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The impact of order batching and picking area zoning on order picking system performance

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ABSTRACT

This paper proposes an approximation model based on queuing network theory to analyze the impact of order batching and picking area zoning on the mean order throughput time in a pick-and-pass order picking system. The model includes the sorting process needed to sort the batch again by order. Service times at pick zones are assumed to follow general distributions. The first and second moments of service times at zones and the visiting probability of a batch of orders to a pick zone are derived. Based on this information, the mean throughput time of an arbitrary order in the order picking system is obtained. Results from a real application and simulation show that this approximation model provides acceptable accuracy for practical purposes. Furthermore, the proposed method is simple and fast and can be easily applied in the design and selection process of order picking systems.

1. Introduction

Order picking, the process of picking products from their storage locations to fill customer orders, is known as the most important activity in warehouses (Tompkins et al., 2003). Recent trends show that customer orders changed from few-and-large orders to many-and-small ones, which arrive late at warehouses but still need to be picked and distributed in short time. These changes require efficient and flexible order picking systems in warehouses for companies to remain competitive.

Main factors impacting order picking system performance include the layout of the warehouse, the storage strategy, the routing policy, the zoning method, and the batching policy. De Koster et al. (2007) give a comprehensive literature review on these topics. Manzini et al. (2005, 2007) present an integrated approach to the development of an expert system based on Artificial Intelligent techniques to support the design and management of an order picking system. The approach combines simulation, meta-heuristic and statistical analysis to study the impact of warehouse layout, product order profile, routing and storage policies on both picker-to-parts and parts-to-picker order-picking system performance with focuses on total travel distance within a certain time period. Chen et al. (2008) follow a similar approach employing data envelopment analysis and additionally focusing on timely delivery.

For a warehouse with a given layout, a predetermined storage strategy and routing policy, zoning and batching are the two major factors influencing the order picking system performance.

Zoning is the problem of dividing the whole picking area into a number of smaller areas (zones) and assigning order pickers to pick requested items within the zone. The analysis on zoning is classified into synchronized zoning, where all zone pickers work on the same batch of orders at the same time, and progressive zoning, where each batch of orders is processed at zone at a time. In progressive zoning, the batch of orders is passed from one zone to the next, which is why such systems are also called pick-and-pass (or sequential) systems. Pick-and-pass order picking systems are widely used in practice. The focus of this paper is on pick-and-pass systems.

For a fixed picking area size and fixed number of order pickers, larger pick zones with more order pickers per zone and hence a smaller number of zones increase service time per zone due to longer travel time in zones. They also tend to increase the picker utilization in pick zones due to higher arrival rates, therefore leading to an increase of mean order throughput time. But on the other hand, more pickers per zone will decrease the utilization of pickers and fewer zones lead to less zone visits of an order or a batch of orders hence less order setup time, which implies a decrease of the mean order throughput time. The zoning problem in an order picking system is to find the right trade-off between these phenomena and hence to find the optimal number of zones minimizing the mean order throughput time.

Compared to other planning issues, the zoning problem has not received much attention yet despite of its importance in order picking system performance. Using simulation, Petersen (2002) shows the zone shape, the number of items on the pick list, the storage policy and the layout of the warehouse (with or without a back cross aisle in the pick zone) have a significant effect on

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the mean travel distance within the zone. Using a mixed-integer programming method, Le-Duc and De Koster (2005) consider the problem of determining the optimal number of zones (for a given pick area) in a pick-and-pack order picking system to minimize the mean order throughput time. Meller and Parikh (2006) propose a mathematical model analogous to the classical dual bin-packing problem, to batch orders in both batch picking and zone picking systems with an objective to minimize work-load imbalance among order pickers and zones. Choe et al. (1992) develop a queuing model to analyze the performance of a pick-and-sort order picking system with synchronized zoning. De Koster (1994) approximates pick-and-pass order picking systems by means of Jackson network modeling and analysis. His model assumes the service time at each pick station is exponentially distributed and customer orders arrive according to a Poisson process. Yu and De Koster (2008) use a general queuing network model to analyze the impact of order batching and pick area zoning of a line-shape pick-and-pass order picking system. Their paper considers a batching policy where the batch size is fixed to two and only those orders containing less than a certain number of lines are batched. They assume order pickers pick one order line (an order line is a certain number of pieces of one article) per picking trip.

Order batching is the process of grouping customer orders together and jointly releasing them for picking. The main objective is to reduce the order picker travel time per order. According to the availability of order information, research on batching for order picking systems is classified into two types: *static batching* and *dynamic (online) batching*. In *static batching*, the order information, i.e., the number of order lines in each order, is known at the beginning of the planning horizon. The batching problem is then to decide the assignment of each order to a batch. *Dynamic batching* takes the stochastic property of the order profile (i.e., the order arrival process and the number of order lines in a batch) into consideration. The batching problem is to determine the batch size or the batch time window (i.e., the time interval used to batch orders) such that the average throughput time of an arbitrary order is minimized. Our paper focuses on dynamic batching.

When orders are batched, they are assigned to a pick bin and then released to the system for picking. Trade-offs exist in the order picking process: if batch sizes increase, the flow rates to pick zones will decrease (fewer bins to zones), leading to lower utilization of the zones and hence reducing the potential waiting time of bins in front of each zone; on the other hand, a larger number of orders in a bin means longer service time at pick zones which tends to increase the mean order throughput time in the system. Also, a larger batch size implies longer queuing time for batch completion and longer processing time in the sorting process at the end of the pick-and-pass system. Therefore finding an optimal batch size is important for the order picking system performance.

Literature on dynamic batching is not abundant. Chew and Tang (1999) model the order batching problem for a single-block warehouse as a queuing model and apply a series of approximations to calculate the lower bound, upper bound, and the mean value of the travel time of a picking tour. They consider the average throughput time of the first order in a batch as the estimation for the average throughput time of individual orders. Le-Duc and De Koster (2007) extend the work of Chew and Tang (1999) into a warehouse with 2-blocks. They perform a direct analysis on the average throughput time of an arbitrary order in the system. Gong and De Koster (2008) use polling models to analyze dynamic order picking, where order pickers travel around the pick area with the aid of RF equipment to pick all outstanding order lines in their pick routes. They show dynamic order picking leads to shorter order throughput time compared with traditional batch picking with optimal batch size described by Le-Duc and De Koster (2007). Van Nieuwenhuyse et al. (2007) model the picking and the sorting processes as a tandem queue. They use a queuing network approach to analyze the factors influencing optimal batch size and the allocation of workers to the picking and the sorting processes with objective of minimizing mean order throughput time.

