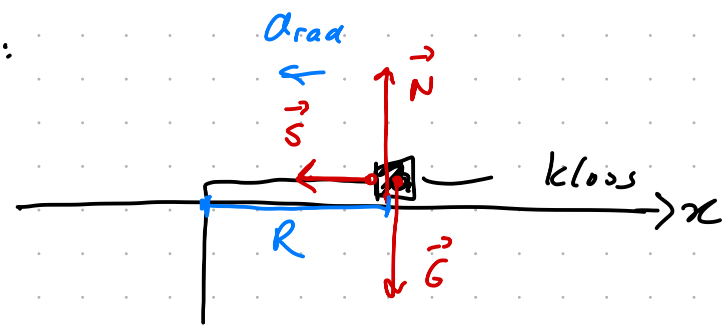
4.1 Snorkraften = kraften som må til for å holde klossen i sirkelbanen.

Kraftdiagram:



Ser på krefter i horisontal retning:

$$\sum F_x = m \cdot a_x \quad , \text{ hvor } a_x \text{ er radiell akselerasjon.}$$

a) $$\underline{S} = m \cdot a_{rad} = m \cdot \frac{v^2}{R} = 0{,}09\,\text{kg} \cdot \frac{\left(0{,}70\,\frac{m}{s}\right)^2}{0{,}40\,m} = \underline{0{,}11\,N}$$

b) $$\underline{S} = m\frac{v^2}{R} = 0{,}09\,\text{kg} \cdot \frac{\left(2{,}8\,\frac{m}{s}\right)^2}{0{,}10\,m} = \underline{7{,}1\,N}$$

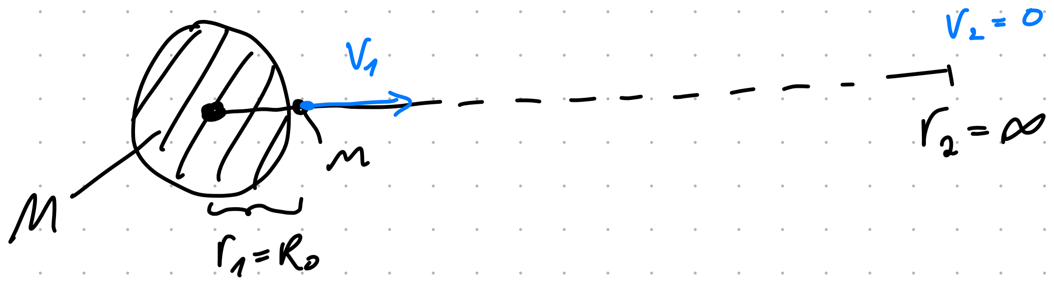
c) Arbeid = endring i kinetisk energi

(kun snoren utfører arbeid)

$$\underline{W} = E_{k2} - E_{k1} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}m\left(v_2^2 - v_1^2\right)$$

$$= \frac{1}{2} \cdot 0{,}9\,\text{kg}\left(\left(2{,}8\,\frac{m}{s}\right)^2 - \left(0{,}7\,\frac{m}{s}\right)^2\right) = \underline{0{,}33\,J}$$

4.2 Dette har vi sett i forelesningen.



$$M = 5{,}97 \cdot 10^{24}\ \text{kg}$$
$$R_0 = 6{,}37 \cdot 10^{6}\ \text{m}$$

Energibevaring i tyngdefelt:

$$\frac{1}{2}mv_1^2 + \underbrace{\left(-\gamma\frac{Mm}{r_1}\right)}_{r_1 = R_0} = \underbrace{\frac{1}{2}mv_2^2}_{v_2 = 0} + \underbrace{\left(-\gamma\frac{Mm}{r_2}\right)}_{\to 0 \text{ når } r_2 \to \infty}$$

