CHAPTER 44

JAMES CLERK MAXWELL, *A TREATISE ON ELECTRICITY AND MAGNETISM*, FIRST EDITION (1873)

F. Achard

This comprehensive mathematical treatise on electricity and magnetism led to a slow spread of Maxwell's theory of the electromagnetic field and of his electromagnetic theory of light, first published in 1865. After Hertz's experiments in 1888, it became recognized as the major source of aether and field physics.

First publication. 2 volumes, Oxford: Clarendon Press (hereafter 'CP'), 1873. xxix + 425; xxiii + 444 pages. Print run: 1,500 copies.

2nd edition. 2 volumes (ed. W.D. Niven), CP, 1881. xxxi + 464; xxiii + 456 pages.

3rd edition. 2 volumes (ed. J.J. Thomson), CP, 1891. xxxii + 506; xxiv + 500 pages. [Photorepr. New York: Dover, 1954; New York: Oxford University Press, 1998.]

German translation of the 2nd ed. Lehrbuch der Electricität und des Magnetismus (trans. B. Weinstein), 2 vols., Berlin: J. Springer, 1883.

French translation of the 2nd ed. Traité d'électricité et de magnétisme (trans. G. Séligmann-Lui, notes by A. Cornu, A. Potier and E. Sarrau), 2 vols., Paris: Gauthier–Villars, 1885–1887. [Photorepr. Sceaux: Jacques Gabay, 1989. Available on the web site of the *Bibliothèque Nationale de France*, as is the *Treatise* (1st ed.).]

Italian translation of the 3rd ed. Trattato di elettricita e magnetismo (trans. E. Agazzi), 2 vols., Turin: UTET, 1973.

Russian translation of the 3rd ed. Traktat ob elektrichestve i magnetizme (trans. B.M. Mikhailovitch, M.L. Levin), 2 vols., Moscow: Nauka, 1989.

Abridged version

Elementary treatise on electricity (ed. W. Garnett), CP, 1881. 2nd ed. 1888.

Landmark Writings in Western Mathematics, 1640–1940
I. Grattan-Guinness (Editor)

German translation. *Die Elektrizitat in elementarer Behandlung* (trans. L. Graetz), Braunschweig: Vieweg, 1883.

French translation. *Traité élémentaire sur l'électricité* (trans. G. Richard), Paris: Gauthier–Villars, 1884.

Manuscripts. Parts of the final draft of the *Treatise* (1st ed.), a few folios from earlier drafts and some proofs annotated by Maxwell and P.G. Tait are held in Cambridge University Archives, Cambridge, United Kingdom. The final draft of the *Elementary treatise on electricity* is also there. See also [Maxwell, 1990–2002], vol. 2.

Related articles: Lagrange on mechanics (§16), Fourier (§26), Green (§30), Hamilton (§35), Thomson and Tait (§40), Heaviside (§49), Rayleigh (§45), Kelvin (§58), Hertz (§52), Lorentz (§60), Einstein (§63).

1 EDUCATION AND CAREER

James Clerk Maxwell was born in Edinburgh on 13 June 1831. His father was descended from a long line of Scottish baronets and had been trained as a lawyer. In 1841 he entered the Edinburgh Academy, where he was a contemporary of Peter Guthrie Tait. In 1847 he entered the University of Edinburgh and followed the lectures of the Professor of Natural Philosophy, James David Forbes, and the Professor of Logic, Sir William Hamilton (not to be confused with the inventor of quarternions). In 1850 he left for Cambridge, and in 1854 came second in the Mathematical Tripos (hereafter, 'MT') and first ex-aequo for the Smith's Prize. In addition to his intensive mathematical education, mainly in the hands of the private coach William Hopkins, he probably came under the influence of George Stokes (1819–1903), the Lucasian Professor of Mathematics; and William Whewell, the Master of Trinity College. In 1850 he also made the acquaintance of William Thomson (1824–1907, later Lord Kelvin), the young Professor of Natural Philosophy at Glasgow, who certainly became a model for him.

Maxwell's career as a teacher began at Cambridge in 1855 as Fellow of Trinity College. In the following year he became Professor of Natural Philosophy at Aberdeen. He had to resign in 1860, following changes in the university system, and was an unsuccessful candidate for the Chair of Natural Philosophy at Edinburgh (obtained by Tait). In the same year, however, he was appointed Professor of Natural Philosophy at King's College, London. He left this position in 1865, probably in order to devote himself fully to scientific research. Over the next few years, he spent most of his time at his ancestral home in Scotland, residing in London in the winter months. It was during this period that he wrote his *Treatise on electricity and magnetism*. In 1871 he was appointed to the new Chair of Experimental Physics at Cambridge and the Directorship of the Cavendish Laboratory. He occupied this position until his death on 5 November 1879.

In 25 years of activity, Maxwell published about a hundred scientific papers. While his main claim to fame lies in his work on electromagnetism and the kinetic theory of gases, he was interested in almost all branches of physics, both mathematical and experimental, and especially in mechanics, geometry and optics. In 1857 his essay on the rings of Saturn earned him the Adams Prize at Cambridge, and in 1860 he won the Rumford Medal of the

Royal Society for his work on colour vision. He was made a Fellow of the Royal Society in 1861, and subsequently became a member of several other learned societies. He also published two books aimed at popularizing advanced topics, the *Theory of heat* (1871) and *Matter and motion* (1877), and a very full edition of the unpublished manuscripts of Henry Cavendish on electricity (1879).

2 MATHEMATICAL THEORIES OF ELECTRICITY AND MAGNETISM IN THE FIRST HALF OF THE 19TH CENTURY

One generally thinks of the mathematical treatment of electric phenomena as beginning with two memoirs on electrostatics by Siméon Denis Poisson (1781–1840) in 1812 and 1813. He based his study of conditions of equilibrium for electricity in a conductor on the Newtonian law of force between two (small) electrified spheres established by Charles Coulomb in the 1780s:

$$F \propto \frac{ee'}{r^2}, \tag{1}$$

where F was the force between the spheres, e and e' their respective electric charges, and r the distance between their centres. Using results of P.S. Laplace and J.L. Lagrange on the theory of gravitation, he found a close agreement between his analytic study and experimental observation. Like Coulomb, he assumed that electricity was being composed of two electric fluids. In 1826 and 1827, he also described a mathematical theory of magnetism based on Coulomb's law for magnetic action and the hypothesis that the magnetization of a body results from the separation of two magnetic fluids in the interior of each molecule [Grattan-Guinness, 1990, 496–514, 948–953, 961–965].

Poisson made frequent use of a function V which was later, under the name 'potential', to play an important role in the work of George Green (§30) and C.F. Gauss. The value of V at a point M is determined from the distribution of electricity by the integral formula

$$V = \iiint \frac{\rho}{r}\, dx\, dy\, dz, \tag{2}$$

where ρ is the volume density of the electric charge at a point M' at a distance r from M. The coordinates of the electric force on a unit positive electric were given by

$$E_x = -\frac{dV}{dx}, \qquad E_y = -\frac{dV}{dy}, \qquad E_z = -\frac{dV}{dz}. \tag{3}$$

In the memoir of 1813, he showed that V satisfied the local differential equation, later called 'Poisson's equation', at every point of the space:

$$\frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{d^2V}{dz^2} = -4\pi\rho. \tag{4}$$

The phenomenon of the action of an electric current on a magnetized body, discovered by Hans Christian Oersted in 1820, attracted the attention of many experts. In the same year, André-Marie Ampère (1775–1836) established by experiment the actions of repulsion and attraction between two conducting wires carrying an electric current, and explained all magnetic actions as deriving from this last phenomenon by supposing magnetic bodies to be composed of molecular electric currents. He coined the terms 'electrostatics' and 'electrodynamics' to distinguish the study of forces on bodies carrying electricity at rest and in motion. Ampère's electrodynamic theory, which finally appeared in a memoir of 1826, was based on a new Newtonian formula expressing the interaction between two elements of current:

$$F = ii'\frac{dl\,dl'}{r^2}\left(\sin\alpha\sin\beta\cos\gamma - \frac{1}{2}\cos\alpha\cos\beta\right), \tag{5}$$

where r was the distance between the elements of current, i and i' the intensities of the currents passing through them, dl and dl' are their lengths, and α, β, γ angles expressing their relative orientation. He also established the equality of the magnetic actions exercised by an electric circuit and a magnetic shell occupying a surface bounded by the circuit [Grattan-Guinness, 1990, 917–968].

