# Skriftlig innlevering 3

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## Oppgave 1

Finner grenser i polarkoordinater:

$$z = 4 - x^2 - y^2 = 4 - r^2 = 3 \Rightarrow r^2 = 1 \Rightarrow r = 1$$

$$m = \int\int\int_T \delta(x,y,z)dV = \int_0^{2\pi}\int_0^1\int_3^{4-r^2} 1 \cdot rdzdrd\theta =$$

$$2\pi \int_0^1 r \cdot [z]_3^{4-r^2}\, dr = 2\pi \int_0^1 r \cdot (4 - r^2 - 3)dr =$$

$$2\pi \int_0^1 (r - r^3)dr = 2\pi \left(\frac{1}{2} - \frac{1}{4}\right) = \frac{\pi}{2}$$

Siden området og massetettheten er uavhengig av $\theta$ vil massesenteret ligge på z aksen ($\bar{x} = 0, \bar{y} = 0$). Vi trenger da bare finne $\bar{z}$:

$$\bar{z} \cdot m = \int\int\int_T z \cdot \delta(x,y,z)dV = \int_0^{2\pi}\int_0^1\int_3^{4-r^2} z \cdot rdzdrd\theta =$$

$$2\pi \int_0^1 r \cdot \frac{1}{2}\left[z^2\right]_3^{4-r^2} dr = \pi \int_0^1 r \cdot \left(\left(4 - r^2\right)^2 - 3^2\right) dr =$$

$$\pi \int_0^1 \left(r^5 - 8r^3 + 7r\right) dr = \frac{5\pi}{3} \Rightarrow \bar{z} = \frac{\frac{5\pi}{3}}{\frac{\pi}{2}} = \frac{10}{3}$$

Massesenteret blir da $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{10}{3}\right)$:

## Oppgave 2

Finner grenser til T i kulekoordinater:

$$x^2 + y^2 + z^2 = 5 \Rightarrow \rho^2 = 5 \Rightarrow \rho = \sqrt{5}$$

$$0 \leq \rho \leq \sqrt{5}$$

$$z = 2\sqrt{x^2 + y^2} = 2r \Rightarrow \phi = \arctan\left(\frac{1}{2}\right) = \arccos\left(\frac{1}{\sqrt{5}}\right)$$

$$0 \leq \phi \leq \arccos\left(\frac{1}{\sqrt{5}}\right)$$

$$0 \leq \theta \leq 2\pi$$

$$\int\int\int_T e^{(x^2+y^2+z^2)^{3/2}} dV = \int_0^{2\pi} \int_0^{\arccos \frac{1}{\sqrt{5}}} \int_0^{\sqrt{5}} e^{\rho^3} \cdot \rho^2 \cdot \sin\phi \cdot d\rho d\phi d\theta =$$

$$2\pi \int_0^{\arccos \frac{1}{\sqrt{5}}} \frac{1}{3}\left[e^{r^3}\right]_0^{\sqrt{5}} \cdot = \frac{2}{3}\pi(e^{5^{3/2}} - 1) \int_0^{\arccos \frac{1}{\sqrt{5}}} \sin\phi d\phi = \frac{2}{3}\pi(e^{5^{3/2}} - 1)\left[-\cos\phi\right]_0^{\arccos \frac{1}{\sqrt{5}}} =$$

$$\frac{2}{3}\pi(e^{5^{3/2}} - 1) \cdot (1 - \frac{1}{\sqrt{5}}) = \frac{2}{15}\pi(e^{5^{3/2}} - 1) \cdot (5 - \sqrt{5})$$

## Oppgave 3

Integrerer planktontettheten over den parametriserte kurven:

$$\int_r f(x,y,z) ds = \int_0^1 \left(8\sqrt{t^2} + 2\left(e^t\right)^2\right) \left|\left[2t, e^t, 1\right]\right| = \int_0^1 \left(8t + 2^{2t}\right) \sqrt{4t^2 + e^{2t} + 1} =$$

$$\int_2^{5+e^2} \sqrt{u} du = \frac{2}{3} u^{\frac{2}{3}} \Big|_2^{5+e^2} = \frac{2}{3}\left(\left(5 + e^2\right)^{\frac{3}{2}} - 2\sqrt{2}\right)$$

## Oppgave 4

$$f(x,y,z) = e^{xz+y} + 2yz \Rightarrow \begin{array}{c} \frac{\partial f}{\partial x} = ze^{xz+y} \\ \frac{\partial f}{\partial y} = e^{xz+y} + 2z \\ \frac{\partial f}{\partial z} = xe^{xz+y} + 2y \end{array}$$

Altså har vektorfeltet en potensialfunksjon og er derfor konservativt og vi kan bruke vektoranalysens fundamentalteorem:

$$\int_C \mathbf{F} \cdot d\mathbf{r} = f\left(r\left(\frac{\pi}{2}\right)\right) - f(r(0))$$

$$r\left(\frac{\pi}{2}\right) = \left(0, 1, \frac{\pi}{2}\right)$$

$$r\left(0\right) = (1, 0, 0)$$

$$f\left(0, 1, \frac{\pi}{2}\right) - f\left(1, 0, 0\right) = e + \pi - e^0 - 0 = e + \pi - 1$$