Tutorial W36-37: Transformers

Submission in Canvas via Matlab Grader. Approved if the score is greater than 40%)

Suggested Solutions

Problem 2-1*: Root Mean Square

Calculate the rms values I_1 and I_2 of currents $i_1(t)$ and $i_2(t)$ respectively with the waveforms shown in Fig.1 where $A = 10$ [A] and $u = \pi/4$ [rad]. The root mean square (rms) of a current i is equal to the rms of one period T_s:

$$I = \sqrt{\frac{1}{T_s}\int_0^{T_s} i_1(t)^2 dt}$$

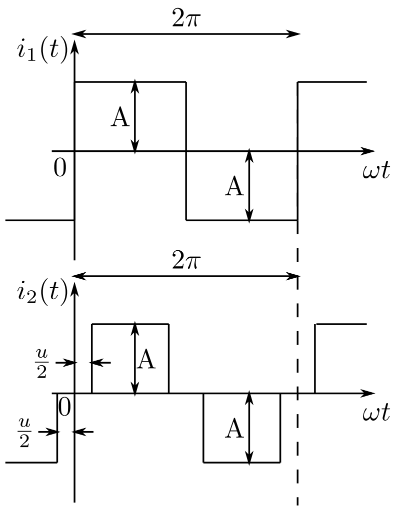


Figure 1: Current waveforms

Solutions
The root mean square (rms) of current i is equal to the rms of one period:

$$I_1 = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_1(t)^2 dt} = A$$

$$I_2 = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_2(t)^2 dt} = \sqrt{\frac{1}{2\pi}\left(\int_{\frac{u}{2}}^{\pi-\frac{u}{2}} i_2(t)^2 dt + \int_{\pi+\frac{u}{2}}^{2\pi-\frac{u}{2}} i_2(t)^2 dt\right)}$$
$$= \sqrt{\frac{1}{2\pi}2A^2(\pi - u)} = A\sqrt{1 - \frac{u}{\pi}}$$

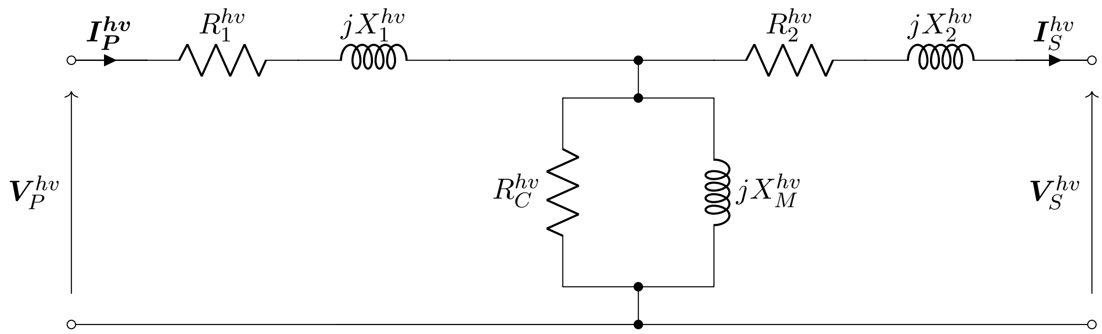


Figure 2: Equivalent circuit referred to high-voltage (hv) side

Problem 2-2 *: Transformer Equivalent Circuit

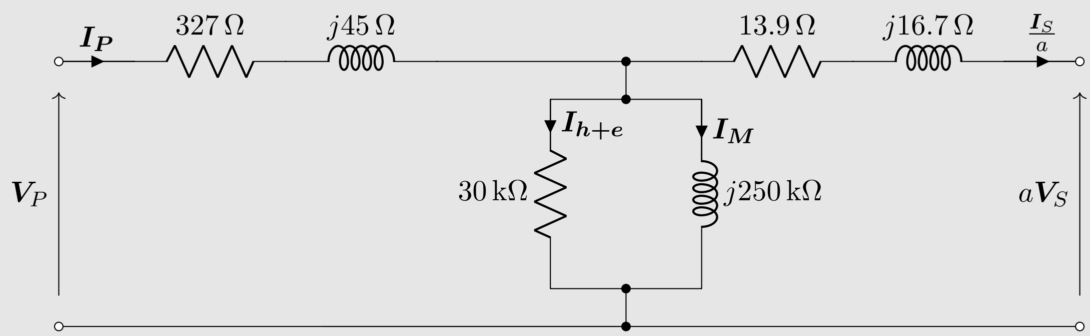
A 20-kVA 8000/480-V distribution transformer has the following resistances and reactances:
$R_1 = 327\,\Omega$; $R_2 = 0.05\,\Omega$
$X_1 = 45\,\Omega$; $X_2 = 0.06\,\Omega$
$X_M = 250$ kΩ; $R_C = 30$ kΩ
The excitation branch impedances are given referred to the high-voltage side of the transformer.

