Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4115 Calculus 3
Spring 2018

Work Sheet Week 3

Remark: Hand in **C.1, C.2** and **C.3**. Optionally, hand in also **C.4**.

A - Reading

SS = Saff-Snider (the part of the textbook that deals with complex numbers)

SS 1.4 The Complex Exponential

SS 1.5 Powers and Roots

B - Finger Exercises

The form $a + \mathrm{i}b$ will always assume a and b real, and the form $r\mathrm{e}^{\mathrm{i}t}$ will always assume r real, non-negative and t real.

B.1

Write the following complex numbers in the form $a + \mathrm{i}b$:

$$z = \frac{e^{1+3\pi\mathrm{i}}}{e^{-1+\mathrm{i}\frac{\pi}{2}}}, \qquad z = \frac{e^{3\mathrm{i}} - e^{-3\mathrm{i}}}{2\mathrm{i}}.$$

B.2

Compute

$$(\sqrt{3} - \mathrm{i})^7 \quad \text{and} \quad (1 + \mathrm{i})^{95},$$

and express the result in the form $a + \mathrm{i}b$

B.3

Find all the values of

$$1^{\frac{1}{5}} \quad \text{and} \quad \left(\frac{2\mathrm{i}}{1+\mathrm{i}}\right)^{\frac{1}{6}},$$

C - Exam Preparation

The form $a + \mathrm{i}b$ will always assume a and b real.

C.1 Use the complex exponential to prove that

$$\sin^2\theta + \cos^2\theta = 1.$$

C.2 Determine which of the following properties of the complex exponential are true (give a proof in case they are true or a counterexample in case they are false):

1. $e^{z+\pi\mathrm{i}} = -e^z$.
2. $\overline{e^z} = e^{\overline{z}}$.
3. e^z is never zero.
4. e^z is a one-to-one function.
5. $e^{-z} = \frac{1}{e^z}$

C.3 Compute the sum and the product of the fifth roots of unity.

C.4 (Optional)

Let $n \in \mathbb{N}$. Assume that $a_0, \ldots, a_{n-1} \in \mathbb{R}$. Use the properties of the conjugate to show that if $z_0 \in \mathbb{C}$ a solution of

$$z^n + a_{n-1}z^{n-1} + \ldots + a_1 z + a_0 = 0$$

then $\overline{z}$ is also a solution.