COMP9020

Foundations of Computer Science

COMP9020 19T3 Staff

Lecturer:	Paul Hunter
Email:	paul.hunter@unsw.edu.au
Lectures:	Mondays and Wednesdays 4-6pm
Consults:	Wednesdays, 2-3pm, Rm204 K17
Research:	Theoretical CS: Algorithms, Formal verification

What you can expect from me

What is this course about?

What is Computer Science?

> *"Computer science no more about computers than astronomy is about telescopes"*
>
> – E. Dijkstra

Course Aims

Computer Science is about exploring the ability, and limitation, of computers to solve problems.

This course aims to increase your level of mathematical maturity to assist with the fundamental problem of **finding, formulating, and proving** properties of programs.

Course Aims

The actual content is taken from a list of subjects that constitute the basis of the tool box of every serious practitioner of computing:

• number theory	week 1
• sets, relations and functions	weeks 2–3
• big-O notation	week 3
• recursion	week 4
• graph theory	week 5
• logic	week 6
• induction	week 7
• combinatorics	week 8
• probability and expectation	week 9

Course Material

All course information is placed on the course website

www.cse.unsw.edu.au/∼cs9020/

Content includes:

- Lecture slides and recordings
- Quizzes and Assignments
- Course Forums
- Practice questions
- Challenge questions

Course Material

Textbook:

- KA Ross and CR Wright: Discrete Mathematics

Supplementary textbook:

- E Lehman, FT Leighton, A Meyer:
  Mathematics for Computer Science

Assessment Summary

60% exam, 30% assignments, 10% quizzes:

- 16 quizzes, worth up to 1 mark each
- 3 assignments, worth up to 10 marks each
- final exam (2 hours) worth up to 60 marks

Quizzes are available for 2 days before each lecture. Assignments due at the end of weeks 4, 7 and 10.

You must achieve 40% on the final exam to pass

Your final score will be taken from your 10 best quiz results, 3 assignments and final exam.

More information

View the course outline at:

www.cse.unsw.edu.au/∼cs9020/outline.html

Particularly the sections on **Student conduct** and **Plagiarism**.

What I will expect from you

Assessments

To achieve good marks in this course you need to demonstrate:

- Your understanding of the material
- Your ability to work with the material

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Mathematical communication

Guidelines for good mathematical writing

Mathematical writing should be:

- Clear
- Logical
- Sensical

Examples

Example

Ex 1 a) 51 b) 72 c) 12

Ex 2: $(A \setminus B) \cup (B \setminus A) = (A \cap B^c) \cup (B \cap A^c) = (A \cup B) \cap (A \cup A^c) \cap (B \cup B^c) \cap (A \cup A^c)$
$= (A \cup B) \cap (A^c \cup B^c) = (A \cup B) \cap (A \cap B)^c = (A \cup B) \setminus (A \cap B)$ by DeM, Dist

Ex 3 a) Yes b) No c) Yes d) No e) Yes

Ex 4 a) True b) False

~~Ex 4 a) True b)~~

Examples

Example

Ex. 2

$$\begin{aligned}
(A \setminus B) \cup (B \setminus A) &= (A \cap B^c) \cup (B \cap A^c) && \text{(Def.)} \\
&= ((A \cap B^c) \cup B) \cap ((A \cap B^c) \cup A^c) && \text{(Dist.)} \\
&= (A \cup B) \cap (B^c \cup B) \\
&\qquad \cap (A \cup A^c) \cap (B^c \cup A^c) && \text{(Dist.)} \\
&= (A \cup B) \cap (A^c \cup B^c) && \text{(Ident.)} \\
&= (A \cup B) \cap (A \cap B)^c && \text{(DeM.)} \\
&= (A \cup B) \setminus (A \cap B) && \text{(Def.)}
\end{aligned}$$

Examples

Example

Ex. 4a

We will show that if R_1 and R_2 are symmetric, then $R_1 \cap R_2$ is symmetric.

Suppose $(a, b) \in R_1 \cap R_2$. Then $(a, b) \in R_1$ and $(a, b) \in R_2$. Because R_1 is symmetric, $(b, a) \in R_1$. Because R_2 is symmetric, $(b, a) \in R_2$. Therefore $(b, a) \in R_1 \cap R_2$. Therefore $R_1 \cap R_2$ is symmetric.

Proofs

A large component of your work in this course is giving **proofs** of **propositions.**

A **proposition** is a statement that is either true or false.

