Øving 4

11.3

14

$$r(t) = \begin{cases} -1 & -\pi < t < 0 \\ \\ 1 & 0 < t < \pi \end{cases} \qquad r(t + 2\pi) = r(t)$$

$$a_n = 0 \quad b_n = \frac{2}{\pi} \int_0^{\pi} \sin nt \; dt =$$
$$\frac{2}{\pi} \left[-\frac{1}{n} \cos nt \right]_0^{\pi} = \begin{cases} \frac{4}{\pi n} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

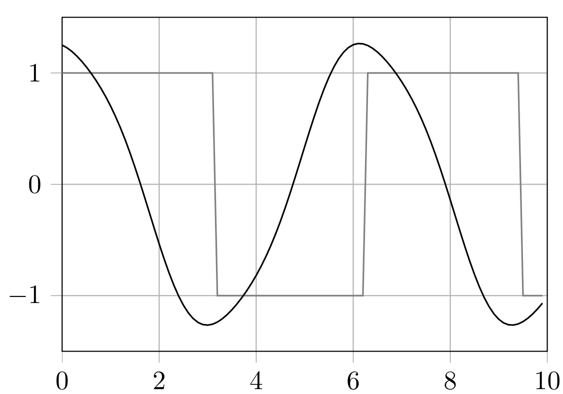
$$r(t) = \sum_{n \text{ odd}}^{\infty} \frac{4}{\pi n} \sin nt$$

$$y = \sum_{n \text{ odd}}^{\infty} A_n \cos nt + B_n \sin nt$$

$$y'' + cy' + y = r(t)$$
$$y_n = A_n \cos nt + B_n \sin nt \quad y_n' = -A_n n \sin nt + B_n n \cos nt \quad y_n'' = -A_n n^2 \cos nt - B_n n^2 \sin nt$$

$$B_n - cA_n n - B_n n^2 = \frac{4}{\pi n} \quad A_n + cB_n n - A_n n^2 = 0$$
$$A_n = \frac{4c}{\pi \left(-n^4 + \left(c^2 + 2\right) n^2 - 1\right)} \quad B_n = \frac{4n^2 - 4}{\pi \left(-n^5 + \left(c^2 + 2\right) n^3 - n\right)}$$

r(t), y(t), c = 1



16

$$r(t) = \begin{cases} t & -\frac{\pi}{2} < t < \frac{\pi}{2} \\ \pi - t & \frac{\pi}{2} < t < \frac{3\pi}{2} \end{cases}$$

$$r(t) \quad \text{odd} \Rightarrow a_n = 0$$

$$b_n = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} t \sin nt \; dt + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (\pi - t) \sin nt \; dt =$$

$$\frac{4}{n^2}(-1)^{\frac{n-1}{2}} \quad \forall \; n \text{ odd}$$

$$y = \sum_{n \text{ odd}}^{\infty} A_n \cos nt + B_n \sin nt$$

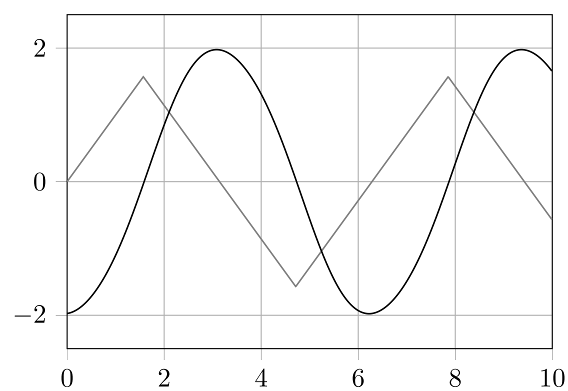
$$y'' + cy' + y = r(t)$$

$$y_n = A_n \cos nt + B_n \sin nt \quad y_n' = -A_n n \sin nt + B_n n \cos nt \quad y_n'' = -A_n n^2 \cos nt - B_n n^2 \sin nt$$