Batching and zoning are closely related issues and are often applied simultaneously in warehouses. However, in most literature they are studied separately. In this paper, the impacts of batching and zoning on the average throughput time of a random order in a pick-and-pass order picking system are studied. This paper extends the work of Chew and Tang (1999) and Le-Duc and De Koster (2007) into a multiple-zones situation. The model described in this paper generalizes the Jackson queuing network modeling of De Koster (1994) by allowing a general order arrival process and general service time distributions, which represent real-life warehouses more accurately and provide a deeper understanding of the pick-and-pass order picking system. It also extends the work of Yu and De Koster (2008) by considering order batching policies with multiple orders. Compared to the work of Yu and De Koster (2008), this paper considers a more general layout of the picking system with a different product storage strategy and picker routing policy. The paper also takes the subsequent sorting process after the picking into consideration. In the paper, we first develop the first and the second moment of service time at each pick zone and sorting station and then model the order picking system as a $G/G/m$ queuing network. The analysis of the queuing network is based on the decomposition method described by Bolch et al. (2006). Compared with simulation, the approach used in this paper provides results, which are sufficiently accurate to be used in design or evaluation of order picking system alternatives.

The paper is organized as follows: the order picking system under consideration in this paper is described at first, and then the description of the approximation model used to analyze the impact of batching and zoning on order picking system performance, followed by an application of the model and numerical examples. Conclusions are drawn after analyzing the numerical examples.

2. The order picking system

The order picking system under consideration is illustrated in Fig. 1. Customer orders, each consisting of a number of order lines, arrive at the warehouse according to a Poisson process and are batched before being sent to the order picking system. When the batch size reaches a certain number of orders, the batch is released for picking to the order picking system with a pick bin assigned to it. The paper assumes the batch size can take any value equal or larger than 1 and the pick bin is sufficiently large to contain all the items of the order batch. The picking area is divided into a number of zones of the same size, each of which consists of a number of picking aisles. Products are stored randomly in the racks along the aisles. One or more order pickers are allocated to a zone. While picking, pickers follow an S-shape travel route as illustrated in Fig. 1. It assumes the aisles are wide enough to allow two-way travel of order pickers. To simplify the analysis, the paper ignores congestion on travel routes when multiple pickers are assigned to a zone.

Roodbergen and Vis (2006) showed the optimal depot position minimizing average travel distance in picking is in the middle of the zone. Still, in practice the depot is often at the zone boundary. Some companies use zone systems with depots alternatively at the left and right boundary, so that two adjacent zones have the depots close to each other. This allows workers to pick orders at two stations at times of low work-load. In line with other warehouse research (Chew and Tang, 1999; Petersen et al., 2004; Rosenwein, 1996; and Gibson and Sharp, 1992), this paper assumes the depot of each zone to be located at the left-most aisle of the zone. This allows a much easier and more transparent analysis and further-

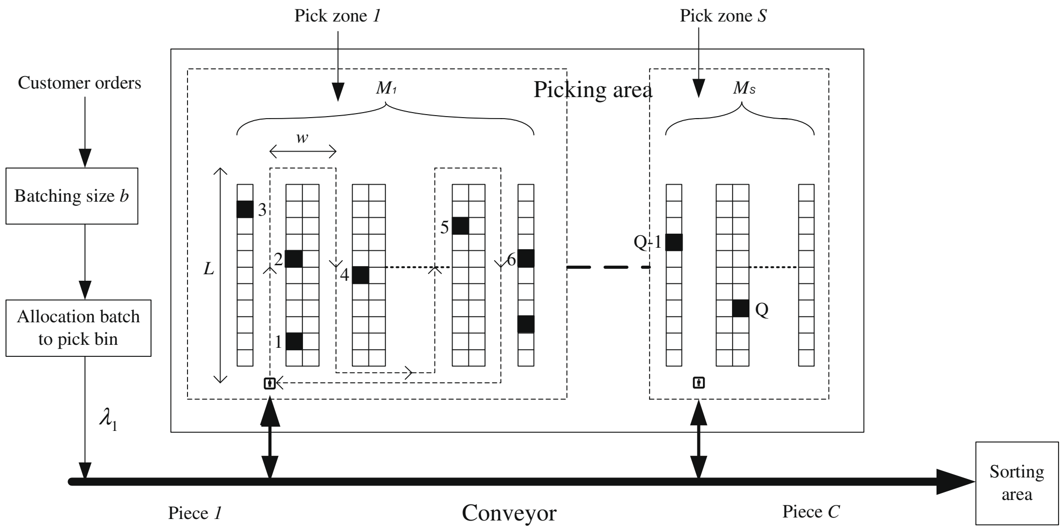

Fig. 1. The order picking system.

more, the position of the depot does not have much impact on the relative performance under different batch sizes and zone structures.

The order picker receives his picking list at the depot and returns to the depot when picking in the zone finishes. Pick zones are connected by conveyor pieces. A pick bin is transferred on the conveyor and will enter a zone if there are items to be picked in the zone. When the picking of a bin has finished completely, it is transferred to the sorting station at the end of the conveyor (no sorting is needed when the batch size is 1). The paper assumes each pick zone has infinite storage capacity (buffer) for pick bins. This assumption is reasonable because in reality the space at the entrance of a zone is large enough to accommodate the arrived pick bins. It also assumes there is a buffer with infinite capacity in front of each conveyor piece, which means that the arrivals will not be lost and pick zones and conveyor pieces can not become blocked because of lack of output capacity. In practice the conveyor pieces can usually contain a sufficiently large number of bins. Fig. 2 gives an illustration of the conveyor system connecting the pick zones. The order picking system is modeled as a $G/G/m$ queuing network with pick zones and conveyor pieces as nodes proceeded by unlimited waiting space in front of them. Fig. 3 gives a schematic illustration of the resulting $G/G/m$ queuing network with 4 pick zones and 5 conveyor pieces. The number of servers at a node equals the capacity of the conveyor piece if the node is a conveyor. If the node is a zone, the number of servers equals the number of pickers working in the zone.

