The Laws of Transdimensional Thermodynamics:

Capacity, Payload, and the Area Quantum at the Ledger in DCT

Marek Hubka *

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Abstract

We formulate and justify three *Transdimensional Thermodynamics* (TDT) laws that govern snap events on the ledger $\mathcal{L}$: (TDT–1) the capacity–payload balance $\Delta S_{\mathcal{L}} = \Delta S_{\text{info}}$; (TDT–2) the area calibration that fixes the one-bit cost $\Delta\mathcal{A}_{\text{one bit}} = 4\ln 2\,\ell_{\text{P}}^2$ in 4D ($\Delta\mathcal{A}_D = {}^{(D)}A_{\text{P}}/\mathcal{T}$ in D dimensions); and (TDT–3) a local, covariant statement that ledger entropy grows only at snaps via a delta-supported source proportional to the exterior entropy influx. These laws are consistent with the black-hole first law and the Clausius relation in local Rindler patches [1, 2, 3, 4, 5], and fit hand-in-glove with the geometric mechanism (NPR) and lossless inner boundary (real–Robin) of DCT/QG parts I-II [6, 7, 8, 9, 10].

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*Independent Researcher, Czech Republic.
Website: tidesofuncertainty.com. Email: marek@tidesofuncertainty.com.

1 Introduction and context

DCT/QG part I fixed the *placement* of the ledger $\mathcal{L}$ [6] by the invariant curvature trigger

$$\mathcal{I} = \mathcal{I}_{\text{crit}} = \frac{12}{\mathcal{T}},$$

which yields $r_{\mathcal{L}} = \sqrt{\mathcal{T}}\,R_{\text{S}}$ and $A_{\mathcal{L}}/A_{\text{H}} = \mathcal{T}$ for the Schwarzschild family [1, 2, 6]. Part II defined the geometric mechanism of a snap, *Null-Pair Removal* (NPR)[7], and justified the *real–Robin* inner boundary that makes the exterior evolution unitary and lossless between snaps [7, 8, 9, 10].

In this paper we add the missing thermodynamic layer: statements that tie the *payload information* written at a snap to *ledger capacity* and a *geometric area tick*. The three boxed laws below are the main deliverables.

2 Setup and notation

We use units $c = \hbar = k_B = 1$, keeping the Newton constant G (or G_D) explicit. We reserve G_D exclusively for the D-dimensional gravitational coupling in the Einstein–Hilbert action. For clarity we denote the D-dimensional Planck "area" unit by ${}^{(D)}A_{\text{P}} \equiv {}^{(D)}\ell_{\text{P}}^{\,D-2}$; in our normalization ${}^{(D)}A_{\text{P}} = G_D$.

Entropy is measured in *nats*; one bit equals $\ln 2$ nats. The Ledger's thermodynamic entropy (capacity) is

$$S_{\mathcal{L}} \;=\; \frac{A_{\mathcal{L}}}{4\,\ell_{\text{P}}^2} \;=\; \frac{A_{\mathcal{L}}}{4\,G} \quad (4\text{D}) \qquad \text{and} \qquad {}^{(D)}S_{\mathcal{L}} \;=\; \frac{{}^{(D)}A_{\mathcal{L}}}{4\,{}^{(D)}A_{\text{P}}} \;=\; \frac{{}^{(D)}A_{\mathcal{L}}}{4\,G_D} \quad (D\text{-dim}).$$

One irreversible bit $\big(\Delta S_{\text{info}} = \ln 2\big)$ consumes

$$\Delta\mathcal{A}_{\text{one bit}} \;=\; 4\ln 2\,\ell_{\text{P}}^2 \quad (4\text{D}), \qquad \Delta\mathcal{A}_D \;=\; 4\ln 2\,{}^{(D)}A_{\text{P}} \quad (D\text{-dim}).$$

Information vs. thermodynamic entropy (definitions). We distinguish two a priori different entropies:

- *Information (Shannon/von Neumann) entropy* S_{info}: a measure of missing information about a variable or quantum state (units: nats unless noted; one bit $= \ln 2$ nats).
- *Thermodynamic (Boltzmann/Gibbs) entropy* S_{th}: the state function entering Clausius $\delta Q = T\,dS_{\text{th}}$ and the Bekenstein–Hawking relation $S_{\text{BH}} = A/(4\ell_{\text{P}}^2)$.

Unless stated otherwise, S_{th} refers to black-hole or Ledger thermodynamic entropy derived from geometry. S_{info} refers to the *payload* written during a snap.

