Øving 1

6.4

11)

$$y'' + 3y' + 2y = u(t-1) + \delta(t-2), \quad y(0) = 0, \quad y'(0) = 1$$

$$s^2Y - s + 3sY + 2Y = \frac{e^{-s}}{s} + e^{-2s}$$
$$Y\left(s^2 + 3s + 2\right) = \frac{e^{-s}}{s} + e^{-2s} + s$$

$$s + 3s + 2 = (s+2)(s+3)$$

$$Y = \frac{e^{-s}}{s(s+2)(s+3)} + \frac{e^{-2s}}{(s+2)(s+3)} + \frac{s}{(s+2)(s+3)}$$
$$Y = \frac{e^{-s}}{6}\left(\frac{1}{s} - \frac{3}{s+2} + \frac{2}{s+3}\right) + e^{-2s}\left(\frac{1}{s+2} - \frac{1}{s+3}\right) - \frac{2}{s+2} + \frac{3}{s+3}$$
$$y(t) = \frac{1}{6}u(t-1)\left(1 - 3e^{-2t+2} + 2e^{-3t+3}\right) +$$
$$u(t-2)\left(e^{-2t+4)} - e^{-3t+6)}\right) - 2e^{-2t} + 3e^{-3t}$$

12)

$$y'' + 2y' + 5y = 25t - 100\delta(t-\pi), \quad y(0) = -2, \quad y'(0) = 5$$

$$s^2Y + 2s - 5 + 2sY + 4 + 5Y = \frac{25}{s^2} - 100e^{-\pi s}$$
$$Y\left(s^2 + 2s + 5\right) = \frac{25}{s^2} - 100e^{-\pi s} - 2s + 1$$
$$Y = \frac{25}{s^2\left(s^2 + 2s + 5\right)} - \frac{100e^{-\pi s}}{s^2 + 2s + 5} - \frac{2s-1}{s^2 + 2s + 5} =$$
$$\frac{5}{s^2} - \frac{2}{s} + \cancel{\frac{2s-1}{s^2 + 2s + 5}} - \frac{100e^{-\pi s}}{(s+1)^2 + 4} - \cancel{\frac{2s-1}{s^2 + 2s + 5}}$$

$$y(t) = 5t - 2 - 50u(t-\pi)e^{-t+\pi}\sin 2(t-\pi) =$$
$$5t - 2 - 50u(t-\pi)e^{\pi - t}\sin 2t$$

6.5

10

$$y(t) - \int_0^t y(\tau)\sin 2(t-\tau)d\tau = \sin 2t$$

$$\int_0^t y(\tau)\sin 2(t-\tau)d\tau = y * \sin 2t$$

$$Y - Y \cdot \frac{4}{s^2+4} = Y\frac{s^2}{s^2+4} = \frac{4}{s^2+4}$$
$$Y = \frac{4}{s^2}$$

$$y(t) = 4t$$

13

$$y(t) + 2e^t \int_0^t y(\tau)e^{-\tau}d\tau = te^t$$

$$e^t \int_0^t y(\tau)e^{-\tau}d\tau = \int_0^t y(\tau)e^t e^{-\tau}d\tau = \int_0^t y(\tau)e^{t-\tau}d\tau = y(t) * e^t$$

$$Y + Y\frac{2}{s-1} = Y\frac{s+1}{s-1} = \frac{1}{(s-1)^2}$$
$$Y = \frac{1}{(s-1)(s+1)} = \frac{1}{s^2-1}$$

$$y(t) = \sinh t$$

22

$$\frac{e^{-as}}{s(s-2)} = \frac{e^{-as}}{s} \cdot \frac{1}{s-2} =>$$
$$f(t) = u(t-a) * e^{2t} = \int_0^t u(\tau-a)e^{2(t-\tau)}d\tau = e^{2t}\int_0^t u(\tau-a)e^{-2\tau}d\tau =$$
$$e^{2t}u(t-a)\int_a^t e^{-2\tau}d\tau = -\frac{1}{2}e^{2t}u(t-a)\left(e^{-2t} - e^{-2a}\right) =$$
$$-\frac{1}{2}u(t-a)\left(1 - e^{2(t-a)}\right)$$

23

$$\mathcal{L}\left(f\right)=\frac{40.5}{s\left(s^{2}-9\right)}=\cdot\frac{4.5}{s}\cdot\frac{9}{s^{2}-9}$$
$$f(t)=4.5*\sinh t=4.5\int_{0}^{t}\sinh\tau d\tau=4.5\left(\cosh t-1\right)$$