The following notations are used throughout this paper:

Data

C	the number of conveyor pieces, with index j, from 1 to C
S	the number of pick zones, with index j, from $C+1$ to $C+S$
J	the total number of nodes (pick zones, conveyor pieces, and the sorting station), with index j, from 1 to $S+C+1$
L	the length of picking aisles (in meters)
w	center-to-center distance between two adjacent aisles (in meters)
v	picker's travel speed in zones (in meters per second)
vl_j	the velocity of conveyor piece j (in bins per second), $j=1,2,\ldots,C$
m_j	the number of servers at node j, $j=1,2,\ldots,C,\ldots,C+S+1$.

The variables are as follows:

b	batch size (given)
λ_j	pick bin input rate to each node (bins/second)
M_j	number of aisles in zone j
q_j	number of order lines to be picked in zone j, $j=C+1,C+2,\ldots,C+S$
CA_j	cross aisle travel time in zone j, given that zone j is visited, and there are q_j lines to be picked in zone j, $j=C+1,C+2,\ldots,C+S$
WA_j	within aisle travel time in zone j, given that zone j is visited, and there are q_j lines to be picked in zone j, $j=C+1,C+2,\ldots,C+S$
tr_j	total travel time in zone j, given that zone j is visited, and there are q_j lines to be picked in zone j, $j=C+1,C+2,\ldots,C+S$
B_j	number of visited aisles in zone j, given that there are q_j lines to be picked
H_j	the farthest visiting aisle in zone j, given that there are q_j lines to be picked, $j=C+1,C+2,\ldots,C+S$
se_j	service time at zone j, given that zone j is visited, and there are q_j lines to be picked, $j=C+1,C+2,\ldots,C+S$
SE'_j	service time at node j, given that node j is visited, and there are Q lines in a batch, $j=1,2,\ldots,C+S+1$

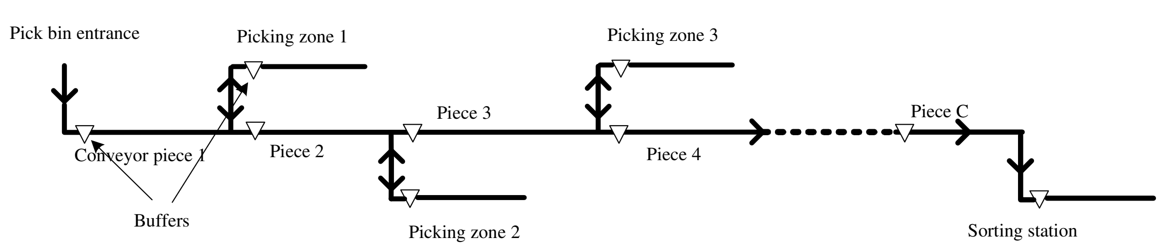

Fig. 2. Illustration of the conveyor connecting the pick zones.

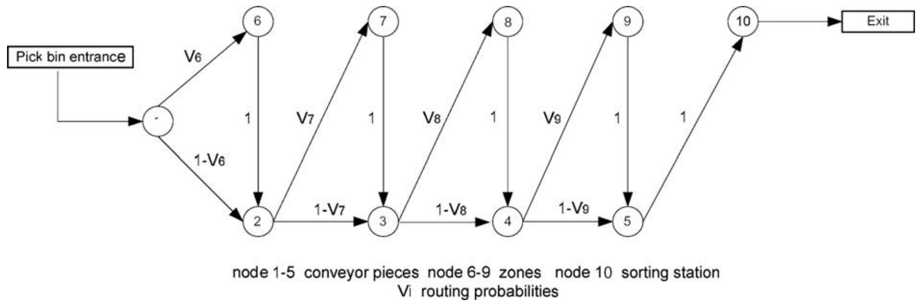

Fig. 3. Schematic illustration of the queuing network with 4 zones.

SE_j service time at node j, given that node j is visited, and there are b orders in a batch, $j = 1,2,\ldots,C+S+1$

c_{sj}^2 squared coefficient of variation (SCV) of service time at node j, $j = 1,2,\ldots,C+S+1$

c_{aj}^2 SCV of internal arrival time of bins to node j, $j = 1,2,\ldots,C+S+1$

t_{ij} transition probability of an order bin from node i to node j, $i,j = 1,2,\ldots,C+S+1$

ρ_j utilization at node j

The next section describes the approximation model used in this paper to analyze the impact of batching and zoning on the order picking system performance.

3. The approximation model

This section first derives the mean and SCV of service time of a pick bin at pick zones, conveyor pieces, and the sorting station. Then it calculates the routing probabilities between nodes followed by the analysis of the mean order throughput time of a random order in the system.

3.1. Mean and SCV of service time at a pick zone

In principle, a pick zone can contain one or more aisles. However, when a zone consists of a single aisle, the storage rack will be located along the conveyor piece. This case is studied by Yu and De Koster (2008). This paper assumes a pick zone consists of at least two aisles. The travel time in zone j, given that zone j is visited and there are q_j lines to be picked, consists of two components: (1) travel time within the aisles WA_j and (2) travel time in the cross aisles CA_j.

Under the assumption that products are randomly stored in slots along the aisles, the number of aisles containing at least one pick location, B_j, can be easily obtained. It has an expected value of

$$E[B_j] = M_j - M_j\left(1 - \frac{1}{M_j}\right)^{q_j} \quad \forall C < j \leqslant C+S, \tag{1}$$

which is also the expected number of aisles to be visited in zone j by an order picker for the pick of an order bin in zone j.

The expected within aisle travel time in zone j can then be stated as

$$E[WA_j] = \frac{L}{\upsilon}E[B_j] + COR \quad \forall C < j \leqslant C+S. \tag{2}$$

The correction factor COR accounts for the extra travel time in the last aisle that is visited. This extra travel time occurs if the number of aisles that has to be visited is an odd number. In this case, the last aisle is both entered and left from the front.

To estimate the correction term, the analysis refers to Roodbergen (2001).

$$COR = \sum_{g\in G}\left[\binom{M_j}{g}\left(\frac{g}{M_j}\right)^{q_j} * X * \left(2 * \frac{L}{\upsilon} * \frac{q_j}{q_j+g} - \frac{L}{\upsilon}\right)\right], \tag{3}$$

where

$G = \{g | 1 \leqslant g \leqslant M_j,\ g \leqslant q_j \text{ and } g \text{ is odd}\}$ and

$$X = 1 - \sum_{i=1}^{g-1}(-1)^{i+1}\binom{g}{g-i}\left(\frac{g-i}{g}\right)^{q_j},$$

which is 1 minus the probability that all of the q_j lines fall into less than $g$aisles.

