Tutorial 07: AC Machinery Fundamentals

Submission in Canvas via Matlab Grader. Approved if the score is greater than 40%

Suggested Solutions

Problem 3-1*

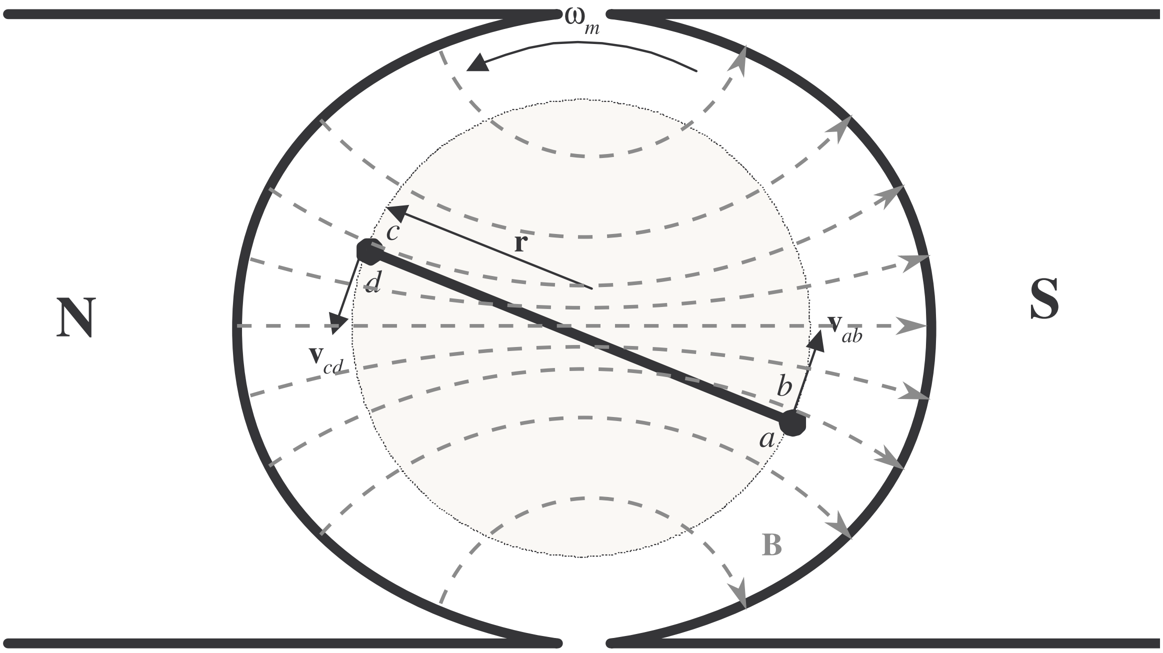

Figure 1: $\boldsymbol{B}$ is a radial uniform magnetic field, aligned as shown.

The simple loop rotating in a radial uniform magnetic field shown in Fig. 1 has the following characteristics: $\boldsymbol{B} = 1.0$ T to the right, $r = 0.1$ m, $l = 0.3$ m, $\omega_m = 377$ rad/s

a) Calculate the voltage $e_{\text{tot}}(t)$ induced in this rotating loop.

b) What is the frequency f in [Hz] of the voltage produced in this loop?

c) Suppose that a 10Ω resistor is connected as a load across the terminals of the loop. Calculate the current that would flow through the resistor.

d) Calculate the magnitude and direction of the induced torque τ_{ind} on the loop for the condition in *c*).

e) Calculate the instantaneous and average electric power, p_{el} and P_{el} respectively, being generated by the loop for conditions in *c*).

f) Calculate the mechanical power P_{mech} being consumed by the loop for the conditions in *c*). How does this number compare to the amount of electric power being generated by the loop?

