Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4115 Calculus 3
Spring 2018

Work Sheet Week 12

Remark: Hand in at least two of the problems listed under **C**.

A - Reading

L ≡ Lay (the part of the textbook that deals with Linear Algebra)

L 5.1 Eigenvectors and Eigenvalues

L 5.2 The Characteristic Equation

L 5.3 Diagonalization

L 5.5 Complex Eigenvalues

B - Finger Exercises

B.1

Find the eigenvalues and the corresponding eigenvectors for the matrix $\begin{bmatrix} -2 & 2 \\ -2 & 3 \end{bmatrix}$.

B.2

Find the eigenvalues and the corresponding eigenvectors for the matrix $\begin{bmatrix} 1 & \mathrm{i} \\ -\mathrm{i} & 1 \end{bmatrix}$.

B.3

Let

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}.$$

Find an invertible matrix S and a diagonal matrix D such that $A = SDS^{-1}$. Compute A^{30}.

B.4

Find a basis of complex 3-vectors and complex eigenvalues for the matrix

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

B.5

Find a basis of $\mathbb{R}^3$ that consist of eigenvectors of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}.$$

C - Exam Preparation

C.1

a) Find the characteristic polynomial of the matrix

$$A_5 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

(Optional) Generalize the previous result to a general $n \times n$ matrix of the form of A_5.

C.2

Assume that $a \neq d$. Find an invertible matrix S such that

$$S \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} S^{-1} = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}.$$

C.3

Let $p(\lambda) = \lambda^2 + a\lambda + b$ be a quadratic polynomial. Find a 2×2-matrix with characteristic polynomial p.

D - Relevant Exercises From the Book

The following roughly divides the exercises from the relevant book chapters into two categories. Note that the book contains answers to odd-numbered exercises.

Exercises to train computing skills:

- Section 5.1, Problems 1-18
- Section 5.2, Problems 1-17
- Section 5.3, Problems 1-20
- Section 5.5, Problems 1-20

Exercises that require understanding, thought and maybe a good idea ;)

- Section 5.1, Problems 19-35

- Section 5.2, Problems 18-27
- Section 5.3, Problems 21-32
- Section 5.5, Problems 21-26