3 The three TDT laws

Information–Thermodynamics Distinction *Conceptual non-identity.* Information entropy S_{info} and thermodynamic entropy S_{th} are distinct notions. In particular,

$$S_{\text{info}} \neq S_{\text{th}} \text{ in general}, \qquad \delta Q = T\, dS_{\text{th}} \text{ (Clausius)}$$

and $S_{\text{BH}} = A/(4\ell_{\text{P}}^2)$ concerns S_{th}.

TDT 1 (Capacity–payload balance)**.** At a snap, DCT imposes the *capacity–payload constraint*

$$\boxed{\Delta S_{\mathcal{L}} \;=\; \Delta S_{\text{info}}}$$

Interpretation. Writing ΔS_{info} nats of *payload* at a snap increases ledger capacity by $\Delta S_{\mathcal{L}}$; reversible metadata (X, Y, Z)[11] carry no capacity cost.
This is a *matching rule* between distinct quantities, not a claim that S_{info} and $S_{\mathcal{L}}$ are the same concept.

TDT 2 (Area calibration)**.**

$$\boxed{\Delta\mathcal{A}_{\text{one bit}} = 4\,\ln 2\,\ell_{\text{P}}^2 \quad \text{(4D)}, \qquad\qquad \Delta\mathcal{A}_D = \frac{{}^{(D)}A_{\text{P}}}{\mathcal{T}} \quad (D\text{-dim}).}$$

for a single irreversible payload bit. Equivalently, $\Delta S_{\mathcal{L}} = \ln 2$ per payload bit.
Units: one bit $= \ln 2$ nats; hence $\Delta\mathcal{A} = 4\ln 2\,\ell_{\text{P}}^2$ matches $S_{\mathcal{L}} = \ln 2$ via $S_{\mathcal{L}} = A_{\mathcal{L}}/(4\ell_{\text{P}}^2)$.

TDT 3 (Episodic growth — local covariant form)**.** Let s^a be a *surface* entropy current on the Ledger worldtube $\mathcal{W}_{\mathcal{L}}$ (induced metric γ_{ab}, volume element $\mathrm{d}V_{\mathcal{W}}$), and let σ_{in} denote the exterior entropy influx density. Then, in the sense of distributions on $\mathcal{W}_{\mathcal{L}}$,

$$\boxed{\nabla_a s^a \;=\; \sum_i J_{(i)}\,\delta_{\Sigma_i}}$$

with $J_{(i)} = \int_{\Sigma_i} \sigma_{\text{in}}\,\mathrm{d}\Sigma$. Equivalently, for any slab $\mathcal{V} \subset \mathcal{W}_{\mathcal{L}}$ crossing N snaps,

$$\int_{\mathcal{V}} \nabla_a s^a\,\mathrm{d}V_{\mathcal{W}} \;=\; \sum_{i=1}^{N} \Delta S_{\mathcal{L}}^{(i)} \;=\; \sum_{i=1}^{N} \int_{\Sigma_i} \sigma_{\text{in}}\,\mathrm{d}\Sigma,$$

and between snaps $\nabla_a s^a = 0$. Along an inward generator with affine parameter λ,

$$\left.\frac{\mathrm{d}S_{\mathcal{L}}}{\mathrm{d}\lambda}\right|_{\text{snap}} \;=\; \sigma_{\text{in}}\Big|_{\mathcal{L}} \geq 0, \qquad \frac{\mathrm{d}S_{\mathcal{L}}}{\mathrm{d}\lambda} = 0 \text{ otherwise.}$$

common/images/p03/tdt_ledger_evolution.pdf

Figure 1: **TDT–3 (episodic growth) as a staircase.** Quiet phases have $\nabla_a s^a = 0$ (flat segments). At snaps, the Ledger writes payload $\Delta S_{\text{info}} = \ln 2$ and increases capacity $\Delta S_{\mathcal{L}} = \ln 2$, producing a jump $\Delta \mathcal{A}_{\mathcal{L}} = 4 \ln 2 \ell_{\text{P}}^2$ via $S_{\text{th}}|_{\mathcal{L}} = A_{\mathcal{L}}/4\ell_{\text{P}}^2$; see TDT-3.

