COMP9020 Assignment 1

# 1.

(a) (R1 ; R2) ; R3 = { (a,b) |∃c, (a,c)∈(R1;R2) ; (c,b)∈R3}

=\{ (a,b) |∃c,∃d, (a,d)∈R1 ; (d,c)∈R2 ; (c,b)∈R3}

=\{ (a,b) |∃d, (a,d)∈R1 ; (d,b)∈(R2;R3)}

= R1 ; (R2 ; R3)

(b) R1 = { (a,b) | (a,b)∈S X S }
= { (a,b) | (a,b) ; (b,b) }    = R1 ; I ( Because I = { (x,x) | x∈S} so (b,b) ⊆ I )
= { (a,b) | (a,a) ; (a,b) }    = I ; R1 ( so on so forth, (a,a) ⊆ I )

(c) If we assign R1 = { (1,2) } ,R2 = { (2,3) } , then (R1 ; R2)$^{\leftarrow}$ = { (3,1) }, and R1$^{\leftarrow}$ **;** R2$^{\leftarrow}$ = $\phi$,
so the statement is not true.

(d) (R1 ∪ R2) ; R3 = { (a,b) | ∃c, (a,c)∈(R1∪R2) ; (c,b)∈R3}

= { (a,b) | ( (a1,c1)∈R1∪(a2,c2)∈R2) ) ; ( (c1,b1)∈R3∪(c2,b2)∈R3) ) },

where {(a1,c1)}∪{(a2,c2)} = {(a,c)} and {(c1,b1)}∪{(c2,b2)} = {(c,b)}.

= { (a,b)| ( (a1,c1)∈R1 ; (c1,b1)∈R3 )∪( (a2,c2)∈R2 ; (c2,b2)∈R3 )}

= (R1 ; R3)∪(R2 ; R3)

(e) If wen assign R1 = { (1,1) , (1,2) } , R2 = { (1,4) , (1,5) } , R3 = { (1,4) , (2,5) } , then
R1 ; (R2∩R3) = { (1,4) } , and (R1 ; R2) ∩ (R1 ; R3) = { (1,4) , (1,5) } , In this case the left is not equal to the right ,the statement is not true.

2.

(a)Base case: $R^i = R^{i+1}$ ,which holds $R^j = R^i$ for $j = i+1$.

Inductive case: Assume $R^{i+n} = R^{i+n+1}$,where $n \in N$.So it is obvious $R^{i+n} = R^{i+n+1} = R^{i+n} \cup (R;R^{i+n})$ .And $R^{i+n+2} = R^{i+n+1} \cup (R;R^{i+n+1}) = R^{i+n} \cup (R;R^{i+n}) = R^{i+n+1}$ .

For each $R^j$ (j>i ),it is equal to the previous one,thus $R^j = R^i$ for all j≥i.

(b)From the formula:$R^{i+1} := R^i \cup (R;R^i)$ i≥0 , we can conclude for every i , $R^i \subseteq R^{i+1}$.So for $k \in [0,i]$,$R^k \subseteq R^i$.And from question (a),we can get if $R^i = R^{i+1}$ , then $R^i = R^j$ for all j≥i ,so $Rj \subseteq Ri$.

In conclusion,$R^k \subseteq R^i$ for all $k \geqslant 0$.

(c)Base case: $R^0 ; R^m = R^m$ (from 1b) $= R^{0+m}$ ,so P(0) holds.

Inductive case: Assume P(n) holds ,which is $R^n;R^m = R^{n+m}$.

$R^{n+1} ; R^m = [R^n \cup (R ; R^n)] ; R^m = (R^n ; R^m) \cup (R ; R^n ; R^m)$ (from 1d)

$R^{n+m+1} = R^{n+m} \cup (R ; R^{n+m}) = (R^n ; R^m) \cup (R ; R^n ; R^m)$ (from 1d)

So $R^{n+1} = R^{n+m+1}$,P(n+1) holds.

Therefore P(n) holds for all $n \in N$.

(d)We assume $(a,b) \in R^{k+1}$ and $(a,b) \notin R^k$.Because $R^{k+1} := R^k \cup (R;R^k )$, $(a,b) \in (R;R^k )$.

Therefore,$\exists (a,c_k) \in R$, $(c_k,b) \in R^k$.And we can also get $(c_k,b) \notin R^i, i \leq k-1$(if $(c_k,b) \in R^{k-1}$,and $(a,c_k) \in R$,we can get $(a,b) \in R^k$).So $(c_k,b) \in R;R^{k-1}$.

Therefore,$\exists (c_k,c_{k-1}) \in R$ ,$(c_{k-1},b) \in R^{k-1}$ , and still $(c_{k-1},b) \notin R^i, i \leq k-2$.

So we get the following result:

(a,ck)$\in$R (ck,b) $\notin R^i, i \leq k-1$

(ck,ck-1)$\in$R (ck-1,b)$\notin R^i, i \leq k-2$

...... ......