In 1831 Michael Faraday (1791–1867) established the existence of the phenomenon of electromagnetic induction, but it was not until 1845 that Franz Neumann gave it a mathematical treatment. Appealing to the electrodynamic theory of Ampère and a qualitative law announced by Emil Lenz in 1834, he established, in the case of two closed circuits in relative motion, an expression for the electromotive force of induction as a function of a 'potential' P. This theory established a connection with the electrodynamic theory, since the electrodynamic force on one of the circuits is obtained by differentiating P with respect to the spatial coordinates [Darrigol, 2000, 45–49, 400–401].

In a memoir published in 1846, Wilhelm Weber attempted to unify all of the electric and magnetic phenomena under a Newtonian formula of interaction between two charged particles. To integrate electrodynamic actions and electromagnetic induction, this formula incorporated the first and second derivatives of their distance with respect to time, so as to give:

$$F = \frac{ee'}{r^2}\left[1 - \frac{1}{C^2}\left(\frac{dr}{dt}\right)^2 + \frac{2r}{C^2}\left(\frac{d^2r}{dt^2}\right)\right], \tag{6}$$

where e and e' were the charges of the two particles, r their distance apart, and C a constant whose importance will appear below. Weber also adopted the hypothesis, proposed by his colleague Gustav Fechner, according to which an electric current is composed of a double flux of positive and negative fluids of equal and opposite rates of flow. Weber was thus able to deduce both Ampère's formula for the interaction between two elements of a circuit and the expression for the electromotive force of induction given by Neumann [Darrigol, 2000, 54–66, 402–405].

3 FARADAY AND THOMSON ON THE NOTION OF FIELD

From 1831 to 1852, Michael Faraday published his 'Experimental researches on electricity and magnetism' in the *Philosophical Transactions of the Royal Society*. These papers contain not only an impressive series of experimental discoveries, but also a collection of heterodox theoretical concepts on the nature of these phenomena expressed in terms of lines of force and fields.

In 1838, in the 11th series of his ‘Experimental researches’, Faraday explicitly rejected the idea of an electrostatic action exercised directly at a distance and attempted to prove that electric induction is propagated by contiguous particles of the insulating medium around the bodies (the ‘dielectric’). Moreover, the electric charges observed on the surface of the conductors resulted, according to him, not from an accumulation or deficiency of electric fluid, but from the polarization of the dielectric. To substantiate these claims, he showed in particular that the inductive action between the two surfaces of a condenser depends on the nature of the dielectric separating them, a property called its ‘specific inductive capacity’ [Gooding, 1978].

In 1845, Faraday announced the discovery of an effect of magnetism on polarized light, today called the ‘Faraday effect’. A few months later he discovered diamagnetism, and devoted the years that followed to the study of this phenomenon and the development of new theoretical concepts. He disagreed with Weber, for whom diamagnetic bodies possess a polarization opposite to that of paramagnetic bodies. For Faraday, on the other hand, the behaviour of different substances resulted from the *local* tendency of the surrounding space, the ‘field’, to minimize the perturbation introduced by the bodies and their capacity to conduct the lines of forces more or less well than the surrounding medium. According to case, they move towards the ‘strongest’ places (where the lines of force were most dense) or the ‘weakest’ [Gooding, 1981].

While Faraday’s experimental discoveries earned him great admiration from his contemporaries, his theoretical ideas were received more with perplexity than enthusiasm. The notions that he developed drew upon visual descriptions of the state of the space surrounding the bodies, which seemed to be purely qualitative and devoid of the precision necessary for a mathematical treatment.

In 1845, the young Thomson published an article in which he showed that Faraday’s experiments on the inductive capacity of dielectrics were compatible with the mathematical theory of electrostatics constructed by Poisson. He deduced Faraday’s results from the hypothesis of a polarization *at a distance* of the dielectric medium under the influence of the electrified surfaces of the condenser, on the model of reasoning used by Poisson for magnetism. However, in another part of his text, he also asserted that Faraday’s physical ideas ‘may be made the foundation of a mathematical theory’ equivalent to the classical theory. His line of argument appealed to a mathematical ‘analogy’ between the propagation of heat and an electrostatic system, published in 1842, to envisage a mathematical theory of electrostatics modelled on the theory of heat of Joseph Fourier (§26) [Smith and Wise, 1989, 203–236].

In the years that followed, Thomson developed new concepts and mathematical methods that converged with the theoretical notions of Faraday. First he interpreted the ponderomotive force on an electric or magnetic body as resulting from the tendency of the system to minimize its ‘mechanical effect’ (or ‘potential energy’). Then he showed, using Green’s theorem (§30.3), that the potential energy of an electrostatic or magnetic system can be regarded as being distributed over the whole space rather than confined to the surface of the bodies. Finally, he developed a programme for reducing electric and magnetic phenomena to the mechanical state of the aether in the wave theory of light, especially by means of numerous dynamical analogies and illustrations. In 1856, he even asserted that the Faraday effect proved that magnetism resulted from rotational motions of the aether with axes coinciding with the lines of magnetic force [Smith and Wise, 1989, 237–281, 402–412].

4 MAXWELL AND THE THEORETICAL REFORM OF ELECTROMAGNETISM

Maxwell began his researches on electromagnetism following the completion of his studies at Cambridge in 1854. They were aimed at constructing, at a theoretical level, a unified mathematical theory of electric and magnetic phenomena that would express the methods and ideas of Faraday as an alternative to the theory of Weber. This programme was announced in his first article, 'On Faraday's lines of force', in 1856 [Maxwell *Papers*, vol. 1, 155–229] and continued in two other major texts, 'On physical lines of force' ('PL') in 1861–1862 [*ibidem,* 451–513] and 'A dynamical theory of the electromagnetic field' ('DT') in 1865 [*ibidem,* 526–597]. According to a famous passage in its preface, the *Treatise* (1873) represented the outcome of this programme.

The reference to Faraday in Maxwell's work has often masked the role played there by the texts of Thomson, and above all the search for the continuity with the mathematical theories of Poisson, Ampère and Neumann. Rather than a 'mathematical translation' of Faraday's texts, Maxwell's theoretical programme comprised a reform of those classical mathematical theories within the theoretical framework constructed by Thomson in the course of the previous decade. Maxwell's originality vis à vis Thomson lay in the systematic implementation of this programme, extending it to electrodynamic phenomena and introducing into the mathematical theory notions of Faraday not used by Thomson, notably the duality of quantity and intensity and the electrotonic state.

Maxwell's publications were pervaded by a tension between the problem of treating new analytic expressions as empirically founded and that of associating them with descriptions of the physical process whereby the medium is supposed to propagate electric and magnetic actions. In 1856, he used an analogy between systems of attraction following the inverse-square law and the motion of an incompressible fluid to introduce new mathematical structures, explicitly avoiding the presentation of a 'physical theory' of the phenomena. In 1861, on the other hand, he presented a hypothetico-deductive argument exhibiting a medium composed of molecular vortices as a possible cause of the phenomena. In 1865 he took a middle course in describing a theory based on a set of eight 'general field equations', which were at the same time introduced as 'deduced from experimental facts' and associated with 'dynamical illustrations'.

Moreover, the article of 1856 contained some problems and gaps that later publications attempted to resolve. On the one hand, he gave no account of the connection between the electrostatic and electrodynamic theories since the treatment of the latter was limited to the magnetic effects of closed circuits. On the other hand, the hypothesis of an electromagnetic medium present in a 'so-called vacuum' raised the problem of its co-existence with the aether in the wave theory of light.

