Tutorial 05: Introduction and Magnetic Circuit

Submission in Canvas via Matlab Grader. Approved if the score is greater than 40%

Suggested Solutions

Problem 1**

Consider the coil in fig.1, which has $N = 25$ turns. The toroid on which the coil is wound has an inside parameter $ID = 5$ cm and an outside diameter $OD = 5.5$ cm. For a current $i = 3$ A, calculate

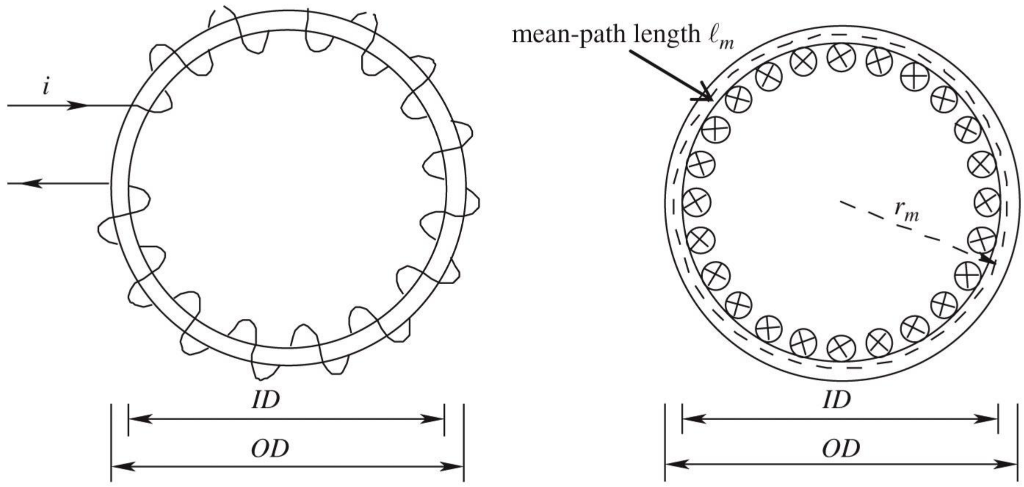


Figure 1: Toroid

the field intensity within the core:

a) very close to the inside diameter. Assign the value to variable H_1.

b) very close to the outside diameter. Assign the value to variable H_2.

c) Compare the results with the field intensity along the mean path: Compute the values $\Delta H_1 = \frac{H_1 - H_m}{H_m}$ and $\Delta H_2 = \frac{H_2 - H_m}{H_m}$, where H_m is the field intensity along the mean-path.

Solutions

a), b) Due to symmetry, the magnetic field intensities H_1 close to the inside diameter and H_2 close to the outside diameter are constant. Close to the inside diameter, the radius is $r_1 = ID/2$, whereas close to the outside diameter the radius is $r_2 = OD/2$. Therefore, the path length are:

$$l_1 = 2\pi r_1 = 2\pi \frac{ID}{2} = \pi ID; \qquad l_2 = 2\pi r_2 = \pi OD$$

From Ampere's law the field intensities are:

$$H_1 = \frac{Ni}{l_1} = \frac{Ni}{\pi ID} = 477.5 \text{ A/m}; \qquad H_2 = \frac{Ni}{l_2} = \frac{Ni}{\pi OD} = 434.1 \text{ A/m}$$

c) Compared with the field intensity along the mean path, $H_m = 454.5$ A/m, the field intensity close to the inside diameter is $(H_1 - H_m)/H_m = 5\%$ higher and the field intensity close to the outside diameter is $(H_m - H_2)/H_m = 4.5\%$ lower.

Problem 2*

In Problem 1, calculate the reluctance in the path of flux lines if $\mu_r = 2000$. The area A_m of the toroid is shown in fig. 2.

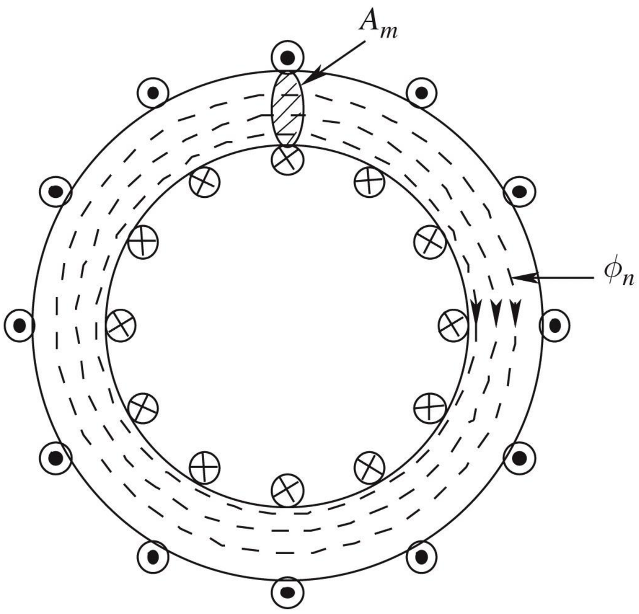


Figure 2: Toroid with flux Φ_m

Solutions
Using the "magnetic Ohm's law":

