Oblig 2c-fasit

1. **Prosesser**

   (a) **Bernoulli-prosess:** MA-223 Eksamen 2020.12: 1-a

   i. $P(X=0) = \binom{3}{0} p^0(1-p)^3 = (1-p)^3 = 0.125$

   $$1-p = \sqrt[3]{0.125} = 0.5$$
   $$p = 1 - 0.5 = \underline{\underline{0.5}}$$

   ii. $P(Y < 6) = P(Y \leq 5) = \text{BIN}_{(13,\,0.5)}(5) = \underline{\underline{0.2905273}}$

   iii. $$\begin{aligned} P(2 < Z < 7) &= P(2 < Z \leq 6) = P(Z \leq 6) - P(Z \leq 2) \\ &= NB_{(4,\,0.5)}(6) - NB_{(4,\,0.5)}(2) \\ &= 0.828125 - 0.34375 = \underline{\underline{0.484375}} \end{aligned}$$

   (b) **Poisson-prosess:** MA-223 Eksamen 2020.12: 1-b

   i. Poisson-prosess. $\lambda = 13.5\text{mail/time}$. Det er også greit med andre enheter, som $\lambda = 0.225\text{mail/minutt}$.

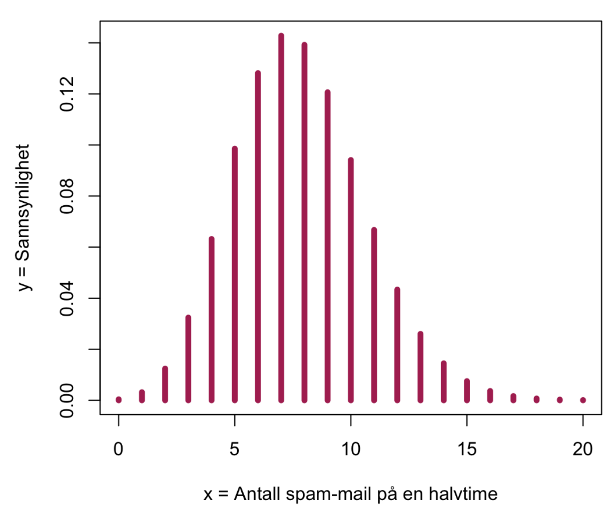
   ii. Antall email i løpet av tiden, la oss kalle antallet A, er Poisson-fordelt med parameter tid$\times$rate, altså enten $1.5 \times 13.5 = 20.25$, eller $0.225 = 20.25$. Så

   $$A \sim pois_{20.25}(x)$$

   Det betyr at
   $$P(A \leq 20) = POIS_{20.25}(20) = \underline{\underline{0.5368952 \approx 53.7\%}}$$

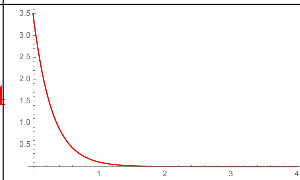
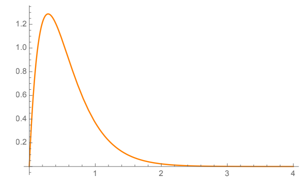
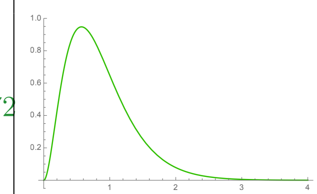
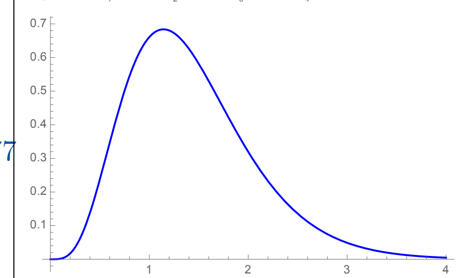
   iii. La B være denne totalen. Raten er $\hat{\lambda} = 13.5 + 2.1 = 15.6$, så da blir parameteren for Poisson-prosessen $\frac{1}{2} \times 15.6 = 7.8$. Da er
   $$B \sim pois_{7.8}$$