Question: Find the parameters numerical values (R_1^{hv}, X_1^{hv}, R_2^{hv}, X_2^{hv}, R_C^{hv} and X_M^{hv}) of the equivalent circuit of this transformer referred to the high-voltage side as shown in Fig.2. Calculate also the variables V_P^{hv}, I_P^{hv}, V_S^{hv} and I_S^{hv} as functions of V_P, I_P, V_S and I_S.

Solutions

The high voltage side (8000 V) is the primary side. The turns ratio of this transformer is $a = 8000/480 = 16.67$. Therefore, the secondary impedances referred to the primary side are:

$$R_2' = a^2 R_2 = 13.9\,\Omega$$
$$X_2' = a^2 X_2 = 16.7\,\Omega$$



Problem 2-3 **: Voltage Regulation and Efficiency

A 2400 / 240 [V], 60 [Hz] transformer has the following parameters in the equivalent circuit of Fig.3: the equivalent leakage impedance is $(2.4 + j4)\,\Omega$), and X_M at the high-side is 1800 Ω. Neglect R_C. The output voltage is 240 [V] (rms) and supplying a load of 1.5 Ω at a power factor of 0.9 (lagging).

a) Calculate the input voltage V_P.

b) Because a real transformer has series impedances within it, the output voltage varies with the load even if the input voltage remains constant. To conveniently compare transformers in this respect, it is customary to define a quantity called *voltage regulation*. Calculate the voltage regulation (VR) defined by the equation:

$$\mathrm{VR} = \frac{V_P/a - V_S}{V_S}$$

c) What is the efficiency of the transformer at this load? The efficiency μ of a device is defined by the equation

$$\mu = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{loss}}}$$

where the losses are the sum of copper losses, hysteresis losses and Eddy current losses.

Solutions

a) The turn ratio of this transformer is $a = 2400/240 = 10$. To calculate V_P use the following equation with rms phasors:

$$\bar{V}_P = a\bar{V}_S + R_{eq}\frac{\bar{I}_S}{a} + jX_{eq}\frac{\bar{I}_S}{a}$$

At PF = 0.9 lagging, current $\bar{I}_S = \dfrac{\bar{V}_S}{|Z_{\text{load}}|\angle\text{acos}(PF)} = \dfrac{240\angle 0^\circ}{1.5\angle 25.84^\circ} = 160\angle - 25.84^\circ$ [A]. Therefore,

$$\begin{aligned}\overline{\boldsymbol{V_P}} &= 10 \cdot 240\angle 0^\circ + 2.4 \cdot \frac{160\angle - 25.84^\circ}{10} + j \cdot 4 \cdot \frac{160\angle - 25.84^\circ}{10} \\ &= 2462.5 + j40.9 = \mathbf{2462.8\angle 0.95^\circ \ [V]}\end{aligned}$$

b) The resulting voltage regulation is:

$$\mathbf{VR} = \frac{V_P/a - V_S}{V_S} = \frac{246.3 - 240}{240} = \mathbf{2.6\%}$$

c) To find the efficiency of the transformer, first calculate its losses. The copper losses are:

$$P_{\text{Cu}} = \left(\frac{I_S}{a}\right)^2 R_{eq} = 614 \text{ [W]}$$

The core losses are neglected.