The expected travel time in cross aisles is twice the expected travel time from the depot to the furthest visited aisle in the zone since order pickers start picking from the depot and return to the depot when picking is finished. Therefore the expect travel time in cross aisles is

$$E[CA_j] = 2 * \frac{w}{\upsilon} * (E[H_j] - 1) \quad \forall C < j \leqslant C+S, \tag{4}$$

where

$$E[H_j] = M_j - \sum_{i=1}^{M_j-1}\left(\frac{i}{M_j}\right)^{q_j} \quad \forall C < j \leqslant C+S \tag{5}$$

is the expected farthest visiting aisles in zone j according to Chew and Tang (1999).

Therefore, by substituting Eqs. (1)–(5) in the following equation, the mean travel time in zone j, given that zone j is visited and q_j lines are to be picked, can be easily obtained,

$$E[tr_j] = E[WA_j] + E[CA_j] \quad \forall C < j \leqslant C+S. \tag{6}$$

The expected service time of a pick bin in zone j, given that q_j lines are to be picked in the zone is the summation of three components: total travel time in the zone, picking time for the order lines, and the setup time for the pick bin (time for starting and finishing the pick list, checking, weighing, labeling, *etc.*):

$$E[se_j] = E[tr_j] + q_j * pt + st \quad \forall C < j \leqslant C+S, \tag{7}$$

where pt is the picking time per order line and st is the setup time per pick bin at a pick zone, both of which are assumed to be constant.

Because travel time is assumed to be independent of the picking time, the expected service time in zone j given that there are Q lines in a batch is calculated as

$$E[SE'_j] = \sum_{q_j=1}^{Q} P_1(q_j,Q) * E[se_j] = \sum_{q_j=1}^{Q} P_1(q_j,Q) * E[tr_j] + \sum_{q_j=1}^{Q} P_1(q_j,Q) * q_j * pt + st \quad \forall C < j \leqslant C+S, \tag{8}$$

where

$$P_1(q_j,Q)=\frac{\binom{Q}{q_j}\left(\frac{1}{S}\right)^{q_j}\left(1-\frac{1}{S}\right)^{Q-q_j}}{1-\left(1-\frac{1}{S}\right)^{Q}}\quad \forall C<j\leqslant C+S \tag{9}$$

is the probability that there are q_j ($q_j > 0$) lines to be picked in zone j, given that there are Q lines in a batch.

The expected service time in zone j given that there are b orders in a batch is calculated as

$$E[\mathrm{SE}_j]=\sum_{Q=b}^{\infty}P_2(b,Q)*E[\mathrm{SE}'_j]\quad \forall C<j\leqslant C+S, \tag{10}$$

where $P_2(b,Q)$ is the probability that a batch has Q lines given that it contains b orders. A customer order contains at least one order line. $P_2(b,Q)$ is a function of batch size b, and the order profile O_n. As an example, if the number of lines in an order follows a shifted Poisson distribution of 1 + Poisson(a), then $Q = b$ + Poisson($a*b$) in distribution and $P_2(b,Q)=\frac{(ab)^{Q-b}}{(Q-b)!}e^{-ab}$.

The second moment of service time in zone j given that there are q_j lines to be picked in that zone is approximated as

$$\begin{aligned}E[\mathrm{se}_j^2]&=E\left[\left(\frac{L}{\upsilon}*B_j+\frac{2w}{\upsilon}*(H_j-1)+q_j*\mathrm{pt}+\mathrm{st}\right)^2\right]\quad \forall C<j\leqslant C+S\\&=\left(\frac{L}{\upsilon}\right)^2*E[B_j^2]+\left(\frac{2w}{\upsilon}\right)^2E[H_j^2]+\left(\frac{4w*L}{\upsilon^2}\right)E[B_jH_j]\\&\quad+\left(\frac{4w*\mathrm{pt}*q_j+4w*\mathrm{st}}{\upsilon}-\frac{8w^2}{\upsilon^2}\right)E[H_j]\\&\quad+\left(\frac{2L*\mathrm{pt}*q_j+2L*\mathrm{st}}{\upsilon}-\frac{4w*L}{\upsilon^2}\right)E[B_j]+(q_j*\mathrm{pt})^2\\&\quad+\left(2\mathrm{pt}*\mathrm{st}-\frac{4w*\mathrm{pt}}{\upsilon}\right)*q_j+\left(\frac{2w}{\upsilon}\right)^2-\frac{4w*\mathrm{st}}{\upsilon}+\mathrm{st}^2,\end{aligned} \tag{11}$$

where

$$E[B_j^2]=M_j\left((M_j-1)\left(1-\frac{2}{M_j}\right)^{q_j}+(1-2M_j)\left(1-\frac{1}{M_j}\right)^{q_j}+M_j\right)\quad \forall C<j\leqslant C+S, \tag{12}$$

$$E[H_j^2]=M_j^2-\sum_{i=1}^{M_j-1}(2i+1)\left(\frac{i}{M_j}\right)^{q_j}\quad \forall C<j\leqslant C+S, \tag{13}$$

and

$$\begin{aligned}E[B_jH_j]&=(M_j+1)\left(M_j-\sum_{i=1}^{M_j}(1-f_i)^{q_j}\right)-\left(M_j-\sum_{i=1}^{M_j-1}\left(\sum_{r=1}^{i}f_r\right)^{q_j}\right)\\&\quad-\sum_{i=1}^{M_j-1}\left(i\left(\sum_{r=1}^{i}f_r\right)^{q_j}-\sum_{s=1}^{i}\left(\sum_{r=1,r\neq s}^{i}f_r+f_{i+1}\right)^{q_j}\right)\\&\quad\forall C<j\leqslant C+S,\end{aligned} \tag{14}$$

according to Chew and Tang (1999). $f_i = 1/M_j$, for $i = 1,2,\ldots,M_j$. $E[B_j]$ and $E[H_j]$ are obtained from Eqs. (1) and (5).