The third Part of PL contains a solution of both of these problems. Maxwell first assumed the existence of a new form of electric current consisting of a variation in the electric polarization (or displacement) in a dielectric and deduced a law of magnetic effects equally applicable to both open and closed circuits. He then gave an argument showing that his electromagnetic medium coincides with the aether in the wave theory of light. He made particular appeal to the close agreement between experimental measurements of the speed of light and a quantity v equal to the ratio $C/\sqrt{2}$, where C is the constant appearing in Weber's formula (6).

In 1865, Maxwell expressed these two solutions in a new form. First, a combination of two of the general field equations implied a new representation of the magnetic effects of closed circuits, according to which the current of conduction in a conducting wire is extended by a current of displacement in the dielectric to form a closed 'total current'. Then he obtained a wave equation from certain field equations from which he again deduced a speed of propagation equal to v. He thus concluded not only that 'light and magnetism are affections of the same substance' but also that 'light is an electromagnetic disturbance propagated through the field according to the electromagnetic laws'. With this last assertion was born the 'electromagnetic theory of light' properly speaking [Siegel, 1991].

5 THE PUBLICATION, FUNCTIONS AND STRUCTURE OF THE *TREATISE*

Around 1860, the rapid development of the telegraphic industry in Britain created an increasing need for knowledge, both theoretical and experimental, of electricity. This is attested by the central role of Thomson in the installation of the submarine telegraphic cable between Britain and the United States, which was completed in 1866 [Smith and Wise, 1989, 649–683]. In 1861, the BAAS created, on Thomson's initiative, a committee charged with defining a standard of electric resistance for use in industry, which Maxwell joined in 1862. From this work he gained the perspective of a precise experimental measurement of v, expressing the ratio of the electromagnetic and electrostatic units of electricity, thereby justifying its conjectured equality with the speed of light [Schaffer, 1992, 1995].

From the middle of the 1860s, several British universities began teaching this new knowledge and scientific practice, creating chairs of experimental physics and associated teaching laboratories [Gooday, 1990]. These innovations also affected the University of Cambridge. In July 1867, it was decided to remodel the MT with a particular view to including the study of electricity, magnetism and heat. Thanks to a donation by the seventh Duke of Devonshire, the Chancellor of the University and a relative of Henry Cavendish, it was decided in 1870 to create a new chair of experimental physics and the now famous Cavendish Laboratory ('CL') [Sviedrys, 1970].

Maxwell played an important part in the reforms at Cambridge. Between 1866 and 1873, he was five times an examiner for the MT and the chief setter of questions on the new subjects. His nomination in March 1871 for the new chair of experimental physics and the directorship of the CL show that he was regarded as the principal British scientific expert on these subjects after Thomson, and also one of the chief architects of the current reforms in Cambridge [Harman, 1995, 33–37].

The publication of the *Treatise on electricity and magnetism* in 1873 was a direct result of these reforms. Maxwell announced his project in 1867, only a few months after the announcement of the reform of the MT, and the book was published in March 1873, just two months after the first session under the new regime. The publication of an advanced work of reference on the subject was an essential ingredient of the success of the reform at Cambridge [Achard, 1998]. The book was also closely connected with the *Treatise on natural philosophy* ('TNP') by Thomson and Tait (§40). Both books were published by the Clarendon Press ('CP'), publishers to the University of Oxford; the arrangement allowed Thomson and Tait to set electromagnetism aside. Throughout the preparation of his work, Maxwell kept up a correspondence with the professors at Glasgow and Edinburgh [Harman, 1995, 24–33].

This context explains at least two functions of the *Treatise*: to describe the chief instruments and methods of measurement of the phenomena, for the benefit of experimenters and engineers; and to give an account of the sophisticated techniques for the mathematical treatment of electricity and magnetism, mainly for the students of the MT. To these must be added a third function, more familiar to us because of the reference to Faraday in the preface: to promote Maxwell's own theoretical ideas, which were still little known, even in Britain. The contents of his book are summarized in Table 1.

This situation leads us to wonder how Maxwell tried hard to reconcile such different aims in his book. It has recently been shown that students and teachers preparing for the MT could study certain chapters of the treatise without first assimilating the theory of the electromagnetic field. The same was true for engineers and experimenters interested in the techniques of electric and magnetic measurement [Warwick, 2003, 286–317]. As indicated by Maxwell in his preface, the chapters dealing with these various 'numerical' and experimental aspects are placed at the end of each of the four parts of the *Treatise*.

Paradoxically enough, Maxwell's theoretical innovations were mainly concerned with the early chapters, on the 'elementary parts of the theory'. But in conformity with the style of his earlier theoretical papers, he introduced the new ideas progressively, without disrupting the exposition of the main results of the classical mathematical theory.

6 MATHEMATICAL STRUCTURES IN THE *TREATISE*

The title of the preliminary chapter applies only to the first six articles, which are devoted to the dimensional theory of physical quantities in an 'absolute' system of units based on the unity of length, time and mass. The rest of the chapter described ideas and mathematical results regarded by Maxwell as representative of his theory of the field.

In his earlier papers, Maxwell made frequent use of vector functions expressing properties of the electromagnetic field. But he expressed their relations in terms of Cartesian coordinates, showing no predilection for the theory of quaternions as studied and developed by Tait since 1857 (§35.4). In November 1870, during his second spell of work on the book, he declared to Tait his intention to 'leaven [his] book with Hamiltonian ideas'. In the following year, he published an article on the 'mathematical classification of the physical quantities', which contained the essentials of his preliminary chapter.

In the *Treatise*, Maxwell distinguished the 'ideas' favoured by the quaternions from their 'operations and methods' (art. 10). For him, they provided a 'primitive and natural' means of expressing the relations between vectorial entities without recourse to Cartesian axes. Thus he insisted on the distinction between scalars and vectors and reserved a special type of symbol for the latter (the 'German letters'). On the other hand, he totally ignored the affiliation between quaternions and complex numbers.

Table 1. Contents by chapters of Maxwell's book.
The second volume starts with Part III. The titles are those placed at the heads of the chapters; sometimes they differ from those in the table of contents (1st edition, and 2nd edition 1st volume). The numbers given are those of the first articles. In the 2nd edition (1881), discussed in section 11 below, I-1 and I-2 contain some alterations; the chapters I-3, I-4, I-5 and I-9 were entirely rewritten.

Chapter(s)	**Subject or/and 'Title(s)' (first art.)**
Preliminary	'On the measurement of quantities' (1).
I	*Electrostatics*
I-1	'Description of phenomena' (27).
I-2 to I-4	Mathematical theory of electrostatics: 'Elementary mathematical theory of statical electricity' (63), 'Systems of conductors'[1] (84), 'General theorems' (95).
I-5	'Mechanical action between electrified bodies'[2] (103).
I-6 to I-8	Geometrical descriptions of the electrostatic field: 'On points and lines of equilibrium' (112), 'Forms of the equipotential surfaces and lines of induction in simple cases' (117) 'Simple cases of electrification' (124).
I-9 to I-12	Analytical procedures: 'Spherical harmonics' (128), 'Confocal quadric surfaces' (147), 'Theory of electric images and electric inversion' (155), 'Theory of conjugate functions in two dimensions' (182).
I-13	'Electrostatic instruments' (207).
II	*Electrokinematics*
II-1 to II-3	Fundamental phenomena and laws: 'The electric current' (230), 'Conduction and resistance' (241), 'Electromotive force between bodies in contact' (246).
II-4 & II-5	Electrolysis: 'Electrolysis' (255), 'Electrolytic polarization' (264).
II-6 to II-8	Mathematical theory of conduction: 'Linear electric currents' (273), 'Conduction in three dimensions' (285), 'Resistance and conductivity in three dimensions' (297).
II-9 & II-10	'Conduction through heterogeneous media' (310), 'Conduction in dielectrics' (325).
II-11 & II-12	Measurements of electric resistance: 'The measurement of electric resistance' (335), 'On the electric resistance of substances' (359).
III	*Magnetism*
III-1	'Elementary theory of magnetism' (371).
III-2 & III-3	Magnetic notions: 'Magnetic force and magnetic induction' (395), 'Magnetic solenoids and shells' (407).
III-4 to III-6	Magnetic induction: 'Induced magnetization' (424), 'Particular problems in magnetic induction' (431), 'Weber's theory of induced magnetism' (442).
III-7 & III-8	Magnetic observations: 'Magnetic measurements' (449), 'On terrestrial magnetism' (465).