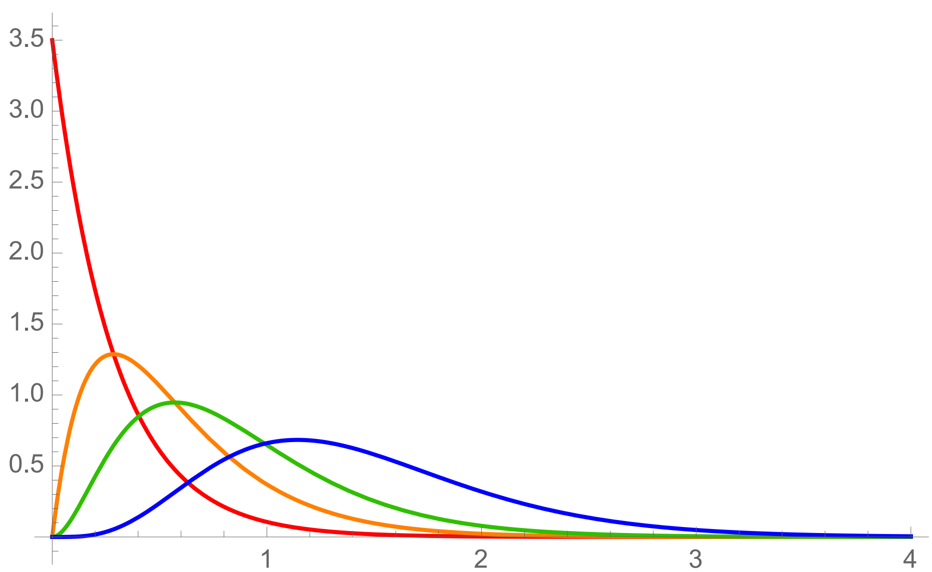
   som ser slik ut:



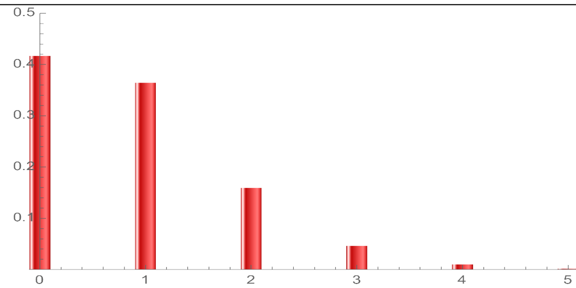
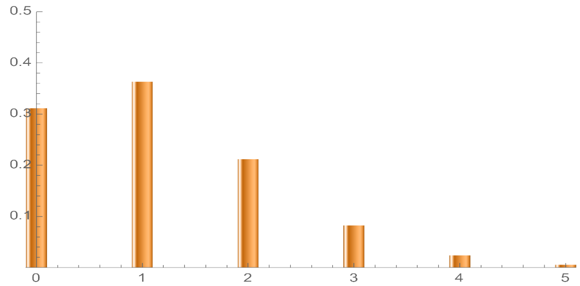
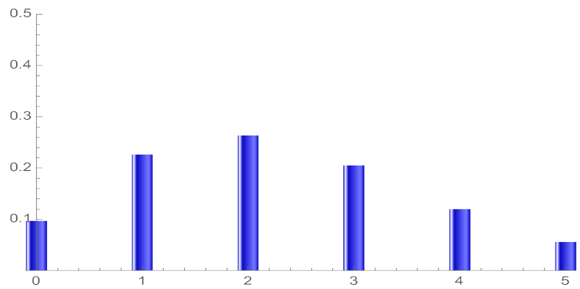
   (c) **Gaussisk prosess:** MA-171 Eksamen 2020.09: 1-c

i. Vi setter det opp i en tabell, og gjentar grafene sammen i samme koordinatsystem under. Begge fremstillinger er godkjent.

k	Fordeling	Uttrykk (optional)	$E[X_k]$	σ_{X_k}	Mini-graf
1	$\gamma_{(1,3.5)}(t)$	$3.5e^{-3.5x}$	0.285714	0.285714	
2	$\gamma_{(2,3.5)}(t)$	$12.25xe^{-3.5x}$	0.571429	0.404061	
3	$\gamma_{(3,3.5)}(t)$	$21.4375x^2e^{-3.5x}$	0.857143	0.494872	
5	$\gamma_{(5,3.5)}(t)$	$21.8841x^4e^{-3.5x}$	1.42857	0.638877	



ii. Vi setter det også her opp i tabell, for oversiktens skyld:

t	Fordeling (optional)	Diagram
$\frac{1}{3}$	$\text{pois}_{2/3}(x) = \frac{\left(\frac{2}{3}\right)^x}{x!} \cdot e^{-2/3}$	
$\frac{1}{2}$	$\text{pois}_1(x) = \frac{1}{x!} \cdot e^{-1}$	
1.2	$\text{pois}_{2.4}(x) = \frac{2.4^x}{x!} \cdot e^{-2.4}$	

2. **Simuleringer**

   (a) *Poisson:*

   i. rgamma(1,1,3)

   ii. t = rgamma(10,1,3) Deretter T = cumsum(T). Den første tallfølgen skal være positiv, og den siste stigende.

   iii. Eksempelkode:

```
t = rgamma(2000,1,rate=3)
T=cumsum(t)
sub40=which(T<=40)
T[sub40]
```