6.6

6

$$\mathcal{L}\left(t^{2}\sin 3t\right)=\frac{d^{2}}{ds^{2}}\mathcal{L}\left(\sin 3t\right)=$$
$$\frac{d^{2}}{ds^{2}}\cdot\frac{9}{s^{2}+9}=54\cdot\frac{s^{2}-3}{\left(s^{2}+9\right)^{3}}$$

14

$$\frac{s}{\left(s^{2}+16\right)^{2}}=\frac{d}{ds}\frac{-1}{2\left(s^{2}+16\right)}$$
$$f(t)=\frac{t}{8}\sin 4t$$

16

$$\frac{2s+6}{\left(s^{2}+6s+10\right)^{2}}=-\frac{d}{ds}\frac{1}{s^{2}+6s+10}$$
$$\frac{1}{s^{2}+6s+10}=\frac{1}{\left(s+3\right)^{2}+1}$$
$$f(t)=te^{-3t}\sin t$$

17

$$\frac{d}{ds}\ln\frac{s}{s-1}=\frac{1}{s(s+1)}$$
$$\mathcal{L}^{-1}\left(\frac{1}{s(s+1)}\right)=1*e^{-t}=\int_{0}^{t}e^{-\tau}d\tau=-\left(e^{-t}-1\right)$$

$$f(t)=\frac{e^{-t}-1}{t}$$

6.7

6

$$y_1' = 5y_1 + y_2, \quad y_2' = y_1 + 5y_2, \quad y_1(0) = 1, \quad y_2(0) = -3$$

$$sY_1 - 1 = 5Y_1 + Y_2, \quad sY_2 + 3 = Y_1 + 5Y_2$$
$$Y_1 (s-5) - 1 = Y_2$$
$$Y_2 (s-5) + 3 = Y_1$$

$$(Y_1 (s-5) - 1)(s-5) + 3 = Y_1$$
$$Y_1 (s-5)^2 - (s-5) + 3 = Y_1$$
$$Y_1 \left((s-5)^2 - 1\right) = s - 8$$
$$Y_1 = \frac{s-8}{(s-5)^2 - 1} = \frac{2}{s-4} - \frac{1}{s-6}$$

$$(Y_2 (s-5) + 3)(s-5) - 1 = Y_2$$
$$Y_2 (s-5)^2 + 3(s-5) - 1 = Y_2$$
$$Y_2 \left((s-5)^2 - 1\right) = -3s + 16$$
$$Y_2 = \frac{-3s+16}{(s-5)^2 - 1} = \frac{-2}{s-4} - \frac{1}{s-6}$$

$$y_1(t) = 2e^{4t} - e^{6t}, \quad y_2(t) = -2e^{4t} - e^{6t}$$

12

$$y_1'' = -4y_1 + 5y_2, \quad y_2'' = -y_1 + 2y_2,$$
$$y_1(0) = 1, \quad y_1'(0) = 0, \quad y_2(0) = 2, \quad y_2'(0) = 0$$

$$s^2 Y_1 - s = -4Y_1 + 5Y_2, \quad s^2 Y_2 - 2s = -Y_1 + 2Y_2$$

$$Y_1 \left(s^2 + 4\right) - s = 5Y_2$$
$$Y_2 \left(s^2 - 2\right) - 2s = -Y_1$$

$$\left(2s - Y_2(s^2 - 2)\right)\left(s^2 + 4\right) - s = 5Y_2$$
$$2s\left(s^2 + 4\right) - Y_2\left(s^2 - 2\right)\left(s^2 + 4\right) - s = 5Y_2$$
$$Y_2\left(\left(s^2 - 2\right)\left(s^2 + 4\right) + 5\right) = 2s\left(s^2 + 4\right) - s$$
$$Y_2 = \frac{2s^3 + 7s}{(s^2+3)(s-1)(s+1)} = \frac{1}{s-1} + \frac{1}{s+1} - \frac{s}{s^2+3}$$

$$Y_1 = \frac{s^3+8s}{(s^2+3)\,(s-1)(s+1)} = Y_2 = \frac{9}{8(s-1)} + \frac{9}{8(s+1)} - \frac{5s}{4\,(s^2+3)}$$

denne