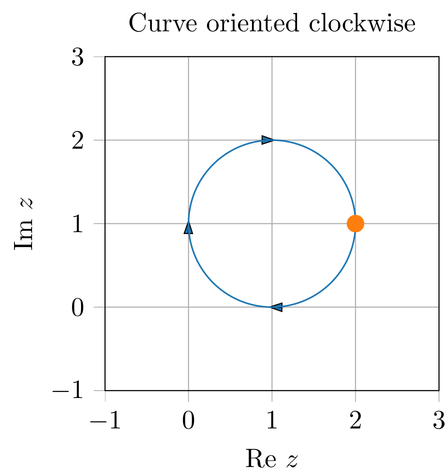
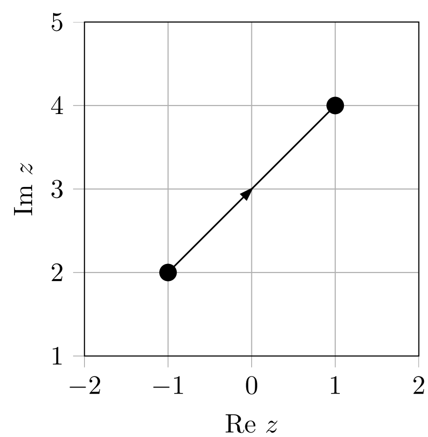
Øving 9

14.1

6



11



$$z(t) = (1+i)t + 3i \qquad -1 \leq t \leq 1$$

22

$$f(z) = \text{Re } z \quad \text{is not analytic, and integration has to be done by line integral:}$$

$$y = 1 + \frac{1}{2}(x-1)^2 \qquad x = t, \quad y = 1 + \frac{1}{2}(t-1)^2 \quad 1 \leq t \leq 3$$

$$z = t + i\left(1 + \frac{1}{2}(t-1)^2\right) \qquad \frac{dz}{dt} = 1 + i$$

$$\int_C f(z)dz = \int_1^3 f(z(t)) \cdot \frac{dz}{dt}dt = \int_1^3 t \cdot (1+i)dt = 4 + 4i$$

25

$$\int_1^i ze^{z^2}dz = \int_1^{-1}\frac{1}{2}e^u du = \left[\frac{1}{2}e^u\right]_1^{-1} = \frac{1}{2}e^{-1} - \frac{1}{2}e = -\sinh 1$$
$$u = z^2 \qquad 2zdz$$

$$z(t_1) = -t \quad -1 \le t_1 \le 0 \qquad z(t_2) = it_2 \quad 0 \le t_2 \le 1$$

$$\int_C f(z)dz = \int_{-1}^0 f(z(t_1))\frac{dz}{dt_1}dt_1 + \int_0^1 f(z(t_2))\frac{dz}{dt_2}dt_2 =$$
$$\int_{-1}^0 -t_1e^{t_1^2}(-1)dt_1 + \int_0^1 it_2e^{-t_2^2}idt_2 = \frac{1}{2} - \frac{1}{2}e - \frac{1}{2} + \frac{1}{2}e^{-1} =$$
$$\frac{1}{2}e^{-1} - \frac{1}{2}e = -\sinh 1$$

29

$$f(z) = \operatorname{Im} z^2 = 2ixy \qquad \text{not analytic}$$
$$z(t_1) = t_1 \quad 0 \le t_1 \le 1 \quad z(z_2)1 - t_2 + it_2 \quad 0 \le t_2 \le 1 \quad z(t_3) = -it_3 \quad -1 \le t_3 \le 0$$
$$f(z(t_1)) = 0 \qquad f(z(t_2)) = 2i(1-t_2)\cdot t_2 \qquad f(z(t_3)) = 0 \qquad \frac{dz}{dt_2} = -1+i$$

$$\int_C f(z)dz = \int_0^1 2i(t_2 - t_2^2)(-1+i) = -(2+2i)\int_0^1 (t_1 - t_2^2)dt_2 =$$
$$-(2+2i)\left[\frac{1}{2}t_2^2 - \frac{1}{3}t_2^3\right]_0^1 = -(2+2i)\left(\frac{1}{2} - \frac{1}{3}\right) = -\frac{1}{3} - \frac{1}{3}i$$

14.2

4

No, the function cannot be analytic in the annulus since the integral over the border would be 6 - 3 = 3 which is not zero.

13

$$f(z) = \frac{1}{z^4 - 1.2} \qquad \text{Analytic for all } z \in \mathbb{C}\backslash\left\{\pm 1.2^{1/4}, \pm i1.2^{1/4}\right\}$$
$$z(t) = e^{it} \qquad 0 \le t \le 2\pi \qquad \frac{dz}{dt} = ie^{it}$$
$$\int_0^{2\pi}\frac{ie^{it}}{e^{4it}} - 1.2dt = \int_{-0.2}^{-0.2}\frac{1}{u}\frac{1}{4e^{3it}}du = \frac{1}{4}\int_{-0.2}^{-0.2}\frac{1}{u^{\frac{7}{4}}}du = 0$$
$$u = e^{4it} - 1.2 \quad du = 4ie^{4it}dt \quad e^{3it} = \left(e^{4it}\right)^{\frac{3}{4}} = u^{\frac{3}{4}}$$
Cauchy's integral theorem applies $\Rightarrow f(x)$ is analytic in the unit circle

22

$$f(z) = \text{Re } z$$
$$z(t_1) = t_1 \quad -1 \le t_1 \le 1 \qquad z(t_2) = e^{it_2} \quad 0 \le t_2 \le \pi$$
$$f(z(t_1)) = t_1 \quad \frac{dz}{dt_1} = 1 \qquad f(z(t_2)) = \cos t_2 \quad \frac{dz}{dt_2} = ie^{it_2}$$

$$\int_C f(z)dz = \int_{-1}^{1} t_1 dt_1 + \int_0^\pi \cos t_2 ie^{it_2} dt_2 =$$
$$0 + \int_0^\pi i\cos t_2(\cos t_2 + i\sin t_2)dt_2 = \int_0^\pi (i\cos^2 t_2 - \cos t_2 \sin t_2)dt_2 =$$
$$\int_0^\pi \left(\frac{1}{2}(1+\cos 2t_2) - \frac{1}{2}\sin 2t_2\right) dt_2 = \int_0^\pi \frac{1}{2} dt_2 = \frac{\pi}{2} \neq 0$$

24

$$\oint_C \frac{dz}{z^2-1} = \oint_C \left(\frac{\frac{1}{2}}{z-1} - \frac{\frac{1}{2}}{z+1}\right) dz = \oint_{C_1} \frac{\frac{1}{2}}{z-1} dz - \oint_{-C_2} \frac{\frac{1}{2}}{z+1} dz =$$
$$2i\pi g(-1) - 2i\pi g(1) = 0, \qquad g(z) = \frac{1}{2}$$

17.1

10