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A study of storage assignment problem for an order picking line in a pick-and-pass warehousing system

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ABSTRACT

Order picking is one of the most crucial factors for efficient warehouse management. Most of previous research in order picking in a warehouse considered only the picker-to-part or part-to-picker system; however, the pick-and-pass system plays an increasingly important role because of the growth of e-commerce and time-based competition. This paper develops an analytical model for the pick-and-pass system by describing the operation of a picker as a Markov Chain for the estimation of the expected distance traveled of the picker in a picking line. Based on the proposed analytical model, this study derives properties of storage assignment and proposes three algorithms that optimally allocate items to storages for the cases of a single picking zone, a picking line with unequal-sized zones, and a picking line with equal-sized zones in a pick-and-pass system.

1. Introduction

Order picking is a major costly constituent of the warehouse operations. According to Coyle, Bardi, and Langley (1996), 50–75% of the total operating costs of a warehouse are attributed to order picking operation. Consequently, order picking has been one of the major study topics over the past several decades and most researches have focused mainly on the development of travel time models or travel distance models for various storage assignments, picking routing and order batching policies.

Storage assignment policies aim to provide ways for allocating items in an order picking warehouse so that order picking time will be reduced. Petersen (1997) divided storage assignment policies into three broad categories: randomized storage, volume-based storage and class-based storage. The random storage policy is widely used in many warehouses because it is simple to use, often requires less space than other storage methods, and results in a better level of utilization of all picking aisles (Petersen and Gerald, 2004).

Volume-based storage policies distribute items based on their demand rates or volumes to locations. Heskett (1963) proposed the COI (cube-per-order index) assignment policy by assigning items with the lowest ration of the required storage space to the order frequency to the locations nearest to the input/output (I/O) point. A few studies examined COI assignment policy in picker-to-part systems. When a picker travels to storage locations to retrieve orders, the system is referred to as a picker-to-part system. Caron, Marchet, and Perego (1998) evaluated and compared different routing policies with storage assignment based on COI. The result demonstrates that in COI-based storage systems the traversal policy outperforms the basic return policy for the range of pick densities and the COI-based ABC curves most frequently found in real-world applications. Caron, Marchet, and Perego (2000) presented an analytical approach to layout design of the picking area in picker-to-part systems using COI-based and random storage policies. The results show that layout preference seems to be strongly affected by decisions concerning the adoption of a COI-based storage policy. Additionally, Kallina and Lynn (1976) discussed some practical conclusions gathered form experience in applying the COI rule in warehouse layout. Hwang et al. (2003) proposed the density-turnover index (DTI) rule with the objective of minimizing the energy consumption of the material handler working in a picker-to-part warehouse system. The DTI rule is based on the weight, space requirement, demand rate of items and the effective distance measure of storage location. Hwang et al. (2003) shows that the DIT rule is substantially better than the COI rule from the view point of human safety with some sacrifice of throughput.

Jarvis and McDowell (1991) developed the necessary and sufficient conditions for optimally allocating items in a class of symmetric warehouses. Simply assigning the most frequently picked items to the nearest aisles will not necessarily minimize the average travel distance if the aisles are not symmetrical. Frazelle and Sharp (1989) showed that the storage assignment problem is in the class of NP-hard problems for which the optimal solution is computationally infeasible to obtain in problems of practical sizes, and thus proposed a heuristic procedure based on the demand dependency between items.

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Nomenclature

n	the number of zones in a picking line
k_z	the number of bins in zone z, $z = 1, 2, \ldots, n$
q	the number of racks in a bin
v_z	the traveling speed of the picker in zone z (bins/unit time)
h_p	the probability that item p to be picked
r_i	the probability that at least one item is picked from bin i
P_{ij}	the probability that a picker moves to bin j from bin i for a pick
P^t_{ij}	the probability that a picker moves to bin j from bin i after finishing t orders
π_i	the stationary probabilities that a picker stays at bin i upon the completion of an order
d_{ij}	the travel distance from bin j to bin i in a tour
d_m	the travel distance given the picker starts a the pickings of an order from bin m in a tour
C	the total travel time of a picker in the picking line
D_z	the travel distance of an individual picker in zone z to complete an order, $z = 1, 2, \ldots, n$
X_t	a random variable denoting the starting position of picking for the tth order, or equivalently, the position of the last item in the $(t-1)$th order

Class-based storage policies divide items into classes and assign a fixed area to each class and then use randomized storage allocation within each class area. Bynzer and Johansson (1996) provided a storage assignment strategy emanating from the product structure to shorten the picking time by using variant characteristics as picking information in the construction of a logical assignment policy. Eynan and Rosenblatt (1994) developed an algorithm involving a one-dimensional search for deriving the boundaries for any desired number of classes in an automated warehouse. Results show that this one-dimensional search procedure is very effective in solving most practical problems.

Schwarz, Graves, and Hausman (1978) inspected the performance of an automatic warehouse system through a specific storage policy that depends on item picking frequency. Hsieh and Tsai (2001) presented a bill of material (BOM) oriented class-based storage assignment method for an automated storage/retrieval system (AS/RS). The proposed method possesses not only the advantage of a class-based storage method, but also the feasibility to integrate an AS/RS into a computer integrated manufacturing (CIM) system. Larson, March, and Kusiak (1997) presented a procedure for warehouse layout by employing the principles of class-based storage to increase floor space utilization and reduce material handling efforts.

