

Derivatives

D_x e^x = e^x
D_x sin(x) = cos(x)
D_x cos(x) = -sin(x)
D_x tan(x) = sec^2(x)
D_x cot(x) = -csc^2(x)
D_x sec(x) = sec(x)tan(x)
D_x csc(x) = -csc(x)cot(x)
D_x sin^-1 = 1/sqrt(1-x^2), x in [-1, 1]
D_x cos^-1 = -1/sqrt(1-x^2), x in [-1, 1]
D_x tan^-1 = 1/(1+x^2), -pi/2 <= x <= pi/2
D_x sec^-1 = 1/(|x|sqrt(x^2-1)), |x| > 1
D_x sinh(x) = cosh(x)
D_x cosh(x) = sinh(x)
D_x tanh(x) = sech^2(x)
D_x coth(x) = -csch^2(x)
D_x sech(x) = -sech(x)tanh(x)
D_x csch(x) = -csch(x)coth(x)
D_x sinh^-1 = 1/sqrt(x^2+1)
D_x cosh^-1 = -1/sqrt(x^2-1), x > 1
D_x tanh^-1 = 1/(1-x^2), -1 < x < 1
D_x sech^-1 = 1/(x*sqrt(1-x^2)), 0 < x < 1
D_x ln(x) = 1/x

Integrals

int 1/x dx = ln|x| + c
int e^x dx = e^x + c
int a^x dx = 1/ln(a) a^x + c
int e^ax dx = 1/a e^ax + c
int 1/sqrt(1-x^2) dx = sin^-1(x) + c
int 1/(1+x^2) dx = tan^-1(x) + c
int 1/x*sqrt(x^2-1) dx = sec^-1(x) + c
int sinh(x) dx = cosh(x) + c
int cosh(x) dx = sinh(x) + c
int tanh(x) dx = ln|cosh(x)| + c
int tanh(x)sech(x) dx = -sech(x) + c
int sech^2(x) dx = tanh(x) + c
int csch(x) coth(x) dx = -csch(x) + c
int tan(x) dx = -ln|cos(x)| + c
int cot(x) dx = ln|sin(x)| + c
int cos(x) dx = sin(x) + c
int sin(x) dx = -cos(x) + c
int 1/sqrt(a^2-u^2) dx = sin^-1(u/a) + c
int -1/(a^2+u^2) dx = 1/a tan^-1(u/a) + c
int ln(x) dx = (xln(x)) - x + c

U-Substitution

Let u = f(x) (can be more than one variable).
Determine: du = f'(x) dx and solve for dx.
Then, if a definite integral, substitute the bounds for u = f(x) at each bound
Solve the integral using u.

Integration by Parts

int udv = uv - int vdu

Fns and Identities

sin(cos^-1(x)) = sqrt(1-x^2)
cos(sin^-1(x)) = sqrt(1-x^2)

sec(tan^-1(x)) = sqrt(1+x^2)
tan(sec^-1(x)) = sqrt(x^2-1) if x >= 1
= (-sqrt(x^2-1) if x <= -1)
sinh^-1(x) = ln(x + sqrt(x^2+1))
sinh^-1(x) = ln(x + sqrt(x^2-1)), x >= 1
tanh^-1(x) = 1/2 ln(x + 1/(1-x)), 1 < x < -1
sech^-1(x) = ln[1+sqrt(1-x^2)/x], 0 < x <= 1
sinh(x) = (e^x - e^-x)/2
cosh(x) = (e^x + e^-x)/2

Trig Identities

sin^2(x) + cos^2(x) = 1
1 + tan^2(x) = sec^2(x)
1 + cot^2(x) = csc^2(x)
sin(x +/- y) = sin(x)cos(y) +/- cos(x)sin(y)
cos(x +/- y) = cos(x)cos(y) +/- sin(x)sin(y)
tan(x +/- y) = (tan(x) +/- tan(y)) / (1 +/- tan(x)tan(y))
sin(2x) = 2sin(x)cos(x)
cos(2x) = cos^2(x) - sin^2(x)
cosh^2(x) - sinh^2(x) = 1
1 + tan^2(x) = sec^2(x)
1 + cot^2(x) = csc^2(x)
sin^2(x) = (1-cos(2x))/2
cos^2(x) = (1+cos(2x))/2
tan^2(x) = (1-cos(2x))/(1+cos(2x))
sin(-x) = -sin(x)
cos(-x) = cos(x)
tan(-x) = -tan(x)

Calculus 3 Concepts

Cartesian coords in 3D

given two points:
(x1, y1, z1) and (x2, y2, z2),
Distance between them:
sqrt((x1-x2)^2 + (y1-y2)^2 + (z1-z2)^2)
Midpoint:
((x1+x2)/2, (y1+y2)/2, (z1+z2)/2)
Sphere with center (h,k,l) and radius r:
(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2

Vectors

Vector: u
Unit Vector: u-hat
Magnitude: ||u|| = sqrt(u1^2 + u2^2 + u3^2)
Unit Vector: u-hat = u/||u||

Dot Product

u . v
Produces a Scalar
(Geometrically, the dot product is a vector projection)
u = <u1, u2, u3>
v = <v1, v2, v3>
u . v = 0 means the two vectors are Perpendicular
theta is the angle between them.
u . v = ||u|| ||v|| cos(theta)
u . v = u1v1 + u2v2 + u3v3
NOTE:
u . v = cos(theta)
||u||^2 = u . u
u . v = 0 when perpendicular
Angle Between u and v:
theta = cos^-1((u . v) / (||u|| ||v||))

