Innlevering 2

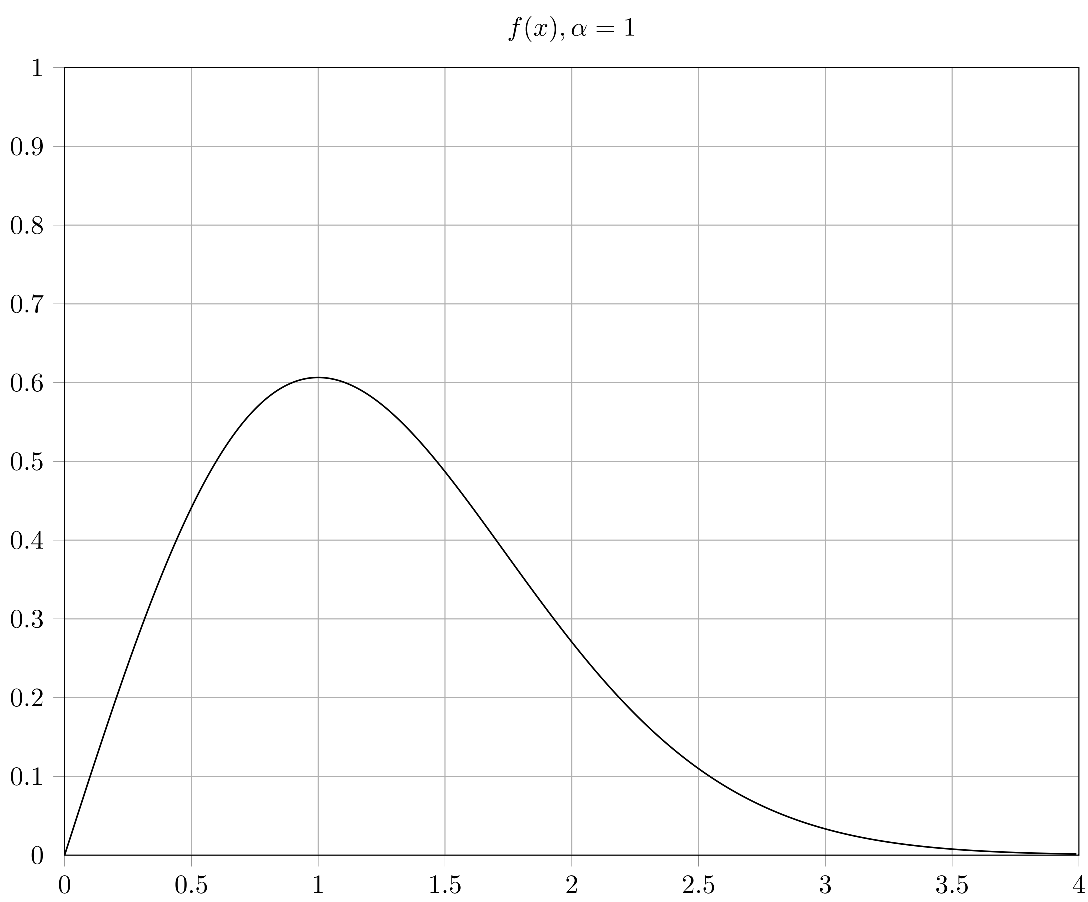
Oppgave 1

a)

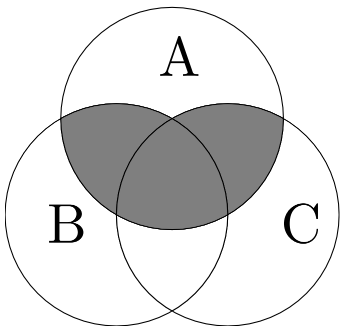
$$f(x) = \frac{d}{dx}F(x) = \frac{d}{dx}\left(1 - e^{-\frac{x^2}{2\alpha}}\right) = \frac{x}{\alpha}e^{-\frac{x^2}{2\alpha}}$$

$$x_{max} = \left\{ x \in (0, \infty) \,\middle|\, \frac{d}{dx}f(x) = 0 \right\}$$

$$\frac{d}{dx}\left(\frac{x}{\alpha}e^{-\frac{x^2}{2\alpha}}\right) = \frac{1}{\alpha}e^{-\frac{x^2}{2\alpha}} - \frac{x^2}{\alpha^2}e^{-\frac{x^2}{2\alpha}} = 0$$
$$-\frac{1}{\alpha^2}\left(x^2 - \alpha\right) = 0 \Rightarrow x = \sqrt{\alpha}$$