Table 1. (*Continued*)

Chapter(s)	**Subject or/and 'Title(s)' (first art.)**
IV	*Electromagnetism*
IV-1 to IV-4	Electromagnetic phenomena: 'Electromagnetic force' (475), 'Ampère's investigation of the mutual action of electric currents' (502), 'On the induction of electric currents' (528), 'On the induction of a current on itself' (546).
IV-5 to IV-8	Dynamical theory of the electromagnetism: 'On the equations of motion of a connected system' (553), 'Dynamical theory of electromagnetism' (568), 'Theory of electric circuits' (578), 'Exploration of the field by means of the secondary circuit' (585).
IV-9 & IV-11	Fundamental equations: 'General equations of the electromagnetic field' (604), 'Dimensions of electric units' (620), 'On energy and stress in the electromagnetic field' (630).
IV-12 to IV-14	Particular cases: 'Current-sheets' (647), 'Parallel currents' (682), 'Circular currents' (694).
IV-15 to IV-19	Electromagnetic instruments and measurements: 'Electromagnetic instruments' (707), 'Electromagnetic observations' (730), 'Comparison of coils' (752), 'Electromagnetic unit of resistance' (758), 'Comparison of the electrostatic with the electromagnetic units' (768).
IV-20 & IV-21	Electromagnetism and light: 'Electromagnetic theory of light' (781), 'Magnetic action on light' (806).
IV-22 & IV-23	Continental theories of the electromagnetism: 'Ferromagnetism and diamagnetism explained by molecular currents' (832), 'Theories of action at a distance' (846).

[1] 2nd ed.: 'On electrical work and energy in a system of conductors'.

[2] 2nd ed.: 'Mechanical action between two electrical systems'.

Maxwell's usage of quaternions in the text reflects this attitude. Such expressions appear occasionally, usually at the end of an article, to express an important formula initially derived in Cartesian form. More rarely, he indicated the possibility of condensing a long argument by the use of quaternions (for example, in art. 522).

Table 2 shows the operations employed by Maxwell. He never wrote a full quaternion, formed as the sum of a scalar part and a vector part; thus he was already very close to the modern usage in vector analysis, which was introduced later by two readers of the *Treatise*, J.W. Gibbs and Oliver Heaviside ([Crowe, 1967, 127–139]; and compare §35.5 and §49).

Another advantage of the quaternions lay in the fact that they could be used to introduce the operator ∇, whose usefulness in mathematical physics had been demonstrated by Tait. This operator is formally defined as follows (art. 17):

$$\nabla = i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}, \tag{7}$$

Table 2. Operations of the quaternion calculus used by Maxwell in the *Treatise*.
α and β are vectors, k is a scalar.

Nature of the operation	**Quaternion notation**	**Vectorial analysis equivalent**
Sum of two vectors.	$\alpha + \beta$	$\alpha + \beta$
Product of a scalar and a vector.	$k\alpha$	$k\alpha$
Scalar part of the product of two vectors.	$S \cdot \alpha\beta$	$-\alpha \cdot \beta$
Vectorial part of the product of two vectors.	$V \cdot \alpha\beta$	$\alpha \times \beta$

Table 3. Operations on scalar and vectorial functions used by Maxwell.
Ψ is a scalar function, σ is a vectorial function.

Name of the operation	**Quaternion notation**	**Vectorial analysis equivalent**
'Slope' of Ψ.	$\nabla\Psi$	$\nabla\Psi$
'Convergence' of σ.	$S \cdot \nabla\sigma$	$-\nabla \cdot \sigma$
'Curl' of σ.	$V \cdot \nabla\sigma$	$\nabla \times \sigma$
'Concentration' of Ψ.	$\nabla^2\Psi$	$-\nabla^2\Psi$

where i, j and k were unit vectors along the three axes of Cartesian coordinates. It can thus be manipulated like a vector. Maxwell went on to list some operations involving ∇ on a scalar or vector function, giving them names that reflect an appropriate geometric property (Table 3).

In what follows, we shall use the notation of modern vector analysis, along with that depicted in Tables 4 and 5 in section 9 below, which summarizes Maxwell's usage in Part IV, ch. 9 of letters to identify the principal equations of the field. The main entities of the field and these equations are listed in Tables 4 and 5, but I shall use them from now on.

Maxwell also made a distinction between two types of 'physical' vector quantities, 'forces' (called 'intensities' in the second edition) and 'flux' (arts. 12–14), constituting a new version of the intensity/quantity duality introduced in 1856. This classification, derived from hydrodynamics, distinguished vector functions respectively expressing a motion of material (the flux through a unit of surface) and the tension that caused it (the gradient of pressure). In electromagnetic theory, it expressed the distinction between, on the one hand, the electric and magnetic forces prevailing at a point of the space and, on the other hand, the electric and magnetic polarizations of the medium or current that they generate at that point. The local numerical relation between a force and the flux it produced depends on the nature of the medium. In the case of an isotropic medium, it is expressed by a relation of vector proportionality (equations (F), (G) and (L)). Finally, Maxwell associated with each type of entity a specific type of mathematical operation. Force was simply integrated along a line to obtain the work effected upon a body. Flux was used in double integrals over a surface to express the quantity that traverses it.

Maxwell went on to describe the properties of line integrals and surface integrals (arts. 16–24). In this last section, he announced two mathematical results that had played a central role in the development of his theory since 1856, especially in the expression of integral laws in the form of local equations, namely, Theorems III and IV (arts. 21 and 24), today called 'Ostrogradsky's theorem' and 'Stokes's theorem' respectively. They form a part of a long sequence of results on multiple integrals going back to the beginning of the century [Cross, 1985]. Reverting to the notation used by Tait, Maxwell also expressed these results in terms of quaternions (art. 25).

7 ELECTROSTATICS AND ELECTROKINETICS

The first chapter in the part on electrostatics contained an empirical introduction to the fundamental concepts and laws of the theory. Maxwell introduced the notion of electricity as a measurable physical quantity (arts. 27–34), independent of the hypothesis of electric fluids (arts. 35–37). He stated Coulomb's law in the same way (arts. 38–43) and defined the key entities of electrostatics (arts. 44–50). At the end of the chapter he described the plan of the chapters that follow, along with his 'theory of electric polarization' on the nature of charge and electric current as an alternative to the theory of fluids (arts. 59–62).

In the second chapter the classical electrostatic results were derived from Coulomb's law, introducing occasionally some of the ideas of field theory. Thus Maxwell defined the notion of electric displacement as 'the quantity of electricity which is forced in the direction of [the electromotive force] across a unit of area', and wrote down equation (F) (art. 68). He also gave Poisson's equation in generalized form:

$$\frac{d}{dx} \cdot K \frac{dV}{dx} + \frac{d}{dy} \cdot K \frac{dV}{dy} + \frac{d}{dz} \cdot K \frac{dV}{dz} + 4\pi\rho = 0, \tag{8}$$

incorporating the coefficient K of capacity of the medium derived from experiments of Faraday (art. 83). This equation is equivalent to equation (J) if one assumes equations (F) and (3).

The fourth chapter was chiefly devoted to two theorems, attributed to Green and Thomson respectively, which Maxwell interpreted physically in accordance with the field theory. He also stated them in the generalized form of equation (8). In the fifth chapter, he set out to *explain* the interactions between electrified fields by a distribution of stress from the surrounding medium rather than by a direct action at a distance and to emphase his agreement with the writings of Faraday (arts. 105–109). Finally, he admitted to not having taken the next step in the search for a physical explanation of the phenomena: to account for this distribution of stress in the medium in terms of 'mechanical considerations' (arts. 110–111).

