COMP9020 Assignment 3

1.

(a) Let A,B,C be the elements in the set of propositional variables,R be the relation $\equiv$.

(R) A is an arbitary propositional element,.From the truth table,it can be seen that $A \equiv A$,so $(A,A) \in R$.

(S)For A,B if $(A,B) \in R$,which is $A \equiv B$,then we get $v(A) = v(B)$.So $v(B) = v(A)$ is also true,and $B \equiv A$,therefore $(B,A) \in R$.

(T)For A,B,C,if $(A,B) \in R,(B,C) \in R$,then we get $A \equiv B, B \equiv C$.List the truth table.

A	B	C
True	True	True
False	False	False

So it can be observed that $v(A) = v(c)$, which is $A \equiv C$.So (T) holds.

In conclusion, the $\equiv$ relation is an equivalence relation on F.

(b) $\neg\top$, $A \wedge \neg A$, $(A \vee \neg A) \wedge (B \wedge \neg B)$, $(A \wedge \neg A) \vee (B \wedge \neg B)$.

(c) (i)

ϕ	ϕ'	$\neg\phi$	$\neg\phi'$
True	True	False	False
False	False	True	True

From truth table,we can observe that $\neg\phi \equiv \neg\phi'$.

(ii)

ϕ	ψ	ϕ'	ψ'	$\phi \wedge \psi$	$\phi' \wedge \psi'$
True	True	True	True	True	True
True	False	True	False	False	False
False	True	False	True	False	False
False	False	False	False	False	False

From truth table,we can observe that $\phi \wedge \psi \equiv \phi' \wedge \psi'$.

(iii)

ϕ	ψ	ϕ'	ψ'	$\phi \vee \psi$	$\phi' \vee \psi'$
True	True	True	True	True	True
True	False	True	False	True	True
False	True	False	True	True	True
False	False	False	False	False	False

From truth table,we can observe that $\phi \vee \psi \equiv \phi' \vee \psi'$.

(d) Let 0:[$\perp$] 1: [$\top$].[A],[B],[C] are three elements of F$\equiv$.(A,B,C are propositional variables,all the '=' operation mean the values equal).

Identity : $[A] \vee 0 = [A] \vee [\perp] = [A \vee \perp] = [A]$, $[A] \wedge 1 = [A] \wedge [\top] = [A \wedge \top] = [A]$, so it holds.

Complementation : $[A] \vee [A]' = [A] \vee [\neg A] = [A \vee \neg A] = [\top] = 1$,

$[A] \wedge [A]' = [A] \wedge [\neg A] = [A \wedge \neg A] = [\perp] = 0$, so it holds.

Commutative : $[A] \vee [B] = [A \vee B] = [B \vee A] = [B] \vee [A]$,

$[A] \wedge [B] = [A \wedge B] = [B \wedge A] = [B] \wedge [A]$, so it holds.

Associative : $([A] \vee [B]) \vee [C] = [A \vee B] \vee [C] = [(A \vee B) \vee C] = [A \vee (B \vee C)] = [A] \vee ([B] \vee [C])$

$([A] \wedge [B]) \wedge [C] = [A \wedge B] \wedge [C] = [(A \wedge B) \wedge C] = [A \wedge (B \wedge C)] = [A] \wedge ([B] \wedge [C])$

So it holds.

Distributive: $[A] \vee ([B] \wedge [C]) = [A] \vee [B \wedge C] = [A \vee (B \wedge C)] = [(A \vee B) \wedge (A \vee C)]$

$= [A \vee B] \wedge [A \vee C] = ([A] \vee [B]) \wedge ([A] \vee [C])$

$[A] \wedge ([B] \vee [C]) = [A] \wedge [B \vee C] = [A \wedge (B \vee C)] = [(A \wedge B) \vee (A \wedge C)]$

$= [A \wedge B] \vee [A \wedge C] = ([A] \wedge [B]) \vee ([A] \wedge [C])$,so it holds.

In summary, F$\equiv$ together with the operations and the defination of 0,1 holds the properties of identity,complementation,commutative,Associative and distributive,so it forms a Boolean Algebra.

2.

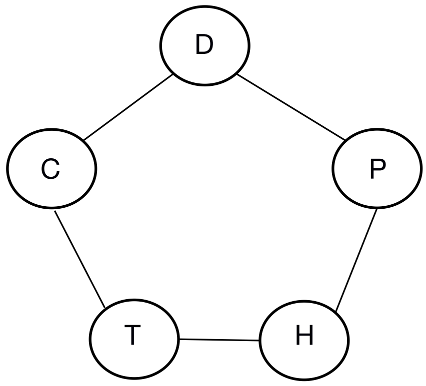
(a)Using the strategy of deleting vertex,edges,replacing a vertex of degree 2 with an edge can not transfer the Petersen graph to K_5,because we can not make the edges of outer-ring shrink,which means deleting the vertex of outer-ring also results the vanish of edges connected to the vertex(e.g deleting the vertex 0 will also make the E(0,5) E(0,4) and E(0,1) disappear),and also there is no vertex of degree 2 in the graph.So the Petersen graph does not contain a subdivision of K_5.