(c2,c1)$\in$R (c1,b) $\notin R^0$

And c1 ≠ c2 ≠ …… ≠ck ,ci$\in$S, |{c1,c2,......,ck}| = k = |S|.So a$\in${c1,c2,......,ck}

$(a,b) \in R^k$,which conflict with hypothesis(assume $(a,b) \notin R^k$).Therefore,$(a,b) \in R^k$,which means for every element in $R^{k+1}$,it is also in $R^k$,so $R^{k+1} = R^k$.

(e)From the conclusion of (c),we can assign n = m = k,which is $R^k;R^k=R^{2k}$.From the conclusion (a) and (d), we can know $R^k = R^{k+1}$,and then $R^k = R^j$ for $j \geqslant k$,so $R^{2k} = R^k$.Therefore $R^k;R^k=R^k$,which indicates if $(a,b) \in R^k$,$(b,c) \in R^k$,then $(a,c) \in R^k$.So $R^k$ is transitive.

(f) F = $(R\cup R^{<-})$

R: Because $F^0 = \{(x,x)|x\in S\}$ and $F^0\subseteq F^i$,so R holds.

S:I can’t really prove it,but I feel it is certainly right,because R and $R^{<-}$ are symmetric,each time they evolve ,they are still symmetric.

T:It is just the same as the problem (e).

So $(R\cup R^{<-})$ is an equivalence relation.

## 3.

(a) a binary tree is either an empty tree represented by a null pointer , or is a single ordered node which contains a data , a left and right pointer and each pointer points to a binary tree.

(b) count(T) = $\begin{cases} 0 & ifT = NULL \\ 1 + count(T->left) + count(T->right) & recursive \end{cases}$

(c) leaves(T) = $\begin{cases} 0 & ifT = NULL \\ 1 & ifT->left = NULL\ and\ T->right = NULL \\ leaves(T->left) + leaves(T->right) & recursive \end{cases}$

(d) internal(T) = $\begin{cases} -1 & ifT = NULL \\ 0 & ifT->left = NULL\ and\ T->right = NULL \\ 1 + internal(T->left) + internal(T->right) & recursive \end{cases}$

(e) **Base case:**If the binary tree is only an empty tree which represents NULL,then from the conclusion c and d,internal(T) = -1 and leaves(T) = 0 ,which holds.And if the binary tree’s left pointer and right pointer each points to NULL,we can also see from the c and d that internal(T) = 0 and leaves(T)=1,which also holds.

**Inductive case:** Assume for a arbitary binary tree,internal(T)=I,leaves(T)=L which satisfies L=I+1.Then we consider the following three conditions.

First:We extend a leaf node,making it to be a fully-internal nodes by adding two leaf nodes to be its successors.This time I’ = I+1,L’ = L-1+2=L+1,so L’=I’+1,which holds.

Second:We extend a leaf node,adding one leaf node to be its either left or right successor.This time I’ = I,L’=L-1+1=L,which holds.

Third:We extend a node which is neither a leaf node nor a fully-internal node,making it to be the fully-internal node by adding a leaf node to it.This time I’=I+1,L’=L+1,which holds.

In conclusion,P(T) holds for all binary trees T.

4.

(a) $h1$ = “Alpha uses channel hi” , $l1$ = “Alpha uses channel lo” ;
$h2$ = “Bravo uses channel hi” , $l2$ = “Bravo uses channel lo”;
$h3$ = “Charlie uses channel hi” , $l3$ = “Charlie uses channel lo”;
$h4$ = “Delta uses channel hi” , $l4$ = “Delta uses channel lo”.

(i) $\phi_1 = (h1 \vee l1) \wedge (h2 \vee l2) \wedge (h3 \vee l3) \wedge (h4 \vee l4)$

(ii) $\phi_2 = \neg(h1 \wedge l1) \wedge \neg(h2 \wedge l2) \wedge \neg(h3 \wedge l3) \wedge \neg(h4 \wedge l4)$

(iii) $\phi_3 = ((h1 \wedge \neg h2) \vee (l1 \wedge \neg l2)) \wedge ((h2 \wedge \neg h3) \vee (l2 \wedge \neg l3)) \wedge ((h3 \wedge \neg h4) \vee (l3 \wedge \neg l4))$

(b)

(i) If we assign h1 = T,h2=F,h3=T,h4=F,l1=F,l2=T,l3=F,l4=T,this time $V(\phi_1 \wedge \phi_2 \wedge \phi_3) = T$.

If we assign h1= T,h2 =T,h3=T,h4=T,l1=F,l2=F,l3=F,l4=F,this time it is quite easy to know $V(\phi_1 \wedge \phi_2 \wedge \phi_3) = F$.

So $\phi_1 \wedge \phi_2 \wedge \phi_3$ is satisfiable for $V(\phi_1 \wedge \phi_2 \wedge \phi_3) = T$ for some truth assignment v.

(ii) First solution : Alpha uses channel hi,Bravo uses channel lo,Charlie uses channel hi,Delta uses channel lo.

Second solution : Alpha uses channel lo,Bravo uses channel hi,Charlie uses channel lo,Delta uses channel hi.