Øving 2

6.4

11)

$$y'' + 3y' + 2y = u(t-1) + \delta(t-2), \quad y(0) = 0, \quad y'(0) = 1$$

$$s^2Y - s + 3sY + 2Y = \frac{e^{-s}}{s} + e^{-2s}$$
$$Y\left(s^2 + 3s + 2\right) = \frac{e^{-s}}{s} + e^{-2s} + s$$

$$s + 3s + 2 = (s+2)(s+3)$$

$$Y = \frac{e^{-s}}{s(s+2)(s+3)} + \frac{e^{-2s}}{(s+2)(s+3)} + \frac{s}{(s+2)(s+3)}$$
$$Y = \frac{e^{-s}}{6}\left(\frac{1}{s} - \frac{3}{s+2} + \frac{2}{s+3}\right) + e^{-2s}\left(\frac{1}{s+2} - \frac{1}{s+3}\right) - \frac{2}{s+2} + \frac{3}{s+3}$$
$$y(t) = \frac{1}{6}u(t-1)\left(1 - 3e^{-2t+2} + 2e^{-3t+3}\right) +$$
$$u(t-2)\left(e^{-2t+4)} - e^{-3t+6)}\right) - 2e^{-2t} + 3e^{-3t}$$

12)

$$y'' + 2y' + 5y = 25t - 100\delta(t-\pi), \quad y(0) = -2, \quad y'(0) = 5$$

$$s^2Y + 2s - 5 + 2sY + 4 + 5Y = \frac{25}{s^2} - 100e^{-\pi s}$$
$$Y\left(s^2 + 2s + 5\right) = \frac{25}{s^2} - 100e^{-\pi s} - 2s + 1$$
$$Y = \frac{25}{s^2\left(s^2 + 2s + 5\right)} - \frac{100e^{-\pi s}}{s^2 + 2s + 5} - \frac{2s-1}{s^2 + 2s + 5} =$$
$$\frac{5}{s^2} - \frac{2}{s} + \cancel{\frac{2s-1}{s^2 + 2s + 5}} - \frac{100e^{-\pi s}}{(s+1)^2 + 4} - \cancel{\frac{2s-1}{s^2 + 2s + 5}}$$

$$y(t) = 5t - 2 - 50u(t-\pi)e^{-t+\pi}\sin 2(t-\pi) =$$
$$5t - 2 - 50u(t-\pi)e^{\pi - t}\sin 2t$$

6.5

10)

$$y(t) - \int_0^t y(\tau)\sin 2(t-\tau)d\tau = \sin 2t$$

$$\int_0^t y(\tau)\sin 2(t-\tau)d\tau = y * \sin 2t$$

$$Y - Y \cdot \frac{4}{s^2+4} = Y\frac{s^2}{s^2+4} = \frac{4}{s^2+4}$$
$$Y = \frac{4}{s^2}$$

$$y(t) = 4t$$

13)

$$y(t) + 2e^t \int_0^t y(\tau)e^{-\tau}d\tau = te^t$$

$$e^t \int_0^t y(\tau)e^{-\tau}d\tau = \int_0^t y(\tau)e^t e^{-\tau}d\tau = \int_0^t y(\tau)e^{t-\tau}d\tau = y(t) * e^t$$

$$Y + Y\frac{2}{s-1} = Y\frac{s+1}{s-1} = \frac{1}{(s-1)^2}$$
$$Y = \frac{1}{(s-1)(s+1)} = \frac{1}{s^2-1}$$

$$y(t) = \sinh t$$

22)

$$\frac{e^{-as}}{s(s-2)} = \frac{e^{-as}}{s} \cdot \frac{1}{s-2} =>$$
$$f(t) = u(t-a) * e^{2t} = \int_0^t u(\tau-a)e^{2(t-\tau)}d\tau = e^{2t}\int_0^t u(\tau-a)e^{-2\tau}d\tau =$$
$$e^{2t}u(t-a)\int_a^t e^{-2\tau}d\tau = -\frac{1}{2}e^{2t}u(t-a)\left(e^{-2t} - e^{-2a}\right) =$$
$$-\frac{1}{2}u(t-a)\left(1 - e^{2(t-a)}\right)$$

23)

$$\mathcal{L}(f) = \frac{40.5}{s(s^2-9)} = \cdot\frac{4.5}{s} \cdot \frac{9}{s^2-9}$$

$$f(t) = 4.5 * \sinh t = 4.5 \int_0^t \sinh \tau d\tau = 4.5(\cosh t - 1)$$

6.6

6)

$$\mathcal{L}\left(t^2 \sin 3t\right) = \frac{d^2}{ds^2}\mathcal{L}(\sin 3t) =$$

$$\frac{d^2}{ds^2} \cdot \frac{9}{s^2+9} = 54 \cdot \frac{s^2-3}{\left(s^2+9\right)^3}$$

