Work sheet week 11

fredag 16. mars 2018 22:39

C.3

This kind of feels like cheating but if

$$P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$Q = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$$

$$P \times Q = \begin{bmatrix} a\,e + b\,g & a\,f + b\,h \\ c\,e + d\,g & c\,f + d\,h \end{bmatrix}$$

Then Geogebra CAS solves the system of equations for P x Q to be stochastic:

1	a + c = 1 → $a + c = 1$
2	b + d = 1 → $b + d = 1$
3	e + g = 1 → $e + g = 1$
4	f + h = 1 → $f + h = 1$
5	x = a e + b g + c e + d g → $x = a\,e + b\,g + c\,e + d\,g$
6	y = a f + b h + c f + d h → $y = a\,f + b\,h + c\,f + d\,h$
7	{$1, $2, $3, $4, $5, $6} Løs: $\{\{x = e + g, y = f + h, a = -c + 1, b = -d + 1, c = c, d = d\}\}$

And comes out with x = e + g = 1 and y = f + h = 1 which means that at least for 2 x 2 matrices the statement holds.

It also makes sence that the product of stochastic matrices is stochastic since stochastic matrices are applied repeatedly in markov chains which is equivalent to raising the matrix to a power and then applying it to the vector.

C.4

P has to be a matrix where only row exchanges has to be done to get E. If v is a vector where all coordinates are equal then this would be a (but not necessarily the only) steady state vector. An example would be:

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P \cdot \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$