv\; \sigma^{(+)}_{ab}\, \sigma^{(-)ab}, \qquad Y \;=\; \mathrm{sgn}\big(S_\sigma\big) \in \{+1, -1\}.$$

Reasoning. In our toy model,

$$\mathcal{L}_{n_+} h_{ab} \;=\; \tfrac{1}{\sqrt{2}}(\partial_t + \partial_r) h_{ab}, \qquad \mathcal{L}_{n_-} h_{ab} \;=\; \tfrac{1}{\sqrt{2}}(\partial_t - \partial_r) h_{ab}.$$

Taking the trace–free parts and forming $\sigma^{(+)} : \sigma^{(-)}$ yields

$$S_\sigma \;\propto\; \big\|(\partial_t h)^{\rm tf}\big\|^2 - \big\|(\partial_r h)^{\rm tf}\big\|^2.$$

Thus:

$$\begin{aligned} \text{time–dominated anisotropy } &\Rightarrow S_\sigma > 0 \Rightarrow Y = +1, \\ \text{radius–dominated anisotropy } &\Rightarrow S_\sigma < 0 \Rightarrow Y = -1. \end{aligned}$$

In static axisymmetry ($\partial_t \varepsilon_+ = 0$), one finds $\sigma^{(-)}_{ab} = -\sigma^{(+)}_{ab}$, so $S_\sigma = -\|\sigma^{(+)}\|^2 \le 0 \Rightarrow Y = -1$.

A.6 Chirality/twist bit Z

Define the Hájíček (normal–bundle) connection and its curvature on $\mathcal{L}$:

$$\omega_a \;\equiv\; -\, n_{-B}\, \nabla_a n_+{}^{B}, \qquad \mathcal{F}_{ab} \;\equiv\; \nabla_a \omega_b - \nabla_b \omega_a,$$

and the pseudoscalar density

$$\Upsilon \;\equiv\; \epsilon^{ab}\, \mathcal{F}_{ab}, \qquad Z \;=\; \mathrm{sgn}(\Upsilon) \in \{+1, -1\},$$

with ϵ^{ab} the Levi–Civita tensor on $\mathcal{L}$. For the axisymmetric toy metric, to first order in Ω,

$$\omega_\phi \;\approx\; \Omega\, r^2 \sin^2\theta, \qquad \omega_\theta \approx 0, \quad \Longrightarrow \quad \Upsilon \;\approx\; \partial_\theta \omega_\phi \;=\; 2\Omega\, r^2 \sin\theta \cos\theta,$$

so at the tile ($\theta_0 \in (0, \frac{\pi}{2})$) we get $Z = \mathrm{sgn}(\Omega)$. At the equator $\Upsilon = 0$ (measure–zero); use a one–ring average or a fixed tie–breaker there.

A.7 The branch/write bit W

Fix once and for all a *model predicate* that turns the local matter/field configuration into a classical branch bit (e.g. "did the normal energy flux through the tile exceed threshold during $\Delta\tau$?"). Then

$$W \in \{0, 1\}, \qquad W = 1 \text{ iff predicate true at the snap.}$$

This is the *only* irreversible bit; by TDT–1/2 a one–bit payload write costs $\Delta S = \ln 2$ and

$$\Delta A \;=\; 4 \ln 2\, \ell_P^2 \quad \text{(in 4D)}.$$

A.8 Boost/gauge invariance (sketch)

Under SO(1, 1) boosts $n_+ \to e^{+\lambda} n_+$, $n_- \to e^{-\lambda} n_-$:

- X: $\varepsilon(e_1, e_2, n_+, n_-)$ rescales by a positive factor, so sgn is unchanged.
- Y: $\sigma^{(\pm)}_{ab} = \frac{1}{2}(\mathcal{L}_{n_+/n_-} h_{ab})^{\mathrm{tf}}$ each rescales oppositely; their full contraction $\sigma^{(+)} : \sigma^{(-)}$ is invariant.
- Z: ω_a shifts by a gauge gradient while $\mathcal{F}_{ab} = \nabla_a \omega_b - \nabla_b \omega_a$ is gauge– and boost–invariant; thus $\Upsilon = \epsilon^{ab} \mathcal{F}_{ab}$ keeps its sign.

Hence (X, Y, Z) are well–defined boost–invariant bits.

