COMP9444 Neural Networks and Deep Learning
Term 3, 2019

Solutions to Exercise 4: Hidden Units and Convolution

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1. **Hidden Unit Geometry**

   Consider a fully connected feedforward neural network with 6 inputs, 2 hidden units and 3 outputs, using tanh activation at the hidden units and sigmoid at the outputs. Suppose this network is trained on the following data, and that the training is successful.

```
Item  Inputs  Outputs
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      123456    123
----  ------  -------
 1.   100000    000
 2.   010000    001
 3.   001000    010
 4.   000100    100
 5.   000010    101
 6.   000001    110
```
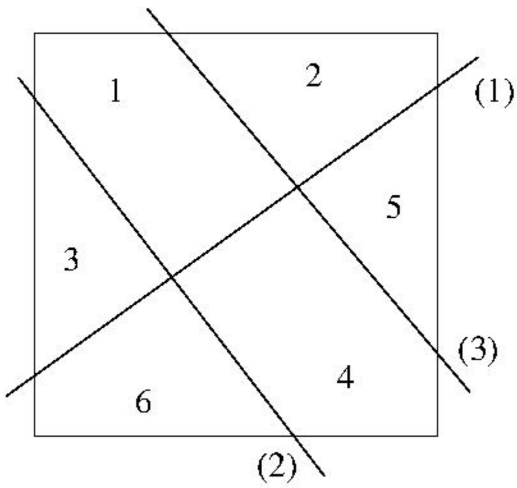

   Draw a diagram showing:
   a. for each input, a point in hidden unit space corresponding to that input, and
   b. for each output, a line dividing the hidden unit space into regions for which the value of that output is greater/less than one half.



2. **Softmax**

   Recall that the formula for Softmax is

   Prob(i) = $\exp(z_i) / \Sigma_j \exp(z_j)$

   Consider a classification task with three classes 1, 2, 3. Suppose a particular input is presented, producing outputs

z_1=1.0, z_2=2.0, z_3=3.0

and that the correct class for this input is Class 2. Compute the following, to two decimal places:

a. Prob(i), for i = 1, 2, 3

Prob(1) = $e^1/(e^1 + e^2 + e^3)$ = 2.718/30.193 = 0.09
Prob(2) = $e^2/(e^1 + e^2 + e^3)$ = 7.389/30.193 = 0.24
Prob(3) = $e^3/(e^1 + e^2 + e^3)$ = 20.086/30.193 = 0.67

b. d(log Prob(2))/dz_j, for j = 1, 2, 3

d(log Prob(2))/dz_1 = d(z_2 - log Σ_j exp(z_j))/dz_1 = -exp(z_1)/Σ_j exp(z_j) = -0.09
d(log Prob(2))/dz_2 = d(z_2 - log Σ_j exp(z_j))/dz_2 = 1 - exp(z_2)/Σ_j exp(z_j) = 1 - 0.24 = 0.76
d(log Prob(2))/dz_3 = d(z_2 - log Σ_j exp(z_j))/dz_3 = -exp(z_3)/Σ_j exp(z_j) = -0.67

Note how the correct class (2) is pushed up, while the incorrect class with the highest activation (3) is pushed down the most.

3. One of the early papers on Deep Q-Learning for Atari games (Mnih et al, 2013) contains this description of its Convolutional Neural Network:

"The input to the neural network consists of an 84 × 84 × 4 image. The first hidden layer convolves 16 8 × 8 filters with stride 4 with the input image and applies a rectifier nonlinearity. The second hidden layer convolves 32 4 × 4 filters with stride 2, again followed by a rectifier nonlinearity. The final hidden layer is fully-connected and consists of 256 rectifier units. The output layer is a fully-connected linear layer with a single output for each valid action. The number of valid actions varied between 4 and 18 on the games we considered."

For each layer in this network, compute the number of

a. weights per neuron in this layer (including bias)
b. neurons in this layer
c. connections into the neurons in this layer
d. independent parameters in this layer

You should assume the input images are gray-scale, there is no padding, and there are 18 valid actions (outputs).

First Convolutional Layer:

J = K = 84, L = 4, M = N = 8, P = 0, s = 4

weights per neuron:	1 + M × N × L = 1 + 8 × 8 × 4 = 257
width and height of layer:	1+(J-M)/s = 1+(84-8)/4 = 20
neurons in layer:	20 × 20 × 16 = 6400

connections:	$20 \times 20 \times 16 \times 257 = 1644800$
independent parameters:	$16 \times 257 = 4112$

Second Convolutional Layer:

$J = K = 20$, $L = 16$, $M = N = 4$, $P = 0$, $s = 2$

weights per neuron:	$1 + M \times N \times L = 1 + 4 \times 4 \times 16 = 257$
width and height of layer:	$1+(J-M)/s = 1+(20-4)/2 = 9$
neurons in layer:	$9 \times 9 \times 32 = 2592$
connections:	$9 \times 9 \times 32 \times 257 = 666144$
independent parameters:	$32 \times 257 = 8224$

Fully Connected Layer:

weights per neuron:	$1 + 2592 = 2593$
neurons in layer:	256
connections:	$256 \times 2593 = 663808$
independent parameters:	663808

Output Layer:

weights per neuron:	$1 + 256 = 257$
neurons in layer:	18
connections:	$18 \times 257 = 4626$
independent parameters:	4626