The 'theory of electric polarization' described at the end of the first and fifth chapters (arts. 60–62, 111) asserted, in conformity with ideas put forward by Faraday, that electric charge manifests the discontinuity between the polarized state of a dielectric and the non-polarized state of a conductor. It also stated that 'the motions of electricity are like those of an incompressible fluid'. This means that electric current always forms closed curves in accordance with the equation

$$\nabla \cdot \mathbf{C} = 0 \tag{9}$$

(which follows from equation (E)). Moreover, Maxwell emphasized that the 'total' electric current is composed of the 'ordinary' current of conduction, which predominates in conductors, and that the variation of electric displacement, which predominates in dielectrics (whence equation (H)). Finally, he stated that the passage of electricity through a medium generated a state of constraint that loosens and reforms at a frequency more or less rapid depending on the nature of the medium. It is this discontinuity of states of constraint at the surface separating two media that is manifest in the electric charge [Buchwald, 1985, 20–40].

The second Part, on 'electrokinematics', was essentially an exposition of the classical mathematical theory of electric current. Maxwell treated electric current as a phenomenon that is empirically observed and quantitatively measured by a galvanometer (art. 240), without any consideration of its physical nature, and mentions Ohm's law (which corresponds locally to equation (G)) and the Joule effect (arts. 241–242). In Chapter 10, the expression (H) of the total current in a dielectric, as consisting of a current of conduction and a variation of displacement, was used to give an account of the phenomenon of electric absorption (arts. 328–334).

8 MAGNETISM AND ELECTROMAGNETISM

The third Part was taken up mainly with a description of the classical theory of magnetism. The early chapters nevertheless contained an introduction to certain ideas and results belonging to field theory, chiefly in the appearance of 'magnetic induction' as a complement of the classical notion of 'magnetic force'. Following Thomson, Maxwell first defined magnetic induction as the magnetic force exercised on a unit magnetic pole lying in an infinitesimal circular cavity. In this case, magnetic induction was expressed in terms of magnetic force by equation (D) (art. 399). He thus deduced, in particular, that the integral of the surface of magnetic induction over a closed surface is always equal to zero (art. 402). Thus

$$\nabla \cdot \mathbf{B} = 0. \tag{10}$$

He also defined a vector $\mathbf{U}$, called the 'potential vector' of magnetic induction, such that, for any surface (S) bounded by a closed curve (C):

$$\iint_{(S)} \mathbf{B} \cdot \mathbf{dS} = \oint_{(C)} \mathbf{U} \cdot \mathbf{dl}. \tag{11}$$

He then used Stokes's theorem to deduce the local equation (A) (art. 405). In the fourth chapter, Maxwell suggested that, according to 'Faraday's method', magnetic induction represents the polarization of a medium under the action of a magnetic force and is expressed, in an isotropic medium, by equation (L) (art. 428).

In the early chapters of the fourth part Maxwell tackled in turn the magnetic effects of electric currents and the phenomenon of induction. But he specifically emphasized the contrast between the second chapter, on the theory of Ampère, and the first and third chapters, which were dominated by 'Faraday's method', that is, the field theory (arts. 493, 502, 528–529). While Ampère's theory proceeded from an initial decomposition of the system, the action on a circuit being calculated as the sum of the actions of each element of current, Faraday's method began with the system taken as a whole, the electromagnetic laws being expressed in terms of properties of the global field. This contrast between mathematical methods reflected that between the physical hypotheses.

In the first chapter, the equivalence of the magnetic effects of an electric circuit and a magnetic sheet enabled Maxwell to express the action on an electric circuit placed in an arbitrary magnetic field (arts. 489–492). At the end of the chapter, he stated a law bearing on the distribution of the magnetic force as a function of that of the electric currents: 'the line-integral of the magnetic force' round a closed curve (C) is equal to $4\pi i$, where i denoted the electric current that flows through an arbitrary surface (S) bounded by (C) (arts. 498–499). That is,

$$\oint_{(C)} \mathbf{H} \cdot \mathbf{dl} = 4\pi i. \tag{12}$$

This law was expressed locally by equation (E). Maxwell stated also that the electric current considered here is made up of both a variation of electric displacement and a current of conduction, so that **C** satisfies equation (H) already used in the second part.

In the third chapter, Maxwell appealed to the work of Faraday and to a series of experiments devised by the Italian physicist Riccardo Felici to state the law of electromagnetic induction: 'the total electromotive force acting around a circuit at any instant is measured by the rate of decrease of the number of lines of magnetic force which pass through it' (arts. 536–541). This last notion was also called 'the magnetic induction through the circuit'. This yields the analytic form of the law of induction,

$$e = -\frac{d}{dt}\left(\iint_{(S)} \mathbf{B} \cdot \mathbf{dS}\right), \tag{13}$$

where (S) was a surface bounded by a closed circuit (C). Although Maxwell attributed the discovery of this law of induction to Faraday, he mentions briefly its 'convergence' with the mathematical theories of induction developed by Neumann, then by Helmholtz, Thomson and Weber (arts. 542–545).

9 THE DYNAMICAL THEORY OF ELECTROKINETIC PHENOMENA, AND THE GENERAL EQUATIONS OF THE ELECTROMAGNETIC FIELD

The fourth chapter of Part IV contains a consideration of the similarities and differences between the phenomenon of self-induction of an electric current and the inertia (or 'momentum') of a fluid in motion in a tube. Maxwell concluded by proposing to deduce the principal structure of the theory of electricity from a dynamical hypothesis that stated that the phenomena were produced by a connected system in motion lying both in the surrounding space and in the conducting bodies (art. 552). According to him, the dynamical theory of Lagrange (§16) made it possible to avoid any more detailed hypothesis on the nature of the motions of this system.

The fifth chapter described the fundamental relations of the Lagrangian theory adapted to the needs of dynamical arguments by Hamilton, then by Thomson and Tait (§40). In the sixth chapter, the state of a system of electric circuits was expressed in terms of variables of the following two types (arts. 568–570). Firstly, the ordinary 'mechanical' variables (x_i) described the form and relative position of the circuits. Secondly, the 'electric' variables (y_i) expressed the position of the electricity in motion in the circuits, and their derivatives ($\dot{y}_i$) with respect to time gave the intensities of the electric current. The kinetic energy of the system is then the sum of three quadratic functions (art. 571): the ordinary kinetic energy T_m, which depends only on the motions of the circuits,

$$T_m = \frac{1}{2}\sum_i A_i \dot{x}_i^2 + \sum_{i<j} B_{ij}\dot{x}_i\dot{x}_j; \tag{14}$$

the electrokinetic energy T_e, which depends only on the electric currents in the circuits,

$$T_e = \frac{1}{2}\sum_i L_i \dot{y}_i^2 + \sum_{i<j} M_{ij}\dot{y}_i\dot{y}_j; \tag{15}$$

and a third term T_{me} which depended upon the products of one mechanical variable and one electric variable,

$$T_{me} = \sum_{i,j} C_{ij}\dot{x}_i\dot{y}_j. \tag{16}$$

Maxwell also showed that the coefficients depended only on the mechanical variables (art. 572). Finally, a series of experiments allowed that the term T_{me} be regarded as negligible (arts. 574–577).