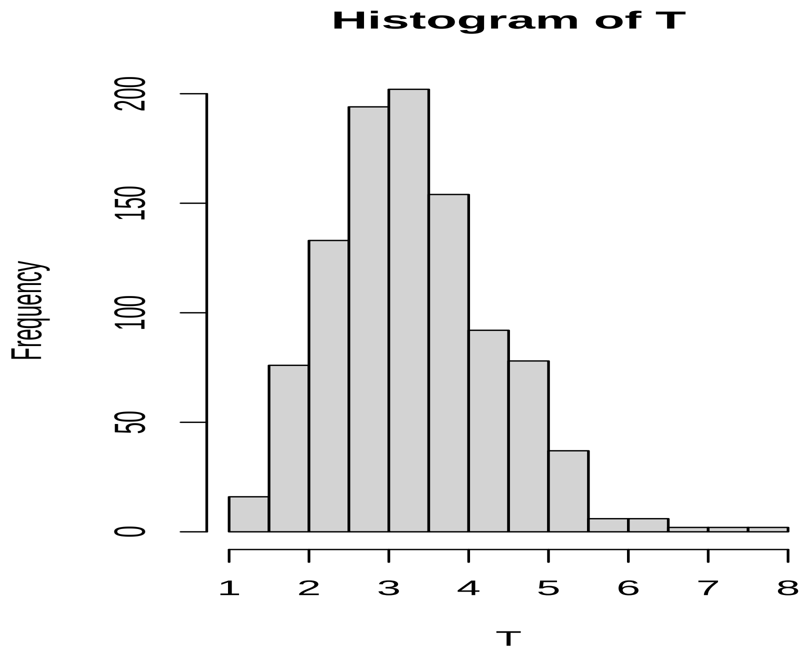
   Output: 0.4411751 0.4975134 0.8666810 1.2799313 1.8236952 2.6661901 3.4556604. . . . 38.8607782 39.2833117

   Siste index er 141, så $K_{+40} = 141$

   iv. Simulér 10 ventetider og regn ut T_{+10} ... gjør dette 1000 ganger, og tegn histogram over T_{+10}-verdiene.

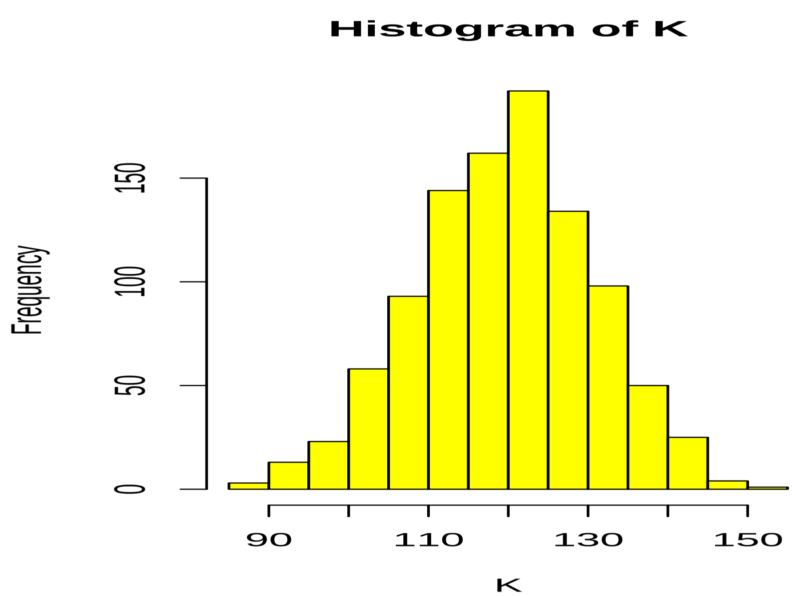
   Snarveien her er at rgamma(1,10,3) simulerer 10 ventinger og gir T_{+10} som output. Raskere enn 10 simuleringer rgamma(10,1,3) med påfølgende summering. Så det kommer an på om man vil ha detaljene. Her skal vi lage histogram, så førstnevnte er raskest. I tillegg skal vi ha 1000 av dem for histogram, så da gjør vi bare slik:

```
T = rgamma(1000,10,3)
hist(T)
```