Solutions

a) The induced voltage on a simple rotating loop is given by

$$e_{\text{ind}}(t) = \pm 2r\omega Bl$$
$$e_{\text{ind}}(t) = \pm 22.62 \text{ V}$$

b) Since this is a two pole machine, the frequency of the voltage produced in this loop is equal to the mechanical frequency of the rotor, i.e. $f = \omega/(2\pi) = 60$ [Hz].

c) If a $R = 10\Omega$ resistor is connected as a load across the terminals of the loop, the current flow would be:

$$i(t) = \frac{e_{\text{ind}}}{R} = \pm 2.262 \text{ A}$$

d) The induced torque would be:

$$\tau_{\text{ind}}(t) = 2rilB$$
$$\tau_{\text{ind}}(t) = 0.136 \text{ N.m, clockwise}$$

e) The instantaneous power generated by the loop is:

$$p(t) = e_{\text{ind}} i = (22.62) \cdot (2.262) = 51.17 \text{ W} \tag{1}$$

The average power generated by the loop is:

$$P_{\text{ave}} = \frac{1}{T}\int_T 51.17 dt = 51.17 \text{ W} \tag{2}$$

f) The mechanical power being consumed by the loop is:

$$P = \tau_{\text{ind}}\omega = (0.136) \cdot 377 = 51.17 \text{ W}$$

Note that the amount of mechanical power consumed by the loop is equal to the amount of electrical power created by the loop. This machine is acting as a generator, converting mechanical power into electrical power.

Problem 3-2**

Assume the field distribution produced by the stator in the machine shown in Fig.2 to be radially uniform. The magnitude of the air gap flux density is B_s, the rotor length is l with radius r, and the rotational speed of the motor is ω_m.

a) Calculate the amplitude of the emf $e_{11'}$ induced in the coil for the values of i_a: 0 A and 10 A.

b) Which plot of $e_{11'}$ versus θ is correct in Fig.3?

c) In the position shown, the current i_a in the coil $11'$ equals I_0. Calculate the torque acting on the coil in this position for two values of instantaneous speed ω_m: 0 rad/s and 100 rad/s.

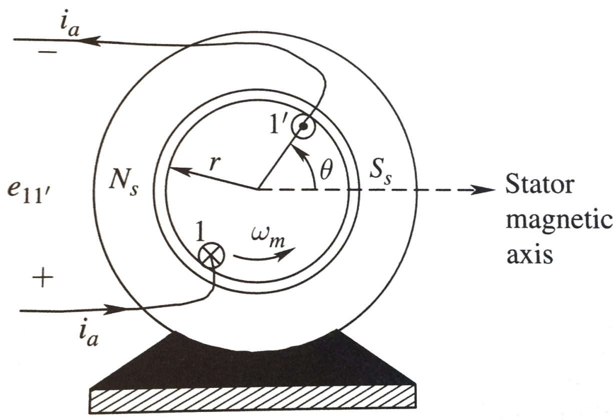

Figure 2: Problem 6.1

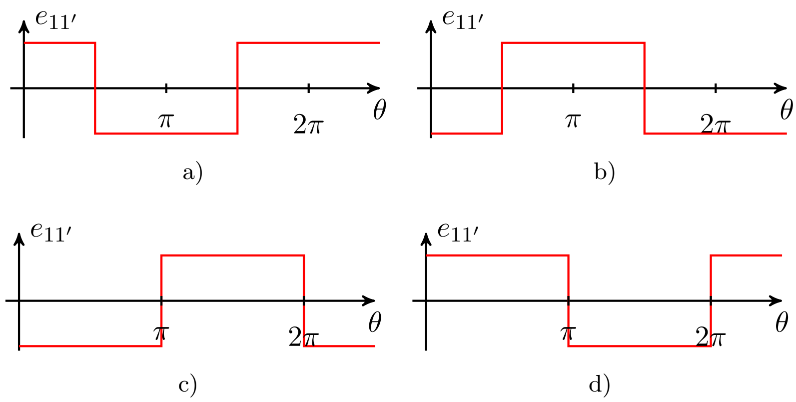

Figure 3: $e_{11'}$ versus θ

Solutions

a) The magnitude of the emf is given by:

$$emf = B_s\, 2l\, u = B_s\, 2l\, r\, \omega_m$$

and is independent of the current i_a.

b) The sign of the emf is positive when a positive charge accumulates on the side indicated with "+" sign on Fig.2. The direction where the positive charge accumulates is given by the cross-product:

$$\vec{f}_q = q(\vec{u} \times \vec{B})$$

Therefore, in the θ position shown in Fig.2, the direction of the positive charge, shown in Fig.4 is toward the "+" sign and the emf is positive.

