Innlevering 5

Oppgave 1

a)

$$P(X < 1467) = P\left(Z < \frac{1467 - 1468}{2}\right) = 0.3085$$

$$P(1467 < \bar{X} < 1469) = P\left(-\frac{\sqrt{8}}{2} < Z < \frac{\sqrt{8}}{2}\right) = 1 - 2 * 0.0793 = 0.8414$$

a)

$$\hat{\mu} = \bar{X} \qquad Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim n(z, 1, 0)$$
$$P\left(-z_{\alpha/2} \leq \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq z_{\alpha/2}\right) = 1 - \alpha = 0.9$$
$$-z_{\alpha/2} \leq \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq z_{\alpha/2}$$
$$-z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \bar{X} - \mu \leq z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$
$$-\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq -\mu \leq -\bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$
$$\bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \geq \mu \geq \bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$
$$\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$\alpha = 0.1, \quad z_{\alpha/2} = 1.645, \quad \sigma = 2$$
$$\left[\bar{X} - 1.645\frac{2}{\sqrt{n}}, \bar{X} + 1.645\frac{2}{\sqrt{n}}\right]$$

$$\bar{X} = 1468.88, \quad n = 5$$
$$\left[1468.88 - 1.645\frac{2}{\sqrt{5}}, 1468.88 + 1.645\frac{2}{\sqrt{5}}\right] =$$
$$[1467.41, 1470.35]$$

Oppgave 2

a)

$$P(X > 185) = P\left(Z > \frac{185 - 179}{6}\right) = 0.1587$$

$$P(X > 185|X > 179) = \frac{P(X > 185 \cap X > 179)}{P(179)} = \frac{P(X > 185)}{0.5} = 2 * 0.1587 = 0.3174$$

b)

$$E(\hat{\beta}) = E\left(\frac{\sum_{i=1}^{n} X_i}{n\mu_M}\right) = E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)\frac{1}{\mu_M} = \frac{\mu_K}{\mu_M} \Rightarrow$$

$\hat{\beta}$ er forventningsrett.

$$\hat{\beta} = \frac{\bar{X}}{\mu_M} \sim n\left(\beta, \frac{\sigma_K}{\sqrt{n}\mu_M}\right)$$

$$Z = \frac{\frac{\bar{X}}{\mu_M} - \beta}{\frac{\sigma_K}{\sqrt{n}\mu_M}} = \frac{\bar{X} - \mu_M\beta}{\sqrt{\frac{\sigma^2}{n}}} \sim n(0,1)$$

$$T = \frac{\bar{X} - \mu_M\beta}{\sqrt{\frac{s^2}{n}}} \sim t_{n-1}$$

95% konfidensintervall:

$$P(-t_{\alpha/2} \leq T \leq t_{\alpha/2}) = 1 - \alpha = 0.95$$

$$-t_{\alpha/2} \leq \frac{\bar{X} - \mu_M\beta}{\sqrt{\frac{s^2}{n}}} \leq t_{\alpha/2}$$

$$-t_{\alpha/2}\sqrt{\frac{s^2}{n}} \leq \bar{X} - \mu_M\beta \leq t_{\alpha/2}\sqrt{\frac{s^2}{n}}$$

$$-\bar{X} - t_{\alpha/2}\sqrt{\frac{s^2}{n}} \leq -\mu_M\beta \leq -\bar{X} + t_{\alpha/2}\sqrt{\frac{s^2}{n}}$$

$$\frac{\bar{X}}{\mu_M} + \frac{t_{\alpha/2}}{\mu_M}\sqrt{\frac{s^2}{n}} \geq \beta \geq \frac{\bar{X}}{\mu_M} - \frac{t_{\alpha/2}}{\mu_M}\sqrt{\frac{s^2}{n}}$$

$$\frac{\bar{X}}{\mu_M} - \frac{t_{\alpha/2}}{\mu_M}\sqrt{\frac{s^2}{n}} \leq \beta \leq \frac{\bar{X}}{\mu_M} + \frac{t_{\alpha/2}}{\mu_M}\sqrt{\frac{s^2}{n}}$$