$$B_n - cA_n n - B_n n^2 = \frac{4}{n^2}(-1)^{\frac{n-1}{2}} \quad A_n + cB_n n - A_n n^2 = 0$$

$$A_n = \frac{4c}{n^5 + (c^2 - 2)\, n^3 + n} \quad B_n = \frac{4\left(n^2 - 1\right)}{n^6 + (c^2 - 2)\, n^4 + n^2}$$

r(t), y(t), c = 2



11.4

3

$$f(x) = |x| \quad -\pi < x < \pi \quad f(x+2\pi) = f(x)$$

$$b_n = 0 \quad a_0 = \frac{1}{\pi}\int_0^\pi x dx = \frac{\pi}{2}$$

$$a_n = \frac{2}{\pi}\int_0^\pi x \cos nx \; dx = \frac{2}{n}\cancel{\sin n\pi} + \frac{2}{\pi n^2}(\cos n\pi - 1) = -\frac{4}{\pi n^2} \quad n \text{ odd}$$

$$F(x) = \frac{\pi}{2} - \frac{4}{\pi}\sum_{n \text{ odd}}^{N} \frac{1}{n^2}\cos nx = \frac{\pi}{2} - \frac{4}{\pi}\cos x - \frac{4}{9\pi}\cos 3x - \frac{4}{25\pi}\cos 5x - ...$$

$$E^* = \int_\pi^\pi x^2 dx - \pi\left[2\left(\frac{\pi}{2}\right)^2 + \sum_{n \text{ odd}}^{N}\left(\frac{-4}{\pi n^2}\right)^2\right] =$$

$$\frac{1}{6}\pi^3 - \frac{16}{\pi}\sum_{n \text{ odd}}^{N}\frac{1}{n^4} = \frac{1}{6}\pi^3 - \frac{16}{\pi}\left(\frac{1}{1} + \frac{1}{81} + \frac{1}{625} + ...\right)$$

$$E^*_{1-5} = 0.0748, 0.0119, 0.00373$$

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$$f(x) = \begin{cases} -1 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases} \quad f(x+2\pi) = f(x)$$

$$a_n = 0 \quad b_n = \frac{2}{\pi}\int_0^\pi \sin nx \; dx = \begin{cases} \frac{4}{\pi n} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

$$F(x) = \frac{4}{\pi}\sum_{n \text{ odd}}^{N}\frac{1}{n}\sin nx = \frac{4}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + ...\right)$$

$$E^* = 2\int_0^\pi dx - \pi\sum_{n \text{ odd}}^{N}\left(\frac{4}{\pi n}\right)^2 = 2\pi - \frac{16}{\pi}\sum_{n \text{ odd}}^{N}\frac{1}{n^2}$$

$$E^*_{1-5} = 1.19, 0.624, 0.421$$

6

Square errors for discontious functons are larger than for continous functions.

11

From 11.4.5 we have

$$E^* = 2\pi - \frac{16}{\pi} \sum_{n \text{ odd}}^{N} \frac{1}{n^2}$$

$$\lim_{N\to\infty} E^* = 0$$

$$2\pi - \frac{16}{\pi} \sum_{n \text{ odd}}^{\infty} \frac{1}{n^2} = 0$$

$$\sum_{n \text{ odd}}^{\infty} \frac{1}{n^2} = 2\pi \cdot \frac{\pi}{16} = \frac{\pi^2}{8}$$

First few partial sums:

$$1, 1.1111, 1.1511, 1.1715, 1.1839$$

Denne

$$\sum_{\substack{n=\infty \\ n\neq 0}}^{\infty} \frac{e^{inx}}{2n^2} = \sum_{n=1}^{\infty} \frac{1}{2n^2} \left(e^{inx} + e^{-inx}\right) =$$

$$\sum_{n=1}^{\infty} \frac{1}{2n^2} (\cos nx + i \sin nx + \cos nx - i \sin nx) =$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \cos nx \quad \Rightarrow \quad a_0 = 0, \quad a_n = \frac{1}{n^2}, \quad b_n = 0$$