Øving 10

14.3

1

$$C : |z+1| = \frac{3}{2}$$

$$\oint_C \frac{z^2}{z^2-1}dz = \oint_C \frac{z^2}{z-1}\frac{1}{z+1}dz = 2\pi i\frac{(-1)^2}{-2} = -\pi i$$

11

Singularity at $2i$ is inside curve

$$\oint_C \frac{1}{z^2+4}dz = \oint_C \frac{1}{z+2i}\frac{1}{z-2i}dz = 2\pi i\frac{1}{2i+2i} = \frac{\pi}{2}$$

13

$$\oint_C \frac{z+2}{z-2}dz = 2\pi i \cdot (2+2) = 8\pi i$$

18

$$\oint_C \frac{\sin z}{4z^2-8iz}dz = \oint_C \frac{\sin z}{4z}\frac{1}{z-2i}dz = 2\pi i\frac{\sin 2i}{4\cdot 2i} = \frac{\pi}{4}i\sinh 2$$

14.4

3

$$\oint_C \frac{e^{-z}}{z^n}dz = 2\pi i(e^{-z})^{(n-1)}(0) = 2\pi i(-1)^{n-1}e^{-0} = 2\pi i(-1)^{n-1}$$

16

$$\oint_C \frac{e^{4z}}{z(z-2i)^2}dz = \left(\frac{e^{4z}}{z}\right)'\Big|_{2i} = \frac{4ze^{4z}-e^{4z}}{z^2}\Big|_{2i} =$$

$$\frac{8ie^{8i}-e^{8i}}{-4} = \frac{1-8i}{4}e^{8i}$$

15.1

18

$$a_n = n^2\left(\frac{i}{4}\right)^n, \qquad \left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+1)^2}{4n^2}$$

$$\lim_{n\to\infty}\frac{(n+1)^2}{4n^2} = \frac{1}{4} < \infty \Rightarrow \text{Series is convergent}$$

19

$$a_n = \frac{i^n}{n^2 - i}, \qquad \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{n^2 - i}{(n+1)^2 - i} \right| = 1 < \infty$$

Series is convergent