On the Necessity of Codimension-2 Reduction in Resolving Spacetime Singularities

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Abstract

This article motivates the foundational axiom of Dimensional Collapse Theory (DCT/QG): that when curvature reaches a universal threshold, spacetime undergoes a local, codimension-2 "snap" which removes a null 2-plane and leaves behind a spacelike $(D-2)$-dimensional surface—the Ledger $\mathcal{L}$. The aim is not to derive this rule from deeper principles, but to show that it is a highly constrained and natural choice once we impose a small set of physical demands: locality, covariance, causality, boost invariance, and compatibility with area-based thermodynamics. We review the singularity problem as a geometric crisis driven by the Raychaudhuri equation, enumerate possible local geometric responses at a curvature threshold, and argue that a codimension-1 or higher codimension reduction fails to provide a stable, thermodynamic boundary. In contrast, deleting exactly the two null directions normal to a collapsing cross-section yields a unique "Goldilocks" option: it halts catastrophic focusing, preserves causal structure, creates a natural area carrier for entropy, and sets the stage for the detailed constructions in Papers I–IV of the DCTQG series.

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1 Singularities as a geometric crisis

The classical singularity theorems show that, under broad conditions, General Relativity predicts geodesic incompleteness and curvature blow-ups [1]. The underlying mechanism is not mysterious: it is the relentless focusing of geodesic congruences.

For a null congruence with tangent k^a, expansion θ, and affine parameter λ, the Raychaudhuri equation in D spacetime dimensions is [2, 3]

$$\frac{\mathrm{d}\theta}{\mathrm{d}\lambda} = -\frac{1}{D-2}\,\theta^2 - \sigma_{ab}\sigma^{ab} - R_{ab}k^a k^b, \qquad \frac{\mathrm{d}\ln\mathcal{A}}{\mathrm{d}\lambda} = \theta, \tag{1}$$

where σ_{ab} is the shear and $\mathcal{A}(\lambda)$ is the cross-sectional area of an infinitesimal bundle. Under the null energy condition (NEC), $R_{ab}k^a k^b \geq 0$, so a negative θ typically runs away to $-\infty$ in finite affine parameter. The area $\mathcal{A}$ shrinks to zero and curvature invariants diverge.

From the perspective of effective field theory, this signals a *geometric crisis*: the classical description runs out before we reach the regime we would like to probe. A satisfactory resolution should obey at least four conditions:

(a) **Locality and covariance:** the rule must be formulated locally in terms of tensorial quantities, without reference to global coordinates or boundary conditions.

(b) **Causality:** the resolution must not introduce superluminal signaling or retro-causal behavior.

(c) **Thermodynamic compatibility:** in the presence of horizons, any resolution should respect the area-entropy connection [4, 5, 6] and the generalized second law [7].

(d) **Mildness:** the modification should be as small and surgical as possible, leaving ordinary GR+QFT intact where curvature is subcritical.

Dimensional Collapse Theory starts from the idea that the breakdown occurs because we are trying to continue the geometry *through* a regime in which the classical dimensionality of spacetime is no longer the correct description. The central proposal is that spacetime carries a simple, local "click rule": when a clean, dimensionless invariant $\mathcal{I}[g]$ crosses a universal threshold $\mathcal{I}_{\rm crit}$ [8], the manifold undergoes a small, discrete change in its effective dimensionality.

2 Local curvature triggers and click rules

The first step is to ensure that the trigger is geometric and local. In DCT we use a curvature scalar of the schematic form

$$\mathcal{I} \;\equiv\; K L^4, \qquad K = R_{ABCD}R^{ABCD}, \tag{2}$$

where L is a macroscopic length scale set by the environment [8]. The details of the choice and the derivation of the universal critical value $\mathcal{I}_{\rm crit}$ are the subject of Paper I. For present purposes we only need three qualitative properties:

(i) $\mathcal{I}$ is scalar and local.

(ii) $\mathcal{I}$ is dimensionless and insensitive to trivial rescalings.

