Øving 5

11.3

2

$$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx = \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} e^{2ix} e^{-i\omega x} dx =$$

$$\frac{1}{\sqrt{2\pi}} \int_{-1}^{1} e^{i(2-\omega)x} dx = \frac{1}{i\sqrt{2\pi}(2-\omega)} \left[e^{i(2-\omega)x} \right]_{-1}^{1} =$$

$$\frac{-i}{\sqrt{2\pi}(2-\omega)} \cdot \frac{2i}{2i} \left(e^{i(2-\omega)} - e^{-i(2-\omega)} \right) = \sqrt{\frac{2}{\pi}} \cdot \frac{1}{2-\omega} \sin(2-\omega)$$

6

$$\sqrt{2\pi} \hat{f}(\omega) = \int_{\infty}^{0} e^{x} e^{-i\omega x} dx + \int_{0}^{\infty} e^{-x} e^{-i\omega x} dx =$$

$$\int_{\infty}^{0} e^{(1-i\omega)x} dx + \int_{0}^{\infty} e^{-(1+i\omega)x} dx =$$

$$\frac{1}{1-i\omega} \left[e^{(1-i\omega)x} \right]_{-\infty}^{0} - \frac{1}{1+i\omega} \left[e^{-(1+i\omega)x} \right]_{0}^{\infty} =$$

$$\frac{1}{1-i\omega}(1-0) - \frac{1}{1+i\omega}(0-1) = \frac{2}{1+\omega^2}$$

$$\hat{f}(\omega) = \sqrt{\frac{2}{\pi}} \cdot \frac{1}{1+\omega^2}$$

9

$$\sqrt{2\pi} \hat{f}(\omega) = \int_{-1}^{0} -x e^{-i\omega x} dx + \int_{0}^{1} x e^{-i\omega x} dx =$$

$$\int_{0}^{1} x e^{i\omega x} dx + \int_{0}^{1} x e^{-i\omega x} dx = 2 \int_{0}^{1} x \frac{1}{2} \left(e^{i\omega x} + e^{-i\omega x} \right) dx =$$

$$2 \int_{0}^{1} x \cos(\omega x) dx = 2 \left[x \frac{1}{\omega} \sin(\omega x) + \frac{1}{\omega^2} \cos(\omega x) \right]_{0}^{1} =$$

$$2 \left(\frac{1}{\omega} \sin\omega + \frac{1}{\omega^2} (\cos\omega - 1) \right) \Rightarrow \hat{f}(\omega) = \sqrt{\frac{2}{\pi}} \left(\frac{1}{\omega} \sin\omega + \frac{1}{\omega^2} (\cos\omega - 1) \right)$$

10

$$\sqrt{2\pi}\hat{f}(\omega) = \int_{-1}^{1} xe^{-i\omega x}dx = \left[\frac{i}{\omega}xe^{-i\omega x} + \frac{1}{\omega^2}\right]_{-1}^{1} =$$

$$\frac{i}{\omega}e^{-i\omega} + \frac{1}{\omega^2}e^{-i\omega} + \frac{i}{\omega}e^{i\omega} - \frac{1}{\omega^2}e^{i\omega} =$$

$$\frac{2i}{\omega}\cdot\frac{1}{2}\left(e^{i\omega}+e^{-i\omega}\right) - \frac{2i}{\omega^2}\cdot\frac{1}{2i}\left(e^{i\omega}-e^{-i\omega}\right) =$$

$$2i\left(\frac{1}{\omega}\cos\omega - \frac{1}{\omega^2}\sin\omega\right) \Rightarrow \hat{f}(\omega) = \sqrt{\frac{2}{\pi}}i\left(\frac{1}{\omega}\cos\omega - \frac{1}{\omega^2}\sin\omega\right)$$

11

$$\sqrt{2\pi}\hat{f}(\omega)\int_{-1}^{0} -e^{-i\omega x}dx + \int_{0}^{1} e^{-i\omega x}dx =$$

$$\int_{0}^{1} -e^{i\omega x}dx + \int_{0}^{1} e^{-i\omega x}dx = -2i\int_{0}^{1}\frac{1}{2i}\left(e^{i\omega x}-e^{-i\omega x}\right)dx =$$

$$-2i\int_{0}^{1}\sin\omega x dx = \frac{2i}{\omega}\left[\cos\omega x\right]_{0}^{1} = \frac{2i}{\omega}\left(\cos\omega - 1\right)$$

Denne

$$\sqrt{2\pi}\cdot\widehat{f(3x+5)}(\omega) = \int_{-\infty}^{\infty} f(3x+5)e^{-i\omega x}dx$$

$$y = 3x+5 \quad x = \frac{y-5}{3} \quad dx = \frac{1}{3}dy$$

$$\int_{-\infty}^{\infty} f(3x+5)e^{-i\omega x}dx = \frac{1}{3}\int_{-\infty}^{\infty} f(y)e^{-i\omega\frac{y-5}{3}}dy =$$

$$\frac{1}{3}\int_{-\infty}^{\infty} f(y)e^{-i\frac{\omega}{3}y}e^{i\frac{5}{3}}dy = \frac{1}{3}e^{i\frac{5}{3}\omega}\int_{-\infty}^{\infty} f(y)e^{-i\frac{\omega}{3}y}dy =$$

$$\frac{1}{3}e^{i\frac{5}{3}\omega}\hat{f}\left(\frac{\omega}{3}\right)\cdot\sqrt{2\pi} \Rightarrow \widehat{f(3x+5)} = \frac{1}{3}e^{i\frac{5}{3}\omega}\hat{f}\left(\frac{\omega}{3}\right)$$