

Innlevering 5

Oppgave 1

a)

$$P(X < 1467) = P\left(Z < \frac{1467 - 1468}{2}\right) = 0.3085$$

$$P(1467 < \bar{X} < 1469) = P\left(-\frac{\sqrt{8}}{2} < Z < \frac{\sqrt{8}}{2}\right) = 1 - 2 * 0.0793 = 0.8414$$

a)

$$\hat{\mu} = \bar{X} \quad Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim n(z, 1, 0)$$
$$P\left(-z_{\alpha/2} \leq \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq z_{\alpha/2}\right) = 1 - \alpha = 0.9$$

$$\begin{aligned} -z_{\alpha/2} &\leq \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq z_{\alpha/2} \\ -z_{\alpha/2} \frac{\sigma}{\sqrt{n}} &\leq \bar{X} - \mu \leq z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \\ -\bar{X} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} &\leq -\mu \leq -\bar{X} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \\ \bar{X} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}} &\geq \mu \geq \bar{X} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \\ \bar{X} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} &\leq \mu \leq \bar{X} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \end{aligned}$$

$$\alpha = 0.1, \quad z_{\alpha/2} = 1.645, \quad \sigma = 2$$

$$\left[\bar{X} - 1.645 \frac{2}{\sqrt{n}}, \bar{X} + 1.645 \frac{2}{\sqrt{n}} \right]$$

$$\bar{X} = 1468.88, \quad n = 5$$

$$\left[1468.88 - 1.645 \frac{2}{\sqrt{5}}, 1468.88 + 1.645 \frac{2}{\sqrt{5}} \right] =$$
$$[1467.41, 1470.35]$$

Oppgave 2

a)

$$P(X > 185) = P\left(Z > \frac{185 - 179}{6}\right) = 0.1587$$

$$P(X > 185 | X > 179) = \frac{P(X > 185 \cap X > 179)}{P(X > 179)} = \frac{P(X > 185)}{0.5} = 2 * 0.1587 = 0.3174$$

b)

$$E(\hat{\beta}) = E\left(\frac{\sum_{i=1}^n X_i}{n\mu_M}\right) = E\left(\frac{1}{n} \sum_{i=1}^n X_i\right) \frac{1}{\mu_M} = \frac{\mu_K}{\mu_M} \Rightarrow$$

$\hat{\beta}$ er forventningsrett.

$$\hat{\beta} = \frac{\bar{X}}{\mu_M} \sim n\left(\beta, \frac{\sigma_K}{\sqrt{n}\mu_M}\right)$$

$$Z = \frac{\frac{\bar{X}}{\mu_M} - \beta}{\frac{\frac{\sigma_K}{\sqrt{n}\mu_M}}{\sqrt{\frac{\sigma^2}{n}}}} = \frac{\bar{X} - \mu_M\beta}{\sqrt{\frac{\sigma^2}{n}}} \sim n(0, 1)$$

$$T = \frac{\bar{X} - \mu_M\beta}{\sqrt{\frac{s^2}{n}}} \sim t_{n-1}$$

95% konfidensintervall:

$$P(-t_{\alpha/2} \leq T \leq t_{\alpha/2}) = 1 - \alpha = 0.95$$

$$-t_{\alpha/2} \leq \frac{\bar{X} - \mu_M\beta}{\sqrt{\frac{s^2}{n}}} \leq t_{\alpha/2}$$

$$-t_{\alpha/2} \sqrt{\frac{s^2}{n}} \leq \bar{X} - \mu_M\beta \leq t_{\alpha/2} \sqrt{\frac{s^2}{n}}$$

$$-\bar{X} - t_{\alpha/2} \sqrt{\frac{s^2}{n}} \leq -\mu_M\beta \leq -\bar{X} + t_{\alpha/2} \sqrt{\frac{s^2}{n}}$$

$$\frac{\bar{X}}{\mu_M} + \frac{t_{\alpha/2}}{\mu_M} \sqrt{\frac{s^2}{n}} \geq \beta \geq \frac{\bar{X}}{\mu_M} - \frac{t_{\alpha/2}}{\mu_M} \sqrt{\frac{s^2}{n}}$$

$$\frac{\bar{X}}{\mu_M} - \frac{t_{\alpha/2}}{\mu_M} \sqrt{\frac{s^2}{n}} \leq \beta \leq \frac{\bar{X}}{\mu_M} + \frac{t_{\alpha/2}}{\mu_M} \sqrt{\frac{s^2}{n}}$$

$$n = 5, \quad t_{4,0.025} = 2.776, \quad \bar{X} = 167.5, \quad s^2 = 5.1^2$$

$$\left[\frac{167.5}{179} - \frac{2.776}{179} \sqrt{\frac{5.1^2}{5}}, \frac{167.5}{179} + \frac{2.776}{179} \sqrt{\frac{5.1^2}{5}} \right] =$$

$$[0.900, 0.971]$$

Oppgave 3

a)