4 Derivations and justification

Justification for TDT-1

TDT-1 is the foundational axiom of the framework's information bookkeeping. It states a direct, one-to-one equivalence between the information-theoretic content of an irreversible payload (ΔS_{info} in nats) and the resulting change in the Ledger's geometric entropy capacity ($\Delta S_{\mathcal{L}}$). Hence

$$\Delta S_{\mathcal{L}} = \Delta S_{\text{info}}.$$

This is motivated by the physical role of the Ledger:

- A snap changes the usable capacity of $\mathcal{L}$ only through the irreversible payload bit W.
- The reversible geometric metadata bits (X, Y, Z) set the local frame and inner phase but, being reversible, can be written and un-written without thermodynamic cost and thus do not consume net capacity[11].

Therefore, the entire capacity increase must be equal to the payload written. The information-theoretic origin of the constant $\mathcal{T}$ derived in part I of the DCT/QG framework[6], based on the minimal 1-in-4 bit structure of a reversible snap.

From BH entropy to TDT–2

Using $S_{\mathcal{L}} = \mathcal{T}\, S_{\text{BH}}$ with $S_{\text{BH}} = A_{\text{H}}/(4\ell_{\text{P}}^2)$ [5, 4, 1], a single payload bit gives

$$\Delta S_{\mathcal{L}} = \ln 2 \quad \Rightarrow \quad \Delta \mathcal{A} = 4\ell_{\text{P}}^2\, \Delta S_{\mathcal{L}} = 4 \ln 2\, \ell_{\text{P}}^2 \quad (4D).$$

In D dimensions, adopt ${}^{(D)}\ell_{\mathrm{P}}^{D-2} \equiv {}^{(D)}A_{\mathrm{P}}$, the area tick is equivalent to

$$\Delta\mathcal{A}_D = \frac{{}^{(D)}A_{\mathrm{P}}}{\mathcal{T}}.$$

This shows that the area quantum in TDT–2 is perfectly consistent with the ledger-horizon relations and TDT–1. A more formal, first-principles derivation of this result from the theory's foundational information axiom is provided in Appendix A.

Local covariant form TDT–3

Define a surface current s^a on the worldtube of $\mathcal{L}$ such that $\int_{\mathcal{C}} s^a \,\mathrm{d}\Sigma_a = \Delta S_{\mathcal{L}}$ for any cross-section $\mathcal{C}$. During lossless evolution with a real–Robin boundary, no energy/entropy crosses $\mathcal{L}$ [8, 10, 9], hence $\nabla_a s^a = 0$. At a snap, couple s^a to the coarse-grained entropy influx Φ_S associated with the predicate that defines W (NPR)[7]; the only allowed source is a delta-supported layer whose strength is fixed by the incoming payload entropy flux. This yields the boxed law in TDT–3[3].

5 Consistency with GR thermodynamics

Horizon response convention at a snap

Two consistent bookkeeping choices exist:

Option H (horizon-locked). Impose $A_{\mathcal{L}}/A_H = \mathcal{T}$ at all times. Then a Ledger tick $\Delta A_{\mathcal{L}} = 4\ln 2\,\ell_P^2$ implies a concurrent horizon tick $\Delta\mathcal{A}_H = \Delta\mathcal{A}_{\mathcal{L}}/\mathcal{T}$, so

$$\delta S_{\mathrm{BH}} = \frac{\Delta\mathcal{A}_H}{4\ell_P^2} = \frac{\ln 2}{\mathcal{T}}, \qquad \delta M = T_H\,\delta S_{\mathrm{BH}} = \frac{T_H\,\ln 2}{\mathcal{T}}.$$

Option L (ledger-local). Treat the snap as an interior write with no instantaneous horizon change: $\Delta\mathcal{A}_{\mathcal{L}} = 4\ln 2\,\ell_P^2$, $\Delta\mathcal{A}_H = 0$ at the event. The ratio $A_{\mathcal{L}}/A_H$ then relaxes back to $\mathcal{T}$ under subsequent GR evolution; no horizon heat crosses at the instant.

Adopted convention. For algebraic continuity with ledger placement in part I of DCT/QG[6] we adopt **Option H** in what follows; **Option L** leads to the same coarse-grained predictions after relaxation.