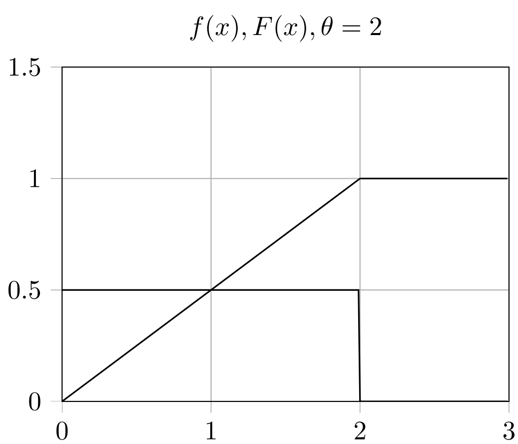
b)



$$P\left(A \cap (B \cup C)\right) = P(A) \cdot \left(P(B) + P(C) - P(B) \cdot P(C)\right)$$

$$P(A) = P(B) = P(C) = 1 - F(2) \qquad F(2) = 1 - e^{-2}$$
$$P\left(A \cap (B \cup C)\right) = e^{-2}\left(e^{-2} + e^{-2} - \left(e^{-2}\right)^2\right) = 2e^{-4} - e^{-6} = 0.0342$$

Oppgave 2



$$F(x) = \int_0^x f(y)dy = \begin{cases} \frac{1}{\theta}x & 0 < x < \theta \\ 1 & x > \theta \end{cases}$$
$$P(X \leq 0.4) = F(0.4) = \frac{0.4}{2} = 0.2$$

Oppgave 3

$$P(X \geq 0) = \sum_{x=0}^{2} f(x) = 0.5 + 0.2 + 0.1 = 0.8$$

$$P(X \geq 0 | X \leq 1) = \frac{P(X \geq 0 \cup X \leq 1)}{P(X \leq 1)} =$$
$$\frac{P(0 \leq X \leq 1)}{P(X \leq 1)} = \frac{\sum_{x=0}^{1} f(x)}{\sum_{x=-2}^{1} f(x)} =$$
$$\frac{0.5 + 0.2}{0.1 + 0.1 + 0.5 + 0.2} = \frac{0.7}{0.9} = \frac{7}{9} = 0.778$$

$$E(X) = \sum_{x=-2}^{2} x f(x) =$$
$$-2 \cdot 0.1 - 1 \cdot 0.1 + 0 \cdot 0.5 + 1 \cdot 0.2 + 2 \cdot 0.1 = 0.1$$

Oppgave 4

$$g(x) = \sum_{y=0}^{2} f(x, y) \qquad h(x) = \sum_{x=-1}^{1} f(x, y)$$

	y = 0	y = 1	y = 2	g(x)
x = -1	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{3}$
x = 0	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{3}$
x = 1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{3}$
h(y)	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y)$$
$$E(X) = -1 \cdot \frac{1}{3} + 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} = 0 \qquad E(Y) = 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} = 1$$
$$Var(Y) = Var(X) = E(X^2) - E(X)^2 = (-1)^2 \cdot \frac{1}{3} + 0 + 1 \cdot \frac{1}{3} - 0 = \frac{2}{9}$$
$$E(XY) = -1 \cdot 1 \cdot \frac{1}{12} - 1 \cdot 2 \cdot \frac{1}{12} + 1 \cdot 1 \cdot \frac{1}{12} + 1 \cdot 2 \cdot \frac{1}{6} = \frac{1}{6}$$
$$\text{Cov}(X, Y) = \frac{1}{6} - 0 \cdot 1 = \frac{1}{6} \neq 0 \Rightarrow \text{X og Y ikke uavhengig}$$

