Work sheet week 4

lørdag 20. januar 2018 01.53

C.1

We can solve the linear system (treating z as a constant) in matrix notation:

$$\begin{bmatrix} 1-z & z & 1 \\ z-1 & z & 0 \end{bmatrix} \to \begin{bmatrix} 1-z & z & 1 \\ 0 & 2z & 1 \end{bmatrix} \to \begin{bmatrix} 1-z & z & 1 \\ 0 & 1 & \frac{1}{2z} \end{bmatrix} \to \begin{bmatrix} 1-z & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2z} \end{bmatrix} \to$$

$$\begin{bmatrix} 1 & 0 & \frac{1}{2(1-z)} \\ 0 & 1 & \frac{1}{2z} \end{bmatrix} \Rightarrow x = \frac{1}{2(1-z)}, \qquad y = \frac{1}{2z}, \qquad z \neq 0 \wedge z \neq 1$$

We see that for z not equal to 0 or 1 we get one well defined solution $\frac{1}{2}\left(\frac{1}{1-z}, \frac{1}{z}\right)$ and for z = 0 or 1 we get no solutions

C.2

We can rearrange the equations to find y as expressions in x:

$ax + by = 0 \Rightarrow y = -\frac{ax}{b}$

$cx + dy = 0 \Rightarrow y = -\frac{cx}{d}$

For the system to be consistent the two y values has to be equal for the same value of x:

$y = -\frac{ax}{b} = -\frac{cx}{d} = y \Rightarrow \frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$

We see that as long as the condition $ad = bc$ holds, the system will be consistent and there will exist an infinite number of non-zero solutions given by the equation $y = -\frac{ax}{b}$

C.3

We can rearrange the new equations to match the format of the equations of the previous exercise:

$ax + by = \lambda x \Rightarrow (a - \lambda)x + by = 0$

$cx + dy = \lambda y \Rightarrow cx + (d - \lambda)y = 0$

If we call these new coefficients for example s and t,

$t = a - \lambda$

$s = d - \lambda$

$tx + by = 0$

$cx + sy = 0$

we know from the previous exercise that if $ts = bc$ then there exists an infinite number of non-zero solutions of the system. If we can find a value for lambda such that this holds, then a solution exists.

$$ts = (a-\lambda)(d-\lambda) = bc \Rightarrow \lambda^2 - (a+d)\lambda + ad - bc = 0 \Rightarrow \lambda = \frac{a + d \pm \sqrt{(a+d)^2 - 4(ad - bc)}}{2}$$

We have now found a value of lambda dependent on a, b, c and d which is defined for the whole complex plane and will always give a sensible value (since there isn't any division by zero or that sort) which gives the linear system an infinite number of non-zero solutions.