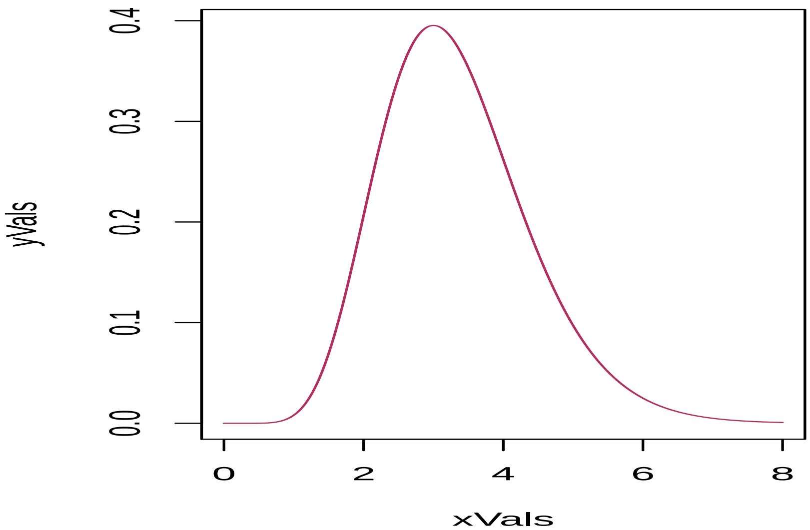


v. Vi kan loope koden over 1000 ganger, men velger å bruke at $K_{+40} \sim \text{pois}_{3\cdot 40}$ for denne fasitens skyld:

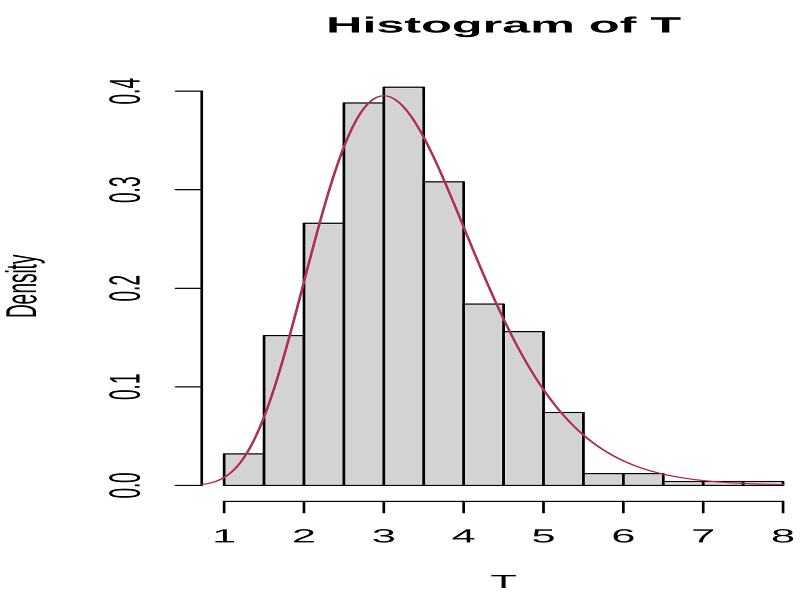
```
K = rpois(1000,120)
hist(K)
```



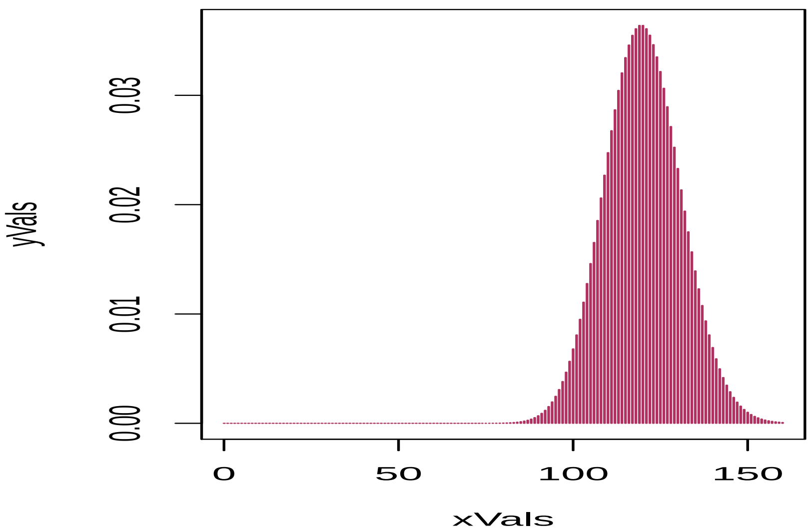
vi. Grafen til $\gamma_{(10,3)}$:



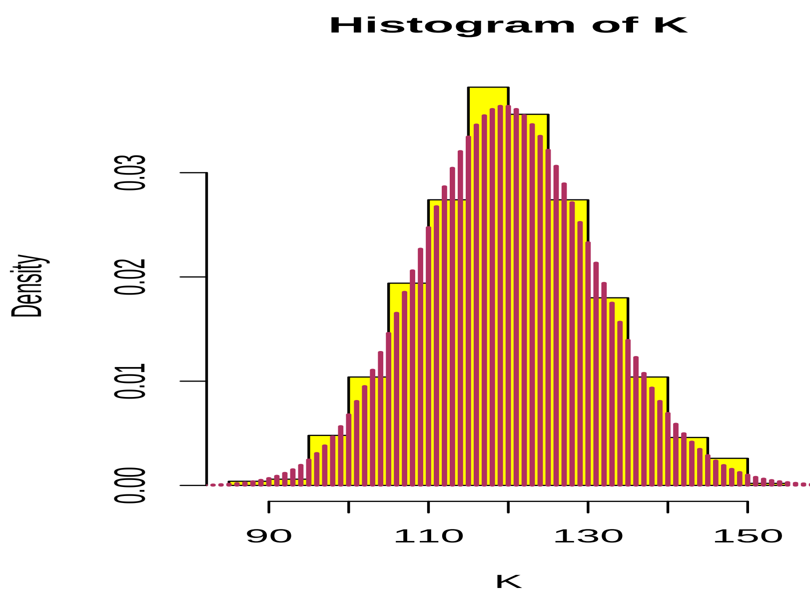
... sammen med histogrammet over T_{+10}-verdiene:



vii. Grafen til $\text{pois}_{3.40}$:



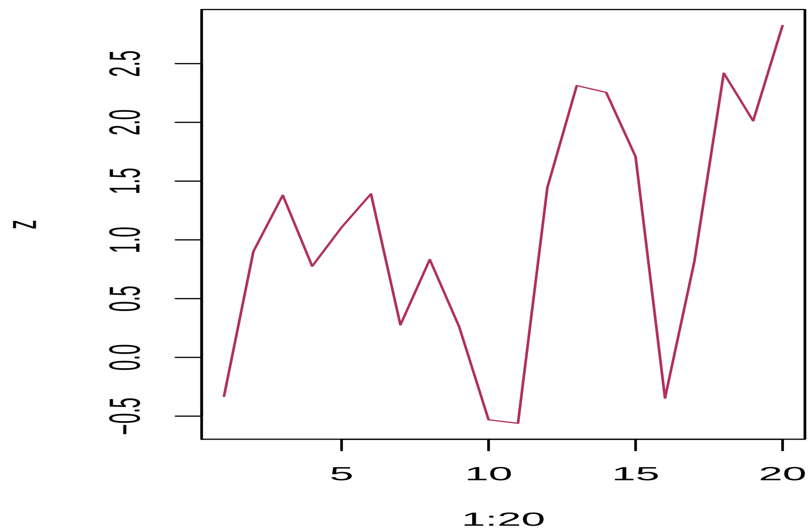
... sammenligner med histogrammet over K_{+40}:



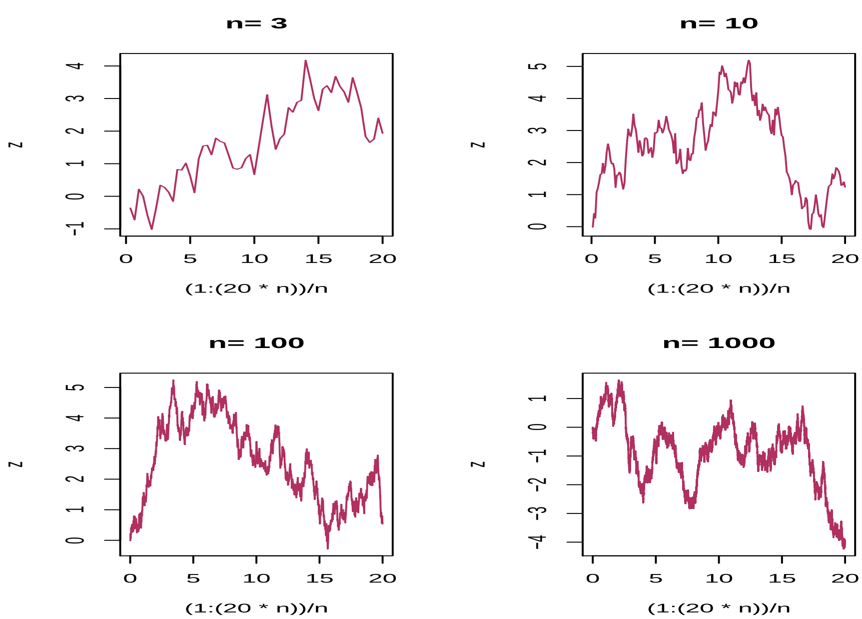
(b) *Gaussisk:*

i. rnorm(1,17,2.4)

