Innlevering 2

Tuesday, February 20, 2018 18:55

1.

$$f(x,y) = \frac{1-2xy}{x^2+y^2}$$

$$\nabla f(x,y) = \left(\frac{-2y(x^2+y^2) - (1-2xy)(2x)}{(x^2+y^2)^2}, \frac{-2x(x^2+y^2) - (1-2xy)(2y)}{(x^2+y^2)^2} \right) =$$

$$\frac{2}{(x^2+y^2)^2}\left(-2yx^2 - 2y^3 - 2x + 4x^2y, 2xy^2 - 2x^3 - 2y + 4xy^2\right) \neq 0$$

Funksjonen har altså ingen kritiske punkter og definisjonsmengden er ubegrenset så den kan ikke ha en største eller minste verdi.

2.

Parametriserer skjæringskurven:

$x^2 + 2y^2 = 1$

$x^2 = \cos^2\theta \wedge 2y^2 = \sin^2\theta, \qquad \theta \in [-\pi, \pi\rangle$

$x(\theta) = \cos\theta \wedge y(\theta) = \frac{\sqrt{2}}{2}\sin\theta$

$z(\theta) = x - 4y \Rightarrow z = \cos\theta - 2\sqrt{2}\sin\theta$

Maksimerer $z(\theta)$:

$z'(\theta) = -\sin\theta - 2\sqrt{2}\cos\theta = 0$

$\sin\theta = -2\sqrt{2}\cos\theta$

$\sin^2\theta = 8\cos^2\theta$

$9\cos^2\theta = 1$

$\cos\theta = \frac{1}{3}$

$\theta = \pm\arccos\frac{1}{3}$

$$z = \frac{1}{3} - 2\sqrt{2}\sin\left(\pm\arccos\frac{1}{3}\right) = \frac{1}{3} \pm 2\sqrt{2}\sin\arccos\frac{1}{3} = \frac{1}{3} \pm 2\sqrt{2}\sqrt{1-\left(\frac{1}{3}\right)^2} = \frac{1}{3} \pm \frac{8}{3}$$

$z = 3 \vee z = -\frac{7}{3}$

3.

Finner jacobi determinanten for substutisjonen
$u = x - y$
$v = x + y$

$$dA = dx\,dy = \left(\frac{\partial(u,v)}{\partial(x,y)}\right)^{-1} du\,dv$$

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$$

$$dA = \frac{1}{2} du\,dv$$

$$\iint_R \sin\left(\frac{x-y}{x+y}\right) dA = \iint_S \sin\frac{u}{v}\ du\,dv$$

Punkter i trapes (A, B, C, D):
$(1,1), (2,2), (2,0), (4,0)$

Ligninger for kanter i trapes:
AB:
$y = x \Rightarrow x - y = 0 \Rightarrow u = 0$

