

## Øving 8

13.3

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$$z = x + iy|z|^2 \operatorname{Im} \left( \frac{1}{z} \right) = (x^2 + y^2) \frac{y}{x^2 + y^2} = y$$
$$z = 0 \Rightarrow y = 0 \Rightarrow |z|^2 \operatorname{Im} \left( \frac{1}{z} \right) = 0 \Rightarrow f(z) \text{ is continuous}$$

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$$\frac{\operatorname{Im} z^2}{|z|^2} = \frac{2xy}{x^2 + y^2}$$

Limit along  $x = y$  :

$$\lim_{x \rightarrow 0} \frac{2x^2}{x^2 + x^2} = 1 \neq 0 \Rightarrow f(z) \text{ is discontinuous}$$

13.4

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$$f(z) = \ln |z| + i \operatorname{Arg} z = \ln r + i\theta = u(r, \theta) + iv(r, \theta)$$
$$u_r = \frac{1}{r} = \frac{1}{r} v_\theta$$
$$u_\theta = 0 = v_r$$

$f(z)$  is therefore analytic

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$$u = x^3 - 3xy^2$$
$$u_{xx} + u_{yy} = 6x - 6x = 0 \Rightarrow u(x, y) \text{ is harmonic}$$
$$u_x = 3x^2 - 3y^2 = v_y$$
$$v = \int (3x^2 - 3y^2) dy = 3x^2 y - y^3 + C(x)$$
$$u_y = -6xy = -v_x$$
$$v = \int 6xy dx = 3x^2 y + C(y)$$
$$v = 3x^2 y - y^3 + C$$

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$$v = e^{-x} \sin 2y v_{xx} + v_{yy} = e^{-x} \sin 2y - 4e^{-x} \sin 2y = -3e^{-x} \sin 2y \neq 0$$

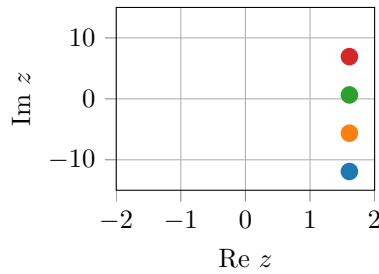
$v(x, y)$  is therefore not harmonic

## 13.5

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$$e^z = 4 + 3i$$

$$z = \ln(4 + 3i) = \ln 5 + i(\operatorname{Arg}(4 + 3i) + 2\pi n)$$



## 13.6

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$$\begin{aligned}\sinh(3 + 4i) &= \frac{1}{2}(e^{3+4i} - e^{-3-4i}) = \frac{1}{2}(e^3 e^{4i} - e^{-3} e^{-4i}) = \\ &= \frac{1}{2}(e^3(\cos 4 + i \sin 4) - e^{-3}(\cos 4 - i \sin 4)) = \\ &= \cos 4 \cdot \frac{1}{2}(e^3 - e^{-3}) + i \sin 4 \cdot \frac{1}{2}(e^3 + e^{-3}) = \\ &= \sinh 3 \cos 4 + i \cosh 3 \sinh 4\end{aligned}$$

$$\cosh(3 + 4i) = \cosh 3 \cos 4 + i \sinh 3 \sin 4$$

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$$\begin{aligned}\sin z = 100 &\Rightarrow \frac{1}{2i}(e^{iz} - e^{-iz}) = 100 \\ e^{iz} - e^{-iz} &= 200i \\ e^{2iz} - 1 &= 200ie^{iz} \Rightarrow (e^{iz})^2 - 200ie^{iz} - 1 = 0 \\ e^{iz} &= \frac{200i \pm \sqrt{(200i)^2 - 4 \cdot 1 \cdot (-1)}}{2} = \frac{200i \pm \sqrt{-39996}}{2} = \\ &= 100i \pm 3i\sqrt{1111} \approx 199.995 \vee 0.005i \\ iz = \ln 199.995i &= \pm \ln 199.995 + \ln i = \pm 5.298 + i\left(\frac{\pi}{2} + 2\pi n\right) \\ z &= \frac{\pi}{2} + 2\pi n \pm 5.298i\end{aligned}$$

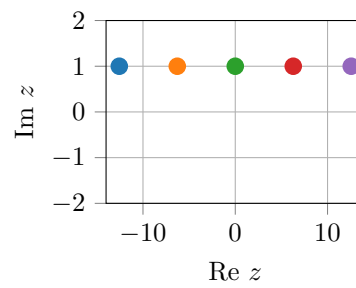
19

$$\begin{aligned}\sinh z = 0 &\Rightarrow \frac{1}{2}(e^z - e^{-z}) = 0 \\ e^{2z} - 1 &= 0 \\ 2z = \ln 1 &= 2\pi ni \\ z &= \pi ni\end{aligned}$$

13.7

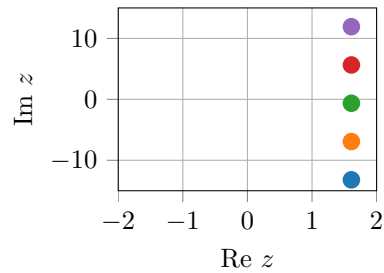
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$$\ln e^i = i \ln e = i - 2\pi n$$



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$$\ln(4 - 3i) = \ln 5 + i \operatorname{Arg}(4 - 3i) = \ln 5 + i \operatorname{atan} \frac{-3}{4} + 2i\pi n$$



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$$i^{\frac{i}{2}} = e^{\frac{i}{2} \ln i} = e^{\frac{i}{2} \cdot i \frac{\pi}{2}} = e^{-\frac{\pi}{4}} \approx 0.4559$$

```
using Plots
R = 0.0821
T = 243.6
isotherm(x) = R * T / x
v = 1 : 0.1 : 10
p = isotherm.(v)
```

```

plot(xlabel = "V[L]", ylabel = "P[atm]", xlims = (0, 15), ylims = (0, 25),
      xticks = 0 : 1 : 15, yticks = 0 : 2 : 25)
plot!(reverse(v), reverse(p);
      color = :red, label = "Trinn 1", arrow = :arrow, linewidth = 2)
plot!([1, 10], [20, 20];
      color = :blue, label = "Trinn 2", arrow = :arrow, linewidth = 2)
plot!([10, 10], [20, 2];
      color = "#007f00", label = "Trinn 3", arrow = :arrow, linewidth = 2)

```