$$\begin{aligned}
 P(X > 0) &= 0.5 \\
 P(X > 0.05) &= P(Z > 0.05/0.03) = 1 - 0.9525 = 0.0475 \\
 Y &= \frac{1}{2}(X_1 + X_2) \\
 \sigma_Y &= \frac{0.03}{\sqrt{2}} = 0.021 \\
 P(-0.05 > Y \cup Y > 0.05) &= 2 \cdot P(Z < -0.05/0.021) = 2 \cdot 0.0087 = 0.0174
 \end{aligned}$$

b)

$$\begin{aligned}
 \hat{\sigma}^2 &= S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2 \\
 \frac{(n-1)S^2}{\sigma^2} &\sim \chi_{n-1}^2 \\
 E(\chi_{n-1}^2) &= n-1 \Rightarrow E\left(\frac{(n-1)S^2}{\sigma^2}\right) = \frac{n-1}{\sigma^2} E(S^2) = n-1 \\
 E(S^2) &= \sigma^2 \\
 \text{Var}(S^2) &= \text{Var}\left(\frac{\sigma^2}{n-1} \chi_{n-1}^2\right) = \frac{\sigma^4}{(n-1)^2} \cdot 2(n-1) = 2\sigma^4/(n-1) \\
 Z &= \frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2 \\
 P(\chi_{n-1,1-\alpha/2}^2 \leq Z \leq \chi_{n-1,\alpha/2}^2) &= 1 - \alpha \\
 \chi_{n-1,1-\alpha/2}^2 &\leq \frac{(n-1)S^2}{\sigma^2} \leq \chi_{n-1,\alpha/2}^2 \\
 \frac{\chi_{n-1,1-\alpha/2}^2}{(n-1)S^2} &\leq \frac{1}{\sigma^2} \leq \frac{\chi_{n-1,\alpha/2}^2}{(n-1)S^2} \\
 \frac{(n-1)S^2}{\chi_{n-1,1-\alpha/2}^2} &\geq \sigma^2 \geq \frac{(n-1)S^2}{\chi_{n-1,\alpha/2}^2} \quad \frac{(n-1)S^2}{\chi_{n-1,\alpha/2}^2} \leq \sigma^2 \leq \frac{(n-1)S^2}{\chi_{n-1,1-\alpha/2}^2} \\
 n = 5, \quad S^2 &= 0.0003905, \quad \alpha = 0.05, \quad \chi_{4,0.025}^2 = 11.143, \quad \chi_{4,1-0.025}^2 = 0.484 \\
 \left[\frac{4 \cdot 0.0003905}{11.143}, \frac{4 \cdot 0.0003905}{0.484} \right] &= [0.00014, 0.00323]
 \end{aligned}$$

Den oppgitte usikkerheten er inneholdt i intervallet som vil si at det er 95% sannsynlig at den faktisk er i dette intervallet.

Oppgave 4

a)

$$P(X > 40) = P\left(Z > \frac{40 - 35}{5}\right) = P(Z > 1) = 0.1587$$

$$P(30 < X < 40) = 1 - 2 \cdot 0.1587 = 0.6826$$

$$Y = X_1 + X_2 \sim n(70, 50)$$

$$P(Y > 80) = P\left(Z > \frac{10}{\sqrt{50}}\right) = P(Z > 1.41) = 0.0793$$

b)

En god estimator må være forventningsrett og ha lavest mulig varians.

$$\hat{\mu} = \bar{X} = \frac{1}{5} \sum_{i=1}^5 X_i$$

$$\text{Var}(\hat{\mu}) = \text{Var}\left(\frac{1}{5} \sum_{i=1}^5 X_i\right) = \frac{1}{25} \sum_{i=1}^5 \text{Var}(X_i) = \frac{1}{5} \sigma^2$$

$$\hat{\sigma}^2 = S^2 = \frac{1}{4} \sum_{i=1}^5 (X_i - \bar{X})^2$$

$$\hat{\sigma}_{\mu}^2 = \frac{1}{5} S^2$$

c)

$$Z = \frac{\bar{X} - \mu}{\sqrt{\frac{S^2}{n}}} \sim t_{n-1}$$

$$P(-t_{n-1, \alpha/2} \leq Z \leq t_{n-1, \alpha/2}) = 1 - \alpha$$

$$-t_{n-1, \alpha/2} \leq \frac{\bar{X} - \mu}{\sqrt{\frac{S^2}{n}}} \leq t_{n-1, \alpha/2}$$

$$-t_{n-1, \alpha/2} \sqrt{\frac{S^2}{n}} \leq \bar{X} - \mu \leq t_{n-1, \alpha/2} \sqrt{\frac{S^2}{n}}$$

$$\bar{X} - t_{n-1, \alpha/2} \sqrt{\frac{S^2}{n}} \leq \mu \leq \bar{X} + t_{n-1, \alpha/2} \sqrt{\frac{S^2}{n}}$$

d)

x