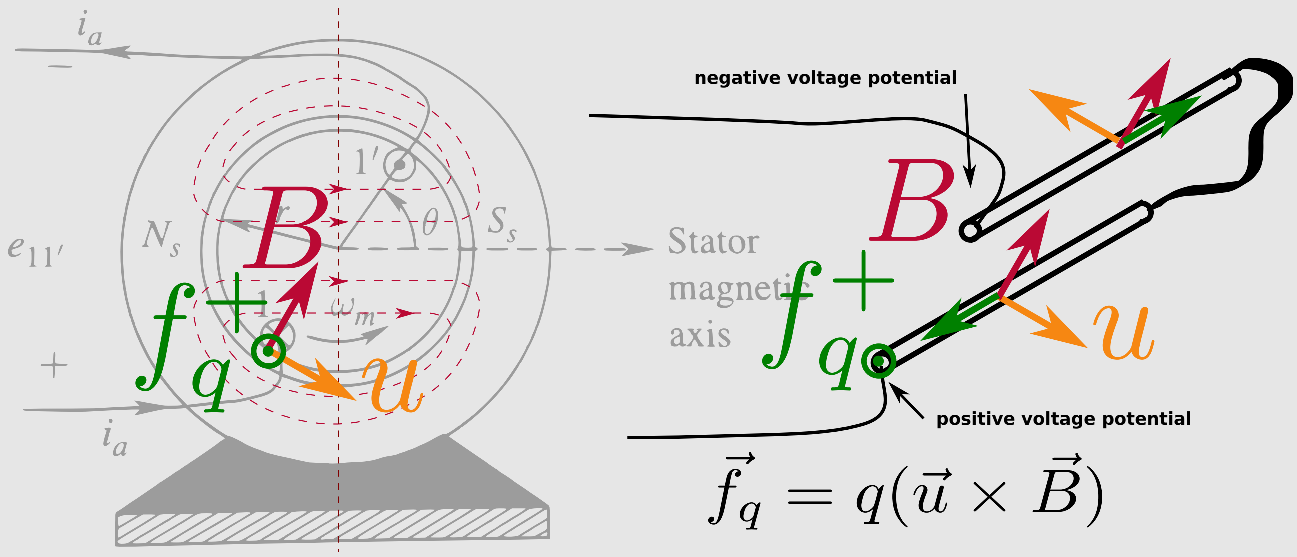

Figure 4: In this θ configuration the emf is positive.

When θ crosses $\frac{\pi}{2}$ or $-\frac{\pi}{2}$, the direction of $\vec{B}$ is reversed and the emf changes sign. Finally, the plot of emf $e_{11'}$ as a function of θ is shown in Fig.5.

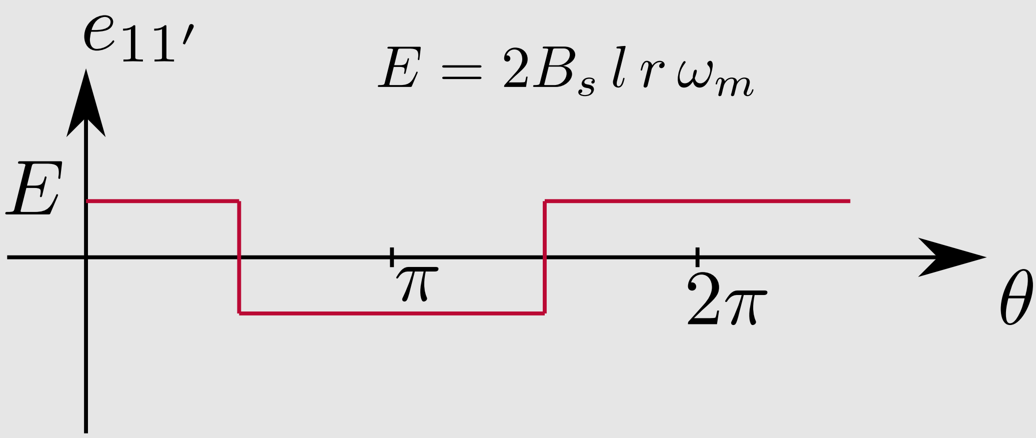

Figure 5: Emf $e_{11'}$ induced in the coil as a function of θ.

c) The torque acting on the coil in the position shown in Fig.2 is given by:

$$T_{\text{em}} = 2B_s\, I_0\, l\, r$$

and is independent of the speed ω_m.