Petersen and Gerald (2004) analyzed the effects of picking, storage and routing decisions on order picker traveling and concluded that the use of either a class-based or volume-based storage policy provides nearly the same level of savings as batching consideration. Gibson and Sharp (1992) demonstrated that significant reductions in distance can be achieved through locating high frequency items close to the I/O point. Ruben and Jacobs (1999) developed batch construction heuristics and found that the methods used for constructing batches of orders and for assigning storage space to individual items have significantly impact on order retrieval efforts in a warehouse.

Most previous research studied the storage policy for AS/RS or picker-to-part systems while few considered the pick-and-pass systems. An order picking line can be divided into several zones and each zone is assigned a picker in it. After finishing the pickings in the zone, the picker hands the container with picked items to the next picker, who continues the assembly of the order. Therefore an order is only finished after having visited all relevant zones. This system is referred to as a pick-and-pass system (De Koster, LeDuc, & Roodbergen, 2007). In general, such a system stores small to medium-sized items such as health and beauty, household, office or food products (Maloney, 2000).

For pick-and-pass systems, De Koster (1994) developed an approximation method to evaluate the effect of changing the layout of the system. Malmborg (1995) studied the problem of assigning products to locations with zoning constraints. Jane (2000) proposed a heuristic algorithm to balance the workloads among all pickers so that the serial pick lane is a smooth one. Petersen and Gerald (2002) showed that the storage policy has a significant effect on the average travel distance within a zone. Jewkes, Lee, and Vickson (2004) developed an efficient dynamic programming algorithm to determine the optimal product allocation and picker locations for an order picking line with multiple pickers in which pickers operate from a home base location and use 'out and back' picking strategy to fill orders. Jewkes et al. (2004) developed an efficient dynamic programming algorithm to determine the optimal product allocation and picker locations for an order picking line with multiple pickers. For a synchronized zone system where all the zones process the pickings for a single order simultaneously, Jane and Laih (2005) proposed a clustering algorithm for item assignment by balancing the workload among all pickers so that the utilization of the order picking system is improved. Yu and De Koster (2007) presented an approximation method based on $G/G/m$ queueing network modeling to analyze pick-and-pass order picking system. The approximation method appears to be sufficiently accurate for practical stations.

The objective of this paper is to present algorithms that find optimal item storage assignments in a picking line for a pick-and-pass system by considering the three different cases of zone (or single picker), a picking line (or multiple pickers) with equal-sized zones, and a picking line with unequal-sized zone. The operation of a picker in a picking line is modeled as a Markov chain in order to calculate the expected distance traveled of the picker in the picking line. The performance of the model are compared and validated with the results generated by simulation modes.

2. Description of the warehouse system and picking operations

This paper considers the layout and picking operations of a pick-and-pass system, as illustrated in Fig. 1. The picking line in the system has n pickers in which each one picks the items of an order located in the bins (vertical shelves) in his/her zone. Each bin has the same number of racks in it and only one item type can be stored in a rack. It is further assumed that there are exactly the same numbers of racks as item types. Dummy items with no demand may be created to assure this relationship holds if there are less item types than racks.

For information support system, a computer aided picking system (CAPS) or electronic paperless pick-to-light system can be implemented in practice in a pick-and-pass system. In a CAPS, light indicator modules are mounted to all racks and these modules automatically guide order pickers quickly to the picking locations, and show the exact quantity of each item to be retrieved. The advantages of the CAPS include effectively improving the picking productivity by 50% or more, and reducing the picking task error

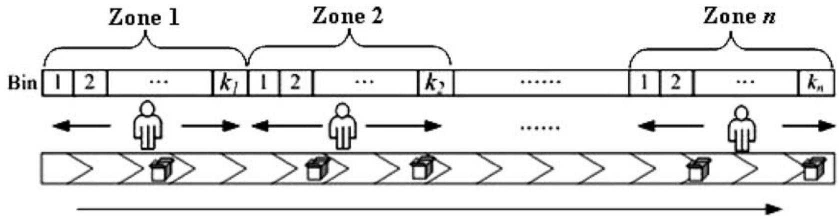

Fig. 1. A pick-and-pass system.

as well (Jane & Laih, 2005). Additionally, CAPS simplifies the training for the pickers, thus cuts down the operational cost.

In such a system, the light indicator modules can only show the data of items to be picked in one order in a picker's assigned zone at a time. The picker picks the item quantity displayed on a rack and then confirms the pick by pressing the lighted button. When he/she finishes the pickings of one order in the assigned zone, CAPS sends the data of next order to all light indicator modules in that zone. For the picking operation, it is reasonable to assume that a picker always finds the shortest travel distance for each order and can pick all items on one order in a zone in one tour where the shortest distance is found as follows.

Algorithm 1. Algorithm for shortest distance determination

Step 0. Let b be the beginning bin number, and r and l be the furthest bin numbers on the right and left sides, respectively, on the order.
Step 1. Determine the first side. If $(r - b) \leqslant (b - l)$, then right side is the first side; otherwise, left side is the first side.
Step 2. The picker goes to the furthest picking bin in the first side and then to the other side to finish the picking operation.

The shortest travel distance is $2 \times \min\{(r-b),\ (b-l)\} + \max\{(r-b), (b-l)\}$. For example, suppose the light indicator modules on bins 1, 2, 3, 4 and 6 are on and the picker is at bin 4 to start the picking operation. According to Algorithm 1, right side is the first side; so after picking the item in bin 4, the picker goes to the right side (bin 6) first. Hence, the sequence of bin traveled is 46,321 and the shortest travel distance is 7 units.