(b)Delete the vertex 9 and the edges connected to it,which are E(9,4),E(9,6),E(9,7).Now vertex 4,6,7 have degree of two.We use the strategy of replacing a vertex of degree 2 with an edge to make vertex 4,6,7 disappear.After that,we get 6 vertexes,each of them is degree 3.Then list all the edges,E(0,1) E(0,5) E(0,3) E(1,8) E(1,2) E(2,3) E(2,5) E(3,8) E(5,8).We can observe that the new graph has two parts,one is vertex 0,2,8 ,the other one is vertex 1,3,5.In each part,every vertex doesn't connect to each other,and every vertex connects to the every vertex in another part.So it is $K_{3,3}$.Therefore Petersen graph contains a subdivision of $K_{3,3}$.

3.

(a) Define Defence against the Dark Arts as the vertex D, Potions as the vertexP, Herbology as the vertexH, Transfiguration as the vertexT, Charms as the vertexC. Also define edges between two vertexes as the two vertexes have clashes. (eg. there is an edge between the vertice D and the vertice P,meaning that Defence against the Dark Arts clashes with Potions).

The model will look like the following undirected graph:



The graph problem here is to find the maximum clique.

(b) From the graph,we can find the maximum clique is 2,so the maximum number of classes Harry can take is two.

The all two course sets he can take is {(D,H),(D,T),(P,T),(P,C),(H,C)}.

4.

(a) It is easy to get T(1) = 1,T(2) = 2 from observation.And for T(3),we can see it recursively,by setting a root and its left and right child will be either T(2),None or T(1),T(1) . So T(3) = T(2)*1 + T(1)*T(1) + T(2)*1 = 5. For T(4), we still apply this method. T(4) = T(3)*1 + T(2)*T(1) + T(1)*T(2) + 1*T(3) = 14.
In conclusion ,T(n) = $\sum_{i=0}^{n-1}$ T(i)*T(n-i-1) (we define T(0) = 1).
So, T(n) = $\begin{cases} 1 & n = 0 \\ \sum_{i=0}^{n-1} T(i) * T(n-i-1) & n >= 1 \end{cases}$

(b) Because every node of full binary tree has either two non-empty children or two empty children,the number of edges of the full binary tree is always even.And because the property of tree: |V| = |E| + 1,the number of nodes is always odd.

(c) The number of internal nodes of a full binary tree of n nodes is (n-1)/2.(From observation,the number of node of full binary tree is always odd).For a certain kind of internal nodes of a tree,it has only one extension which can form a full binary tree,so the number of full binary trees with n nodes is equal to the number of internal nodes with (n-1)/2 nodes,which is B(n) = T((n-1)/2),n>0.

(d) See the number of propositional variables this question,which is n as the leaves' number,therefore the number of internals is n-1.
B(n+n-1) = B(2n-1) is the number of frame which illustrates the number of combination of leaves(propositional variables).
For each leaf,it has either ¬ in front of it or no.So together it has 2^n possible condition.
For each internal node,it represents either ∧ or ∨,so together it has 2^{n-1} possible condition.
Also it has n! possible permutations of propositional variables.
Altogether,F(n) = B(2n-1)*n!*2^n*2^{n-1}.

5.

(a) From the graph and the uniform distribution of each vertice,we can get the following:

$P_1(n+1) = (1/3)*P_2(n) + (1/3)*P_4(n)$

$P_2(n+1) = (1/2)*P_1(n) + (1/2)*P_3(n) + (1/3)*P_4(n)$

$P_3(n+1) = (1/3)*P_2(n) + (1/3)*P_4(n)$

$P_4(n+1) = (1/2)*P_1(n) + (1/3)*P_2(n) + (1/2)*P_3(n)$

(b) As pi(n+1) = pi(n),we can turn the equations into the following:

$P_1(n) = (1/3)*P_2(n) + (1/3)*P_4(n)$

$P_2(n) = (1/2)*P_1(n) + (1/2)*P_3(n) + (1/3)*P_4(n)$

$P_3(n) = (1/3)*P_2(n) + (1/3)*P_4(n)$

$P_4(n) = (1/2)*P_1(n) + (1/3)*P_2(n) + (1/2)*P_3(n)$

It can be found that $P_1(n) = P_3(n)$,$P_2(n) = P_4(n)$,and $P_1(n) = (2/3)*P_2(n)$,and also $P_1(n) + P_2(n) + P_3(n) + P_4(n) = 1$.So $P_1(n) = P_3(n) = 2/10$, $P_2(n) = P_4(n) = 3/10$.

(c) E(v1) = 0*(2/10) + 1*(3/10) + 1*(3/10) + 2*(2/10) = 1