Oppgave 5

a)

$$P(A) = P((A \cap B) \cup (A \cap B^c)) = P(A \cap B) + P(A \cap B^c) = 0.05 + 0.15 = 0.200$$

$$P(B) = 0.1 + 0.05 = 0.15$$
$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.05}{0.15} = \frac{1}{3} = 0.333$$
$$P(A|B^c) = \frac{P(A \cap B^c)}{P(B^c)} = \frac{0.15}{1 - 0.15} = \frac{3}{17} = 0.176$$

$$P(A|B) = 0.33 \neq 0.20 \Rightarrow \text{A og B ikke uavhengige}$$

b)

$$E(R_1) = 0.8 \cdot (-100) + 0.2 \cdot 400 = 0$$

$$E(R_1|B) = P(A|B) \cdot 1000 + P(A^c|B) \cdot (-100) = \frac{1}{3} \cdot 1000 + \frac{2}{3} \cdot (-100) = 66.7 \text{ millioner}$$

$$E(R_1|B^c) = \frac{3}{17} \cdot 400 + \frac{14}{17} \cdot (-100) = -11.76$$

$$P(B|A) = \frac{P(B \cup A)}{P(A)} = \frac{0.05}{0.2} = \frac{1}{4}$$
$$P(B|A^c) = \frac{P(B \cup A^c)}{P(A^c)} = \frac{0.1}{0.8} = \frac{1}{8}$$

$$E(R_2|A) = \frac{1}{4} \cdot 1000 + \frac{3}{4} \cdot (-100) = 175$$
$$E(R_2|A^c) = \frac{1}{8} \cdot 1000 + \frac{7}{8} \cdot (-100) = 37.5$$

$$E(R_1, R_2) = E(R_1) + P(A) \cdot \max(0, E(R_2|A)) + P(A^c) \cdot \max(0, E(R_2|A^c)) =$$
$$0 + 0.2 \cdot 175 + 0.8 \cdot 37.5 = 65$$

$$E(R_2, R_1) = 65 + 0.15 \cdot 66.7 + 0.85 \cdot 0 = 75$$

Altså er letestrategien der det først letes i felt 2
og så i felt 1 dersom det var olje i felt 2
Dette vil gi en forventet fortjeneste på 75 millioner

Oppgave 6

$$F(y) = P(Y \leq y) = 1 - P(Y \geq y) = 1 - \left(\frac{k}{y}\right)^{\theta}$$
$$f(y) = \frac{d}{dy}F(y) = \frac{d}{dy}\left(1 - \frac{k^{\theta}}{y^{\theta}}\right) = \frac{\theta k^{\theta}}{y^{\theta+1}}$$

$$E(Y) = \int_k^{\infty} y f(y) dy = \int_k^{\infty} y \frac{\theta k^{\theta}}{y^{\theta+1}} dy = \theta k^{\theta} \int_k^{\infty} y^{-\theta} dy =$$
$$\theta k^{\theta} \left[-\frac{1}{\theta - 1} y^{-\theta+1}\right]_k^{\infty} = \frac{\theta}{\theta - 1} k^{\theta} k^{-\theta+1} = k \frac{\theta}{\theta - 1}$$

$$Var(Y) = E(Y^2) - E(Y)^2 \qquad E(Y^2) = \int_k^{\infty} y^2 f(y) dy = \int_k^{\infty} y^2 \frac{\theta k^{\theta}}{y^{\theta+1}} dy =$$
$$\theta k^{\theta} \int_k^{\infty} y^{-\theta+1} dy = \theta k^{\theta} \left[-\frac{1}{\theta - 2} y^{\theta+2}\right]_k^{\infty} = \theta k^{\theta} \frac{1}{\theta - 2} k^{-\theta+2} = k^2 \frac{\theta}{\theta - 2}$$