The following assumptions on the warehouse and the picking operations under study are made in the paper:

(1) A container is sufficiently large to handle all the items in a picking tour.
(2) The storage racks are long and low, i.e., the vertical travel time is negligible.
(3) The speed of a picker is constant.
(4) The time to pick an item from a rack is constant.
(5) All items of all orders have the same sizes and weights.
(6) No orders can be spread, i.e., strict picking order policy is implemented.
(7) A picker always chooses the shortest travel distance for each order.
(8) Upon the completion of current order, a picker stays at the bin of the last item on the order and starts next order from that location.
(9) Each item is independent of the others within an order.

3. Evaluation of travel distance

The picking operations of a picker in a pick-and-pass system can be modeled as a Markov chain. Since the starting position of pickings for each order in a zone is a random variable, it is necessary to calculate the stationary probability that a picker stays at bin i in a zone upon the completion of an order. First, we find the probability that a picker reaches bin j from bin i in the zone for an order fulfillment so that the transition matrix of a picker can be established. Using the transition matrix, we can find the steady probability of a picker staying at the bin i and the expected distance traveled per order can be computed accordingly.

3.1. The transition probability for order picking

Let $B_i = \{p|\text{item } p \in \text{bin } i\}$, then the probability that at least one item be picked in bin i can be expressed as

$$r_i = 1 - \prod_{p \in B_i} (1 - h_p) \tag{1}$$

The probability that X_t is at bin i, $i \in$ zone z given that the starting position of the $(t-1)$th order, X_{t-1}, is at bin m, $m \in$ zone z can be found as follows.

The variable X_t is equal to X_{t-1} if and only if all the items to be picked on the $(t-1)$th order are located in bin m. Thus,

$$P_{mm} = P\{X_t = m|X_{t-1} = m\} = \prod_{\substack{j=1 \\ j \neq m}}^{k_z} (1 - r_j) \quad \text{for } m = 1, 2, \ldots, k_z \tag{2}$$

When X_t is not equal to X_{t-1}, it is necessary to consider the position where the picker stays at the end of the $(t-1)$th order. Since the picker will certainly chooses the shortest route if ever moves to the next bin for picking, there are three situations in zone z and they can be discussed below.

Case 1. $m < k_z/2$ For $X_{t-1} = m < (k_z/2)$ and $X_t = i$ where $1 \leqslant i \leqslant 2m - 1$, the picker will not reach bin j, $j < i$ or $j > 2m - i$, because he/she always selects the shortest route. This implies that $(t-1)$th order does not include any item in bin j. When the items in bin $(2m - i)$ are listed in the $(t-1)$th order, the probability that the picker first goes to bin $(2m - i)$ then proceeds to bin i is 1/2 since the distance between bin $(2m - i)$ and bin m is equal to the distance between bin i and bin m. Thus,

$$\begin{aligned} P\{X_t = i|X_{t-1} = m\} &= r_i(1 - r_{2m-i}) \prod_{j \notin [i, 2m-i]} (1 - r_j) + r_i \frac{1}{2} r_m \prod_{j \notin [i, 2m-i]} (1 - r_j) \\ &= r_i \left(1 - \frac{1}{2} r_{2m-i}\right) \prod_{j \notin [i, 2m-i]} (1 - r_j) \quad \text{for } 1 \leqslant i \leqslant 2m - 1 \end{aligned} \tag{3}$$

If $X_{t-1} = m < (k_z/2)$ and $X_t = i$ for $2m \leqslant i \leqslant k_z$, then no matter whether there are items to be picked in bin j, $j < m$, the picker finally goes to bin i. So,

$$P\{X_t = i|X_{t-1} = m\} = r_i \prod_{j > i} (1 - r_j) \quad \text{for } 2m \leqslant i \leqslant k_z \tag{4}$$

Case 2. $m = k_z/2$. When the number of bins is odd and $X_{t-1} = (k_z/2)$, the probability that a picker first goes to bin i then goes to bin $(2m - i)$ is 1/2 for the tth order; hence,

$$P\{X_t = i | X_{t-1} = m\} = r_i\left(1 - \frac{1}{2}r_{2m-i}\right)\prod_{j \notin [i, 2m-i]}(1 - r_j) \quad \text{for } 1 \leqslant i \leqslant k_z \tag{5}$$

Case 3. $m > k_z/2$

This situation is contrary to Case 1. So, it follows that

$$P\{X_t = i | X_{t-1} = m\} \tag{6}$$

$$= \begin{cases} r_i \prod_{j<i}(1 - r_j) & 1 \leqslant i \leqslant 2m - k_z \\ r_i\left(1 - \frac{1}{2}r_{2m-i}\right)\prod_{j \notin [i, 2m-i]}(1 - r_j) & 2m - k_z < i \leqslant k_z \end{cases} \tag{7}$$