Problem 3-3*: Three-Phase Circuits

A balanced three-phase inductive load is supplied in steady state by a balanced three-phase voltage source with a phase voltage of 120 V rms. The load draws a total of 10 kW at a power factor of 0.85.

a) Calculate the rms value of the phase currents.

b) Calculate the magnitude of the per-phase load impedance, assuming a wye-connected load.

c) Which of the following phasor diagrams shown in Fig.6 is correct?

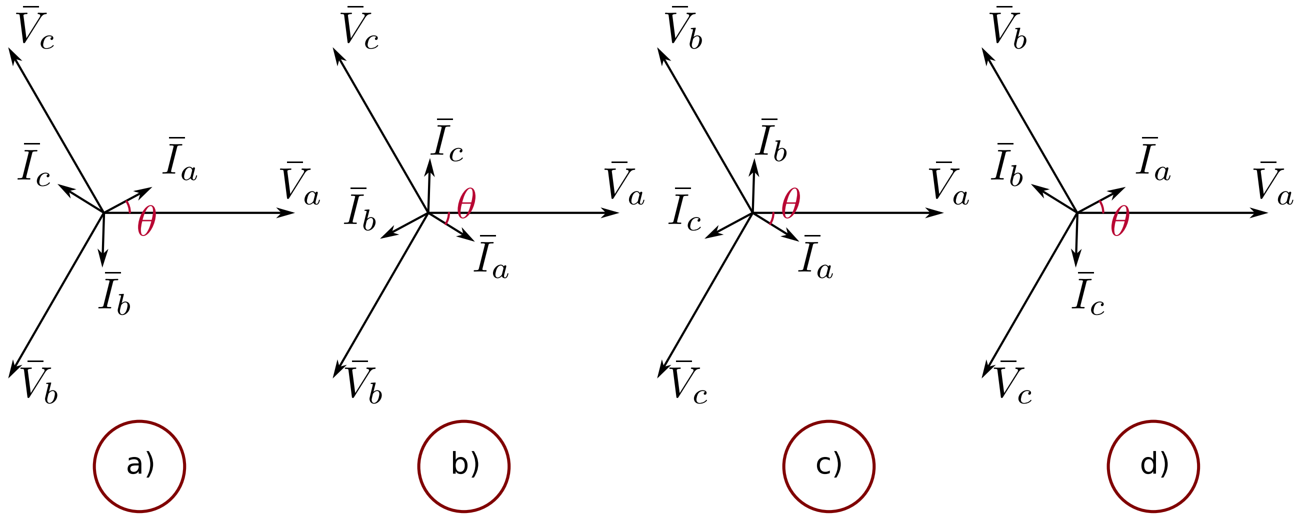

Figure 6: Phasor diagram

Solutions
using a per-phase analysis:

$$|S| = \frac{P}{\text{PF}} = \frac{10 * 10^3}{0.85} = 11765 \text{ VA}$$

$$|S| = 3VI \quad \Rightarrow \quad I = \frac{|S|}{3V} = 32.68 \text{ A}$$

$$|S| = 3|Z|I^2 \quad \Rightarrow \quad |\boldsymbol{Z}| = \frac{|S|}{3I^2} \boldsymbol{= 3.68\,\Omega}$$

With $\theta = \arccos(\text{PF}) = 31.8°$ the phasor diagram 2 shown in Fig.6 is correct because:

- The current is delayed compared to the voltage since the load is inductive.
- Phasors in phase b are $-120°$ from phasors in phase a. The positive direction is counter-clockwise.

Problem 3-4*: Space vectors

In a three-phase, 2-pole ac machine with $N_s = 100$, $l_g = 1$ mm, assume that the neutral of the wye-connected stator windings is accessible. The rotor is electrically open-circuited. The phase-a is applied a current $i_a(t) = 10\sin(\omega t)$.

a) Calculate $\vec{B}_a$ at the following instants of ωt: 0, 90, 135, and 210 degrees.

b) Which of the following two $B_a(\theta)$ distributions shown in Fig.7 at these instants is correct?