$$n = 5, \quad t_{4,0.025} = 2.776, \quad \bar{X} = 167.5, \quad s^2 = 5.1^2$$

$$\left[\frac{167.5}{179} - \frac{2.776}{179}\sqrt{\frac{5.1^2}{5}}, \frac{167.5}{179} - \frac{2.776}{179}\sqrt{\frac{5.1^2}{5}}\right] =$$

$$[0.900, 0.971]$$

Oppgave 3

a)

$$P(X > 0) = 0.5$$
$$P(X > 0.05) = P(Z > 0.05/0.03) = 1 - 0.9525 = 0.0475$$
$$Y = \frac{1}{2}(X_1 + X_2)$$
$$\sigma_Y = \frac{0.03}{\sqrt{2}} = 0.021$$
$$P(-0.05 > Y \cup Y > 0.05) = 2 \cdot P(Z < -0.05/0.021) = 2 \cdot 0.0087 = 0.0174$$

b)

$$\hat{\sigma}^2 = S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$$
$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$
$$E(\chi^2_{n-1}) = n - 1 \Rightarrow E\left(\frac{(n-1)S^2}{\sigma^2}\right) = \frac{n-1}{\sigma^2} E(S^2) = n - 1$$
$$E(S^2) = \sigma^2$$

$$\mathrm{Var}\left(S^2\right) = \mathrm{Var}\left(\frac{\sigma^2}{n-1}\chi^2_{n-1}\right) = \frac{\sigma}{\sqrt{n-1}} \cdot 2(n-1) = 2\sigma\sqrt{n-1}$$

$$Z = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$
$$P(\chi^2_{n-1,1-\alpha/2} \leq Z \leq \chi^2_{n-1,\alpha/2}) = 1 - \alpha$$
$$\chi^2_{n-1,1-\alpha/2} \leq \frac{(n-1)S^2}{\sigma^2} \leq \chi^2_{n-1,\alpha/2}$$
$$\frac{\chi^2_{n-1,1-\alpha/2}}{(n-1)S^2} \leq \frac{1}{\sigma^2} \leq \frac{\chi^2_{n-1,\alpha/2}}{(n-1)S^2}$$
$$\frac{(n-1)S^2}{\chi^2_{n-1,1-\alpha/2}} \geq \sigma^2 \geq \frac{(n-1)S^2}{\chi^2_{n-1,\alpha/2}} \qquad \frac{(n-1)S^2}{\chi^2_{n-1,\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)S^2}{\chi^2_{n-1,1-\alpha/2}}$$
$$n = 5, \quad S^2 = 0.0003905, \quad \alpha = 0.05, \quad \chi^2_{4,0.025} = 11.143, \quad \chi^2_{4,1-0.025} = 0.484$$
$$\left[\frac{4 \cdot 0.0003905}{11.143}, \frac{4 \cdot 0.0003905}{0.484}\right] = [0.00014, 0.00323]$$

Den oppgitte usikkerheten er inneholdt i intervallet som vil si at det er 95% sannsynelig at den faktisk er i dette intervallet.

Oppgave 4

a)

$$P(X > 40) = P\left(Z > \frac{40-35}{5}\right) = P(Z > 1) = 0.1587$$
$$P(30 < X < 40) = 1 - 2 \cdot 0.1587 = 0.6826$$
$$Y = X_1 + X_2 \sim n(70, 50)$$
$$P(Y > 80) = P\left(Z > \frac{10}{\sqrt{50}}\right) = P(Z > 1.41) = 0.0793$$

b)

En god estimator må være forventningsrett og ha lavest mulig varians.

$$\hat{\mu} = \bar{X} = \frac{1}{5}\sum_{i=1}^{5} X_i$$
$$\operatorname{Var}(\hat{\mu}) = \operatorname{Var}\left(\frac{1}{5}\sum_{i=1}^{5} X_i\right) = \frac{1}{25}\sum_{i=1}^{5}\operatorname{Var}(X_i) = \frac{1}{5}\sigma^2$$
$$\hat{\sigma}^2 = S^2 = \frac{1}{4}\sum_{i=1}^{5}(X_i - \bar{X})^2$$
$$\hat{\sigma}_\mu^2 = \frac{1}{5}S^2$$

c)