Let $\mathbf{P}$ denote the matrix of one-step transition probabilities P_{ij} where $i = 1, 2, \ldots, k_z, j = 1, 2, \ldots, k_z$, then the long-term proportion of time that a picker will be at bin i at the completion of an order is the unique non-negative solution of

$$\pi_i = \lim_{t \to \infty} p_{ji}^t = \sum_{j=1}^{k_z} \pi_j P_{ji} \quad \text{for } i = 1, 2, \ldots, k_z$$

$$\sum_{i=1}^{k_z} \pi_i = 1$$

Hence,

$$[\pi_1 \cdots \pi_k]\mathbf{P} = \begin{bmatrix} \pi_1 \\ \vdots \\ \pi_k \end{bmatrix}$$

$$\sum_{i=1}^{k} \pi_i = 1 \tag{8}$$

3.2. The expected travel distance

There are two factors that determine the travel distance of a picker in a pick-and-pass system. One is the location of starting bin of each order. The distribution of this variable can be described by Eq. (8). Another factor is the picking locations in an order. The travel distance is zero if and only if a picker's starting location is equal to the final picking location in the order. When the picker starts to process order from bin m and the terminal position is bin i, for $i \neq m$, the picker either goes to bin i while no items need to be picked except those between bin i and bin m, or to bin a where a is the farthest position in the opposite direction of bin i. Thus,

$$\begin{aligned} &\mathrm{E}[\text{travel distance from bin } m \text{ to bin } i] = \mathrm{E}[d_m i] \\ &= \sum_{a \in [L,m)}\left[(|m - i| + 2 \times |m - a|) \times r_a \times \prod_{b \in (a,L]}(1 - r_b)\right] \\ &\quad + |m - i| \times \left(\prod_{b \in [L,m)}(1 - r_b)\right) \\ &= |m - i| + 2 \times \sum_{a \in [L,m)}\left[|m - a| \times r_a \times \prod_{b \in (a,L]}(1 - r_b)\right] \end{aligned} \tag{9}$$

where

$$L = \begin{cases} \max\{(m - |m - i|), 1\} & \text{for } i > m \\ \min\{(m + |m - i|), k_z\} & \text{for } i < m \end{cases}$$

Consequently, the expected travel distance given the starting position is bin m in a tour can be expressed as

$$\mathrm{E}[d_m] = \sum_{i=1}^{k_z} \mathrm{E}[d_{mi}] \times P\{X_t = i | X_{t-1} = m\} \text{ for } m = 1, 2, \ldots, k_z \tag{10}$$

In the long-run, the average travel distance of a picker to complete an order in zone z is

$$\mathrm{E}[D_z] = \sum_{m=1}^{k_z} \mathrm{E}[d_m] \times \pi_m \tag{11}$$

Consequently, in terms of the travel distance of an individual picker in zone z to complete an order, the expected travel time of the picking line can be expressed as

$$\mathrm{E}[C] = \sum_{z=1}^{n} \frac{1}{v_z}\mathrm{E}[D_z] \tag{12}$$

Example 1. Consider an order picking line with 57 bins having eight pickers in a pick-and-pass system. The related data are listed in Table 1. A simulation model of the pick-and-pass system based on the derived analytical model is implemented in Flexsim (2007), as shown in Fig. 2. The FlexSim is a 3D object-oriented simulation environment for modeling discrete-event flow processes. A set of simulation experiments was conducted in order to test the accuracy of the analytical model. The simulation model calculates the travel distance of each randomly generated picking location according to the probabilities that a picker travels to bins. The model was run 1000 times with 500 orders for each picker in the line. The mean travel distance and standard deviation for each picker are recorded, and the width of the confidence interval for the travel distance was assessed at a significance level of 1%. The differences between the values predicted analytically and the simulation results are displayed in Table 2, where the difference is expressed as l (the

Table 1
The data of Example 1.

Zone z	Speed of picker, v_z	The probability of picking an item in a bin Bin no.							
		1	2	3	4	5	6	7	8
1	2	0.7	0.5	0.8	0.3	0.2	0.6	0.5	
2	1	0.5	0.6	0.7	0.2	0.6	0.4		
3	1.5	0.1	0.9	0.5	0.7	0.6	0.3	0.7	0.5
4	2	0.2	0.3	0.5	0.7	0.1	0.8	0.9	0.5
5	1	0.1	0.1	0.3	0.3	0.5	0.5	0.9	
6	1.5	0.9	0.5	0.5	0.3	0.3	0.1	0.1	
7	2	0.5	0.1	0.3	0.1	0.5	0.3	0.9	
8	1	0.1	0.3	0.5	0.9	0.5	0.3	0.1	

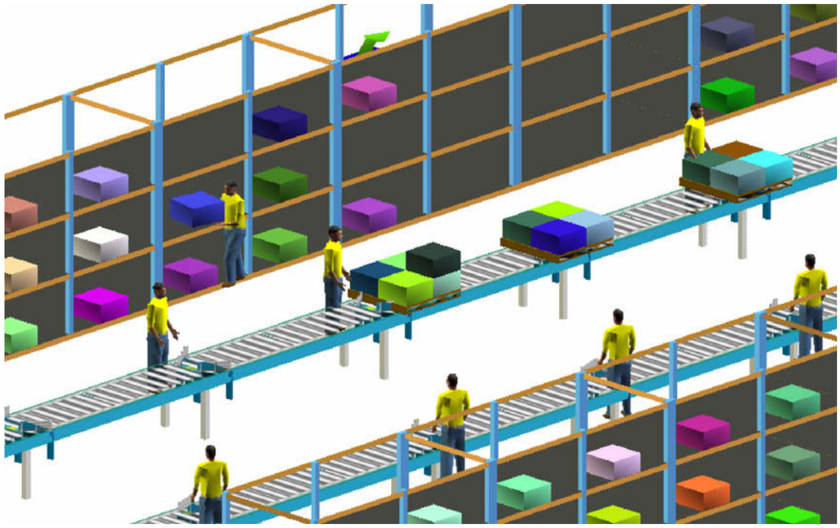

Fig. 2. The pick-and-pass system modeled by FlexSim Simulator.

Table 2
The results of Example 1.