ii.
```
x = rnorm(20,0,1)
z = cumsum(x)
plot(1:20,z,type=''l'')
```



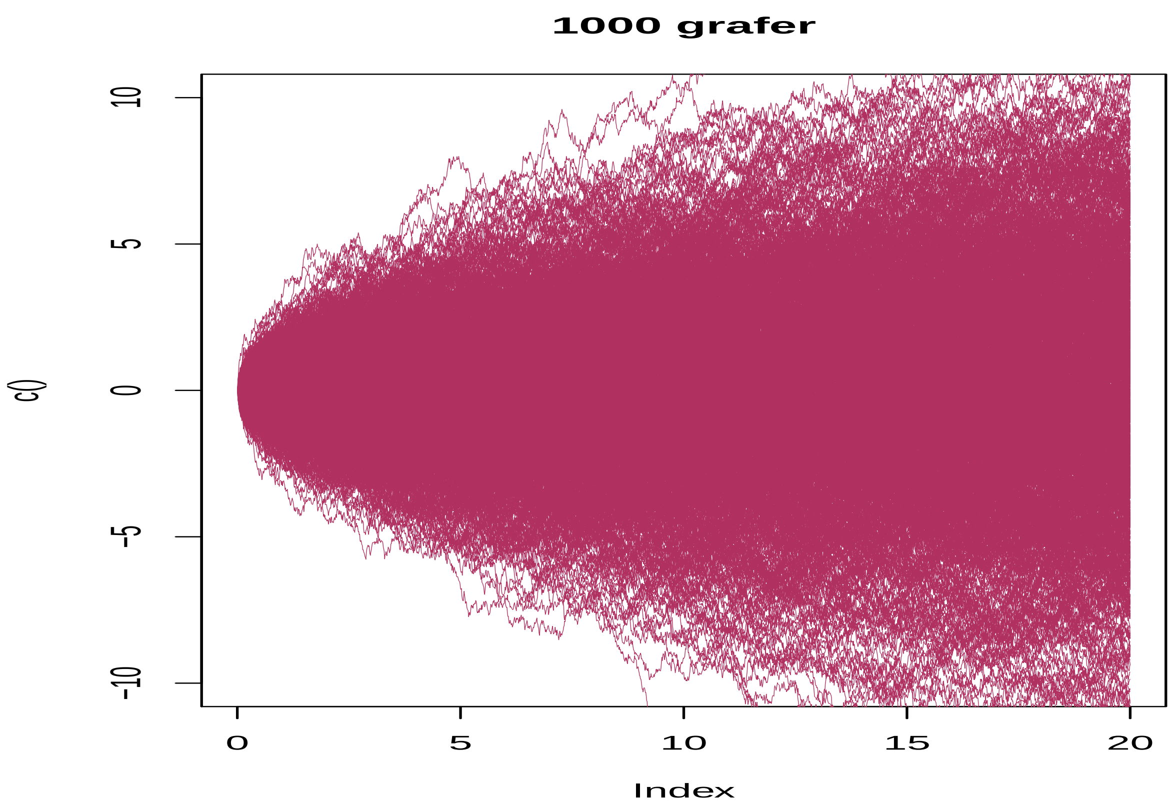
iii.
```
for (n in c(3,10,100,1000)) {
  x = rnorm(20*n,0,1/sqrt(n))
  z = cumsum(x)
  plot((1:(20*n))/n,z,type="l",col="maroon",main=paste("n=",n))
}
```



iv. Gjør oppgaven over for $n = 100$, men plott 10000 simuleringer i samme plott. Hvordan synes du dette ser ut?

```
par(mfrow=c(2,2))
n=100
plot(c(),c(),xlim=c(0,20),ylim=c(-10,10),main="1000 grafer")
for (i in 1:1000) {
  x = rnorm(20*n,0,1/sqrt(n))
```

```
  z = cumsum(x)
  lines((1:(20*n))/n,z,type="l",col="maroon",main=paste("n=",n),lwd=0.07)
}
par(mfrow=c(1,1))
```



v. Ta sluttverdien fra de 10000 simuleringene, og lag et histogram over disse (i **R**, bruk hist med probability=TRUE). Sammenlign med $\phi_{(0,\sqrt{20})}(x)$.

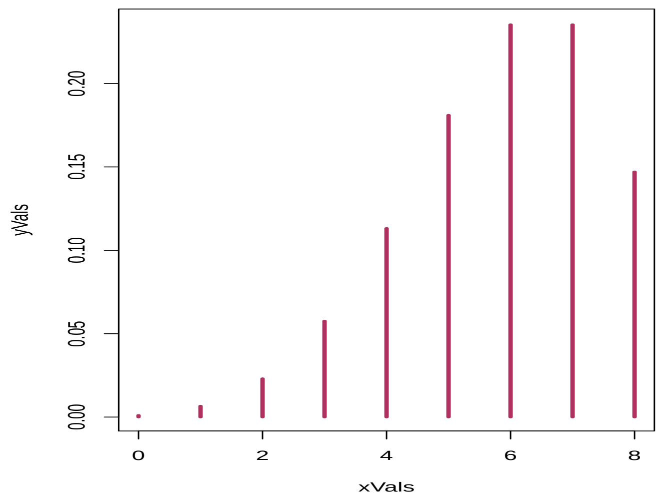
```
n=100
ends = c()
for (i in 1:10000) {
  x = rnorm(20*n,0,1/sqrt(n))
  z = cumsum(x)
  ends=c(ends,z[20*n])
}
hist(ends, probability = TRUE)
```