The second moment of service time at zone j, given that zone j is visited and there are Q lines in a batch is

$$\begin{aligned}E[\mathrm{SE}_j'^2]&=\sum_{q_j=1}^{Q}E[\mathrm{se}_j^2]*P_1(q_j,Q)\\&=\sum_{q_j=1}^{Q}E[\mathrm{se}_j^2]*\frac{\binom{Q}{q_j}\left(\frac{1}{S}\right)^{q_j}\left(1-\frac{1}{S}\right)^{Q-q_j}}{1-\left(1-\frac{1}{S}\right)^{Q}}\quad \forall C<j\leqslant C+S.\end{aligned} \tag{15}$$

The second moment of service time in zone j, given that there are b orders in a batch is

$$E[\mathrm{SE}_j^2]=\sum_{Q=b}^{\infty}P_2(b,Q)*E[\mathrm{SE}_j'^2]\quad \forall C<j\leqslant C+S. \tag{16}$$

The SCV of service time at zone j, given that there are b orders in a batch is

$$c_{sj}^2=\frac{E[\mathrm{SE}_j^2]-E[\mathrm{SE}_j]^2}{E[\mathrm{SE}_j]^2}\quad \forall j>C. \tag{17}$$

3.2. Mean and SCV of service time at a conveyor piece

A conveyor piece j can contain k_j pick bins and is assumed to have constant speed, vl_j. This paper approximates it as k_j servers in parallel, each of which has constant service rate of $\frac{\mathrm{vl}_j}{k_j}$. This means when all the k_j servers are busy (i.e., conveyor piece j is full of pick bins) the output rate of the conveyor piece j is vl_j. In the approximation, the output rate of a conveyor piece is proportional to the number of bins on it. Since the speed of a conveyor piece is constant and we do not consider conveyor failures (which are rare in practice), the SCV of service time of a conveyor piece is zero.

$$c_{sj}^2=0\quad \forall j\leqslant C. \tag{18}$$

The mean service time of servers on a conveyor piece j is the reciprocal of its service rate,

$$E[\mathrm{SE}_j]=\frac{k_j}{\mathrm{vl}_j}\quad \forall j\leqslant C. \tag{19}$$

3.3. Mean and SCV of service time at the sorting station

When orders are batched, they need to be sorted again by order upon completion of the pick process. This paper assumes a manual sorting process, where the service time of a pick bin at the sorting station is modeled as a constant setup time plus sorting time. We assume sorting time is linearly proportional to the number of lines in the bin and sorting time per line is constant. The service time of a pick bin, given that there are Q lines in a batch is

$$\mathrm{SE}'_j=\mathrm{sc}+\mathrm{so}*Q\quad j=S+C+1, \tag{20}$$

where sc is the pick bin setup time and so is the sorting time per order line.

The expected service time of a pick bin at the sorting station given that there are b orders in a batch is then calculated as

$$E[\mathrm{SE}_j]=\mathrm{sc}+\mathrm{so}*E[Q]=\mathrm{sc}+\mathrm{so}*\sum_{Q=b}^{\infty}Q*P_2(b,Q)\quad j=S+C+1. \tag{21}$$

The second moment of service time of a pick bin at the sorting station, given that there are b orders in a batch is

$$E[\mathrm{SE}_j^2]=\mathrm{sc}^2+2\mathrm{sc}*\mathrm{so}*E[Q]+\mathrm{so}^2E[Q^2]\quad j=S+C+1, \tag{22}$$

where

$$E[Q^2]=\sum_{Q=b}^{\infty}Q^{2*}P_2(b,Q). \tag{23}$$

The SCV of service time of a pick bin at the sorting station can be obtained from Eqs. (21), (22) and (17).

3.4. Mean throughput time of a pick bin of b orders in the system

The calculation of the mean order throughput time in the order picking system is based on the $G/G/m$ queuing network approximation model described by Bolch et al. (2006) (refer to Appendix). The mean pick bin throughput time is the summation of the mean waiting time and the service time at each node. The approximation analysis uses two parameters to characterize the arrival processes and the service times at each node, one to describe the rate and the other to describe the variability. The two parameters for service time are $E[\mathrm{SE}_j]$, the mean, and c_{sj}^2, the SCV.

For the arrival processes, the parameters are λ_j, the arrival rate, which is the reciprocal of the mean inter-arrival time between two pick bins at each node, and c_{aj}^2, the SCV of the inter-arrival times of pick bins at a node. Customer orders arrive at the system according to a Poisson process with rate $\bar{\lambda}_{01}$. After batching b orders, the batched orders are assigned with a pick bin, which is sent to the order picking system at conveyor piece 1 for picking. Therefore, the input of the order picking system has *Erlang* distribution with parameters $(b, \bar{\lambda}_{01})$. The input rate is

$$\lambda_1 = \frac{\bar{\lambda}_{01}}{b}. \tag{24}$$

The SCV of the pick bin inter-arrival time is

$$c_{a1}^2 = \frac{1}{b}. \tag{25}$$

To calculate the internal arrival rates and the SCVs of inter-arrival time at each node, the transition probabilities of pick bins after the service at each node to another node is needed.

3.5. Transition probabilities between nodes and throughput time calculation

Because of the random storage policy in the picking area, the probability that an order line is stored in zone j, is

$$p_j = \frac{1}{S} \quad \forall C < j \leqslant C + S. \tag{26}$$

The probability that a pick bin enters zone j, given that there are Q lines in the batch of orders, equals the probability that there is at least one order line to be picked at zone j

$$P_j = 1 - (1 - p_j)^Q \quad \forall C < j \leqslant C + S. \tag{27}$$

The probability that a pick bin has to enter zone j, given that it contains b orders is calculated as

$$V_j = \sum_{Q=b}^{\infty} P_2(b, Q) * P_j \quad \forall C < j \leqslant C + S. \tag{28}$$

At the end of a conveyor piece, a pick bin is either transferred on a subsequent conveyor piece for transportation or pushed into a zone for picking. The transition probabilities between these nodes depend on the layout of the system. For the layout sketched in Fig. 2, they are given by

$$t_{jj+C} = V_{j+C} \quad \forall j < C, \tag{29}$$
$$t_{jj+1} = 1 - V_{j+C} \quad \forall j < C, \tag{30}$$
$$t_{jj-S} = 1 \quad \forall C < j \leqslant C + S, \tag{31}$$
$$t_{jj+S+1} = 1 \quad j = C. \tag{32}$$

From the transition probabilities between nodes, the internal traffic rates λ_j and the SCV of the inter-arrival time between two bins at each node can be obtained (refer to Appendix).