In subsequent chapters, Maxwell considered only phenomena depending on the electrokinetic energy T_e. He also defined the concept of the *electrokinetic momentum p_i associated with each circuit* (A_i) by setting

$$p_i = \frac{dT_e}{d\dot{y}_i} = L_i\dot{y}_i + \sum_{k\neq i} M_{ki}\dot{y}_k. \tag{17}$$

Using formulae from the fifth chapter, he then expressed the external forces applied to the system by differentiating the electrokinetic energy (arts. 579–580) and applied these results in the case of a system compound of two circuits (arts. 581–584). By differentiating with respect to y_i, Maxwell obtained an expression for the electromotive force Y_i' applied to a circuit (A_i) and not compensated by the resistance of the circuit (art. 579). That is,

$$Y_i' = \frac{d}{dt}\left(\frac{dT_e}{d\dot{y}_i}\right) - \frac{dT_e}{dy_i} = \frac{dp_i}{dt} \tag{18}$$

with

$$Y_i' = E_i - R_i\dot{y}_i, \tag{19}$$

where E_i is the external electromotive force and R_i the resistance of (A_i). According to Maxwell, the internal electromotive force e_i of induction corresponds to the opposite of this expression. Thus, in the case of two circuits, the electromotive force of induction in the circuit (A_2) is given by

$$e_2 = -\frac{dp_2}{dt} = -\frac{d}{dt}(M\dot{y}_1 + L_2\dot{y}_2). \tag{20}$$

Likewise, the external mechanical forces applied to the circuits were obtained by differentiating T_e with respect to the mechanical variables x_i (art. 580):

$$X'_j = \frac{d}{dt}\frac{dT_e}{d\dot{x}_j} - \frac{dT_e}{dx_j} = -\frac{dT_e}{dx_j}. \tag{21}$$

The electrodynamic force X_j on a circuit appeared as the opposite of this expression, and in the case of two rigid circuits he obtained the formula

$$X_j = \dot{y}_1\dot{y}_2\frac{dM_{12}}{dx_j}. \tag{22}$$

In the eighth chapter, Maxwell used the electrokinetic moment p of a circuit (C) to define two vector functions **B** and **U**, *the electrokinetic momentum at a point*, by the equations

$$p = \oint_{(C)} \mathbf{U}\cdot\mathbf{dl} = \iint_{(S)} \mathbf{B}\cdot\mathbf{dS}. \tag{23}$$

He immediately identified these new vectorial entities with the notions of magnetic induction and vector-potential respectively, which were introduced in Part III and were related by the local equation (A) (art. 592). Maxwell finally derived local expressions for the electromotive force of induction and the mechanical force as functions of **U** and **B**, and thus obtained equation (B) for the electromotive force in a conductor moving with a speed **v** and equation (C) for the mechanical force on a conductor traversed by a current of density **C** (arts. 595–603).

The ninth chapter covered the 'general equations of the electromagnetic field', already introduced in earlier parts of the treatise. Without any recourse to the earlier dynamical reasoning, Maxwell then stated equations from (D) to (L) in turn (arts. 605–614). Finally, he obtained an expression for the vector-potential as a function of the distribution of the electric currents according to Ampère's theory of magnetism and under the following condition on **U**, today called a 'gauge condition' (arts. 615–617):

$$\nabla\cdot\mathbf{U} = 0. \tag{24}$$

The principal entities and field equations collected at the end of the chapter are shown in Tables 4 and 5.

Chapter 11 contained expressions for the three types of energy: electrostatic, magnetic and electrokinetic (arts. 630–636). Appealing to the theory of Ampère, Maxwell proposed

Table 4. The principal quantities of the electromagnetic field (art. 618). Vectors are denoted by bold letters, instead of the German letters used by Maxwell.

Vectors	
Electromagnetic momentum at a point (or potential-vector).	$\mathbf{U}\ (F, G, H)$
Magnetic induction.	$\mathbf{B}\ (a, b, c)$
Total electric current.	$\mathbf{C}(u, v, w)$
Electric displacement.	$\mathbf{D}\ (f, g, h)$
Electromotive force.	$\mathbf{E}\ (P, Q, R)$
Mechanical force.	$\mathbf{F}\ (X, Y, Z)$
Velocity of a point.	$\mathbf{v}\ (\dot{x}, \dot{y}, \dot{z})$
Magnetic force.	$\mathbf{H}\ (\alpha, \beta, \gamma)$
Intensity of magnetization.	$\mathbf{I}\ (A, B, C)$
Current of conduction.	$\mathbf{R}\ (p, q, q)$
Scalars	
Electric potential.	Ψ
Magnetic potential.	Ω
Electric density.	e
Density of magnetic ‘matter’.	m
Physical properties of the medium (isotropic media)	
Conductivity for electric currents.	C
Dielectric inductive capacity.	K
Magnetic inductive capacity.	μ

to regard the energy of the field as divided over the whole space into two fundamental forms (arts. 637–638): electrostatic (potential) energy,

$$W = \frac{1}{2} \iiint (\mathbf{D} \cdot \mathbf{E})\, d\tau; \tag{25}$$

and electrokinetic (kinetic) energy,

$$T = \frac{1}{8\pi} \iiint (\mathbf{B} \cdot \mathbf{H})\, d\tau. \tag{26}$$

The chapter ended with a consideration of the possibility of explaining magnetic action by a state of stress in the surrounding medium, in line with the corresponding study of electrostatic action in part I, chapter 5 (arts. 639–646).

10 THE ELECTROMAGNETIC THEORY OF LIGHT

As in 1865, the main argument in favour of the electromagnetic theory of light lay in deriving from the general field equations (A), (B), (E), (F), (G), (H) and (L) an expression which reduced, in the case of a dielectric medium, to the following wave equation for the vector potential **U** (arts. 783–784):

Table 5. The general equations of the electromagnetic field (art. 619).

Name of the equation (when given)	Expression in vectorial analysis	
Equation of magnetic induction.	$\mathbf{B} = \nabla \times \mathbf{U}$	(A)
Equation of electromotive force.	$\mathbf{E} = \mathbf{v} \times \mathbf{B} - \dot{\mathbf{U}} - \nabla\Psi$	(B)
Equation of mechanical force.	$\mathbf{F} = \mathbf{C} \times \mathbf{B} - e\nabla\Psi - m\nabla\Omega$	(C)[1]
Equation of magnetization.	$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{I}$	(D)
Equation of electric currents.	$4\pi\mathbf{C} = \nabla \times \mathbf{H}$	(E)
Equation of electric displacement.	$\mathbf{D} = \frac{1}{4\pi} K\mathbf{E}$	(F)
Equation of the current of conduction.	$\mathbf{R} = C\mathbf{E}$	(G)
Equation of the total current.	$\mathbf{C} = \mathbf{R} + \dot{\mathbf{D}}$	(H)
	$e = \nabla \cdot \mathbf{D}$	(J)[2]
Equation of induced magnetization.	$\mathbf{B} = \mu\mathbf{H}$	(L)
	$m = -\nabla \cdot \mathbf{I}$	
	$\mathbf{H} = -\nabla\Omega$	

[1]In the third edition, this equation was rewritten

$$\mathbf{F} = \mathbf{C} \times \mathbf{B} + e\mathbf{E} - m\nabla\Omega$$

following a correction proposed by G.F. FitzGerald in 1883.
[2]This expression is consistent with the Cartesian expression given in art. 612 and with similar ones used elsewhere in the *Treatise* (e.g. art. 82); but it is the opposite of the quaternion expression given in art. 619. The consistent quaternion expression is $e = -S \cdot \nabla\mathbf{D}$.

$$K\mu \frac{d^2\mathbf{U}}{dt^2} - \nabla^2\mathbf{U} = \mathbf{0}. \tag{27}$$

Maxwell thereby deduced the existence of 'electromagnetic disturbances' whose speed of propagation was given by

$$V = \frac{1}{\sqrt{K\mu}}. \tag{28}$$

In the case of air, he showed that this number coincided with v, 'the number of electrostatics units of electricity in one electromagnetic unit'.

To establish that 'light is an electromagnetic disturbance', Maxwell compared the various experimental measurements of v, given in the preceding chapter, with those of the speed V_L of light (Table 6). He concluded that his theory, which implied that these two quantities were equal, 'is certainly not contradicted by the comparison of these results such as they are' (arts. 786–787).

As in 1865, Maxwell claimed that light is a special kind of electromagnetic perturbation, whose law of propagation is expressed by (27). This reasoning allowed him to justify a strong thesis with minimal hypotheses, since it rests solely on the equality of v and V_L, along with an acceptance of the general equations of the electromagnetic field. Although,

Table 6. Comparison of the ratio of electric units with the velocity of light (art. 787).

