Øving 1

6.1

4)

$$\mathcal{L}\left(\cos^2 \omega t\right) = \mathcal{L}\left(\frac{1}{2}\left(1+\cos 2\omega t\right)\right) = \frac{1}{2s} + \frac{\omega}{s^2+4\omega^2}$$

13)

$$f(t) = \begin{cases} 1 & x < 1 \\ -1 & 1 < x < 2 \\ 0 & x > 2 \end{cases} =$$

$$1 - 2u\left(t-1\right) + u\left(t-2\right) =>$$

$$\mathcal{L}\left(f\right)\left(s\right) = \frac{1}{s}\left(1 - 2e^{-s} + e^{-2s}\right)$$

26)

$$\mathcal{L}\left(f\right)\left(s\right) = \frac{5s+1}{s^2-25} = \frac{5s+1}{\left(s-5\right)\left(s+5\right)} = \frac{A}{s-5} + \frac{B}{s+5}$$

$$A = 2.6, B = -2.4 => \mathcal{L}\left(f\right)\left(s\right) = \frac{2.6}{s-5} - \frac{2.4}{s+5} =>$$

$$f\left(t\right) = 2.6e^{5t} - 2.4e^{-5t}$$

32)

$$\mathcal{L}\left(f\right)\left(s\right) = \frac{1}{\left(s+a\right)\left(s+b\right)} = -\frac{1}{\left(b-a\right)\left(s+a\right)} - \frac{1}{\left(a-b\right)\left(s+b\right)} =>$$

$$f\left(t\right) = -\frac{e^{at}}{b-a} - \frac{e^{bt}}{a-b}$$

40)

$$\mathcal{L}\left(f\right)\left(s\right) = \frac{1}{s^2-2s-3} = \frac{1}{\left(s-3\right)\left(s+1\right)} = \frac{1}{4\left(s-3\right)} - \frac{1}{4\left(s+1\right)} =>$$

$$f(t) = \frac{e^{-3t}}{4} - \frac{e^{t}}{4}$$

6.2

5)

$$y'' - \frac{1}{4}y = 0, \quad y(0) = 0, \quad y'(0) = 1$$
$$Y = \mathcal{L}\left(y\right)$$

$$s^2 Y - s - \frac{1}{4}Y = 0$$
$$Y\left(s^2 - \frac{1}{4}\right) = s$$
$$Y = \frac{s}{s^2 - \frac{1}{4}} = \frac{s}{\left(s - \frac{1}{2}\right)\left(s + \frac{1}{2}\right)} = \frac{\frac{1}{2}}{s - \frac{1}{2}} + \frac{\frac{1}{2}}{s + \frac{1}{2}} =>$$

$$y(t) = \frac{1}{2}\left(e^{-\frac{t}{2}} + e^{\frac{t}{2}}\right)$$

10)

$$y'' + 0.04y = 0.02t^2, \quad y(0) = -25, \quad y'(0) = 0$$
$$Y = \mathcal{L}\left(y\right)$$

$$s^2 Y + 25 + 0.04Y = \frac{0.04}{s^3}$$
$$Y\left(s^2 + 0.04\right) = \frac{0.04}{s^3} - 25$$
$$Y = \frac{0.04}{s^3\left(s^2 + 0.04\right)} - \frac{25}{s^2 + 0.04}$$

$$\frac{0.04}{s^3\left(s^2 + 0.04\right)} = \frac{1}{s^3} - \frac{25}{s} + \frac{25}{s^2 + 0.04}$$
$$Y = \frac{1}{s^3} - \frac{25}{s} + \frac{25}{s^2 + 0.04} - \frac{25}{s^2 + 0.04} = \frac{1}{s^3} - \frac{25}{s}$$
$$y(t) = \frac{1}{2}t^2 - 25$$