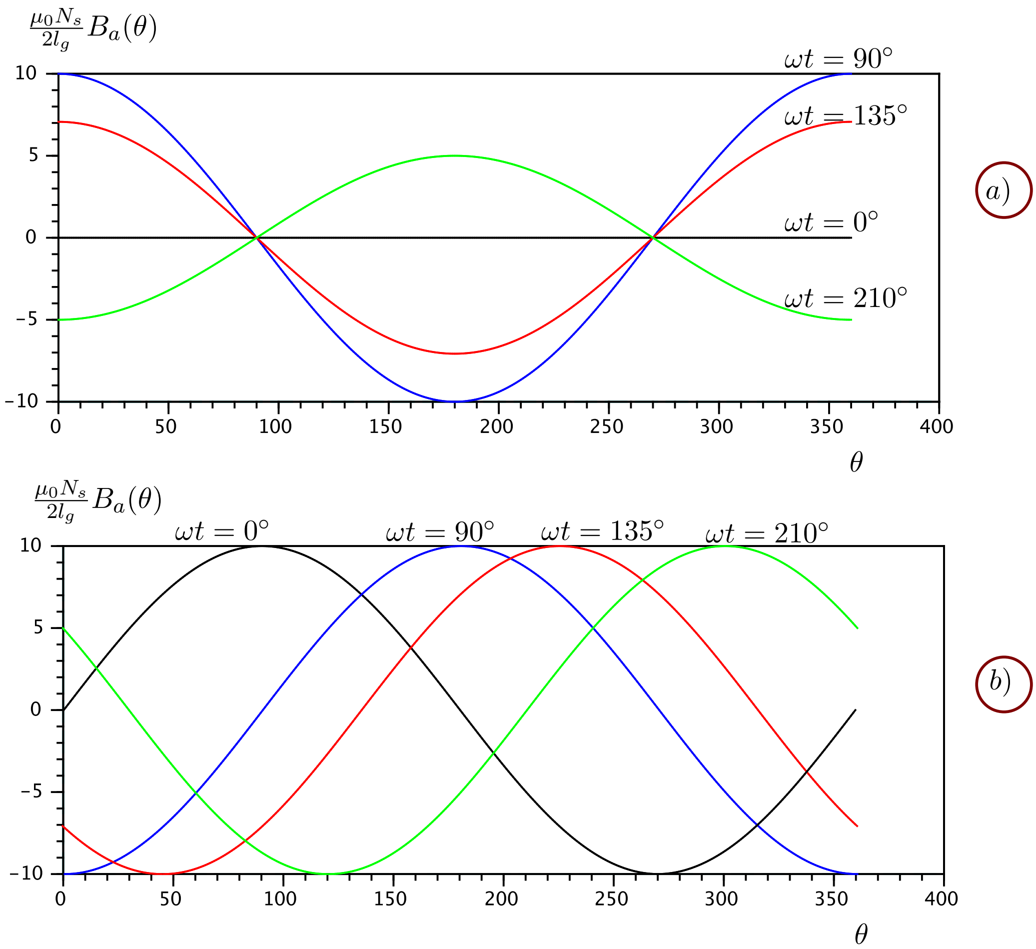

Figure 7: Field distributions at different instants.

Solutions

$$\vec{B}_a(t) = \frac{\mu_0 N_s}{2l_g} i_a(t) \angle 0° = \frac{\mu_0 N_s}{2l_g} 10 \sin(\omega t) \angle 0°$$

$$\begin{cases} \omega t = 0° : & \vec{B}_a(t) = 0 \\ \omega t = 90° : & \vec{B}_a(t) = 10 \frac{\mu_0 N_s}{2l_g} \angle 0° \\ \omega t = 135° : & \vec{B}_a(t) = 7.07 \frac{\mu_0 N_s}{2l_g} \angle 0° \\ \omega t = 210° : & \vec{B}_a(t) = -5 \frac{\mu_0 N_s}{2l_g} \angle 0° \end{cases}$$

The $B_a(\theta)$ distributions at these instants are shown in Fig.8.

