TMA 4100 Skriftlig innlevering 2

1

$x^2(t) + y^2(t) = l^2(t)$

Hvor l er avstanden mellom bilen og bussen.

$x(0) = 3$
$y(0) = 4$
$l(0) = \sqrt{3^2 + 4^2} = 5$
$x'(0) = -80$
$y'(0) = 50$

$2x(t)x'(t) + 2y(t)y'(t) = 2l(t)l'(t)$
$2 \cdot 3 \cdot (-80) + 2 \cdot 4 \cdot 50 = 2 \cdot 5 \cdot l'(0)$
$$l'(0) = \frac{200 - 240}{5} = -8$$

Avstanden mellom bilene er avtagende og I tidspunktet beskrevet I oppgaven minker den med 8 km/h

2

Hvis
$$f(x) = \frac{1}{(2+x)\ln(2+x)}$$
Så vil
$$\sum_{i=1}^{n} \frac{1}{n\left(2 + \frac{i}{n}\right)\ln\left(2 + \frac{i}{n}\right)}$$
Være en riemannsum for f på intervallet $[0, 1]$.

For å finne grenseverdien
$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n\left(2 + \frac{i}{n}\right)\ln\left(2 + \frac{i}{n}\right)}$$
Kan vi heller utrykke det som et integral fra 0 til 1 av f(x)
$$\int_0^1 \frac{1}{(2+x)\ln(2+x)}dx = \int_{\ln 2}^{\ln 3} \frac{1}{(2+x)u}(2+x)du = \int_{\ln 2}^{\ln 3} \frac{1}{u}du = [\ln|u|]_{\ln 2}^{\ln 3}$$
$$= \ln(\ln 3) - \ln(\ln 2) = \ln\frac{\ln 3}{\ln 2}$$
$u = \ln(2+x)$
$$\frac{du}{dx} = \frac{1}{2+x}$$
$dx = (2+x)du$

$u_{upper} = \ln(2+1) = \ln 3$
$u_{lower} = \ln(2+0) = \ln 2$

3
$$\int_2^\infty \frac{x^2+9}{x^4+3x^2-4}dx$$

$x^4 + 3x^2 - 4 = \left(x^2+4\right)\left(x^2-1\right) = \left(x^2+4\right)(x-1)(x+1) \Rightarrow$

$$\frac{x^2+9}{x^4+3x^2-4} = \frac{Ax+B}{x^2+4} + \frac{C}{x-1} + \frac{D}{x+1} \Rightarrow$$

$$x^2+9 = (Ax+B)(x^2-1) + C(x^2+4)(x+1) + D(x^2+4)(x-1) =$$
$$Ax^3 - Ax + Bx^2 - B + Cx^3 + Cx^2 + 4Cx + 4C + Dx^3 - Dx^2 + 4Dx - 4D =$$
$$(A+C+D)x^3 + (B+C-D)x^2 + (-A+4C+4D)x - B + 4C - 4D \Rightarrow$$

$A+C+D=0$
$B+C-D=1$
$-A+4C+4D=0$
$-B+4C-4D=9$

$A=0, B=-1, C=1, D=-1$

$$\frac{x^2+9}{x^4+3x^2-4} = -\frac{1}{x^2+4} + \frac{1}{x-1} - \frac{1}{x+1}$$

$$\int_2^\infty \frac{x^2+9}{x^4+3x^2-4}dx = -\int_2^\infty \frac{1}{x^2+4}dx + \int_2^\infty \frac{1}{x-1}dx - \int_2^\infty \frac{1}{x+1}dx =$$
$$-\frac{1}{2}\left[\arctan\frac{x}{2}\right]_2^\infty + [\ln|x-1| - \ln|x+1|]_2^\infty = -\frac{1}{2}\left[\arctan\frac{x}{2}\right]_2^\infty + \left[\ln\left|\frac{x-1}{x+1}\right|\right]_2^\infty$$
$$-\frac{1}{2}\lim_{x\to\infty}\arctan\frac{x}{2} + \frac{1}{2}\arctan\frac{2}{2} + \lim_{x\to\infty}(\ln|x-1| - \ln|x+1|) - (\ln|2-1| - \ln|2+1|) =$$
$$-\frac{1}{2}\lim_{x\to\infty}\arctan\frac{x}{2} + \lim_{x\to\infty}(\ln|x-1| - \ln|x+1|) + \frac{\pi}{8} - \ln\frac{1}{3}$$