$$\mathscr{R} = \frac{\text{MMF}}{\Phi_m} = \frac{Ni}{B_m A_m} = \frac{Ni}{\mu_m H_m A_m} = \frac{Ni}{\mu_r \mu_0 \frac{Ni}{l_m} A_m} = \frac{l_m}{\mu_r \mu_0 A_m}$$

where the area A_m shown in Fig.2 is equal to:

$$A_m = \frac{\pi}{4} \left(\frac{OD - ID}{2} \right)^2$$

Finally:

$$\mathscr{R} = \frac{\pi \left(\frac{OD+ID}{2} \right)}{\mu_r \mu_0 \frac{\pi}{4} \left(\frac{OD-ID}{2} \right)^2} = 13.37 \cdot 10^6 \text{ A/Wb}$$

Problem 3 ***

Consider the core of dimensions given in Problem 2. The coil requires an inductance of 25μH. The maximum current is 3 A and the maximum flux density is not to exceed 1.3 T. Calculate the number of turns N (an integer!) and the relative permeability μ_r of the magnetic material that should be used.

Solutions

The inductance L_m of the coil is the ratio between the flux linkage λ_m and the current i:

$$L_m = \frac{\lambda_m}{i} = \frac{N\Phi_m}{i} = \frac{NA_mB_m}{i} = \frac{NA_m\mu_mH_m}{i} = \frac{NA_m\mu_m\frac{Ni}{l_m}}{i} = A_m\mu_m\frac{N^2}{l_m} \tag{1}$$

The maximum flux density $B_{\max}$ is reached when the current is maximum, $i = i_{\max}$:

$$B_{\max} = \mu_m\frac{Ni_{\max}}{l_m} \tag{2}$$

$B_{\max} \leq 1.3$ T and $L_m = 25\mu$H furnishes the following two equations - two unknowns (N and μ_m) system:

$$\begin{cases} \mu_m\frac{Ni_{\max}}{l_m} & \leq 1.3 \text{ T} \\ A_m\mu_m\frac{N^2}{l_m} & = 25\,\mu\text{H} \end{cases}$$

Solving this system gives:

$$N \geq 11.75 \qquad \text{and} \qquad \mu_r = \frac{668451}{N^2}$$

For the following values, all the requirements are met:

$$N = 12 \qquad \text{and} \qquad \mu_r = 4642$$

or for example:

$$N = 25 \qquad \text{and} \qquad \mu_r = 1070$$

Problem 4*

In the rectangular toroid of Fig. 3, $w = 5$ mm, $h = 15$ mm, the mean path length $l_m = 18$ cm, $\mu_r = 5000$, and $N = 100$ turns. Calculate the maximum current beyond which the flux density in the core will exceed 1.3 T.

Solutions

The flux density in the core is:

$$B_m = \mu_r\mu_0\frac{Ni}{l_m}$$

The maximum current $i_{\max}$, is given when the flux density is equal to $B_m = B_{\max} = 1.3$ T:

