Maple 12, 5

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Finn summen til rekken:

$$\sum_{n=2}^{\infty} \frac{2n(n-1)}{6^{n-2}}$$

Slenger på en x^n og evaluerer funksjonen I x=1

$$f(x) = \sum_{n=2}^{\infty} \frac{2n(n-1)}{6^{n-2}} x^n$$

$$f(1) = \sum_{n=2}^{\infty} \frac{2n(n-1)}{6^{n-2}} 1^n = \sum_{n=2}^{\infty} \frac{2n(n-1)}{6^{n-2}}$$

Faktoriserer ut en x for å kunne integrere vekk n

$$\frac{f(x)}{x} = \sum_{n=2}^{\infty} \frac{2n(n-1)}{6^{n-2}} x^{n-1}$$

$$\int \frac{f(x)}{x} dx = \sum_{n=2}^{\infty} \frac{2n(n-1)}{6^{n-2}} \int x^{n-1} = \sum_{n=2}^{\infty} \frac{2n(n-1)}{6^{n-2}} \cdot \frac{x^n}{n} = \sum_{n=2}^{\infty} \frac{2(n-1)}{6^{n-2}} x^n$$

Faktoriserer så ut x^2 for å kunne gjøre det samme med (n - 1)

$$\frac{\int \frac{f(x)}{x} dx}{x^2} = \sum_{n=2}^{\infty} \frac{2(n-1)}{6^{n-2}} x^{n-2}$$

$$\int \frac{\int \frac{f(x)}{x} dx}{x^2} dx = \sum_{n=2}^{\infty} \frac{2}{6^{n-2}} x^{n-1}$$

Faktoriserer så ut enda en x for at 6^{n-2} og x^{n-2} skal ha samme eksponent

$$\frac{\int \frac{\int \frac{f(x)}{x} dx}{x^2} dx}{x} = \sum_{n=2}^{\infty} \frac{2}{6^{n-2}} x^{n-2} = 2 \sum_{n=2}^{\infty} \frac{x^{n-2}}{6^{n-2}} = 2 \sum_{n=2}^{\infty} \left(\frac{x}{6}\right)^{n-2} = 2 \sum_{n=0}^{\infty} \left(\frac{x}{6}\right)^{n}$$

Dette gjennkjenner vi som den geometriske rekken med konstant $k = \frac{x}{6}$

$$2 \sum_{n=0}^{\infty} \left(\frac{x}{6}\right)^{n} = 2 \cdot \frac{1}{1 - \frac{x}{6}} = 2 \cdot \frac{6}{6 - x} = -\frac{12}{x - 6} = \frac{\int \frac{\int \frac{f(x)}{x} dx}{x^2} dx}{x}$$

Vi jobber oss så bakover for å "Kle av" f(x)

$$\int \frac{\int \frac{f(x)}{x} dx}{x^2} dx = -\frac{12x}{x-6}$$

$$\frac{\int \frac{f(x)}{x} dx}{x^2} = \left(-\frac{12x}{x-6}\right)' = -\frac{12(x-6) - 12x^2}{(x-6)^2} = -\frac{12}{x-6} + \frac{12x}{(x-6)^2}$$

$$\int \frac{f(x)}{x} dx = -\frac{12x^2}{x-6} + \frac{12x^3}{(x-6)^2}$$

$$\frac{f(x)}{x} = -\frac{24x}{x-6} + \frac{48x^2}{(x-6)^2} - \frac{24x^3}{(x-6)^3}$$

$$f(x) = -\frac{24x^2}{x-6} + \frac{48x^3}{(x-6)^2} - \frac{24x^3}{(x-6)^3}$$

$$f(1) = \frac{24}{5} + \frac{48}{25} + \frac{24}{125} = \frac{864}{125}$$