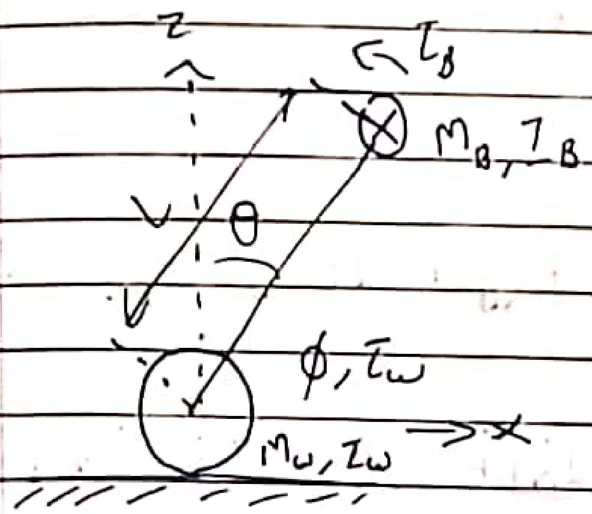
Self balancing Robot (LQR).



$x = r\phi$

COG new co-ordinates

$x = x + l\sin\theta$
$z = l\cos\theta$

$V_{COG} = \sqrt{\dot{x}_{CG} + \dot{z}_{CG}}$

K.E of body $= \frac{1}{2} m_b V_{CG}^2 + \frac{1}{2} I_b \dot{\theta}^2$

$= \frac{1}{2} m_b (\dot{x}^2 + 2\dot{x}l\cos\theta\,\dot{\theta} + l^2\dot{\theta}^2)$

$+ \frac{1}{2} I_b \dot{\theta}^2$ ——— (1)

K.E of wheel $= \frac{1}{2} m_w \dot{x}^2 + \frac{1}{2} I_w \dot{\phi}^2$

$= \frac{1}{2} m_w \dot{x}^2 + \frac{1}{2} \frac{I_w}{r^2} \dot{x}^2$ ——— (2)

P.E of body = $M_b g l \cos\theta$ — (3)

P.E of wheel = 0 — (4)

Lagrange

$$L = E_K - E_P = E_{KB} + 2E_{Kw} - E_{PB} - 2E_{Pw}$$

$$L = \frac{1}{2}\left[m_b \dot{x}^2 + 2\dot{x} l \dot{\theta}\cos\theta + l^2 \dot{\theta}^2\right] + \frac{1}{2} I_b \dot{\theta}^2$$
$$+ M_w \dot{x}^2 + \frac{I_w \dot{x}^2}{r^2} - M_b g l \cos\theta \quad (5)$$

For $q_i = x$,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = -\left(\frac{\tau_R + \tau_L}{r}\right)$$

$$\ddot{x}\left[M_b + 2m_w + 2\frac{I_w}{r^2}\right] + M_b l \cos\theta\, \ddot{\theta} - M_b l \sin\theta\, (\dot{\theta})^2 = -\frac{(\tau_R + \tau_L)}{r}$$

(6)

For $q_i = \theta$:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = -(T_R + T_L)$$

$$M_b L \cos\theta\, \ddot{x} + (M_b L^2 + I_b)\ddot{\theta} - M_b g L \sin\theta = -(T_R + T_L)$$

——— ⑦

$$\dot{x} = Ax + Bu$$

$$x = \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix}, \quad u = \begin{bmatrix} T_R \\ T_L \end{bmatrix}, \quad \dot{x} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix}$$

After linearization, using Jacobian

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & val & 0 \end{bmatrix} \qquad val = \frac{M_b g L}{M_b L^2 + I_b}$$

$$B = \begin{bmatrix} 0 & 0 \\ b_1 & b_1 \\ 0 & 0 \\ b_2 & b_2 \end{bmatrix} \qquad b_1 = \frac{-1}{r_w\left[M_b + 2M_w + 2\, I_w/r^2\right]}$$

$$b_2 = -1/(M_b L^2 + I_b)$$

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

will balance the system.

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