Velocity of light (metres/second)		**Ratio of electric units (metres/second)**	
Fizeau	314 000 000	Weber	310 740 000
Aberration, &c	308 000 000	Maxwell	288 000 000
Foucault	298 360 000	Thomson	282 000 000

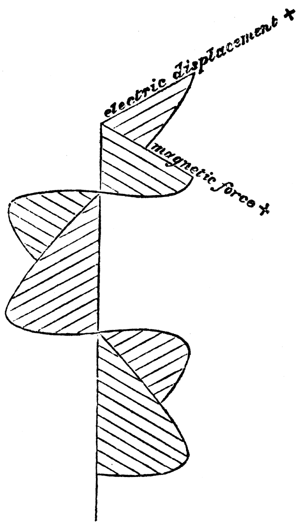

Figure 1. Image of an electromagnetic wave (art. 791). It represents 'the values of the magnetic force and of the electromotive force at a given instant in different points of the ray [...] for the case of a simple harmonic disturbance in one plane'. The 'magnetic and electric disturbances' are transverse to the direction of propagation and perpendicular to each other. 'This corresponds to a ray of plane-polarized light'.

for Maxwell as for his contemporaries, the idea of propagation of 'electromagnetic disturbances' required the acceptance of the hypothesis of a material support, he made no further assumptions about its configuration.

The chapter ended by suggesting various paths that experimental research might take in order to confirm the electromagnetic nature of light by studying the correlation between the optical and electric properties of various substances, such as relationships between the refractive index and inductive capacity of transparent dielectric media (arts. 788–789), the conductivity and opacity of media (arts. 798–805), and also the optical and electric properties of crystalline media (arts. 794–797). Maxwell also presented a study of plane waves and polarized light (arts. 790–791). He showed that, in the case of a plane, the electric and magnetic forces were perpendicular to the direction of propagation of the wave, agreeing with the hypothesis that light waves vibrate transversely (see Figure 1).

Maxwell did not discuss how electromagnetic perturbations might be produced by an electric device, suggesting that he saw no easy way of doing this. Moreover, by choosing in his *Treatise* to derive a wave equation involving the 'electrokinetic moment' **U** rather than the magnetic force **H** as in DT, he seems to be exercising a preference for this entity. This choice introduced a weakness into his reasoning, since to obtain equation (27) he had to assume that Poisson's equation can also be invoked in the non-stationary case of a phenomenon of propagation (art. 783). As remarked by G.F. FitzGerald (1851–1901) in 1890, this involved the assumption that electrostatic potential propagated itself instantaneously. It then turned out that this choice was closely connected with the choice of a 'gauge condition' on the vector-potential [Hunt, 1991, 117–118].

In the next chapter, Maxwell treated the phenomenon of magnetic action on light. Inspired by PL, he tried to account for the Faraday effect by supposing that a medium subjected to a magnetic influence was the seat of molecular vortices whose axes of rotation coincide with the lines of magnetic force. By adding a term to the expression for the kinetic energy of the system and using an equation of Lagrange given in the fifth chapter, he arrived at an expression for the angle of rotation of the plane of polarization of the light as a function of various parameters, including the intensity of the magnetism and the length of the ray of light passing through the medium (arts. 822–831). As Maxwell was to remark in 1879, this line of reasoning constituted a 'hybrid' theory, since it explains the magneto-optical phenomenon by supposing that light and magnetism comprise motions of the aether that interact according to mechanical laws rather than by directly invoking the electromagnetic theory of light.

11 THE MAXWELLIANS

Maxwell published no theoretical articles on electromagnetism after 1873. But in 1872 the CP suggested that he write a less mathematical work on electricity and magnetism based on his great treatise, in the style of Thomson and Tait in their *Elements of natural philosophy*. This task, begun around 1874, remained unfinished at the time of his death in November 1879. As well as this, the CP advised him in 1877 that, thanks to the popularity of the *Treatise*, it was already time to start work on a second edition. Table 1 above indicates the changes that he envisaged at that time. Both works were finally edited in 1881 by two of Maxwell's collaborators at Cambridge, W.D. Niven and William Garnett. The unfinished manuscript of the *Elementary treatise on electricity* was completed using sections from the great *Treatise* to cover the material in the first volume (see the publication history at the head of this article).

Maxwell's presence at Cambridge did not, however, immediately result in the formation of a school of research in electromagnetism based on his ideas. Neither his lectures on electricity and magnetism, aimed at students of the MT and the Natural Science Tripos, nor his experimental research conducted at the CL, continuing the metrological work begun by the committee of the BAAS, were chiefly concerned with his field theory. Some graduates, especially those motivated by theoretical research, progressively assimilated the subtleties of the field by following intercollegiate courses delivered by W.D. Niven [Warwick, 2003, 286–356].

Speaking generally, the *Treatise* probably owed its commercial success in Britain to its value in mathematical and experimental education rather than to a rapid acceptance of its theoretical innovations. As early as 1865, William Thomson had reservations about Maxwell's theory, and publicly opposed it in his Baltimore lectures in 1884 (§58). In his review of the *Treatise*, Tait [1873] remarked on the opposition to the idea of action at a distance and the thesis of a 'connection between radiation and electric phenomena', but he ignored essential features such as the physical theories of electric charge and of the displacement current. This was true also of George Chrystal's review of the second edition [Chrystal, 1882]. Nevertheless, the success of the work certainly contributed to the dissemination of the field theory.

From the end of the 1870s, several dozen 'Maxwellians' began to publish their research on electromagnetism. Many of these came from MT and the CL, notably Joseph John Thomson (1856–1940) and John Henry Poynting (1852–1914), who were both students of Niven. But some of them were related to other British (and American) universities: notably FitzGerald, former student and professor at Trinity College, Dublin; and Oliver Lodge (1851–1940), former student of University College London and professor of physics at Liverpool. The most remarkable case is certainly that of Oliver Heaviside (1850–1925), self-taught by reading works such as the TNP and, above all, Maxwell's *Treatise* (§49).

One of the first extensions of Maxwell's work lay in explaining electric and magnetic phenomenon by a mechanical state of the aether. Some authors, such as Lodge, presented mechanical models to illustrate the production of certain phenomena by the aether [Hunt, 1991, 73–104]. Another direction, particularly favoured by the physicists at Cambridge, consisted of using the Lagrangian theory and the principle of least action to establish a mechanical basis for the phenomena without stating precise hypotheses on the arrangement of the mechanism supposed to produce them [Buchwald, 1985, 54–64].

Although FitzGerald worked on the mechanical models and discussed their merits with Lodge, he also used Lagrangian methods, notably in an influential article published in 1879. A correspondence between Maxwell's theory and the optical theory of MacCullagh led to the incorporation in the electromagnetic theory of light, not only of the phenomena of reflection and refraction of light, but also of the magneto-optical phenomena of Faraday, and of John Kerr discovered in 1876 [Hunt, 1991, 15–23; Buchwald, 1985, 73–129].

From 1879, Lodge sought to confirm experimentally the electromagnetic theory of light by producing light waves by purely electric means. After first asserting the fruitlessness of such an exercise, FitzGerald retracted in 1882 and thought about conditions necessary to produce observable electromagnetic waves. Lodge announced the production of electromagnetic waves in electric wires in 1888, but his triumph was eclipsed by the experiments of Hertz [Hunt, 1991, 24–48, 146–151].

Finally, in 1884, Poynting presented his famous 'theorem'. He deduced from the fundamental equations of field theory a mathematical equality interpreted as expressing that the variation of the energy contained in a given volume per unit time is due to the flux of energy across the surface that bounds it. According to this interpretation, during the passage of current in an electric wire, energy moves, not along the wire, perpendicular to it from the dielectric to the conductor, where it is converted into heat. This interpretation favours Maxwell's ideas on electric current rather than the traditional image of an imponderable fluid in motion [Buchwald, 1985, 41–54; Hunt, 1991, 109–114].