(iii) There exists a universal critical value $\mathcal{I}_{\text{crit}}$ such that, in astrophysical black holes, $\mathcal{I} \to \mathcal{I}_{\text{crit}}$ just inside the event horizon.

The click rule is:

$$\mathcal{I} = \mathcal{I}_{\text{crit}} \quad \Longrightarrow \quad \text{spacetime performs a local } \textit{snap}. \tag{3}$$

The rest of the DCTQG series works out “what the snap does” in detail. This paper focuses on one key aspect: *how many dimensions are removed* and *why*.

3 A menu of geometric responses

Suppose that in some small neighborhood the curvature grows and $\mathcal{I} \to \mathcal{I}_{\text{crit}}$. What can the geometry do, locally and covariantly, to avoid the catastrophic focusing implied by Eq. (1)?

At an abstract level, there are three broad classes of response:

(a) **Modify the dynamics:** change the right-hand side of the Raychaudhuri equation by violating the energy conditions, introducing new repulsive forces, or changing the Einstein equations themselves.

(b) **Impose nonlocal structure:** introduce global matching conditions, wormholes, or other topological identifications which redirect the geodesics elsewhere.

(c) **Modify the manifold structure:** change the dimensionality of the effective spacetime in the region where the crisis occurs.

Options (a) and (b) are widely explored in the literature. DCT instead pursues option (c): keep the field equations and local energy conditions intact as long as possible, and let spacetime itself take a small, discrete step in dimensionality when the click rule fires. The question then becomes sharp: *what codimension ΔD is admissible?*

4 Why codimension one is not enough

A seemingly mild modification is to remove a single null direction, i.e. to replace a D-dimensional region by a $(D-1)$-dimensional null hypersurface when $\mathcal{I} = \mathcal{I}_{\text{crit}}$. This effectively projects the dynamics onto a lightfront and might appear to “cap off” the focusing.

However, codimension-one reduction fails on two fronts:

(1) Focusing persists within the null hypersurface. A null hypersurface still admits null congruences tangent to it, with their own expansions and shears. The Raychaudhuri equation continues to drive $\mathcal{A} \to 0$ *within* the reduced geometry. One has simply moved the problem from the bulk to the boundary, not removed it.

(2) Null surfaces do not carry a robust area entropy. Bekenstein–Hawking entropy and the generalized entropy functionals used in quantum extremal surface (QES) constructions are built on $(D-2)$-dimensional spacelike surfaces with well-defined area [4, 5, 6, ?]. The "area" of a patch on a null hypersurface depends on an arbitrary choice of cross-section and affine parameter; it cannot serve as a stable, frame-independent measure of thermodynamic entropy. Any theory that aims to preserve the area–entropy connection in the spirit of Jacobson's derivation of Einstein's equations [9] must treat null sheets as *carriers* of flux, not as the fundamental storage surface for entropy.

For DCT, which explicitly uses area quanta on a codimension-2 Ledger to account for information payloads [10], a codimension-1 reduction would be a poor foundation.

5 Codimension-two removal as the "Goldilocks" option

DCT instead adopts a codimension-two operation: a local removal of the normal null 2-plane spanned by (n_+, n_-). Concretely, pick null directors ${n_+}^A, {n_-}^A$ satisfying

$$n_+ \cdot n_- = -1, \tag{4}$$

and define the tangential projector

$$P^A{}_B = \delta^A{}_B + {n_+}^A n_{-B} + {n_-}^A n_{+B}. \tag{5}$$

The Null–Pair Removal (NPR) map of Paper II [11] contracts every index of every tensor with $P^A{}_B$, deleting components along the normal null 2-plane while preserving the intrinsic $(D-2)$-dimensional geometry.

This choice satisfies the constraints listed in Sec. 1 in a remarkably tight way:

(a) **Minimality.** Removing exactly the two null directions normal to a collapsing cross-section is the smallest possible change that directly targets the focusing problem in Eq. (1). Higher-codimension removals would overkill the geometry and destroy more structure than necessary.