First law check (Kerr–Newman form)

The first law reads [5, 4, 1, 2]

$$\mathrm{d}M \;=\; T_H\,\mathrm{d}S_{\mathrm{BH}} + \Omega_H\,\mathrm{d}J + \Phi_H\,\mathrm{d}Q, \qquad S_{\mathrm{BH}} = \frac{A_{\mathrm{H}}}{4\ell_{\mathrm{P}}^2}.$$

We check consistency at fixed (J, Q) ($\mathrm{d}J = \mathrm{d}Q = 0$). At a *snap* (Option H), one payload bit gives

$$\Delta S_{\mathrm{info}} = \ln 2, \qquad \Delta S_{\mathcal{L}} = \ln 2 \quad (\text{TDT–1}).$$

Using the placement relation $S_{\mathcal{L}} = \mathcal{T}\, S_{\rm BH}$, the horizon change is

$$\Delta S_{\rm BH} \;=\; \frac{\Delta S_{\mathcal{L}}}{\mathcal{T}} \;=\; \frac{\ln 2}{\mathcal{T}}.$$

Interpreting the first law across this finite step as the energetic cost of writing this one bit to the black hole is therefore

$$\Delta M \;=\; T_H\big|_{\rm snap}\, \Delta S_{\rm BH} \;=\; T_H\big|_{\rm snap}\, \frac{\ln 2}{\mathcal{T}}.$$

This matches the area quantum (TDT–2): $\Delta A_{\mathcal{L}} = 4\ln 2\, \ell_{\rm P}^2$, hence

$$\Delta A_{\rm H} \;=\; \frac{\Delta A_{\mathcal{L}}}{\mathcal{T}}, \qquad \Delta S_{\rm BH} \;=\; \frac{\Delta A_{\rm H}}{4\ell_{\rm P}^2} \;=\; \frac{\ln 2}{\mathcal{T}}.$$

All relations are internally consistent.

Clausius sketch (local Rindler)

In Jacobson's argument, imposing $\delta Q = T\,\delta S$ across local Rindler horizons with $\delta S \propto \delta A$ yields Einstein's equations [3]. On $\mathcal{L}$ the entropy functional satisfies $S_{\mathcal{L}} = A_{\mathcal{L}}/(4\ell_{\rm P}^2)$, so formally

$$\mathrm{d}S_{\mathcal{L}} \;=\; \frac{1}{4\ell_{\rm P}^2}\, \mathrm{d}A_{\mathcal{L}}.$$

However, by the lossless boundary (real–Robin) condition, between snaps

$$\delta Q\big|_{\mathcal{L}} = 0 \;\Rightarrow\; \mathrm{d}S_{\mathcal{L}} = 0, \qquad \nabla_a s^a = 0.$$

At snaps the change is *discrete* and must be written with uppercase jumps:

$$\Delta S_{\mathcal{L}} = \ln 2, \qquad \Delta A_{\mathcal{L}} = 4\ln 2\, \ell_{\rm P}^2.$$

When comparing Ledger and horizon responses, the placement ratio $\frac{A_{\mathcal{L}}}{A_H} = \mathcal{T}$ implies $\Delta S_{\rm BH} = \Delta S_{\mathcal{L}}/\mathcal{T}$, reproducing the factor $1/\mathcal{T}$ seen in the first-law check.

6 Dynamics interface: NPR, Robin, and the impulse

NPR part of DCT shows snaps are instantaneous maps comprising (i) state dephasing $\rho \mapsto \sum_r \Pi_r \rho \Pi_r$ (writing W) [12, 13], (ii) NPR deletion of normal components, and (iii) a focused Raychaudhuri impulse that ticks area by $\Delta\mathcal{A}$, ensuring finite focusing:

$$\int_{\lambda^-}^{\lambda^+} \theta\, \mathrm{d}\lambda \;=\; \ln\Big(1 + \frac{\Delta\mathcal{A}}{A}\Big) \;\approx\; \frac{\Delta\mathcal{A}}{A}.$$

Between snaps, real–Robin implies unit-modulus reflectivity and self-adjoint exterior dynamics [8, 9, 10].

7 D-dimensional generalization

With ${}^{(D)}\ell_{\mathrm{P}}^{D-2} = {}^{(D)}A_{\mathrm{P}}$, the capacity and tick generalize to

$$S_{\mathcal{L}} = \frac{{}^{(D)}A_{\mathcal{L}}}{4\,{}^{(D)}A_{\mathrm{P}}}, \qquad \Delta\mathcal{A}_D = \frac{{}^{(D)}A_{\mathrm{P}}}{\mathcal{T}}.$$

From part I, the placement/area ratios along the Tangherlini ladder satisfy

$$\frac{{}^{(D)}r_{\mathcal{L}}}{{}^{(D)}r_{\mathrm{H}}} = \mathcal{T}^{\frac{1}{2(D-3)}}, \qquad \frac{{}^{(D)}A_{\mathcal{L}}}{{}^{(D)}A_{\mathrm{H}}} = \mathcal{T}^{\frac{D-2}{2(D-3)}},$$

so the *entropy* cost per snap is D-independent (one bit $\Rightarrow \ln 2$ nats of payload), while areas/radii scale with D as above [2, 6].