The output power of the transformer at this power factor is:

$$\begin{aligned}P_{\text{out}} &= V_S I_S \cos(\theta) \\ &= 240 \cdot 160 \cdot 0.9 = 34560 \text{ [W]}\end{aligned}$$

Therefore, the efficiency of the transformer at this condition is

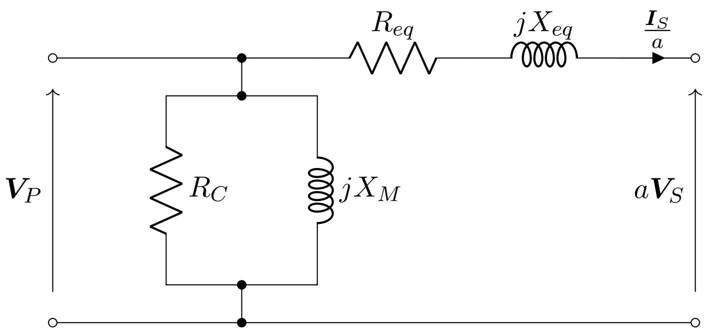
$$\boldsymbol{\mu} = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{loss}}} = \mathbf{98.25\%}$$

Problem 2-4**: Determining Transformer Model Parameters

A single-phase, 25 kVA, 2200/220 V transformer has been tested to determine its equivalent circuit.

Open-circuit test (on secondary side)	**Short-circuit test (on primary side)**
$V_{OC} = 220$ V	$V_{SC} = 72.74$ V
$I_{OC} = 0.88$ A	$I_{SC} = 11.36$ A
$P_{OC} = 107.56$ W	$P_{SC} = 516.2$ W

Find the equivalent circuit (values of parameters R_C, X_M, R_{eq} and X_{eq}) of this transformer referred to the high-voltage side (primary) of the transformer as shown in Fig. 3.



$$R_{eq} = R_P + a^2 R_S$$
$$X_{eq} = X_P + a^2 X_S$$

Figure 3: Equivalent circuit of the transformer referred to its primary side.

Solutions

a) **Open circuit test:** referred to the low-voltage or secondary side.

$$\left\| \frac{1}{Z_e} \right\| = \left\| \frac{a^2}{R_C} - j\frac{a^2}{X_M} \right\| = \frac{I_{OC}}{V_{OC}}$$
$$\Rightarrow \left\| \frac{1}{Z_e} \right\| = \frac{0.88}{220} = 0.004\ \Omega^{-1}$$

The angle θ of the complex impedance is found by the following method:

$$\text{PF} = \cos\theta = \frac{P_{OC}}{V_{OC} I_{OC}}$$
$$\Rightarrow \theta = \cos^{-1}\left(\frac{P_{OC}}{V_{OC} I_{OC}}\right) = \cos^{-1}\left(\frac{107.56}{220 * 0.88}\right) = 56.25^\circ$$

Therefore:

$$\frac{a^2}{R_C} - j\frac{a^2}{X_M} = \frac{I_{OC}}{V_{OC}}\angle - \theta = 0.004\angle - 56.25^\circ = 0.00222 - j0.00333\ \Omega^{-1}$$

$$\Rightarrow \begin{cases} \boldsymbol{R_C} & = \frac{10^2}{0.00222} \boldsymbol{= 44998\ \Omega} \\ \boldsymbol{X_M} & = \frac{10^2}{0.00333} \boldsymbol{= 30067\ \Omega} \end{cases}$$

Short circuit test: referred to the high voltage or primary side.

$$\|Z_{eq}\| = \|R_{eq} + jX_{eq}\| = \frac{V_{SC}}{I_{SC}}$$

$$\Rightarrow \|Z_{eq}\| = \frac{72.74}{11.36} = 6.403\ \Omega$$

The angle θ_2 of the complex impedance is found by the following method:

$$\cos\theta_2 = \frac{P_{SC}}{V_{SC}I_{SC}}$$

$$\Rightarrow \theta_2 = \cos^{-1}\left(\frac{P_{SC}}{V_{SC}I_{SC}}\right) = \cos^{-1}\left(\frac{516.2}{72.74 * 11.36}\right) = 51.34^\circ$$

Therefore:

$$R_{eq} + jX_{eq} = \frac{V_{SC}}{I_{SC}}\angle\theta_2 = 6.403\angle 51.34^\circ = 4 + j5\ \Omega$$

$$\Rightarrow \begin{cases} \boldsymbol{R_{eq}} & \boldsymbol{= 4\ \Omega} \\ \boldsymbol{X_{eq}} & \boldsymbol{= 5\ \Omega} \end{cases}$$