(b) **Boost invariance.** A single null direction has no invariant normalization; rescalings $k^a \to e^\lambda k^a$ are an exact local symmetry. The pair (n_+, n_-), by contrast, carries an invariant boost class under $SO(1,1)$: $n_+ \to e^\lambda n_+$, $n_- \to e^{-\lambda} n_-$. The projector $P^A{}_B$ is built to be invariant under this symmetry. Any local rule that singles out one null direction over the other would explicitly break this gauge freedom; NPR does not.

(c) **Thermodynamic canvas.** The image of $P^A{}_B$ is a spacelike $(D-2)$-dimensional surface $\mathcal{L}$ with intrinsic metric h_{ab} and area $A(\mathcal{L})$. This provides a natural "canvas" for area-based entropy. In Paper I, this surface is placed at a universal radius inside a Schwarzschild black hole via the curvature trigger $\mathcal{I} = \mathcal{I}_{\text{crit}}$ [8], and in Paper III its area is quantized by the rules of Transdimensional Thermodynamics (TDT) [10].

(d) **Information accounting and reversibility.** Removing a null 2-plane is not purely topological: it deletes geometric data (orientation, shear alignment, twist) that can, in principle, be recorded. This is precisely what the Infinity bits $\{W, X, Y, Z\}$ of Paper IV are designed to capture [12]. NPR plus these bits make the snap reversible up to smooth gauge, in the same way that recording a small set of discrete choices restores reversibility to a coarse-grained map.

In this sense, codimension-two reduction is a "Goldilocks" operation: any less and the singularity problem persists; any more and one loses the geometric structure needed to define a thermodynamic boundary.

6 How the axiom organizes the DCTQG series

The codimension-2 snap is the only genuine axiom of DCT/QG. Everything else in the series is derived from, or calibrated against, this geometric choice.

- **Part I.** Given the click rule $\mathcal{I} = \mathcal{I}_{\text{crit}}$, the location of $\mathcal{L}$ in Schwarzschild and more general black-hole geometries is computed explicitly, and the universal constant $\mathcal{T}$ is derived from the requirement that Ledger and horizon entropies remain consistent with Bekenstein–Hawking and QES bounds [8].
- **Part II.** NPR is worked out in detail: the precise form of $P^A{}_B$, its boost invariance, and its action on fields of arbitrary rank are specified. The Raychaudhuri equation is updated to a form that remains finite in the presence of snaps [11].
- **Part III.** Transdimensional Thermodynamics is constructed on $\mathcal{L}$: one bit of payload entropy costs a fixed quantum of area, and the Ledger entropy grows in a staircase fashion at snaps while remaining constant between them [10].
- **Part IV.** The Infinity bits $\{W, X, Y, Z\}$ are introduced as a minimal, boost-invariant register that records discrete kinematic information about the removed null 2-plane, making the NPR operation reversible up to smooth gauge and allowing a clean information-theoretic interpretation of snaps [12].

Seen from this vantage point, the codimension-2 axiom is analogous to the Schrödinger equation in quantum mechanics: it is not derived from something deeper within the framework, but it proves to be the unique simple postulate from which a large, tightly constrained structure grows.

7 Conclusion

The dimensional-collapse axiom of DCT/QG can be stated in one line: *when a local curvature invariant $\mathcal{I}$ reaches a universal threshold $\mathcal{I}_{\text{crit}}$, spacetime undergoes a codimension-2 snap that removes the normal null 2-plane and leaves a spacelike Ledger surface $\mathcal{L}$ behind.* This paper has argued that, once we insist on locality, covariance, causality, boost invariance, and compatibility with area-based thermodynamics, this is not a whimsical choice among many, but a highly constrained option.

A codimension-1 reduction fails to halt focusing and cannot carry a stable entropy. Higher-codimension reductions destroy too much structure. Only the codimension-2 NPR operation both disarms the singularity mechanism and provides a natural canvas for entropy, information accounting, and reversible reconstruction. The success of DCT/QG as a whole—from Ledger placement and TDT to Infinity bits and echoes phenomenology—is, in this sense, the primary evidence in favor of this simple axiom.

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