Zone z	Proposed model	Simulation model			
	Expected travel distance	Mean travel distance	Std. dev.	Confidence interval width (at 1%)	Difference (%)
1	5.40	5.39	0.06	0.31	0.24
2	4.08	4.08	0.06	0.31	0.01
3	5.85	5.83	0.05	0.26	0.29
4	7.83	7.82	0.07	0.36	0.13
5	3.88	3.91	0.09	0.46	0.83
6	3.90	3.82	0.09	0.46	2.00
7	5.04	5.07	0.09	0.46	0.58
8	3.34	3.30	0.08	0.41	1.32
Travel time	26.94	26.87	0.16	0.82	0.24

distance calculated from the analytical model – the distance generated from simulation l)/the distance calculated from the analytical model $\times$ 100%. Table 2 indicates that the proposed analytical model provides a relatively good approximation of the travel distance for the pick-and-pass system under study since the differences in general are less than 2%. Despite the large number of simulation runs, the width of the confidence interval stays within 0.26–0.46 of the mean travel distance at a significance of 1%.

4. The proposed algorithms

In this section, we develop storage assignment algorithms to minimize the travel distance of a picker within a zone such that the total travel time of all picking line will be minimized. First, we discuss several properties of the pick-and-pass system from the analytical model.

4.1. An optimal assignment policy for a zone

In a pick-and-pass system with CAPS, the expected travel time of a picker at a bin is an increasing function of travel distance as shown in the following lemma.

Lemma 1. *Given k bins in a zone and a picker is at bin m at the beginning of an order, then it follows that* $\mathrm{E}[d_{m1}] > \cdots > \mathrm{E}[d_{m(m-1)}]$ *and* $\mathrm{E}[d_{m(m+1)}] < \cdots < \mathrm{E}[d_{mk}]$.

Proof. Suppose a picker goes to bin i from bin m, for $i > m$. Let $T = \mathrm{E}[d_{mi}]$, $T' = \mathrm{E}[d_{m(i+1)}]$ and $r'_j = (1 - r_j)$ for $j = 1, 2, 3, \ldots, k$. Applying Eq. (9), we have

$$T = (i-m) + 2 \times \left[1 \times r_{m-1} \times \prod_{j=m-2}^{m-i} r'_j + 2 \times r_{m-2} \times \prod_{j=m-3}^{m-i} r'_j + \cdots + (i-m) \times r_{m-i}\right] \equiv (i-m) + 2R$$

and

$$\begin{aligned} T' &= (i-m+1) + 2 \times \left[1 \times r_{m-1} \times \prod_{j=m-2}^{m-i-1} r'_j + 2 \times r_{m-2} \times \prod_{j=m-3}^{m-i-1} r'_j + \cdots + (i-m) \times r_{m-i} \times r'_{m-i-1} + (i-m+1) \times r_{m-i-1}\right] \\ &= (i-m+1) + 2 \times r'_{m-i-1} \times \left[1 \times r_{m-1} \times \prod_{j=m-2}^{m-i} r'_j + 2 \times r_{m-2} \times \prod_{j=m-3}^{m-i} r'_j + \cdots + (i-m) \times r_{m-i}\right] + 2 \times (i-m+1) \times r_{m-i-1} \\ &\equiv (i-m+1) + 2 \times r'_{m-i-1} \times R + 2 \times (i-m+1) \times r_{m-i-1} \end{aligned}$$

Then, since $0 \leqslant R \leqslant (i-m)$, we have

$$T' - T = 1 + 2 \times r_{m-i-1}[(i-m+1) - R] > 0$$

Similarly, it follows that $\mathrm{E}[d_{m(m+1)}] < \cdots < \mathrm{E}[d_{mk}]$. $\square$

Jewkes et al. (2004) advocated the use of the following well-known result for the assignment of the item locations to minimize the expected travel distance for a pick-and-pass system.

Lemma 2. *If a=(a_1, a_2, ..., a_k) and b=(b_1, b_2, ..., b_k) are non-negative and $0 \leqslant a_1 \leqslant a_2 \leqslant \ldots \leqslant a_k$, then the sum $\sum_{j=1}^{k} b_{\sigma(j)} a_j$ is minimized over permutations σ of b by the ordering $b_{\sigma(1)} \geqslant b_{\sigma(2)} \geqslant \ldots \geqslant b_{\sigma(k)}$.*

Lemma 1 and Lemma 2 indicate that an optimal item assignment can be obtained by making the picking probability at the starting bin of an order as large as possible. Next, we show that if the picking probability at the bins closer to the starting bin is greater, then the probabilities that the picker moves to the farther bins for picking will be reduced.

Lemma 3. *Given a picker is at bin m in a zone and $r_a > r_b > 0$, for some $a \in B_i$, $b \in B_j$ for $m < i < j$, then an exchange between the items in bin i and bin j will increase P_{mj} and decrease P_{mi}. On the other hand, if $a \in B_i$, $b \in B_j$ for $m > i > j$, then an interchange between the items in bin i and bin j will increase P_{mj} and decrease P_{mi}.*

Proof. We have

$$P_{mi} = r_a\left(1 - \frac{1}{2}r_{2m-i}\right)\prod_{j \notin [i,2m-i]}(1 - r_j) \equiv r_a(1 - r_b)R_i$$

and $P_{mj} \equiv r_b R_j$ for $m < i < j$. Now, let P'_{mi} and P'_{mj} be the one-step transition probabilities after the exchange. Since $1 > r_a > r_b > 0$, we have

$$P_{mi} - P'_{mi} = R_i[r_a(1 - r_b) - r_b(1 - r_a)] > 0$$

and

$$P_{mj} - P'_{mj} = R_j(r_b - r_a) < 0$$

The case of $a \in B_i$, $b \in B_j$ for $m > i > j$can be proved in a similar way. □

Following the discussion on the assignment of bin locations, we will develop a property that can be used to allocate items to a bin.