AD:
$y = -x + 2 \Rightarrow x + y = 2 \Rightarrow v = 2$

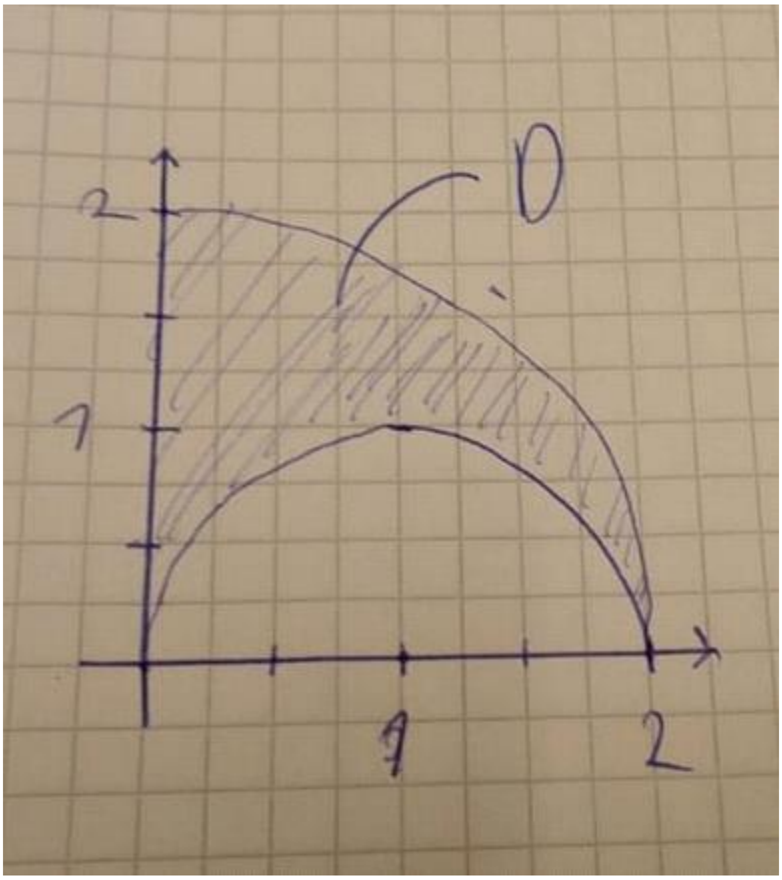
BC:
$y = -x + 4 \Rightarrow x + y = 4 \Rightarrow v = 4$

DC:
$y = 0$
$$\left.\begin{matrix} y = x - u \\ x = v - y \end{matrix}\right\} \Rightarrow y = v - y - u \Rightarrow 2y = v - u = 0 \Rightarrow u = v$$

$2 \leq v \leq 4 \wedge 0 \leq u \leq v$
Integral:
$$\iint_R \sin\left(\frac{x-y}{x+y}\right) dA = \int_2^4 \int_0^v \sin\frac{u}{v}\ \frac{1}{2} du\,dv =$$

$$\frac{1}{2}\int_2^4 \left[-v\cos\frac{u}{v}\right]_0^v dv = \frac{1}{2}\int_2^4 (-v\cos 1 + v\cos 0) dv = \frac{1}{2}(1-\cos 1)\int_2^4 v\,dv$$
$$= \frac{1}{2}(1-\cos 1)\cdot\left[\frac{1}{2}v^2\right]_2^4 =$$
$$\frac{1}{4}(1-\cos 1)\cdot(16-4) = 3(1-\cos 1)$$

4.



$y = \sqrt{4\ - x^2} \Rightarrow r = 2$
$y = \sqrt{2x - x^2} \Rightarrow y^2 + (x-1)^2 = 1 \Rightarrow r = 2\cos\theta$

$$\iint_D \sqrt{4 - x^2 - y^2}dxdy = \int_0^{\frac{\pi}{2}}\int_{2\cos\theta}^{2} \sqrt{4 - r^2}\cdot rdrd\theta = \int_0^{\frac{\pi}{2}}\int_{r=2\cos\theta}^{r=2} -\frac{1}{2}\sqrt{u}\,dud\theta$$
$$= -\int_0^{\frac{\pi}{2}} -\frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_{r=2\cos\theta}^{r=2} d\theta =$$
$u = 4 - r^2$
$du = -2r\,dr \Rightarrow rdr = -\frac{du}{2}$

$$\int_0^{\frac{\pi}{2}} -\frac{1}{3}\left[(4-r^2)^{\frac{3}{2}}\right]_{2\cos\theta}^{2} d\theta = -\frac{1}{3}\int_0^{\frac{\pi}{2}}\left(-\left(4 - 4\cos^2\theta\right)^{\frac{3}{2}}\right)d\theta = \frac{8}{3}\int_0^{\frac{\pi}{2}}\sin^3\theta\, d\theta$$
$$= \frac{8}{3}\int_0^{\frac{\pi}{2}}\left(1 - \cos^2\theta\right)\sin\theta\ \ d\theta$$
$v = \cos\theta$
$dv = -\sin\theta\, d\theta$

$$= -\frac{8}{3}\int_1^0 (1 - v^2)dv = \frac{8}{3} - \frac{8}{3}\left[\frac{1}{3}v^3\right]_0^1 = \frac{16}{9}$$