Siden tan(x) har en vertikal asymptote I $x = \frac{\pi}{2}$ hvor $x \to \frac{\pi}{2}^+ \Rightarrow y \to \infty$ vil da:

$$\lim_{x\to\infty}\arctan\frac{x}{2} = \frac{\pi}{2}$$

$$\lim_{x\to\infty}(\ln|x-1| - \ln|x+1|) = \lim_{x\to\infty}\ln\frac{|x-1|}{|x+1|} = \lim_{x\to\infty}\ln\left|\frac{x-1}{x+1}\right| = \lim_{x\to\infty}\ln\left|\frac{x\left(1-\frac{1}{x}\right)}{x\left(1+\frac{1}{x}\right)}\right|$$

$$= \lim_{x\to\infty}\ln\left|\frac{1-\frac{1}{x}}{1+\frac{1}{x}}\right| =$$

$$\ln\left|\frac{1}{1}\right| = \ln|1| = \ln 1 = 0$$
$$-\frac{1}{2}\lim_{x\to\infty}\arctan\frac{x}{2} + \lim_{x\to\infty}(\ln|x-1| - \ln|x+1|) + \frac{\pi}{4} - \ln\frac{1}{3} = -\frac{\pi}{4} + 0 + \frac{\pi}{8} - \ln\frac{1}{3} = -\frac{\pi}{8} + \ln 3$$

4.

$$l = \int_0^1 \sqrt{1 + \left(y'(x)\right)^2}dx$$
$$y'(x) = \frac{2}{3}\cdot\frac{3}{2}\cdot x^{\frac{1}{2}} = \sqrt{x}$$
$$l = \int_0^1 \sqrt{1+\sqrt{x}^2}dx = \int_0^1 \sqrt{1+x}\,dx = \int_0^1 (1+x)^{\frac{1}{2}} = \left[\frac{2}{3}(1+x)^{\frac{3}{2}}\right]_0^1 = \frac{2}{3}2^{\frac{3}{2}} - \frac{2}{3} = \frac{2}{3}\left(\sqrt{2^3}-1\right)$$
$$= \frac{2}{3}\left(2\sqrt{2}-1\right) = \frac{4\sqrt{2}-2}{3} = \frac{4}{3}\sqrt{2} - \frac{2}{3}$$

Buelengden til kurven er $\frac{4}{3}\sqrt{2} - \frac{2}{3} = \frac{2}{3}\left(2\sqrt{2}-1\right) \approx 1.22$

$$S = 2\pi \int_0^1 x\sqrt{1+\sqrt{x}^2}\,dx = 2\pi \int_0^1 x\sqrt{1+x}\,dx = \left[u \cdot v - \int v\,du\right]_0^1$$

$$= \left[\frac{2}{3}x(1+x)^{\frac{3}{2}} - \frac{2}{3}\int (1+x)^{\frac{3}{2}}\,dx\right]_0^1 =$$

$$\left[\frac{2}{3}x(1+x)^{\frac{3}{2}} - \frac{4}{15}(1+x)^{\frac{5}{2}}\right]_0^1 = \frac{2}{3}2^{\frac{3}{2}} - \frac{4}{15}2^{\frac{5}{2}} + \frac{4}{15} = \frac{4}{3}\sqrt{2} - \frac{16}{15}\sqrt{2} + \frac{4}{15} =$$

$$\left(\frac{4}{3} - \frac{16}{15}\right)\sqrt{2} + \frac{4}{15} = \frac{4}{15}\sqrt{2} + \frac{4}{15} = \frac{4}{15}\left(\sqrt{2}+1\right)$$

$u = x$

$du = dx$

$dv = \sqrt{1+x}\,dx$

$v = \frac{2}{3}(1+x)^{\frac{3}{2}}$

Overflatearealet som dannes er da:

$\frac{4}{15}\left(\sqrt{2}+1\right) \approx 0.64$