3. **Ekstra sannsynlighetsfordelinger**

(a) Beta-binomisk fordeling.

```
library(extraDistr)
xVals=0:8
yVals=dbbinom(xVals,8,8,3)
plot(xVals,yVals,type="h")
```

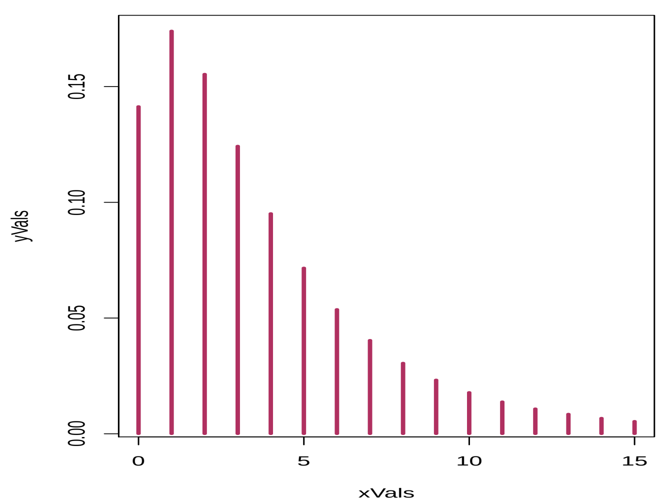
```
pbbinom(5,8,8,3)
```



$\mathrm{BB}_{(8,3,8)}(5) = 0.3823529$

(b) Beta negativ-binomisk fordeling.

```
library(extraDistr)
xVals=0:15
yVals=dbnbinom(xVals,4,5,4)
plot(xVals,yVals,type="h")
pbnbinom(7,4,5,4)
```

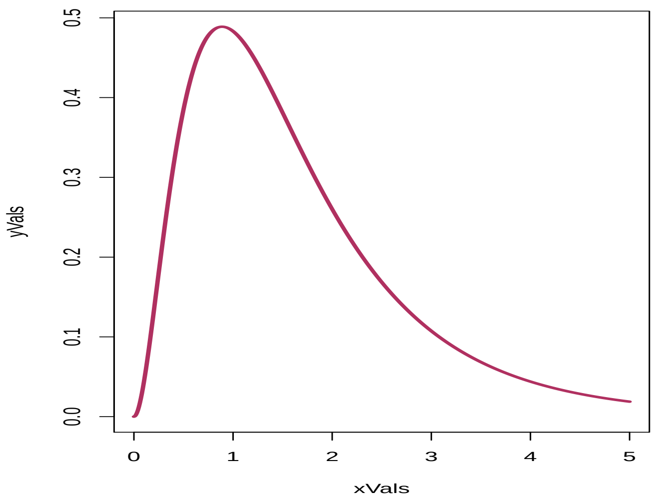


$\mathrm{BNB}_{(5,4,4)}(7) = 0.8561959$

(c) Gamma-gamma-fordeling.

```
library(extraDistr)
xVals=seq(0,5,0.01)
yVals=dbetapr(xVals,3,8,4)
```

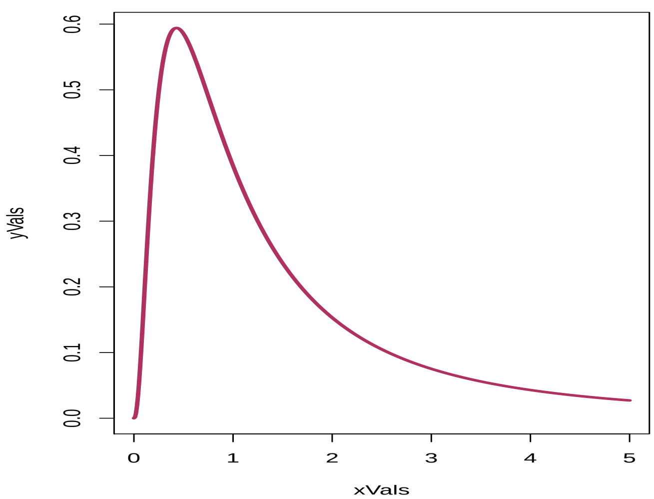
```
plot(xVals,yVals,type="l",lwd=3,col="maroon")
pbetapr(3,3,8,4)
```