Projection of u onto v:
pr_v u = (u . v / ||v||^2) v
Cross Product
u x v
Produces a Vector
(Geometrically, the cross product is the area of a parallelogram with sides ||u|| and ||v||)
u = <u1, u2, u3>
v = <v1, v2, v3>

u x v = | i j k |
 u1 u2 u3
 v1 v2 v3

u x v = 0 means the vectors are parallel

Lines and Planes

Equation of a Plane
(x0, y0, z0) is a point on the plane and
<A, B, C> is a normal vector

A(x-x0) + B(y-y0) + C(z-z0) = 0
<A, B, C> . <x-x0, y-y0, z-z0> = 0
Ax + By + Cz = D where
D = Ax0 + By0 + Cz0

Equation of a line
A line requires a Direction Vector
u = <u1, u2, u3> and a point (x1, y1, z1)
then,
a parameterization of a line could be:
x = u1t + x1
y = u2t + y1
z = u3t + z1

Distance from a Point to a Plane
The distance from a point (x0, y0, z0) to a plane Ax+By+Cz=D can be expressed by the formula:
d = |Ax0+By0+Cz0-D| / sqrt(A^2+B^2+C^2)

Coord Sys Conv

Cylindrical to Rectangular

x = r cos(theta)
y = r sin(theta)
z = z

Rectangular to Cylindrical

r = sqrt(x^2 + y^2)
tan(theta) = y/x
z = z

Spherical to Rectangular

x = rho sin(phi) cos(theta)
y = rho sin(phi) sin(theta)
z = rho cos(phi)

Rectangular to Spherical

rho = sqrt(x^2 + y^2 + z^2)
tan(phi) = y/x
cos(phi) = z / sqrt(x^2+y^2+z^2)

Spherical to Cylindrical

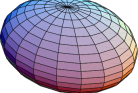
r = rho sin(phi)
theta = theta
z = rho cos(phi)

Cylindrical to Spherical

rho = sqrt(r^2 + z^2)
theta = theta
cos(phi) = z / sqrt(r^2+z^2)

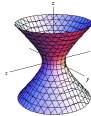
Surfaces

Ellipsoid
x^2/a^2 + y^2/b^2 + z^2/c^2 = 1



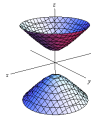
Hyperboloid of One Sheet

x^2/a^2 + y^2/b^2 - z^2/c^2 = 1
(Major Axis: z because it follows -)



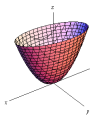
Hyperboloid of Two Sheets

z^2/a^2 - x^2/b^2 - y^2/c^2 = 1
(Major Axis: Z because it is the one not subtracted)



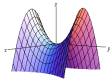
Elliptic Paraboloid

z = x^2/a^2 + y^2/b^2
(Major Axis: z because it is the variable NOT squared)



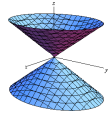
Hyperbolic Paraboloid

(Major Axis: Z axis because it is not squared)
z = y^2/b^2 - x^2/a^2



Elliptic Cone

(Major Axis: Z axis because it's the only one being subtracted)
x^2/a^2 + y^2/b^2 - z^2/c^2 = 0



Cylinder

1 of the variables is missing
OR
(x-a)^2 + (y-b)^2 = c
(Major Axis is missing variable)

Partial Derivatives

Partial Derivatives are simply holding all other variables constant (and act like constants for the derivative) and only taking the derivative with respect to a given variable.

Given z=f(x,y), the partial derivative of z with respect to x is:

f_x(x,y) = z_x = partial z / partial x = partial f(x,y) / partial x

likewise for partial with respect to y:

f_y(x,y) = z_y = partial z / partial y = partial f(x,y) / partial y

Notation

For f_xyy, work "inside to outside" f_x then f_xy, then f_xyy
f_xyy = partial^3 f / partial^2 y partial x,
For partial^3 f / partial^2 y partial x, work right to left in the denominator

Gradients

The Gradient of a function in 2 variables is nabla f = <f_x, f_y>
The Gradient of a function in 3 variables is nabla f = <f_x, f_y, f_z>

Chain Rule(s)

Take the Partial derivative with respect to the first-order variables of the function times the partial (or normal) derivative of the first-order variable to the ultimate variable you are looking for summed with the same process for other first-order variables this makes sense for. Example:

let x = x(s,t), y = y(t) and z = z(x,y).
z then has first partial derivative:
partial z / partial x and partial z / partial y
x has the partial derivatives:
partial x / partial s and partial x / partial t
and y has the derivative:
dy/dt

In this case (with z containing x and y as well as x and y both containing s and t), the chain rule for partial z / partial s is partial z / partial s = partial z / partial x * partial x / partial s

The chain rule for partial z / partial t is partial z / partial t = partial z / partial x * partial x / partial t + partial z / partial y * dy/dt
Note: the use of "d" instead of "partial" with the function of only one independent variable

Limits and Continuity

Limits in 2 or more variables
Limits taken over a vectorized limit just evaluate separately for each component of the limit.

Strategies to show limit exists
1. Plug in Numbers, Everything is Fine
2. Algebraic Manipulation

-factoring/dividing out
-use trig identities
3. Change to polar coords
if(x,y) -> (0,0) <=> r -> 0

Strategies to show limit DNE
1. Show limit is different if approached from different paths (x=y, x=y^2, etc.)
2. Switch to Polar coords and show the limit DNE.

Continuity
A fn, z = f(x,y), is continuous at (a,b) if

f(a,b) = lim(x,y) -> (a,b) f(x,y)
Which means:
1. The limit exists
2. The fn value is defined
3. They are the same value