8 Capacity, used space, and remnants

Define the total capacity (in bits) and the cumulative count of irreversible payload writes:

$$C(M) \equiv \frac{A_{\mathcal{L}}}{4\ln 2\,\ell_{\mathrm{P}}^2}, \qquad N_{\mathrm{irr}}(t) \text{ non-decreasing (snaps only).}$$

Then

$$C(M(t)) \;\geq\; N_{\mathrm{irr}}(t),$$

with equality at a stabilized remnant where evaporation has reduced C to the used payload N_{irr}. In 4D, this gives a parametric floor

$$M_{\mathrm{rem}} \;\sim\; \frac{\ln 2}{\sqrt{\pi}}\sqrt{N_{\mathrm{irr}}}\,M_{\mathrm{P}},$$

modulo *kinematic* floors from mode structure[11].

9 Worked mini-examples

One-bit snap (4D). Given $\Delta S_{\mathrm{info}} = \ln 2$, TDT–1 yields $\Delta S_{\mathcal{L}} = \ln 2$ and TDT–2 gives $\Delta\mathcal{A} = 4\ln 2\,\ell_{\mathrm{P}}^2$.

Two separated snaps. Between snaps, $\nabla_a s^a = 0$ (lossless Robin), so the total jump in $S_{\mathcal{L}}$ is the sum of the two impulses, each proportional to the respective Φ_S at that event.

***D*-dimensional tick.** With G_D fixed, a single payload bit costs $\Delta\mathcal{A}_D = {}^{(D)}A_{\mathrm{P}}/\mathcal{T}$ irrespective of D; geometric scaling enters only through the placement/area ratios.

10 Discussion and outlook

TDT fixes the ledger's thermodynamic bookkeeping: how much capacity must be provisioned when a payload bit is written, how that translates into an area tick, and why ledger entropy grows only episodically. It *does not* decide *when* a snap occurs—that depends on the curvature trigger, NPR, and decoherence predicates [6, 7]. Upcoming in the DCT/QG series: we will formalize the reversible (X, Y, Z) bits; derive the focused Raychaudhuri impulse; develop GW echo phenomenology with inner phases set by this metadata.

A Formal Derivation of the Area Quantum

This appendix provides the rigorous, first-principles derivation of second law of TDT, the universal area quantum. This derivation serves as the formal proof of Postulate P1 in part I of the DCT/QG framework, which was used in the ledger placement and derivation of $\mathcal{T}$ constant. The argument proceeds from the theory's most fundamental axiom regarding the information structure of a snap.

1. **The Foundational Information Axiom:** The theory's starting point, justified heuristically in Section 4.1, is that a snap event has a minimal four-bit structure with only one irreversible payload bit (W). This fixes the payload fraction to $p = 1/4$ and thereby determines the universal constant $\mathcal{T} = 1/(4 \ln 2)$ through the full QES analysis in part I.

2. **Entropy Cost per Bit (TDT–1):** The first law of TDT, states that the ledger capacity increase (in nats) equals the payload information written. For a single payload bit, this cost is:
$$\Delta S_{\mathcal{L}} = \Delta S_{\text{info}} = \ln 2.$$

3. **Deriving the Area Cost:** The ledger's entropy is defined by the standard geometric formula, $S_{\mathcal{L}} = A_{\mathcal{L}}/(4\ell_{\text{P}}^2)$. A change in the ledger's entropy must therefore correspond to a change in its area according to $\Delta A_{\mathcal{L}} = 4\ell_{\text{P}}^2 \, \Delta S_{\mathcal{L}}$. By substituting the entropy cost for one bit from the previous step, we derive the precise area quantum:
$$\Delta \mathcal{A}_{\text{one bit}} = 4\ell_{\text{P}}^2 \times (\ln 2) = 4 \ln 2 \, \ell_{\text{P}}^2.$$

This result is TDT–2 in four dimensions. This derivation demonstrates that the specific area cost per bit is not an independent postulate but is a direct and necessary consequence of TDT–1 and the fundamental information-theoretic axiom that fixes $\mathcal{T}$. This completes the proof of Postulate P1[6].

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