The utilization of each node is given by

$$\rho_j = \begin{cases} \lambda_j / \mathrm{vl}_j & \forall j \leqslant C, \\ \lambda_j E[\mathrm{SE}_j]/m_j & \forall j > C. \end{cases} \tag{33}$$

The expected sojourn time of a bin at node j is given by

$$E[T_j] = E[\mathrm{vt}_j] * (E[W_j] + E[\mathrm{SE}_j]) \quad \forall 1 \leqslant j \leqslant C + S + 1, \tag{34}$$

where $E[W_j]$ is the expected waiting time of a pick bin in front of node j as calculated by (A.6), and $E[\mathrm{vt}_j]$ is the expected number of visits to node j of a pick bin. The probability mass function of vt_j is given by

$$\mathrm{vt}_j = \begin{cases} 0 \text{ with probability } 1 - V_j \\ 1 \text{ with probability } V_j \end{cases} \quad \forall j. \tag{35}$$

Hence

$$E[\mathrm{vt}_j] = 0 * (1 - V_j) + V_j = V_j \quad \forall j. \tag{36}$$

The expected throughput time of a pick bin is the summation of the expected sojourn time at each node,

$$E[\mathrm{tpt}] = \sum_{j=1}^{C+S+1} E[T_j]. \tag{37}$$

3.6. Mean throughput time of an arbitrary order in the order picking system

The mean throughput time of an arbitrary order in the system has two components: the mean waiting time to form a batch, $E[W_b]$, and the mean throughput time of the pick bin in the system, $E[\mathrm{tpt}]$, as derived in the previous sub-section. Therefore, the mean throughput time of an arbitrary order in the order picking system is

$$E[\mathrm{TPT}] = E[W_b] + E[\mathrm{tpt}], \tag{38}$$

where $E[W_b]$ is approximated as

$$E[W_b] = \frac{\sum_{i=0}^{b-1} i * E[Y]}{b} \tag{39}$$

and $E[Y]$, the mean customer order inter-arrival time, equals $1/\lambda_{01}$.

4. An application of the approximation model

This section discusses the application of the approximation model to the order picking system in the bulky storage area at the parts distribution center (DC) of an international motor production company. The discussion will show that the approximation method gives a fairly accurate estimation of the order picking system performance and therefore it is appropriate for practical uses.

Table 1
Data and comparison with results of the real order picking system

Parameter	Value
Number of stations	4
Number of order pickers per station	1
Number of order lines to pick per order	Empirical distribution (mean 2.5, stdv 1.9)
Order inter-arrival time to the system (second)	Empirical distribution (mean 28.9, stdv 52.4)
Service time at station A (second)	Empirical distribution (mean 40.1, stdv 41.6)
Service time at station B (second)	Empirical distribution (mean 51.0, stdv 51.1)
Service time at station C (second)	Empirical distribution (mean 54.1, stdv 48.0)
Service time at station D (second)	Empirical distribution (mean 38.8, stdv 35.0)
Probability to enter station A	0.385
Probability to enter station B	0.254
Probability to enter station C	0.271
Probability to enter station D	0.435
MOTT from G/G/m approximation model (second)	302.1
MOTT from WMS (second)	321.7
Relative error	6.1%

The bulky storage area of the DC stores in total 240 products. The whole area is divided into four pick stations connected by conveyor pieces. Each station has one order picker. In this picking system, orders are not batched therefore the sorting time is zero in this case. Through analyzing the log file from the *Warehouse Management System* (WMS) for one picking day, which was chosen to be a representative for the typical picking process, we obtained the data for the order arrival process to the system, the service times at pick stations, and the routing probabilities of order bins to enter a station, which are listed in Table 1. The capacities of conveyor pieces and their moving speeds are also measured. These data is input into our approximation model. The resulting mean order throughput time (MOTT) is compared with the MOTT obtained from the warehouse management system.

Table 1 shows the relative error of the approximation model is around 6%. In the next section, this approximation method is further validated and is used to evaluate the impact of batching and zoning on system performance.

Table 2
Parameters used in the example

Parameter	Value
Total number of aisles	36
Number of order pickers	12
Number of sorters	15
Length of an aisle	25 meters
Center-to-center distance between 2 aisles	3.5 meters
Size of pick bins ($L * W * H$)	$60 * 40 * 35$ centimeters
Conveyor speed	0.7 bins per second (0.1 meters minimum space between 2 bins)
Conveyor length in total	220 bins
Picking time per line	12 seconds
Sorting time per line	12 seconds
Picking setup time in a zone	20 seconds
Sorting setup time at sorting station	20 seconds
Picker's travel speed	1 meter per second
Distribution of the number of lines in a customer order	1 + Poisson(4).

5. Numerical examples

This section uses an example to illustrate the application of the approach described in the previous sections to analyze the impact of order batching and zoning on the mean throughput time of an

