TMA 4100 Skriftlig innlevering 1

1:

$$-1 \leq \cos\frac{\pi}{x-2} \leq 1$$

$$-\lim_{x\to 2}(x-2)^2 \leq \lim_{x\to 2}(x-2)^2 \cdot \cos\frac{\pi}{x-2} \leq \lim_{x\to 2}(x-2)^2$$

$$0 \leq \lim_{x\to 2}(x-2)^2 \cdot \cos\frac{\pi}{x-2} \leq 0$$

$$\lim_{x\to 2}(x-2)^2 \cdot \cos\frac{\pi}{x-2} = 0 \Rightarrow A = 0$$

2:

$$x^3 + y^3 = 6\,x\,y$$
$$\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}(6\,x\,y)$$
$$3x^2 + 3y^2\frac{dy}{dx} = 6y + 6x\frac{dy}{dx}$$
$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$$

$$x = 3, y = 3$$

$$\frac{dy}{dx} = \frac{6\cdot 3 - 3\cdot 9}{3\cdot 9 - 6\cdot 3} = -1 = m$$

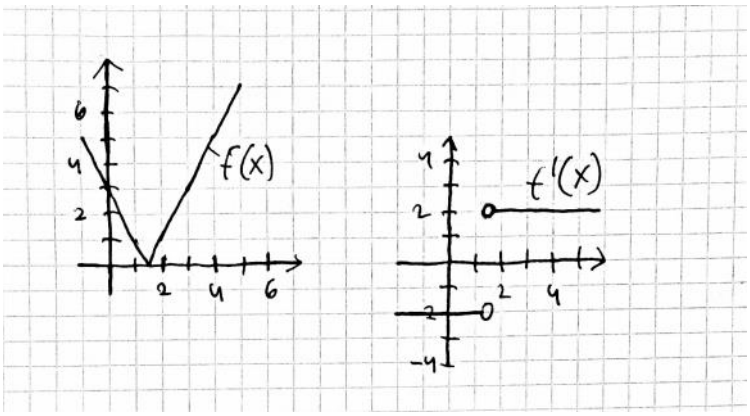
$$\frac{y - y_0}{x - x_0} = m$$
$$\frac{y-3}{x-3} = -1$$

$$y = -x + 6$$

3:

$$f(x) = |2x - 3| \Rightarrow f(x) = \begin{cases} 2x - 3 & x > \frac{3}{2} \\ 0 & x = \frac{3}{2} \\ -2x + 3 & x < \frac{3}{2} \end{cases}$$

$$f'(x) = \begin{cases} 2 & x > \frac{3}{2} \\ -2 & x < \frac{3}{2} \end{cases}$$



4:

$$-1 \leq \sin\frac{1}{x} \leq 1$$

$$-\left|\lim_{x\to 0} \ln\left(1 + \sqrt{|x|}\right)\right| \leq \lim_{x\to 0} \ln\left(1 + \sqrt{|x|}\right) \cdot \sin\frac{1}{x} \leq \left|\lim_{x\to 0} \ln\left(1 + \sqrt{|x|}\right)\right|$$

$$0 \leq \lim_{x\to 0} \ln\left(1 + \sqrt{|x|}\right) \cdot \sin\frac{1}{x} \leq 0$$

$\lim_{x\to 0} \ln\left(1 + \sqrt{|x|}\right) \cdot \sin\frac{1}{x} = 0$ Q.E.D.