Example

Propositions:

- $3 + 5 = 8$
- All integers are either even or odd
- There exist a, b, c such that $1/a + 1/b + 1/c = 4$

Not propositions:

- $3 + 5$
- x is even or x is odd
- $1/a + 1/b + 1/c = 4$

Proposition structure

Common proposition structures include:

If A then B	$(A \Rightarrow B)$
A if and only if B	$(A \Leftrightarrow B)$
For all x, A	$(\forall x.A)$
There exists x such that A	$(\exists x.A)$

$\forall$ and $\exists$ are known as **quantifiers**.

Proofs

A large component of your work in this course is giving **proofs** of **propositions.**

A proof of a proposition is an argument to convince the reader/marker that the proposition is true.

A **proof** of a proposition is a finite sequence of logical steps, starting from base assumptions (**axioms** and **hypotheses**), leading to the proposition in question.

Proofs

Example

Prove: $3 \times 2 = 2 \times 3$

$$
\begin{aligned}
3 \times 2 &= (2+1) \times 2 \\
&= (2 \times 2) + (1 \times 2) \\
&= (1 \times 2) + (2 \times 2) \\
&= 2 + (2 \times 2) \\
&= (2 \times 1) + (2 \times 2) \\
&= 2 \times (1+2) \\
&= 2 \times 3.
\end{aligned}
$$

Proofs: pitfalls

Starting from the proposition and deriving true **is not valid**.

Example

Prove: $1 = -1$

$$\begin{array}{lrcll} & 1 & = & -1 & \\ \text{So} & (1)^2 & = & (-1)^2 & \\ \text{So} & 1 & = & 1 & \text{which is true.} \end{array}$$

Does this mean that $1 = -1$?

Proofs: pitfalls

Make sure each step is logically valid: for example, $x = y$ implies $x^2 = y^2$ but $x^2 = y^2$ does not imply $x = y$.

Example

Suppose $a = b$. Then,

$$
\begin{array}{lrcl}
 & a^2 & = & ab \\
\text{So} & a^2 - b^2 & = & ab - b^2 \\
\text{So} & (a-b)(a+b) & = & (a-b)b \\
\text{So} & a+b & = & b \\
\text{So} & a & = & 0
\end{array}
$$

This is true no matter what value a is given at the start, so does that mean everything is equal to 0?

Proofs: pitfalls

For propositions of the form $\forall x.A$ where x can have infinitely many values:

- You cannot enumerate infinitely many cases in a proof.
- Only considering a finite number of cases is not sufficient.

Example

$$\text{For all } n,\ n^2 + n + 41 \text{ is prime}$$

True for $n = 0, 1, 2, \ldots, 39$. Not true for $n = 40$.

Proofs: pitfalls

The order of quantifiers matters when it comes to propositions:

Example

- For every number x, there is a number y such that y is larger than x
- There is a number y such that for every number x, y is larger than x

Proof strategies: direct proof

Proposition form	You need to do this
$A \Rightarrow B$	Assume A and prove B
$A \Leftrightarrow B$	Prove "If A then B" and "If B then A"
$\forall x.A$	Show A holds for every possible value of x
$\exists x.A$	Find a value of x that makes A true

Proof strategies: contradiction

To prove A is true, assume A is false and derive a contradiction.
That is, start from the negation of the proposition and derive false.

Example

Prove: $\sqrt{2}$ is irrational
Proof: Assume $\sqrt{2}$ is rational ...

Negating propositions

Proposition form	Its negation
A and B	
A or B	
$A \Rightarrow B$	
$A \Leftrightarrow B$	
$\forall x.A$	
$\exists x.A$	

Negating propositions

Proposition form	Its negation
A and B	not A or not B
A or B	
$A \Rightarrow B$	
$A \Leftrightarrow B$	
$\forall x.A$	
$\exists x.A$	

Negating propositions

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A and B	not A or not B
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$\exists x.A$	$\forall x.$ not A

Proof strategies: contrapositive

To prove a proposition of the form "If A then B" you can prove "If not B then not A"

Example

Prove: If $m + n \geq 73$ then $m \geq 37$ or $n \geq 37$.

Proof strategies: dealing with $\forall$

How can we check infinitely many cases?

- Choose an **arbitrary** element: an object with no assumptions about it (may have to check several cases)
- Induction (see week 7)

Example

Prove: For every integer n, n^2 will have remainder 0 or 1 when divided by 4.

Note: "Arbitrary" is not the same as "random".