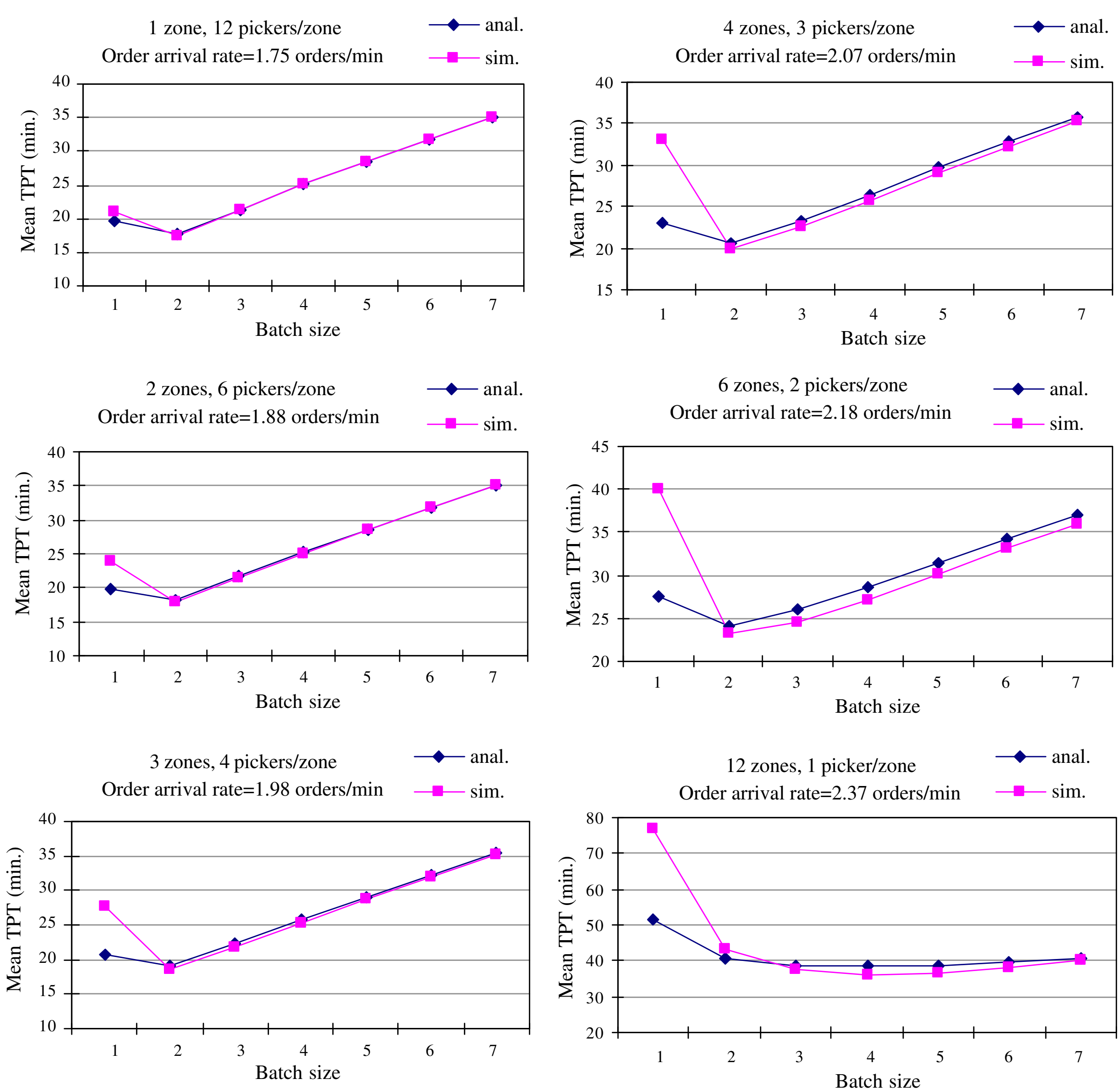

Fig. 4. The impact of batch size on mean order throughput time in different zone settings.

arbitrary order in the order picking system. The parameters used in the example are listed in Table 2. This paper assumes there are enough workers at the sorting station so that the waiting time in front of the sorting station is negligible. The number of pickers and the number of storage aisles are chosen to be integral multiples of the number of zones, since otherwise the system will become imbalanced, leading to increased throughput times. Imbalanced systems can always be improved by balancing (see Yu and De Koster, 2008). For every experiment with a given number of zones, the storage aisles and the order pickers are equally divided over the zones. This allows comparison of various batch and zone sizes. We take the order arrival rate, for which the system is just stable when the batch size is 1. The results obtained from the approximation model are compared with simulation results. The simulation model is built in AUTOMOD 10.0. For each particular setting, the simulation results were obtained from runs of more than 10,000 pick batches so that the 95% confidence intervals are within 1% of the mean order throughput time in the system. The same orders are taken across experiments.

Table 3
Relative approximation errors compared with simulation for varying number of zones and worker utilizations

Utilization	0.55	0.65	0.75	0.85	0.94
1 Zone	0.1%	0.1%	0.0%	0.7%	2.3%
2 Zones	0.7%	0.8%	1.2%	2.0%	13.8%
3 Zones	1.1%	1.4%	2.1%	4.4%	25.2%
4 Zones	1.2%	1.7%	2.7%	6.7%	30.2%
6 Zones	1.3%	2.0%	3.8%	8.4%	31.2%
12 Zones	1.2%	2.5%	4.9%	10.6%	29.4%

Fig. 4 shows the impact of batch size on mean order throughput time with different zone settings in comparison to simulation results. It appears that the average mean order throughput time is a convex function of batch size under different zone settings. This result is consistent with the findings of Chew and Tang (1999) who consider a single-block warehouse and of Le-Duc and De Koster (2007) who consider a two-block warehouse with a single pick zone. The reason is as follows. The mean order throughput time has three components, the waiting time to form a batch, waiting time in front of nodes for service, and service time at nodes. When the batch size is small, the batch-forming time and the service time