Lemma 4. *Given a picker is at bin m in a zone and $r_i > r_j > 0$ for $m < i < j$ and $h_a > h_b$ for some $b \in B_i$, $a \in B_j$, then an exchange between the locations of items a and b will increase P_{mi} and decrease P_{mj}. On the other hand, if $b \in B_i$, $a \in B_j$ for $m > i > j$, then an exchange between the locations of items a and b, will increase P_{mi} and decrease P_{mj}.*

Proof. Let $r_i = 1 - \prod_{p \in B_j}(1 - h_p) \equiv 1 - (1 - h_b)P_i$, $r_j \equiv 1-(1-h_a)P_j$, $P_{mi} \equiv r_i(1 - r_j)R_i$ and $P_{mj} \equiv r_b R_j$.Now, let r'_i and r'_j be the new bin probabilities with the item *a* and *b* exchanged, $r'_i = 1 - (1 - h_a)P_i$ and $r'_j = 1 - (1 - h_b)P_j$. Clearly, $r'_i > r_i$ and $r'_j < r_j$. Let P'_{mi} and P'_{mj} denote the one-step transition probability after the exchange; then,

$$P'_{mi} - P_{mi} = R_i[r'_i(1 - r'_j) - r_i(1 - r_j)] > 0$$

and

$$P'_{mj} - P_{mj} = R_j(r'_j - r_j) < 0$$

The case of $b \in B_i$, $a \in B_j$ for $m > i > j$can be proved in a similar way. □

The above lemmas lead to the following theorem.

Theorem. *Given a picker is at bin m, then the expected travel distance of the picker is minimized by placing the first q highest demand items in bin m, the second q highest demand items in a bin next to bin m, and the third q highest demand items next to a bin that has already been allocated items and so on, so that the items with the lowest demands are placed in the farthest bin.*

These properties suggest an algorithm for developing an optimal storage assignment for single zone in a pick-and-pass system. The algorithmic procedure can be stated as follows:

Algorithm 2. Algorithm for a zone

Step 1. Sort all items in the descending order of picking frequencies.
Step 2. Group the first q items into group G_1, the next q items into group G_2, and so on, so that the last q items are group into G_k.
Step 3. Let Z^* be an empty set. Assign G_1 and G_2 into Z^* such that $Z^* = \{G_1, G_2\}$.
Step 4. Assign next group into Z^* by considering:
  Case 1: Put the group at the last position so that $Z_1 = \{Z^*, G\}$.
  Case 2: Put the group at the first position so that $Z_2 = \{G, Z^*\}$.
Step 5. Calculate the expected travel distance in the zone, E[D_z], for these two cases by using Eqs. (1)–(11). If the E[D_z] for case 1 is smaller than the E[D_z] for case 2 then $Z^* = Z_1$. Else, $Z^* = Z_2$.
Step 6. Repeat Step 4 and Step 5 until all groups are assigned. Z^* is an optimal assignment for the zone.

4.2. An optimal assignment policy for a picking line

Bynzer & Johansson (1995) described a pick-and-pass system with multiple pickers in it. In such a system, an important issue is to balance the workloads among all pickers. Similar to a manufacturing flow line, imbalance can cause serious deterioration of order throughput time.

In addition, the zone sizes, or the number of bins, in a picking line could be fixed (equal-sized) or varied (unequal-sized). The algorithm for a picking line with unequal-sized zones could be stated as follows:

Algorithm 3. Algorithm for a picking line with unequal-sized zones

Step 1 and Step 2: Sort and group all items as Step 1 and Step 2 of Algorithm 1.
Step 3. Let Z_z^* be an empty set for zone z, z = 1, 2, ..., n. Assign G_1 and G_{n+1} into the zone with the picker who has the highest traveling speed, G_2 and G_{n+2} into the zone with the picker who has the second highest traveling speed and so on, such that every zone has two groups.
Step 4. Calculate the expected travel time, E[D_z]/v_z, for each zone, for i = 1, 2,..., n.
Step 5. Assign next group into the zone with the lowest expected travel time.
Step 6. Determine the position of the group in the zone as Step 4 and Step 5 of Algorithm 1.
Step 7. Repeat Step 4, Step 5 and Step 6 until all groups are assigned. Z_z^* is an optimal assignment for zone z and $\{Z_z^*, z = 1,2,\ldots,n\}$ is an optimal assignment for the picking line.

Example 2. Consider eight zones in an order picking line which has 40 bins. Each bin has three racks such that there are 120 items in the line. The demand rates of the items and the data of each group are listed in Table 3. The speeds of pickers in the order picking line are listed in Table 4. The optimal total travel time is 18.1477 time units/order and an optimal assignment is shown in Table 5. Some picking lines may adopt the policy of having equal number of bins in each zone. An algorithm that finds an optimal storage assignment for such a line can be described below.

Table 3
The data of Example 2.

