TMA 4100 Skriftlig innlevering 4

mandag 6. november 2017 20.20

1

a)

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{-t^2} = \sum_{n=0}^{\infty} \frac{(-t^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot t^{2n}}{n!}$$

$$f(x) = \int_0^x e^{-t^2} dt = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int_0^x t^{2n} dt = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,(2n+1)} x^{2n+1}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,(2n+1)} x^{2n+1} = x - \frac{x^3}{1! \cdot 3} + \frac{x^5}{2! \cdot 5} - \frac{x^7}{3! \cdot 7} + \cdots$$

b)

$$f(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,(2n+1)} = \sum_{n=0}^{\infty} a_n = S$$

$$S_n = \sum_{n=0}^{N} a_n$$

$a_n \cdot a_{n+1} < 0 \Rightarrow \text{alternerende rekke}$
$\Rightarrow |S - S_N| < |a_{N+1}|$

$|a_n| < 0.0005$

$$\frac{1}{n!\,(2n+1)} < 0.0005$$

$$\left.\begin{array}{r} n!\,(2n+1) > 2000 \\ 5!\,(2 \cdot 5 + 1) = 1320 \\ 6!\,(2 \cdot 6 + 1) = 9360 \end{array}\right\} \Rightarrow n \geq 6$$

$|S - S_N| < |a_6| < 0.0005 \Rightarrow N = 6 - 1 = 5 \Rightarrow 6 \text{ ledd av taylor rekken}$

2

a)

Bruker forholdstest:

$$\left.\begin{array}{l}\displaystyle\sum_{n=1}^{\infty} a_n\, x^n \\ \left|\dfrac{a_{n+1}}{a_n}x\right| < 1\end{array}\right\} \Rightarrow konvergens$$

$$\sum_{n=1}^{\infty} \frac{n}{(n-1)!} x^n$$

$$a_n = \frac{n}{(n-1)!}$$

$$\frac{a_{n+1}}{a_n}x = \frac{(n+1)(n-1)!}{n \cdot n!}x = \frac{(n+1)(n-1)!}{n \cdot (n-1)! \cdot n}x = \frac{n+1}{n^2}x \to \lim_{n\to\infty} \frac{n+1}{n^2}x = 0, \qquad x \in \mathbb{R}$$

$$|0| < 1 \Rightarrow konvergens\ for\ x \in \mathbb{R}$$

b)

$$f(x) = \sum_{n=1}^{\infty} \frac{n}{(n-1)!} x^n$$

$$\frac{f(x)}{x} = \sum_{n=1}^{\infty} \frac{n}{(n-1)!} x^{n-1}$$

$$\int \frac{f(x)}{x} dx = \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \int n x^{n-1} dx = \sum_{n=1}^{\infty} \frac{x^n}{(n-1)!}$$

$$\frac{\int \frac{f(x)}{x} dx}{x} = \sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)!} = \sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$$

$$\int \frac{f(x)}{x} dx = x e^x$$

$$\frac{f(x)}{x} = e^x (x+1)$$

$$f(x) = e^x \left(x^2 + x\right)$$

3
a)

$$T'(t) = k\,\Delta T = k\big(20 - T(t)\big)$$

$$T' + k\,T = 20\,k$$

$$T'e^{kt} + k\,Te^{kt} = 20\,ke^{kt}$$
$$\left(Te^{kt}\right)' = 20\,ke^{kt}$$
$$Te^{kt} = \int 20\,ke^{kt}dt = 20\,e^{kt} + C$$
$$T = 20 + Ce^{-kt}$$

$$T(0) = 25 = 20 + Ce^{-k\cdot 0}$$
$$C = 5$$

$$T(t) = 20 + 5e^{-kt}$$

b)

$$T(3) = 22 = 20 + 5e^{-k\cdot 3}$$
$$e^{-k\cdot 3} = \frac{2}{5}$$
$$k = \frac{\ln\frac{5}{2}}{3}$$

$$T(t) = 20 + 5e^{-\frac{\ln\frac{5}{2}}{3}\cdot t}$$

$$T(t) = 21 = 20 + 5e^{-\frac{\ln\frac{5}{2}}{3}\cdot t}$$
$$e^{-\frac{\ln\frac{5}{2}}{3}\cdot t} = \frac{1}{5}$$
$$\frac{\ln\frac{5}{2}}{3}\cdot t = \ln 5$$
$$t = \frac{3\ln 5}{\ln\frac{5}{2}} \approx 5.27 = 5h\ 16m\ 10s$$

Det tar 5 timer 16 minutter og 10 sekunder før bunnen er 21 grader

4

$f(x) = x \ln x$

$$h = \frac{3-1}{4} = \frac{1}{2}$$

$$S_4 = \frac{1}{2 \cdot 3}\left(f(1) + 4f\left(1 + \frac{1}{2}\right) + 2f(1+1) + 4f\left(1 + \frac{3}{2}\right) + f(1+2)\right) \approx 2.944$$

$K \geq f^{(4)}(x), \qquad x \in [1, 3]$

$f'(x) = \ln x + 1$

$f''(x) = \frac{1}{x}$

$f'''(x) = -\frac{1}{x^2}$

$f^{(4)}(x) = \frac{1}{x^3}$

$f^{(4)}(1) = 1 \geq f^{(4)}(x), \qquad x \in [1, 3]$

$K \geq 1$

$$\left|\int_1^3 f(x)dx - S_n\right| \leq \frac{K(3-1)^5}{180n^4} \leq 10^{-4}, \qquad n = 2m, \qquad m \in \mathbb{N}$$

$$n \geq \sqrt[4]{\frac{2^5}{180 \cdot 10^{-4}}} \approx 6.5 \Rightarrow n = 8$$

n lik 8 gir en feil på mindre enn 10^{-4}