12 THE EXPERIMENTS OF HERTZ AND THEIR IMPACT

Maxwell's treatise was also read in continental Europe. Towards the end of the 1870s, it aroused sufficient interest to warrant translations into German and French, which appeared in 1883 and 1885 respectively. While the treatise disseminated the existence of Maxwell's theoretical ideas, it seems that they did not exercise a strong influence on the majority of continental research into electromagnetism up to the middle of the 1880s. A theoretical article of Helmholtz published in 1870, even before the appearance of the *Treatise*, constitutes the main and most remarkable exception.

Helmholtz presented a theory of electromagnetic action covering the case of open circuits and taking account of the absence of experimental data on this situation. An expression for the vector potential involved a parameter k whose value expressed the various possible options, three different values corresponding to the theories of Neumann, Maxwell and Weber respectively. All the mathematical laws in Maxwell's theory can be recovered by supposing that space is infinitely polarizable [Buchwald, 1985, 177–186].

In July 1879, Helmholtz worked on a problem for a Prize of the Berlin Academy that was explicitly aimed at testing the validity of Maxwell's prediction about open circuits [Darrigol, 2000, 233–234]. From that date, he encouraged a brilliant student, Heinrich Hertz (1857–1894), to seek a solution of these problems. Hertz provisionally abandoned this project, but in 1884 he published a theoretical account affirming the superiority of Maxwell's theory.

In 1886, Hertz began some experimental research that led to an impressive series of articles published between 1887 and 1889. First he described a device capable of producing extremely rapid electric oscillations and detecting the electromotive forces that they generate. He then emphasized the electrodynamic effect generated by the variable polarization of a dielectric. Then he showed that the propagation of this effect in an electric wire or in air generated progressive or stationary waves according to the experimental configuration. In the latter case, the disposition of the sinks and nodes enabled him to show that the speed of propagation in air is close to that of light. He concluded this study by describing the spatial distribution of waves, with emphasis on their reflection and refraction [Buchwald, 1994].

These experiments, soon repeated by many physicists, assured the triumph of Maxwell's theory and its adherents in Britain. They also had the effect of drawing attention to the work of Heaviside [Hunt, 1991, 158–168]. Inspired in part by this work, Hertz proposed a new formulation of Maxwell's theory in 1890. He also studied the implications of field theory in the rest of physics by developing his ideas on the foundations of mechanics (§52).

In Germany, there was a spectacular surge of interest in Maxwell's theory. In the course of the following decade, several German authors put forward accounts of electromagnetic field theory which abandoned the traditional representations of charge and electric current generally in favour of an agnostic attitude [Darrigol, 2000, 253–262].

13 LARMOR AND THE NOTION OF ELECTRON

In 1894, John Joseph Larmor (1857–1942) published a theoretical article on magnetism that formed one of the sources heralding the advent of the modern notion of electron (see §60 on Lorentz). Although originally conceived as an extension of Maxwell's theory, it diverged in certain central aspects from the Maxwellian approach.

Larmor's theory consisted of supposing that matter is made up of 'isolated singularities of aether', carrying an elementary positive or negative charge, which he called 'electrons'. He conceived of electric current as being formed by the convection of these electrons, and of properties of matter as resulting from their disposition. Thus, not only did Larmor abandon the Maxwellian representations of electric charge and current, but he also renounced at the same time the macroscopic approach characteristic of Maxwell's work. However, Larmor's theory did preserve the idea that electric and magnetic actions were propagated via the aether [Buchwald, 1985, 131–173; Hunt, 1991, 209–239; Darrigol, 2000, 332–343].

Larmor published the second and third parts of his article in 1895 and 1897. Probably influenced by the work of Lorentz, which he had discovered in the meantime, he developed his theory of electrons to explain the principal magneto-optical phenomena (the Faraday, Kerr and Zeeman effects) and the optical paradoxes of bodies of motion. Finally, he gave a synthetic account of his theory in *Aether and Matter* (1900), which exercised in the next decade an important influence on electromagnetic research in Britain, especially in Cambridge [Warwick, 2003, 357–398].

BIBLIOGRAPHY

Achard, F. 1998. 'La publication du *Treatise on Electricity and Magnetism* de James Clerk Maxwell', *La revue de synthèse*, *119*, 511–544.

Buchwald, J.Z. 1985. *From Maxwell to microphysics*: *aspects of electromagnetic theory in the last quarter of the nineteenth century*, Chicago: University of Chicago Press.

Buchwald, J.Z. 1994. *The creation of scientific effects*: *Heinrich Hertz and electric waves*, Chicago: University of Chicago Press.

Campbell, L. and Garnett, W. 1882. *The life of James Clerk Maxwell*, London: Macmillan. [Repr. New York: Johnson Reprint, 1969. 2nd ed. 1884.]

Chrystal, G. 1882. 'Clerk Maxwell's "Electricity and Magnetism" ', *Nature*, *25*, 237–240. [Review of the second edition and of the *Elementary treatise on electricity*.]

Cross, J.J. 1985. 'Integral theorems in Cambridge mathematical physics, 1830-55', in P.M. Harman (ed.), *Wranglers and physicists*, Manchester: Manchester University Press, 112–148.

Crowe, M.J. 1967. *A history of vector analysis*: *the evolution of the idea of a vectorial system*, Notre Dame: University of Notre Dame Press.

Darrigol, O. 2000. *Electrodynamics from Ampère to Einstein*, Oxford: Oxford University Press.

Gooday, G. 1990. 'Precision measurement and the genesis of physics teaching laboratories in Victorian Britain', *British journal for the history of science*, *23*, 25–51.

Gooding, D. 1978. 'Conceptual and experimental bases of Faraday's denial of electrostatic action at a distance', *Studies in the history and philosophy of science*, *9*, 117–149.

Gooding, D. 1981. 'Final steps to the field theory: Faraday's study of magnetic phenomena, 1845–1850', *Historical studies in the physical sciences*, *11*, 232–275.

Grattan-Guinness, I. 1990. *Convolutions in French mathematics, 1800–1840*, 3 vols., Basel: Birkhäuser; Berlin: Deutscher Verlag der Wissenschaften.

Harman, P.M. 1995. 'Introduction' to [Maxwell, 1990–2002], vol. 2, 1–37.

Hunt, B.J. 1991. *The Maxwellians*, Ithaca: Cornell University Press.

Maxwell, J.C. *Papers*. *Scientific papers* (ed. W.D. Niven), 2 vols., Cambridge: Cambridge University Press. [Repr. Paris: Hermann, 1927; New York: Dover, 1952.]

Maxwell, J.C. 1990–2002. *Scientific letters and papers* (ed. P.M. Harman), 3 vols., Cambridge: Cambridge University Press. [Vol. 1 1846–1862, vol. 2 1862–1873, vol. 3 1874–1879.]

Schaffer, S. 1992. 'Late Victorian metrology and its instrumentation: a manufactory of Ohms', in R. Bud and S.E. Cozzens (eds.), *Invisible connections*: *Instruments, institutions and science*, Bellingham: SPIE Optical Engineering Press, 23–56.

Schaffer, S. 1995. 'Accurate measurement is an English science', in N.M. Wise (ed.), *The values of precision*, Princeton: Princeton University Press, 135–172.

Siegel, D.M. 1991. *Innovation in Maxwell's electromagnetic theory*: *Molecular vortices, displacement current, and light,* Cambridge: Cambridge University Press.

Smith, C. and Wise, M.N. 1989. *Energy and Empire*: *A biographical study of Lord Kelvin*, Cambridge: Cambridge University Press.

Sviedrys, R. 1970. 'The rise of physical science at Victorian Cambridge', *Historical studies in the physical sciences*, *2*, 127–151.

Tait, P.G. 1873. 'Clerk-Maxwell's Electricity and Magnetism', *Nature*, *7*, 478–480. [Review of the first edition.]

Warwick, A. 2003. *Masters of theory*: *Cambridge and the rise of mathematical physics*, Chicago: University of Chicago Press.