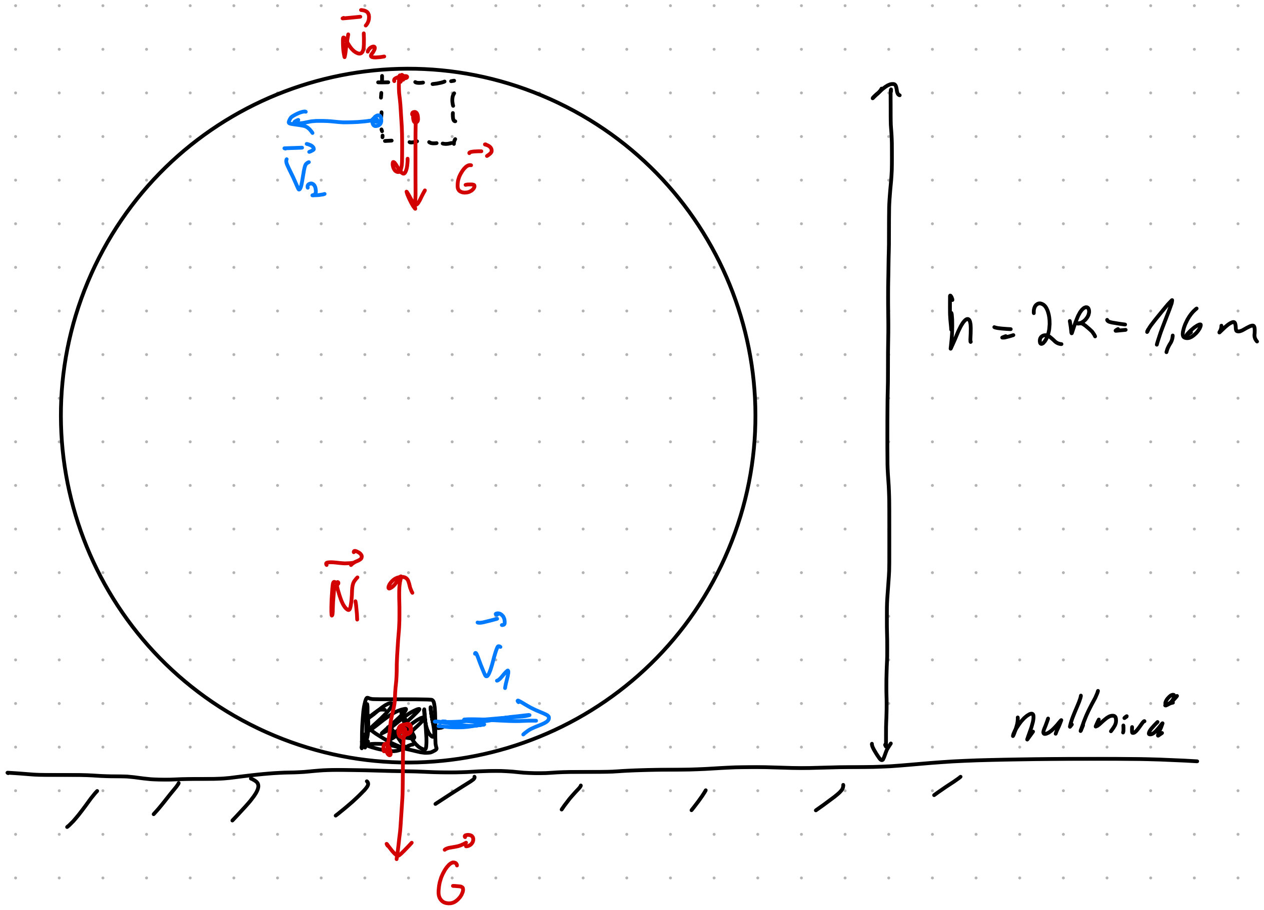
$$\frac{1}{2}\cancel{m}v_1^2 = \gamma\frac{M\cancel{m}}{R_0}$$

$$v_1 = \sqrt{\frac{2\gamma M}{R_0}} = \sqrt{\frac{2 \cdot 6{,}67 \cdot 10^{-11}\,\text{N}\frac{\text{m}^2}{\text{kg}^2} \cdot 5{,}97 \cdot 10^{24}\,\text{kg}}{6{,}37 \cdot 10^{6}\,\text{m}}}$$

$$= 1{,}118 \cdot 10^{4}\ \frac{\text{m}}{\text{s}}$$

$$v_1 = 11{,}2\ \frac{\text{km}}{\text{s}}$$ (uavhengig av massen til legemet)

4.3



$\vec{N} \perp \vec{v} \Rightarrow$ kun tyngdekraften gjør et arbeid.

$$E_{K1} + U_1 = E_{K2} + U_2$$

$$E_{K1} = \frac{1}{2} m v_1^2$$

?

$$N_1 - G = m \cdot a = m \frac{V_1^2}{R}$$

$$V_1^2 = \frac{(N_1 - mg) \cdot R}{m}$$

$$\frac{1}{2}mv_1^2 + U_1 = \frac{1}{2}mv_2^2 + U_2$$

$$\frac{1}{2}\cdot \cancel{m}\frac{(N_1 - mg)\cdot R}{\cancel{m}} + \underbrace{\cancel{mgh_1}}_{=0} = \frac{1}{2}mv_2^2 + mg\underbrace{h_2}_{=2\cdot R}$$

$$\frac{1}{\cancel{2}}mv_2^2 = \frac{R}{2}(N_1 - mg) - 2mgR = \frac{N_1R}{\cancel{2}} - \frac{5}{\cancel{2}}mgR$$

$$V_2^2 = \frac{N_1R}{m} - 5gR = R\left(\frac{N_1}{m} - 5g\right)$$

$$N_2 + G = m\frac{V_2^2}{R} = \frac{m}{\cancel{R}}\cdot \cancel{R}\left(\frac{N_1}{m} - 5g\right) = N_1 - 5mg$$

$$\underline{N_2} = N_1 - 6mg = 3{,}4\,N - 6\cdot 0{,}05\,kg\cdot 9{,}81\,\tfrac{m}{s^2} = \underline{0{,}457\,N}$$

Merkelig at radiusen ikke spiller inn?