26)

$$\mathcal{L}(f(t))(s) = F(s) = \frac{1}{s^4 - s^2} = \frac{1}{s^2} \cdot \frac{1}{s^2-1} = \frac{1}{s^2} \cdot G(s)$$

$$G(s) = \frac{1}{s^2-1} = \frac{1}{(s-1)(s+1)} = \frac{1}{2(s-1)} - \frac{1}{2(s+1)} =>$$

$$g(t) = \frac{e^t}{2} - \frac{e^{-t}}{2} = \sinh t$$

$$f(t) = \int_0^t \int_0^t \sinh t dt^2 = \int_0^t (\cosh t - 1)\, dt = \sinh t - t$$

6.3

21)

$$y'' + 4y = g(t) = \begin{cases} 4\cos t & 0 < t < \pi \\ 0 & \pi < t \end{cases} = 4(1 - u(t-\pi))\cos t$$

$$g(t) = 4(1 - u(t-\pi))\cos t = 4\cos t - 4\cos(t)u(t-\pi) = \\ 4\cos t + 4\cos(t-\pi)u(t-\pi)$$

$$\mathcal{L}(g(t))(s) = \frac{4s}{s^2+1} + e^{-\pi s}\frac{4s}{s^2+1} = \left(1 + e^{-\pi s}\right)\frac{4s}{s^2+1} \\ \mathcal{L}(y) = Y$$

$$s^2Y - s - \frac{8}{3} + 4Y = \left(1 + e^{-\pi s}\right)\frac{4s}{s^2+1}$$
$$Y\left(s^2+4\right) = \left(1 + e^{-\pi s}\right)\frac{4s}{s^2+1} + s + \frac{8}{3}$$
$$Y = \left(1 + e^{-\pi s}\right)\frac{4s}{(s^2+1)(s^2+4)} + \frac{s}{s^2+4} + \frac{8}{3(s^2+4)} =$$
$$\frac{4}{3}\left(1 + e^{-\pi s}\right)\left(\frac{s}{s^2+4} - \frac{s}{s^2+1}\right) + \frac{s}{s^2+4} + \frac{8}{3(s^2+4)} =$$
$$\frac{4}{3}\left(\frac{s}{s^2+4} - \frac{s}{s^2+1} + e^{-\pi s}\left(\frac{s}{s^2+4} - \frac{s}{s^2+1}\right)\right) + \frac{s}{s^2+4} + \frac{8}{3(s^2+4)} =$$
$$\frac{7s}{3(s^2+4)} + \frac{8}{3(s^2+4)} - \frac{4s}{3(s^2+1)} + \frac{4}{3}e^{-\pi s}\left(\frac{s}{s^2+4} - \frac{s}{s^2+1}\right)$$

$$y = \frac{7}{3}\cos 2t + \frac{4}{3}\sin 2t - \frac{4}{3}\cos t + \frac{4}{3}u(t-\pi)\,(cos2t + cost)$$

38)

$$R = 4\Omega, \quad L = 1H, \quad C = 0.05F, v(t) = \begin{cases} 34e^{-t} & 0 < t < 4 \\ 0 & 4 < t \end{cases}$$
$$i'(t) + 4i(t) + 20\int_0^t i(\tau)d\tau = v(t)$$
$$\mathcal{L}\left(i(t)\right)(s) = I(s)$$

$$sI + 4I + \frac{20}{s}I = \left(1 - e^{-4}e^{-4s}\right)\frac{34}{s+1}$$
$$\left(s^2 + 4s + 20\right)I = \left(1 - e^{-4}e^{-4s}\right)\frac{34s}{s+1}$$
$$I = \left(1 - e^{-4}e^{-4s}\right)\frac{34s}{(s+1)\left(s^2+4s+20\right)} = \left(1 - e^{-4}e^{-4s}\right)\left(\frac{2s+40}{s^2+4s+20} + \frac{2}{s+1}\right)$$

$$i(t) = 2e^{-2t}\cos 4t + 9e^{-2t}\sin 4t + 2e^{-t}$$
$$-u(t-4)e^{-4}\left(2e^{-2(t-4)}\cos 4(t-4) + 9e^{-2(t-4)}\sin 4(t-4) + 2e^{-(t-4)}\right)$$