Work sheet week 2

tirsdag 9. januar 2018 14:13

C.1:
The function can be shown to not be of the form $f(z) = z + w$ by a counter example:
If we let $z = 1 + i$ then $f(z) = 1 + 2i$
This means that for $f(z)$ to equal $z + w$ then w has to equal i and can therefore not be independent from z

Similarly you can show that the function cannot be of the form $f(z) = z \cdot w$:
$z = 1 + i \Rightarrow f(z) = 1 + 2i = z \cdot w$
$1 + 2i = (1 + i) \cdot w$
$$w = \frac{1 + 2i}{1 + i} = \frac{1}{2}(3 + i)$$
This is again not independent from z and the function cannot be of this form.

C.2:
The solutions of a second order polynomial can be found using the quadratic formula:

$$z^2 - t\,z + 1 = 0 \Rightarrow z = \frac{t \pm \sqrt{t^2 - 4}}{2}$$
This has only one solution when the discriminant (radicand of the expression above) is zero since both positive and negative 0 is the same number. This happens when $t = 2 \wedge t = -2$ since $2^2 - 4 = (-2)^2 - 4 = 0$
The solution of the polynomial is then $\frac{t}{2}$ which will be 1 and -1 for the respective possible values of t. These values are not strictly complex as they have an imaginary component equal to zero, but they are nevertheless contained in the set of all complex numbers.

For values of t with an absolute value greater than 2 the polynomial has 2 distinct real solutions and for values of t with absolute value less than 2 the polynomial has 2 conjugate complex solutions.

C.3:
A quadratic polynomial with real coefficients will either yield to real solutions, one real solution or to conjugate complex solutions, and since all of these are contained in the set of all complex numbers any coefficients would yield solutions contained in the complex plane.

The generic solution to the polynomial is found with the quadratic equation:
$$w = \frac{-p \pm \sqrt{p^2 - 4q}}{2}$$
To have strictly complex roots the discriminant has to be negative so $p^2 < 4q$ has to hold.
If for example $p = 2$ and $q = 3$ then the solutions are:

$$w = \frac{-2 \pm \sqrt{2^2 - 6}}{2} = -1 \pm \sqrt{2}i$$