Group i (G_i)	Item no.	h_i	Prob. of G_i being picked	Group i (G_i)	Item no.	h_i	Prob. of G_i being picked
1	1	0.99737		11	31	0.76886	
	2	0.98119			32	0.75945	
	3	0.98105	0.99999		33	0.74782	0.98597
2	4	0.97385		12	34	0.73904	
	5	0.97045			35	0.73755	
	6	0.96855	0.99997		36	0.73231	0.98166
3	7	0.96624		13	37	0.72983	
	8	0.95293			38	0.72893	
	9	0.94641	0.99991		39	0.72586	0.97992
4	10	0.94397		14	40	0.71904	
	11	0.92674			41	0.71866	
	12	0.91622	0.99965		42	0.69822	0.97614
5	13	0.90550		15	43	0.68843	
	14	0.90168			44	0.67214	
	15	0.88564	0.99893		45	0.67061	0.96635
6	16	0.87688		16	46	0.66778	
	17	0.85468			47	0.64873	
	18	0.85143	0.99734		48	0.64658	0.95875
7	19	0.85096		17	49	0.63990	
	20	0.84602			50	0.63975	
	21	0.83385	0.99618		51	0.63258	0.95233
8	22	0.83211		18	52	0.61026	
	23	0.82764			53	0.60319	
	24	0.82483	0.99493		54	0.59561	0.93745
9	25	0.81656		19	55	0.57972	
	26	0.81326			56	0.56375	
	27	0.78985	0.99280		57	0.55426	0.91827
10	28	0.78504		20	58	0.53846	
	29	0.78144			59	0.52495	
	30	0.77707	0.98952		60	0.51947	0.89464
21	61	0.50464		31	91	0.23052	
	62	0.50287			92	0.20851	
	63	0.46312	0.86778		93	0.20417	0.51531
22	64	0.45748		32	94	0.19957	
	65	0.45230			95	0.18907	
	66	0.44998	0.83656		96	0.17869	0.46689
23	67	0.43623		33	97	0.17660	
	68	0.43562			98	0.17494	
	69	0.42815	0.81805		99	0.16570	0.43321
24	70	0.42381		34	100	0.14354	
	71	0.42372			101	0.13607	
	72	0.41545	0.80590		102	0.13362	0.35894
25	73	0.40696		35	103	0.12866	
	74	0.40277			104	0.12772	
	75	0.40001	0.78749		105	0.09980	0.31579
26	76	0.39739		36	106	0.09484	
	77	0.37732			107	0.09394	
	78	0.37451	0.76529		108	0.06864	0.23616
27	79	0.37035		37	109	0.06318	
	80	0.35687			110	0.06076	
	81	0.34361	0.73419		111	0.06053	0.17336
28	82	0.30044		38	112	0.05666	
	83	0.29218			113	0.05539	
	84	0.28725	0.64706		114	0.04275	0.14700
29	85	0.27479		39	115	0.04238	
	86	0.27417			116	0.03568	
	87	0.27038	0.61594		117	0.03427	0.10819
30	88	0.26645		40	118	0.03293	
	89	0.25159			119	0.01712	
	90	0.23676	0.58098		120	0.00119	0.05062

Table 4
The speed of pickers of Example 2.

Zone z	1	2	3	4	5	6	7	8
Speed of picker v_z	1	1.5	1.5	2	1	2	1	1.5

Algorithm 4. Algorithm for a picking line with equal-sized zones

Steps 1, 2, 3 and 4 are identical to the first four steps of Algorithm 2.
Step 5. Assign next group into the zone with unfilled bins which has the lowest expected travel time.
Step 6. Determine the position of the group in the zone as Step 6 of Algorithm 2.
Step 7. Repeat Steps 4, 5 and 6 until all groups are assigned. Z_z^* is an optimal assignment for zone z and $\{Z_z^*, z = 1,2,\ldots,n\}$ is an optimal assignment for the picking line.

Example 3. Re-do Example 2 for an order picking line which has equal-sized zones. The optimal total travel time is 18.1485 time units/order and an optimal assignment is depicted in Table 6.

Table 5
An optimal item-bin assignment for Example 2.

Zone z	Group i (G_i) in a bin Bin no.					
	1	2	3	4	5	6
1	G38	G24	G11	G12		
2	G36	G21	G5	G6	G27	
3	G35	G20	G7	G8	G28	
4	G31	G18	G1	G2	G25	G32
5	G39	G23	G13	G14	G40	
6	G30	G17	G3	G4	G26	G33
7	G37	G22	G15	G16		
8	G34	G19	G9	G10	G29	

Table 6
An optimal item-bin assignment for Example 3.

Zone z	Group i (G_i) in a bin Bin no.				
	1	2	3	4	5
1	G39	G24	G11	G12	G36
2	G34	G21	G5	G6	G27
3	G33	G20	G7	G8	G28
4	G31	G18	G1	G2	G25
5	G38	G23	G13	G14	G37
6	G30	G17	G3	G4	G26
7	G40	G22	G15	G16	G35
8	G32	G19	G9	G10	G29

Table 7
An optimal item-bin assignment using Jewkes's algorithm for Example 2.

Zone z	Group i (G_i) in a bin Bin no.					
	1	2	3	4	5	6
1	G40	G19	G1	G20	G39	
2	G30	G13	G2	G14	G29	
3	G31	G15	G3	G16	G32	
4	G35	G22	G9	G4	G10	G21
5	G23	G5	G24			
6	G37	G25	G11	G6	G12	G26
7	G27	G7	G28			
8	G33	G17	G8	G18	G34	

Applying the algorithm of Jewkes et al. (2004) to Example 2 yields a travel time of 19.4852 time units/order with the assignment shown in Table 7. The results illustrate that for a picking line with either unequal-sized or equal-sized zones, the proposed algorithms generate shorter total travel times than that of Jewkes et al. (2004) which adopts the home base policy.