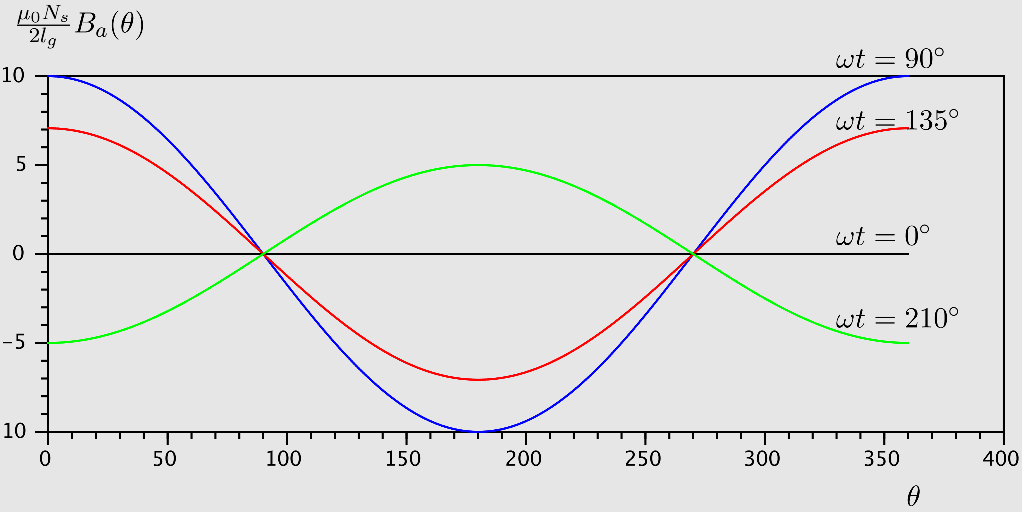

Figure 8: Field distributions at different instants.

Problem 3-5**: Stator Sinusoidally-distributed Windings in AC Machines

In a 2-pole, three-phase ac machine, $l_g = 1.5$ mm and $N_s = 100$. During a balanced, sinusoidal, 60 Hz steady state with the rotor electrically open-circuited, the peak of the magnetising current in each phase is 10 A. Assume that at $t = 0$, the phase-a current is at its positive peak.

a) Calculate the flux-density distribution space vector as a function of time.

b) What is the speed of its rotation in [rpm]?

c) what would be the speed of rotation if the machine had 6 poles?

Solutions

a) flux-density: $\vec{B}_{ms}$ 'm' subscript indicates magnetising flux-density.

$$\vec{B}_{ms} = \frac{\mu_0 N_s}{2 l_g} \frac{3}{2} \hat{I}_m \angle \omega t = 0.628 \angle 377t$$

b) The rotational speed:

$$\omega_{\text{sm}} = \omega = 2\pi f = 377 \text{ [rad/s]}$$

or, in rpm:

$$n_{\text{sm}} = \omega * 60/(2\pi) = 3600 \text{ [rpm]}$$

If it were to assume that the phase-a current is at its positive peak at time $t = \frac{\alpha}{\omega}$:

$$\vec{B}_{ms} = \frac{\mu_0 N_s}{2 l_g} \frac{3}{2} \hat{I}_m \angle (\omega t - \alpha)$$

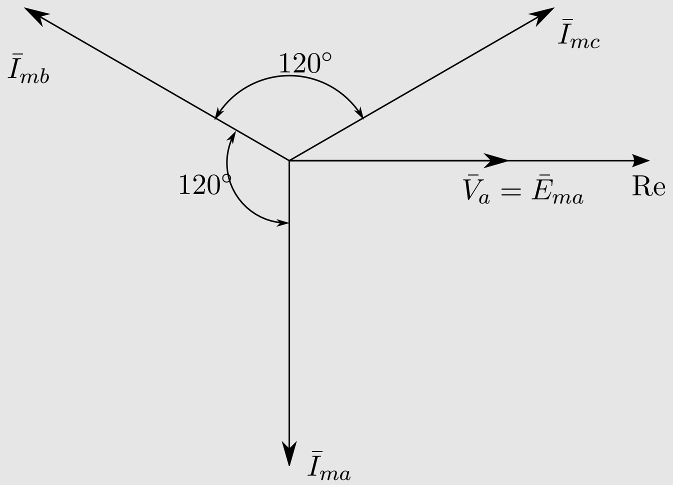

Figure 9: Current phasors

c)

$$\omega_{\text{sm}} = \frac{\omega}{P/2} = \frac{377}{6/2} = 125.7 \text{ rad/s}$$
$$\Rightarrow n_{sm} = 1200 \text{ rpm}$$