$$i_{\max} = \frac{B_{\max}l_m}{N\mu_r\mu_0} = 0.372 \text{ A}$$

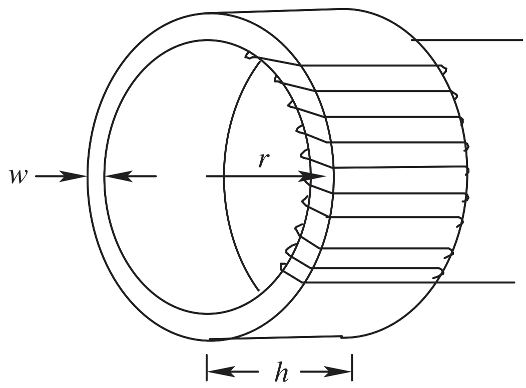
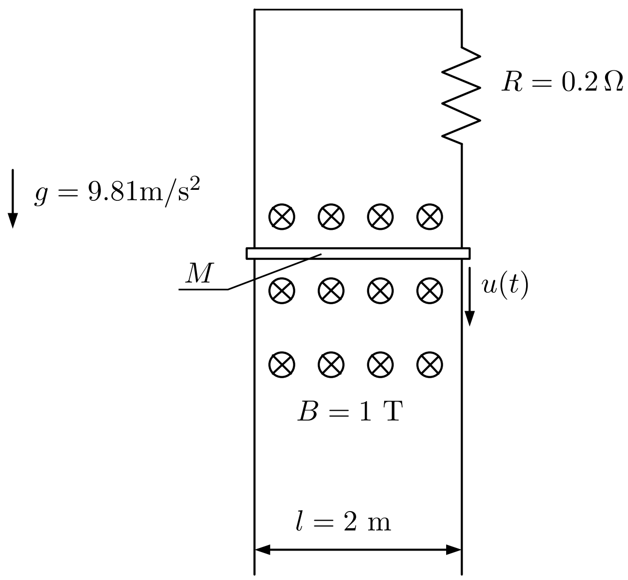


Figure 3: Rectangular toroid

Problem 5 **: Basic Electromagnetic Principles

Consider the system below, which shows a conducting rod in free fall in a uniform magnetic field. The rod mass $M = 1$ kg is free to slide (no friction) on the two conducting rails and has an initial speed equal to zero: $u(0) = 0$. The two rails are connected together on their upper end and the equivalent resistance of the rod and the rails is $R = 0.2\,\Omega$ as shown in the figure.



a) Calculate the current $i(t)$ that flows through the rod as a function of the speed $u(t)$ and show its direction (from left to right or from right to left).

b) Considering the action of gravity g and the electromagnetic force f_{em}, write a differential equation with state $u(t)$ that describes the movement of the rod.

c) Find the vertical speed $u(t)$ of the rod. What is the value of the speed when it has reached a steady state?

Solutions

a) A movement of the rod in the uniform magnetic field will induce a back emf e in the circuit:

$$e = B\,l\,u$$

The orientation of the positive voltage potential is given by the orientation of the vectorial product $\vec{u} \times \vec{B}$, i.e. towards the right. The current i is hence directed towards the right and its value is:

$$i = \frac{e}{R} = \frac{B\,l\,u(t)}{R}$$
$$\Rightarrow \boldsymbol{i} = \frac{1 * 2 * u(t)}{0.2} \boldsymbol{= 10u(t)}$$

b) Using Newton's law:

$$M\dot{u} = Mg - f_{em} = Mg - B\,l\,i$$
$$M\dot{u} + B\,l\,\frac{B\,l}{R}u = Mg$$
$$\Rightarrow \dot{u}(t) + 1 * 2 * 10 * u(t) = 9.81$$
$$\boldsymbol{\dot{u}(t) + 20u(t) = 9.81}$$

c) Therefore, solving this first order differential equation, the vertical speed of the mass is:

$$u(t) = \frac{9.81}{20}\left(1 - e^{-20t}\right) + u(0)e^{-20t}$$

Since initially the speed is zero:

$$\Rightarrow \boldsymbol{u(t) = 0.49\left(1 - e^{-20t}\right)}$$

When the system is in steady state, the speed is $u(t) = 0.49$ m/s.

Problem 6 *

Express the following voltages as phasors:

a) $v_1(t) = \sqrt{2} \times 120 \cos(\omega t - 30°)$ V

b) $v_2(t) = \sqrt{2} \times 120 \cos(\omega t + 30°)$ V

Solutions

a) $\boldsymbol{\bar{V}_1 = 169.7\angle - 30°}$ **V**

b) $\boldsymbol{\bar{V}_2 = 169.7\angle 30°}$ **V**

Problem 7**

The series R-L-C circuit of Fig.4 is in a sinusoidal steady state at a frequency of 60 Hz, $V = 120$ V, $R = 1.3\ \Omega$, $L = 20$ mH and $C = 100\ \mu$F. Calculate $i(t)$ using the phasor-domain analysis.