5. Conclusions and suggestions for future research

This paper investigates the storage assignment problem for a picking line in a pick-and-pass warehousing system. In order to find the expected travel distance of a picker for the fulfillment of an order, this study proposes to describe the picking operation of the picker as a Markov Chain and subsequent simulation results validate the accuracy of this analytical model. Since a picking line in a pick-and-pass system can be classified into the cases of a single zone, a picking line with unequal number of bins in each zone and a picking line with equal number of bins in each zone, this paper develops properties and algorithms that determine the optimal positions of items in the bins with the objective of minimizing the total travel time for these three cases.

This paper provides a basic analytical model for the system under study by assuming that the picking line is smooth with no delay between two consecutive zones (Jewkes et al., 2004). However congestion may occur and hence it is a natural extension to consider the waiting times between two pickers for future study. Another issue is the starting position of a container in a zone. Since this starting position may have impact on the performance of picking operation by evaluating the travel distance of pickers, one might want to find the optimal starting position for different picking policies, for example, the storage assignment and routing policies, and then derives a control mechanism for it in practice.

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References

Bynzer, H., & Johansson, M. I. (1995). Design and performance of kitting and order picking systems. *International Journal of Production Economics, 41*, 115–125.
Bynzer, H., & Johansson, M. I. (1996). Storage location assignment: Using the product structure to reduce order picking times. *International Journal of Production Economics*, 595–603.
Caron, F., Marchet, G., & Perego, A. (1998). Routing policies and COI-based storage policies in picker-to-part systems. *International Journal of Production Economics, 36*, 713–732.
Caron, F., Marchet, G., & Perego, A. (2000). Optimal layout in low-level picker-to-part systems. *International Journal of Production Economics, 38*, 101–117.
Coyle, J. J., Bardi, E. J., & Langley, C. J. (1996). *The management of business logistics*. MN: West: St. Paul.
De Koster, R. (1994). Performance approximation of pick-to-belt order picking systems. *European Journal of Operational Research, 72*, 558–573.
De Koster, R., Le-Duc, T., & Roodbergen, K. J. (2007). Design and control of warehouse order picking: A literature review. *European Journal of Operational Research, 182*, 481–501.
Eynan, A., & Rosenblatt, M. J. (1994). Establishing zones in single-command class-based rectangular AS/RS. *IIE Transactions, 26*, 38–46.
Flexsim. (2007). Flexsim simulation software user guide 4, Flexsim Software Products, Inc.
Frazelle, E. H., & Sharp, G. P. (1989). Correlated assignment strategy can improve any order-picking operation. *Industrial Engineering, 21*, 33–37.
Gibson, D. R., & Sharp, G. P. (1992). Order batching procedures. *European Journal of Operational Research, 58*(1), 57–67.
Heskett, J. L. (1963). Cube-per-order index – A key to warehouse stock location. *Transportation and distribution Management, 3*, 27–31.
Hsieh, S., & Tsai, K. C. (2001). A BOM oriented class-based storage assignment in an automated storage/retrieval system. *The International Journal of Advanced Manufacturing Technology, 17*, 683–691.
Hwang, H., Oh, Y. H., & Cha, C. N. (2003). A stock location rule for a low level picker-to-part system. *Engineering Optimization, 35*, 285–295.
Jane, C. C. (2000). Storage location assignment in a distribution center. *International Journal of Physical and Logistics Management, 30*, 55–71.
Jane, C. C., & Laih, Y. W. (2005). A clustering algorithm for item assignment in a synchronized zone order picking system. *European Journal of Operational Research, 166*, 489–496.
Jarvis, J. M., & McDowell, E. D. (1991). Optimal product layout in an order picking warehouse. *IIE Transactions, 23*, 93–102.
Jewkes, E., Lee, C., & Vickson, R. (2004). Product location, allocation and server home base location for an order picking line with multiple servers. *Computers & Operations Research, 31*, 623–626.
Kallina, C., & Lynn, J. (1976). Application of the cube-per-order index rule for stock location in distribution warehouse. *Interfaces, 7*, 37–46.
Larson, T. N., March, H., & Kusiak, A. (1997). A heuristic approach to warehouse layout with class-based storage. *IIE Transactions, 29*, 337–348.
Malmborg, C. J. (1995). Optimization of cubic-per-order index layout with zoning constraints. *International Journal of Production Research, 33*(2), 465–482.
Maloney, D. (2000). The new corner drugstore. In *Modern Materials handling*, pp. 58–64.
Petersen, C. G. II, (1997). An evaluation of order picking routing policies. *International Journal of Operations & Production Management, 17*, 1098–1111.
Petersen, C. G., & Gerald, A. (2002). Considerations in order picking zone configuration. *International Journal of Operations & Production Management, 27*, 793–805.
Petersen, C. G., & Gerald, A. (2004). A comparison of picking, storage, and routing policies in manual order picking. *International Journal of Production Economics, 92*, 11–19.
Ruben, R. A., & Jacobs, F. R. (1999). Batch construction heuristics and storage assignment strategies for walk/ride and pick systems. *Management Science, 45*, 575–596.
Schwarz, L. B., Graves, S. C., & Hausman, W. H. (1978). Scheduling policies for automatic warehousing systems: Simulation results. *AIIE Transactions, 10*, 260–270.
Yu, M., & De Koster R. (2007). Performance approximation and design of pick-and-pass order picking systems. ERIM report series reference no. ERS-2007-082-LIS. Available at SSRN: <http://ssrn.com/abstract=1069328>.