Husk at vi ikke fikk oppgitt startfarten.
Den regnet vi ut v.h.a. $V_1^2 = \frac{(N_1 - G)\cdot R}{m}$.

Dvs. større R gir større V_1 slik at klossen kommer seg til toppen.

4.4 $F_x(x) = -\alpha x - \beta x^2$

$\alpha = 60{,}0\ \frac{N}{m}$

$\beta = 18{,}0\ \frac{N}{m^2}$

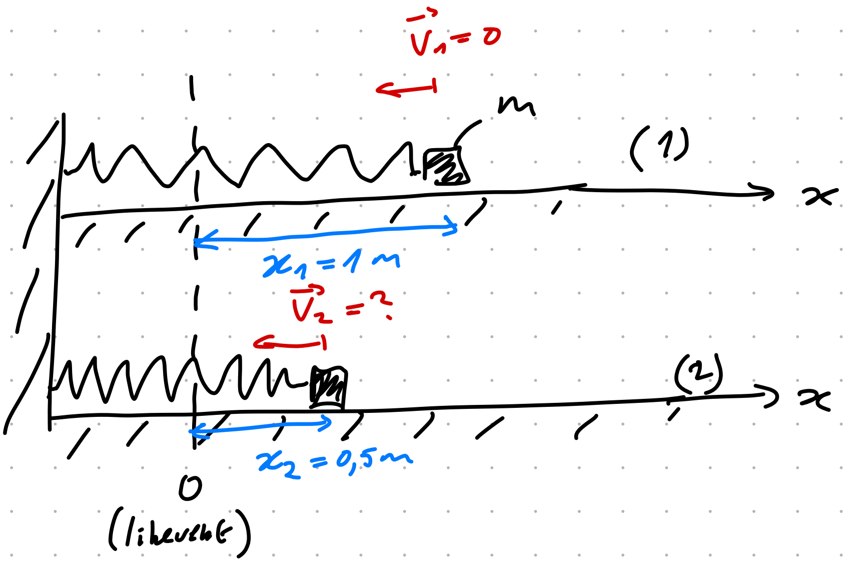
a) $F_x(x) = -\frac{\partial U(x)}{\partial x}$ $\quad (x_0 = 0 \ , \ U(x_0) = 0)$

$$U(x) = -\int_0^x F_x(x)\,dx = \int_0^x \alpha x + \beta x^2\,dx$$

$$= \left[\frac{1}{2}\alpha x^2 + \frac{1}{3}\beta x^3\right]_0^x$$

$$U(x) = \frac{1}{2}\alpha x^2 + \frac{1}{3}\beta x^3$$

b)



Kun fjæren gjør arbeid.

$$E_{K1} + U_{el.1} = E_{K2} + U_{el.2} \qquad E_{K1} = 0$$

$$\frac{1}{2}mv_2^2 = U_{el.1} - U_{el.2}$$

$$V_2 = \sqrt{\frac{2}{m} \cdot (U_{el.1} - U_{el.2})}$$

$$= \sqrt{\frac{2}{m}\left(\frac{1}{2}\alpha x_1^2 + \frac{1}{3}\beta x_1^3 - \frac{1}{2}\alpha x_2^2 - \frac{1}{3}\beta x_2^3\right)}$$

$$= \sqrt{\frac{2}{0{,}9kg} \cdot \left(\frac{60\frac{N}{m}}{2} \cdot (1m)^2 + \frac{18\frac{N}{m^2}}{3}(1m)^3 - \frac{60\frac{N}{m}}{2}(0{,}5m)^2 - \frac{18\frac{N}{m^2}}{3}(0{,}5m)^3\right)}$$

$$= \sqrt{\frac{2}{0{,}9kg}\left(30 N \cdot m + 6 N \cdot m - 7{,}5 Nm - 0{,}75 N \cdot m\right)}$$

$$= \sqrt{\frac{2 \cdot 27{,}75 N \cdot m}{0{,}9\, kg}} = \sqrt{\frac{55{,}5}{0{,}9} \frac{\cancel{kg} \frac{m}{s^2} \cdot m}{\cancel{kg}}}$$

$$\boxed{V_2 = 7{,}85 \frac{m}{s}}$$