14)

$$\frac{s}{\left(s^2+16\right)^2} = \frac{d}{ds}\frac{-1}{2(s^2+16)}$$

$$f(t) = \frac{t}{8}\sin 4t$$

16)

$$\frac{2s+6}{\left(s^2+6s+10\right)^2} = -\frac{d}{ds}\frac{1}{s^2+6s+10}$$

$$\frac{1}{s^2+6s+10} = \frac{1}{\left(s+3\right)^2+1}$$

$$f(t) = te^{-3t}\sin t$$

17)

$$\frac{d}{ds}\ln\frac{s}{s-1} = \frac{1}{s(s+1)}$$

$$\mathcal{L}^{-1}\left(\frac{1}{s(s+1)}\right) = 1 * e^{-t} = \int_0^t e^{-\tau} d\tau = -\left(e^{-t}-1\right)$$

$$f(t) = \frac{e^{-t}-1}{t}$$

6.7

6)

$$y_1' = 5y_1 + y_2, \quad y_2' = y_1 + 5y_2, \quad y_1(0) = 1, \quad y_2(0) = -3$$

$$\begin{gathered}
sY_1 - 1 = 5Y_1 + Y_2, \quad sY_2 + 3 = Y_1 + 5Y_2 \\
Y_1(s-5) - 1 = Y_2 \\
Y_2(s-5) + 3 = Y_1
\end{gathered}$$

$$\begin{gathered}
(Y_1(s-5) - 1)(s-5) + 3 = Y_1 \\
Y_1(s-5)^2 - (s-5) + 3 = Y_1 \\
Y_1\left((s-5)^2 - 1\right) = s - 8 \\
Y_1 = \frac{s-8}{(s-5)^2 - 1} = \frac{2}{s-4} - \frac{1}{s-6}
\end{gathered}$$

$$\begin{gathered}
(Y_2(s-5) + 3)(s-5) - 1 = Y_2 \\
Y_2(s-5)^2 + 3(s-5) - 1 = Y_2 \\
Y_2\left((s-5)^2 - 1\right) = -3s + 16 \\
Y_2 = \frac{-3s+16}{(s-5)^2 - 1} = \frac{-2}{s-4} - \frac{1}{s-6}
\end{gathered}$$

$$y_1(t) = 2e^{4t} - e^{6t}, \quad y_2(t) = -2e^{4t} - e^{6t}$$

12)

$$\begin{gathered}
y_1'' = -4y_1 + 5y_2, \quad y_2'' = -y_1 + 2y_2, \\
y_1(0) = 1, \quad y_1'(0) = 0, \quad y_2(0) = 2, \quad y_2'(0) = 0
\end{gathered}$$

$$s^2 Y_1 - s = -4Y_1 + 5Y_2, \quad s^2 Y_2 - 2s = -Y_1 + 2Y_2$$

$$\begin{gathered}
Y_1\left(s^2 + 4\right) - s = 5Y_2 \\
Y_2\left(s^2 - 2\right) - 2s = -Y_1
\end{gathered}$$

$$\begin{gathered}
\left(2s - Y_2(s^2 - 2)\right)\left(s^2 + 4\right) - s = 5Y_2 \\
2s\left(s^2 + 4\right) - Y_2\left(s^2 - 2\right)\left(s^2 + 4\right) - s = 5Y_2 \\
Y_2\left(\left(s^2 - 2\right)\left(s^2 + 4\right) + 5\right) = 2s\left(s^2 + 4\right) - s \\
Y_2 = \frac{2s^3 + 7s}{(s^2+3)(s-1)(s+1)} = \frac{1}{s-1} + \frac{1}{s+1} - \frac{s}{s^2+3}
\end{gathered}$$

$$Y_1 = \frac{s^3+8s}{(s^2+3)(s-1)(s+1)} = Y_2 = \frac{9}{8(s-1)} + \frac{9}{8(s+1)} - \frac{5s}{4(s^2+3)}$$

$$y_1(t) = \frac{9}{8}e^t + \frac{9}{8}e^{-t} - \frac{5}{4}\cos\sqrt{3}t$$
$$y_2(t) = e^t + e^{-t} + \cos\sqrt{3}t$$

denne)

$$y(t) = e^t\left\{1 + \int_0^t e^{-\tau}y(\tau)d\tau\right\}, \quad t \geq 0$$

$$\int_0^t e^{-\tau}y(\tau)d\tau = e^{-t}\int_0^t e^{t-\tau}y(\tau)d\tau = e^{-t}\left(e^t * y(t)\right)$$

$$y(t) = e^t\left\{1 + e^{-t}\left(e^t * y(t)\right)\right\} = e^t + e^t * y(t)$$

$$Y = \frac{1}{s-1} + Y\frac{1}{s-1}$$
$$Y\left(1 - \frac{1}{s-1}\right) = \frac{1}{s-1}$$
$$Y\frac{s-2}{s-1} = \frac{1}{s-1}$$
$$Y = \frac{1}{s-2}$$

$$y(t) = e^{2t}$$