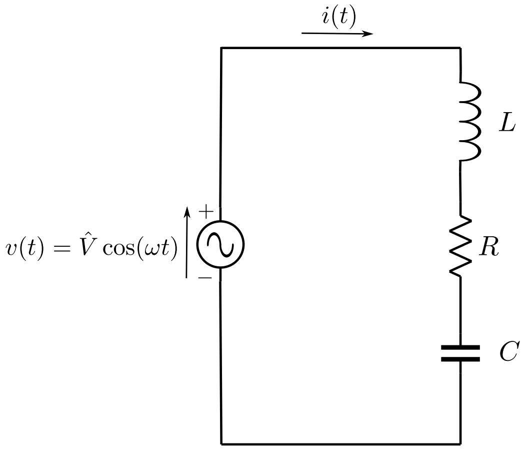


Figure 4: R-L-C circuit

Solutions
The impedance Z of the series-connected elements is obtained by:

$$Z = R + jX_L - jX_C$$

where $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$.
Therefore:

$$Z = |Z|\angle\theta$$

where:

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2} \qquad \text{and} \qquad \theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$\Rightarrow \begin{cases} |Z| = \sqrt{1.3^2 + \left(2\pi * 60 * 20 * 10^{-3} - \frac{1}{2\pi * 60 * 100 * 10^{-6}}\right)^2} \\ \theta = \tan^{-1}\left(\frac{2\pi * 60 * 20 * 10^{-3} - \frac{1}{2\pi * 60 * 100 * 10^{-6}}}{1.3}\right) \end{cases}$$

$$\Rightarrow Z = 19.03\angle -86.08^\circ$$

Finally:

$$\bar{I} = \frac{\bar{V}}{Z} = \frac{120\sqrt{2}\angle 0^\circ}{19.03\angle -86.08^\circ} = 8.917\angle 86.08^\circ$$

$$\Rightarrow \boldsymbol{i(t) = 8.92\cos\left(\omega t + 86.08^\circ\right)}\ \text{A}$$

Problem 8**

An inductive load connected to a 120 V (rms), 60 Hz ac source draws 1 kW at a power factor of 0.8. Calculate the capacitance required in parallel with the load in order to bring the combined power factor to 0.95 (lagging).

Solutions

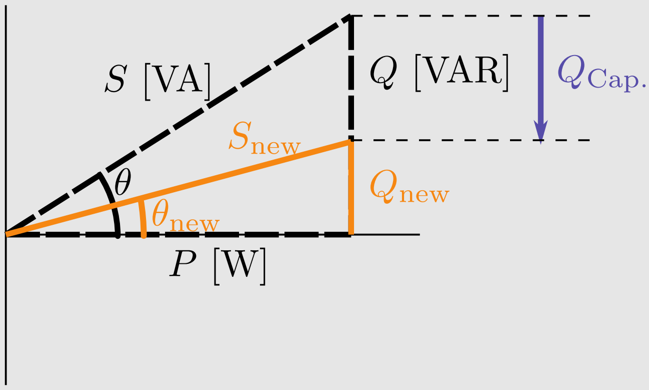


Figure 5: Power triangle

Adding a capacitance only changes the reactive power Q, while the real power P remains constant. Looking at Fig.5, the reactive power with or witout capacitance is:

$$\begin{cases} \text{Without Capacitance:} & \mathrm{PF} = \cos(\theta), \qquad \mathrm{Q} = \mathrm{P}\tan(\theta) \\ \text{With Capacitance:} & \mathrm{PF}_{\mathrm{new}} = \cos(\theta_{\mathrm{new}}), \qquad \mathrm{Q}_{\mathrm{new}} = \mathrm{P}\tan(\theta_{\mathrm{new}}) \end{cases}$$

Therefore, the reactive power which must be substract by using a capacitance is:

$$\begin{aligned} Q_{\mathrm{Cap}} &= Q - Q_{new} = P\tan(\arccos(\mathrm{PF})) - P\tan(\arccos(\mathrm{PF}_{new})) \\ &= 10^3 * (\tan(\arccos(0.8)) - \tan(\arccos(0.95))) = 421.3 \text{ VAR} \end{aligned}$$

Since $\bar{V}_{\mathrm{Cap}} = -jX_c\bar{I}_{\mathrm{Cap}}$:

$$\begin{aligned} &Q_{\mathrm{Cap}} = |V_{\mathrm{Cap}}I_{\mathrm{Cap}}| = \frac{V^2}{X_C} = \omega C V^2 \\ &\Rightarrow \boldsymbol{C} = \frac{Q_{\mathrm{Cap}}}{\omega V^2} = \frac{421.3}{2\pi 60 * 120^2} = \mathbf{77.6\,\mu F} \end{aligned}$$