$G\Gamma_{(3,8,4)}(3) = 0.8744847$

(d) Fischer-Snedecor-fordeling (”F”-fordeling).

```
xVals=seq(0,5,0.01)
yVals=df(xVals,7,3)
plot(xVals,yVals,type="l",lwd=3,col="maroon")
pf(2,7,3)
```



$\mathrm{F}_{(7,3)}(2) = 0.6940364$

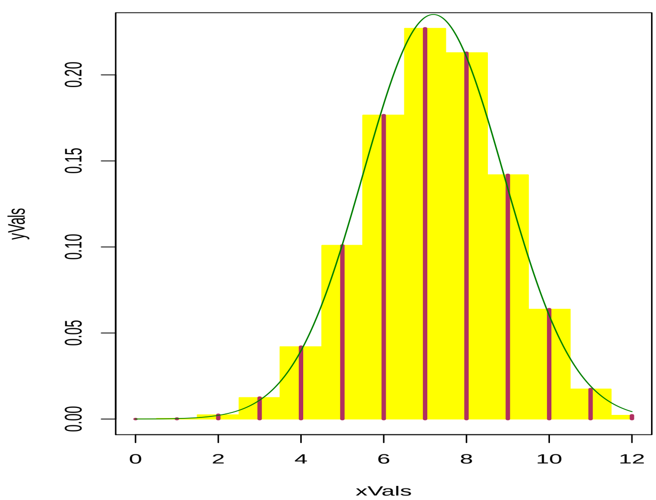
4. **Diverse:**

   (a) *Normaltilnærming:*

      i. Finn normaltilnærmingen til $\mathrm{bin}_{(12,\,0.6)}(x)$.

- $\mu = 12 \cdot 0.6 = 7.2$
- $\sigma = \sqrt{12 \cdot 0.6 \cdot (1-0.6)} = 1.697056$

```
mu=12*.6
sigma=sqrt(12*.6*.4)
xnVals=seq(0,12,0.01)
yVals=dbinom(round(xnVals),12,0.6)
plot(xVals,yVals,type="h",lwd=1,col="yellow")
xVals=0:12
yVals=dbinom(xVals,12,0.6)
lines(xVals,yVals,type="h",lwd=3,col="maroon")
ynVals=dnorm(xnVals,mu,sigma)
lines(xnVals,ynVals,type="l",col=rgb(0,.5,0,1))
pbinom(7,12,0.6)
pnorm(7.5,mu,sigma)
```

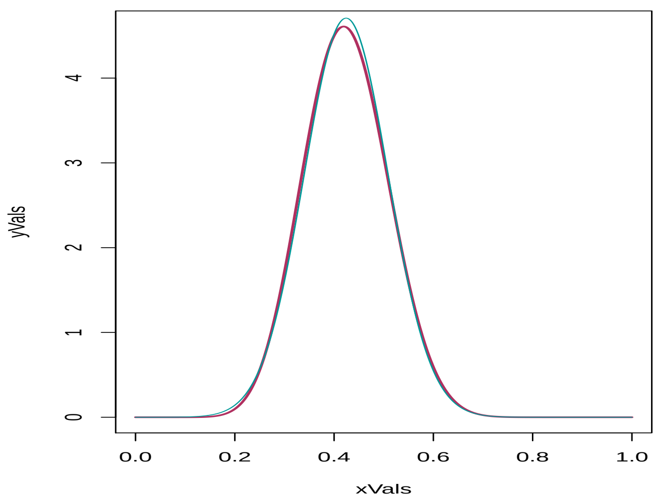


$P(X \leq 7) = \text{BIN}_{(12,\,0.6)}(7) = 0.5618218$

Normaltilnærming: $P(X \leq 7) \approx \Phi_{(7.2,1.697056)}(7 + \frac{1}{2}) = 0.5701581$

ii. Finn normaltilnærmingen til $\beta_{(14,19)}(x)$. Tegn normaltilnærmingen sammen med $\beta_{(14,19)}(x)$. Regn ut $P(X \leq 0.4)$ både ved direkte regning på $\beta_{(14,19)}(x)$ og ved å regne med normaltilnærmingen.

```
mu=14/(14+19)
sigma=sqrt((14*19)/((14+19)^2*(14+19+1)))
xVals=seq(0,1,0.001)
yVals=dbeta(xVals,14,19)
plot(xVals,yVals,type="l",col="maroon",lwd=2)
yValsN=dnorm(xVals,mu,sigma)
lines(xVals,yValsN,type="l",col=rgb(0,0.6,0.6,1),lwd=1)
pbeta(0.4,14,19)
pnorm(0.4,mu,sigma)
```



$P(X \leq 0.4) = I_{(14,19)}(0.4) = 0.3961404$
Normaltilnærming: $P(X \leq 0.4) \approx \Phi_{(0.4242424, 0.08475931)}(0.4) = 0.3874334$