A.9 Finite–difference recipe from metric data (practical extraction)

On a numerical tile (or analytic worksheet):

1. Build h_{ab} from g_{AB} by restriction to $\mathcal{L}$; construct a right–handed dyad $\{e_1, e_2\}$. Compute X from the 4–volume sign.
2. Estimate $(\partial_t h)^{\mathrm{tf}}$, $(\partial_r h)^{\mathrm{tf}}$ by centered differences in a small star around the tile; form

$$S_\sigma \;\propto\; \|(\partial_t h)^{\mathrm{tf}}\|^2 - \|(\partial_r h)^{\mathrm{tf}}\|^2,$$

   and set $Y = \mathrm{sgn}(S_\sigma)$.
3. Compute ω_a by projecting $\nabla_a n_+$ along n_- (or equivalently $\nabla_a n_-$ along n_+); then $\mathcal{F}_{ab} = \partial_a \omega_b - \partial_b \omega_a$ (on $\mathcal{L}$), and $Z = \mathrm{sgn}(\epsilon^{ab} \mathcal{F}_{ab})$.
4. Evaluate the fixed predicate to get W.

A.10 Worked numerical sample (at a glance)

At $r = r_{\mathcal{L}}$, $\theta_0 = \pi/4$ take

$$\Omega \ = \ +3 \times 10^{-3}/r_{\mathcal{L}}, \qquad \partial_r \varepsilon_+ \ = \ +2 \times 10^{-2}/r_{\mathcal{L}}, \qquad \partial_t \varepsilon_+ = 0.$$

Then:

$$\begin{aligned} W &= \text{predicate outcome.} \\ X &= +1 \text{ (by dyad choice)}, \\ Y &= -1 \text{ (radius–dominated shear)}, \\ Z &= +1 \text{ (co–rotating twist)}, \end{aligned}$$

If a transient wave makes $|\partial_t \varepsilon_+| \gg |\partial_r \varepsilon_+|$ momentarily, the same procedure flips Y to $+1$ while X, Z are unchanged.

A.11 Tie–breakers and robustness

Measure–zero cases (exact zeros) can be resolved by fixed defaults (e.g. $+1$) or by a one–ring majority among neighboring tiles. Small–noise stability is controlled by the margins:

$$\left\|(\partial_t h)^{\text{tf}}\right\|^2 - \left\|(\partial_r h)^{\text{tf}}\right\|^2 \ \gtrless \ 0 \quad \Rightarrow \quad \text{flip only if perturbations exceed the signed gap.}$$

Since X, Y, Z depend on *signs* of boost–invariant scalars, they are robust against smooth gauge changes and small metric noise.

A.12 Why these four bits suffice (local reversibility sketch)

Given (X, Y, Z) on a tile and its neighbors, smoothness plus the boost gauge on the normal plane fix how $\mathcal{L}$ was embedded in the pre–snap 4D geometry up to diffeomorphisms; W is the only irreversible bit (the payload). Thus the quadrupole $\{W, X, Y, Z\}$ is the minimal reversible record consistent with the NPR map and TDT bookkeeping.

B Future directions and open work

This paper establishes the local, boost-invariant definition of the Infinity bits $\{W, X, Y, Z\}$, their extraction recipe, and the link to TDT capacity accounting. Here we summarize technical directions that will be developed in subsequent work.

Global reversibility and detailed proofs

Locally, Theorem 4 (idempotent reversibility) shows that NPR plus the metadata (X, Y, Z) fixes discrete kinematic ambiguities up to diffeomorphisms and the $SO(1,1)$ boost gauge. A full, global constructive proof on arbitrary Ledger topologies—including an explicit smoothing theorem for gluing tiles with consistent (X, Y, Z) and nontrivial normal-bundle holonomy $\oint \omega$—is left open. A future technical note will give complete, coordinate-level proofs of boost invariance, dyad behavior, and minimality, extending the sketches in Sec. 4 and Appendix A.

Tie-breakers, stability, and numerics

Measure-zero cases where the defining scalars vanish ($S_\sigma = 0$, $\epsilon^{ab}\mathcal{F}_{ab} = 0$) are currently resolved by fixed conventions or one-ring majorities. A systematic analysis of stability under numerical noise and coarse graining—including thresholds for flipping X, Y, Z and error budgets in finite-difference extractions—is deferred to a numerical implementation paper.

Explicit tetrad constructions in standard spacetimes

The worked Schwarzschild and Kerr remarks in Sec. 8 can be extended to full tetrad constructions in Eddington–Finkelstein, Kruskal, and Kerr (Boyer–Lindquist or horizon-adapted) frames. These will provide explicit tables of (X, Y, Z) across the Ledger for benchmark spacetimes, and will be used to calibrate the inner phase branches $S_\ell(\omega)$ in the echoes phenomenology paper.

Relation to isolated horizons and Gauss–Codazzi data

The normal-bundle connection ω_a and curvature $\mathcal{F}_{ab}$ used in the definition of Z are standard in isolated-horizon technology [3]. A future note will connect the Infinity bits to the Gauss–Codazzi decomposition of data on $\mathcal{L}$, clarifying how NPR removes normal components while preserving intrinsic curvature and normal-bundle information, and how this structure fits within the broader isolated-horizon framework.

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