Problem 3-6**

In a three-phase ac machine, $\bar{V}_a = 120\sqrt{2}\angle 0°$ V. The magnetising inductance $L_m = 75$ mH. Calculate and draw the three magnetising current phasors. Assume a balanced, sinusoidal, three-phase steady-state operation at 60 Hz. Neglect the resistance and the leakage inductance of the stator phase windings.

Solutions

$$\bar{I}_{ma} = \frac{\bar{E}_{ma}}{j\omega L_m} = \frac{\bar{V}_a}{j\omega L_m} = \frac{120\sqrt{2}}{2\pi * 60 * 75 * 10^{-3}}\angle -90° = 4.24\sqrt{2}\angle -90° \text{ A}$$

The current phasors are shown in Fig.9.

Problem 3-7***: 3-Phase Generators

A three-phase, Y-connected, four-pole winding is installed in 24 slots on a stator. There are 40 turns of wire in each slot of the windings. All coils in each phase are connected in series. The flux per pole in the machine is 0.060 Wb, and the speed of rotation of the magnetic field is 1800 rpm.

a) What is the frequency of the voltage produced in this winding?

b) What are the resulting phase and terminal voltages of this stator?

Solutions

a) The frequency of the voltage produced in this winding is:

$$f_{se} = \frac{n_{sm}P}{120} = \frac{1800 * 4}{120} = 60 \text{ Hz}$$

b) There are 24 slots on this stator, with 40 turns of wire per slot. Since this is a four-pole machine, there are four sets of coils (in 8 slots) associated with each phase. The voltage in the coils in *one pair* of slots is

$$E_A = \sqrt{2}\pi N_C \phi f = \sqrt{2} * \pi * 40 * 0.06 * 60 = 640V$$

There are eight slots associated with each phase, and all the coils in each phase are connected in series, so the total phase voltage is:

$$V_{ph} = 4 * 640 = 2560 \text{ V}$$

Since the machine is Y-connected, $V_{LL} = \sqrt{3}V_{\phi} = 4434$ V.

Problem 3-8*: Induced Torque

If an ac machine has the rotor and stator magnetic fields shown in Fig.10.

a) What is the direction of the induced torque in the machine?

b) Is the machine acting as a motor or a generator?

Solutions
Since $\tau_{\text{ind}} = k(\vec{B}_R \times \vec{B}_{net})$, the induced torque is clockwise, opposite the direction of motion. The machine is acting as a generator.

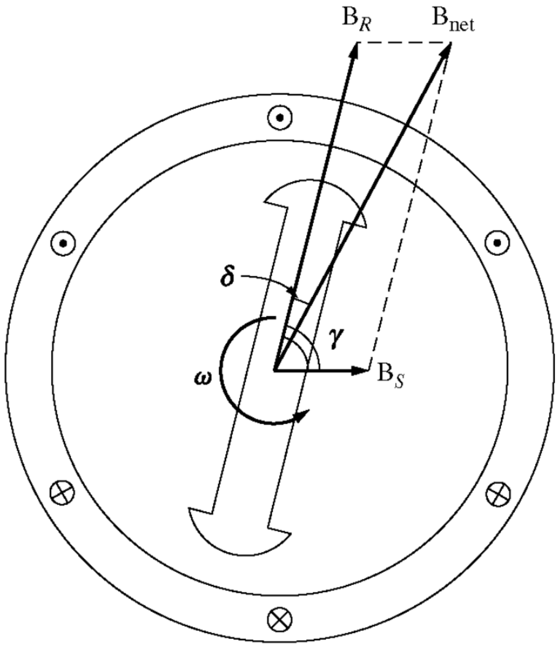

Figure 10: The ac machine of Problem 6