Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4115 Calculus 3
Spring 2018

Work Sheet Week 2

Remark: Only hand in 2 of 3 exercises (of your own choice) from part **C**.

A - Reading

SS = Saff-Snider (the part of the textbook that deals with complex numbers)

SS 1.1 The Algebra of Complex Numbers

SS 1.2 Point Representation of Complex Numbers

SS 1.3 Vectors and Polar Form

B - Finger Exercises

The form $a + \mathrm{i}b$ will always assume a and b real, and the form $r\mathrm{e}^{\mathrm{i}t}$ will always assume r real, non-negative and t real.

B.1

Compute $(2 + \mathrm{i}) + (3 + 4\mathrm{i})$ and $(2 + \mathrm{i}) \cdot (3 + 4\mathrm{i})$.

B.2

Compute

$$\mathrm{e}^{\mathrm{i}\frac{\pi}{2}} - \mathrm{e}^{-\mathrm{i}\frac{\pi}{2}}$$

and

$$\frac{\mathrm{e}^{\mathrm{i}\frac{\pi}{2}}}{\mathrm{e}^{-\mathrm{i}\frac{\pi}{2}}}.$$

B.3

Locate the complex numbers in **B.1-B.2** in the plane.

B.4

Write

$$\frac{1}{1 + \mathrm{i}}$$

in the form $a + \mathrm{i}b$, in the form $r\mathrm{e}^{\mathrm{i}t}$, and locate it in the plane.

B.5

Find all solutions of

$$z^2 - z + 1 = 0$$

in the form $a + \mathrm{i}b$ and $r\mathrm{e}^{\mathrm{i}t}$. Locate them in the plane.

C - Exam Preparation

The form $a + \mathrm{i}b$ will always assume a and b real.

C.1

Is the function

$$f : \mathbb{C} \to \mathbb{C},$$

$$z = a + \mathrm{i}b \mapsto f(z) = a + \mathrm{i}(a + b)$$

of the form $f(z) = z + w$ where w does not depend on z? Is it of the form $z \cdot w$ (where w does not depend on z)?

C.2

Let t be a real parameter. For which values of t does the polynomial

$$z^2 - tz + 1$$

only have one complex root? If you can't find them, can you at least explain why there are only finitely many of them?

C.3

Let $z = a + \mathrm{i}b$ be a complex number. Find real numbers p and q such that z is a solution of

$$z^2 + pz + q = 0.$$

Find all complex numbers w such that w and i are solutions of a polynomial $z^2 + pz + q$ with p and q real.