$$Z = \frac{\bar{X} - \mu}{\sqrt{\frac{S^2}{n}}} \sim t_{n-1}$$
$$P(-t_{n-1,\alpha/2} \le Z \le t_{n-1,\alpha/2}) = 1 - \alpha$$
$$-t_{n-1,\alpha/2} \le \frac{\bar{X} - \mu}{\sqrt{\frac{S^2}{n}}} \le t_{n-1,\alpha/2}$$
$$-t_{n-1,\alpha/2}\sqrt{\frac{S^2}{n}} \le \bar{X} - \mu \le t_{n-1,\alpha/2}\sqrt{\frac{S^2}{n}}$$
$$\bar{X} - t_{n-1,\alpha/2}\sqrt{\frac{S^2}{n}} \le \mu \le \bar{X} + t_{n-1,\alpha/2}\sqrt{\frac{S^2}{n}}$$

d)

$$X_0 - \bar{X} \sim n(0, \frac{\sigma^2}{n} + \sigma^2)$$

$$Z = \frac{X_0 - \bar{X}}{\sqrt{\frac{\sigma^2}{n} + \sigma^2}} = \frac{X_0 - \bar{X}}{\sqrt{\sigma^2\left(\frac{1}{n} + 1\right)}} \sim n(0,1)$$

$$T = \frac{X_0 - \bar{X}}{\sqrt{S^2\left(\frac{1}{n} + 1\right)}} \sim t_{n-1}$$

$$P\left(-t_{n-1,\alpha/2} \leq T \leq t_{n-1,\alpha/2}\right) = 1 - \alpha$$

$$-t_{n-1,\alpha/2} \leq \frac{X_0 - \bar{X}}{\sqrt{S^2\left(\frac{1}{n} + 1\right)}} \leq -t_{n-1,\alpha/2}$$

$$\bar{X} - t_{n-1,\alpha/2}\sqrt{S^2\left(\frac{1}{n} + 1\right)} \leq X_0 \leq \bar{X} + t_{n-1,\alpha/2}\sqrt{S^2\left(\frac{1}{n} + 1\right)}$$

$$\bar{X} = 284, \quad n = 5, \quad \alpha = 0.05, \quad S^2 = 585.5, \quad t_{4,0.025} = 2.776$$

$$\left[284 - 2.776\sqrt{585.5\left(\frac{1}{5} + 1\right)}, 284 + 2.776\sqrt{585.5\left(\frac{1}{5} + 1\right)}\right] =$$

$$[210.4, 357.6]$$

Dette intervallet er bredere siden det er snakk om usikkerheten til en enkeltmåling som i beste fall vil ha det reelle standardavviket, og blir enda bredere når en estimerer standardavviket fra 5 andre målinger. konfidensintervallet fra c) beskriver usikkerheten til gjennomsnittet som vil gå mot null ved flere målinger.

Oppgave 5

a)

$$P(Y > 110) = P\left(Z > \frac{10}{15}\right) = 0.2514$$

$$P(90 < Y < 110) = 1 - P(Y > 110) - P(Y < 90) = 1 - 2 \cdot 0.2514 = 0.4972$$

$$X = \frac{Y_1}{Y_2}$$

$$f_X(x) = \frac{1}{225} \cdot \frac{e^{-\frac{400}{9}}}{\frac{\pi}{225}(x^2+1)} +$$

$$\frac{1}{225}\left(\frac{4}{9}x + \frac{4}{9}\right)\left(\frac{1}{2}\text{erf}\left(\frac{15}{2}\sqrt{2}\frac{\frac{4}{9}x + \frac{4}{9}}{\sqrt{x^2+1}}\right) - \frac{1}{2}\text{erf}\left(-\frac{15}{2}\sqrt{2}\frac{\frac{4}{9}x + \frac{4}{9}}{\sqrt{x^2+1}}\right)\right) \cdot \frac{e^{\frac{-200x^2+400x-200}{9x^2+9}}}{\sqrt{2\pi}\left(\frac{1}{15}\sqrt{x^2+1}\right)}$$

$$F(0.5) + (1 - F(2)) = 0.002869$$

Var koselig å lese på cauchy fordeling og forholdsfordeling i et par timer, men føler det burde være en enklere måte å løse siste punktet i a...