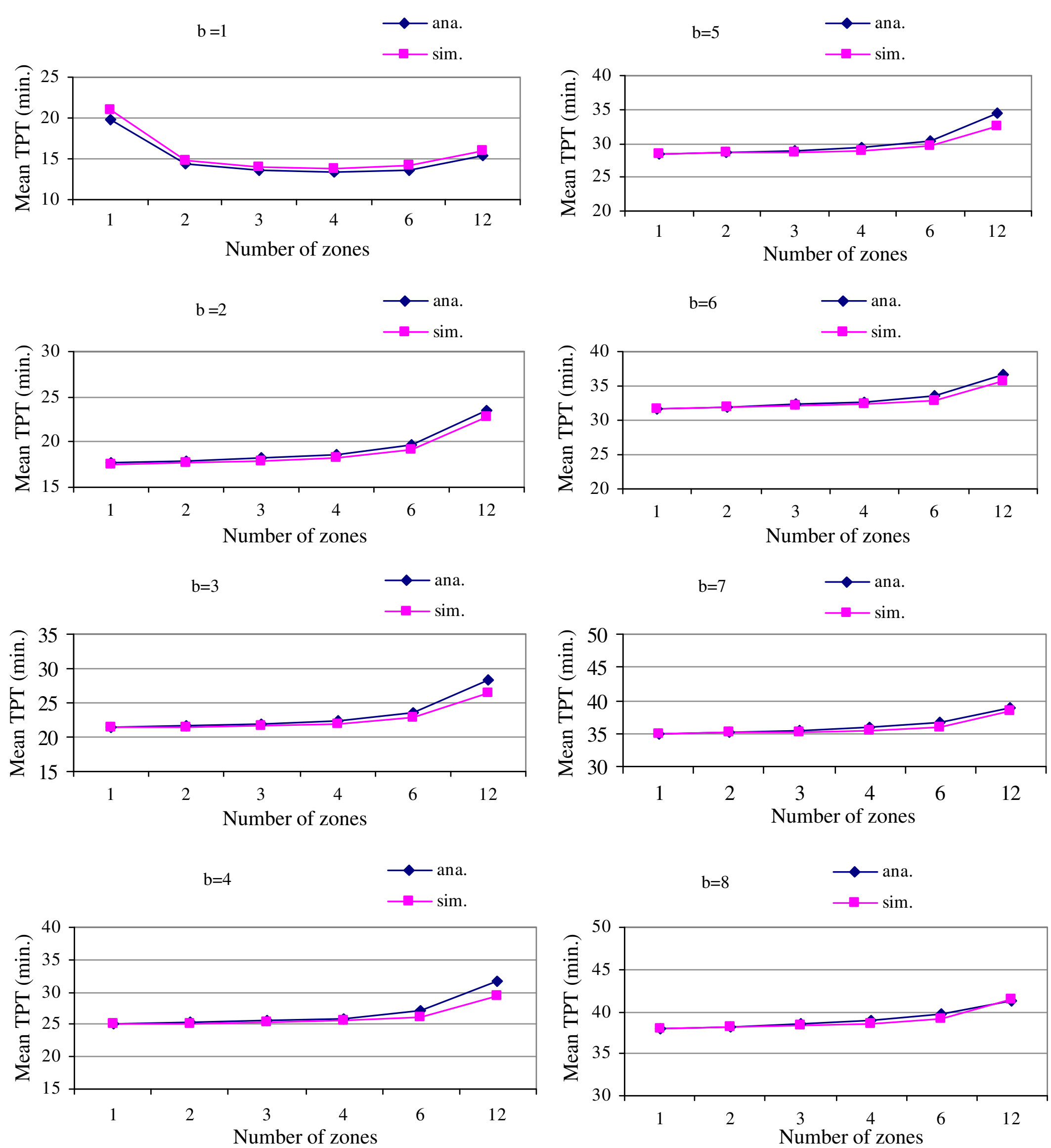

Fig. 5. The impact of zoning on mean order throughput time under the different batch sizes with order arrival rate of 1.75 orders/min.

at nodes are small, but the arrival rates of flows to nodes are high, leading to high utilization of servers and long waiting time for service because of the limited number of servers at nodes. This tradeoff indicates an optimal batch size exists. The results in Fig. 4 and other experiments carried out show the quality of the approximation method is good with a maximum relative error less than 10% compared with simulation when the batch size is larger than 1, in which case the utilizations at nodes are smaller than 0.9. The relative error increases up to the maximum of 30% when the batch size is 1 and the utilizations at nodes are larger than 0.9. The order picking system tends to become unstable when the utilizations are larger than 0.9. In order to further investigate the impact of utilization and the number of zones on the quality of the approximation, experiments are carried out based on the system of Table 2, with batch size 1. The worker utilizations are varied between 0.55 and 0.94 by varying the order arrival rates. Results are shown in Table 3.

Fig. 4 also shows that for a given number of order pickers and given storage space size, the mean order throughput time increases significantly with an increasing batch size beyond the minimum point when there are few zones with multiple servers in a zone, but the increase is less obvious when there are many zones with a single server per zone. The reason for this phenomenon can be explained as follows. In the first situation, the utilization at zones and hence the waiting time in front of zones decreases sharply with an increasing of batch size and soon reaches a small value. Beyond the minimum point, the increase of service time and batch-forming time is much larger than the decrease of waiting time at zones, leads to a significant increase of mean order throughput time. In the second situation, the utilization at zones and consequently the waiting time of pick bins in front of zones decreases gradually with an increasing batch size. Beyond the minimum points, the increase of service time and batch-forming time is only slightly larger than the decrease of waiting time. Therefore, the increase of mean order throughput time over batch sizes is less obvious in this case.

Fig. 4 shows zoning impacts the optimal batch size b^* (it varies from $b^* = 4$ for 12 zones to $b^* = 2$ for other zone configurations) with different order arrival rates. Keeping the mean order arrival rates unchanged, this paper also investigates three different two-stage hyperexponential order arrival distributions with squared coefficient of variation of 1.22, 1.50, and 1.72. The results show the mean order throughput times change only marginally for these different arrival processes. The optimal batch size remains the same as for Poisson order arrivals.

Fig. 5 shows the effect of zoning with different batch sizes on system performance. It shows when the batch size is small ($b = 1$, in the example), zoning tends to reduce the mean order throughput time. When the batch size is larger, zoning has minor negative effect on mean order throughput time. The mean order throughput time starts to increase when the number of zones keeps on increasing. This phenomenon is explained as follows. For a fixed storage area size, increasing the number of pick zones will decrease the arrival rate of flows to each zone and the service time at each zone, leading to a decrease in utilization at each zone, therefore decreasing the waiting time in front of each zone. On the other hand, more zones imply more pick bin visits to zones and hence more setup time, leading to increasing mean order throughput time. When the batch size is small, the flow arrival rates to zones and the server utilizations are high, waiting time accounts for a large part of the mean order throughput time, therefore reducing waiting time by zoning leads to significant improvement of mean order throughput time. When the batch size is large, waiting time in front of each zone is negligible due to a low arrival rate of flows. The reduction on service time per zone is compensated by the increased number of setups and hence the impact of zoning on mean order throughput time is small when the batch size is large. As the number of zones increases, the increased number of setups due to the increased number of visits to zones gradually compensates and exceeds the reduction on waiting time and service time at zones. Therefore the mean order throughput time will increase when the number of zones is large.

With the same mean order arrival rate, the model is also tested by hyperexponential order arrivals with different parameters. The mean order throughput time changes slightly only and the trends shown in Fig. 5 remain. We carried out a sensitivity analysis on the setup time in pick zones varying from 5 to 30 seconds with a step size of 5 seconds. This has an impact on optimal batch sizes, but the trends shown in Fig. 5 still hold. Approximation errors are comparable to those in Fig. 5.

6. Conclusions

Order picking is regarded as the most important warehousing activity. Mean throughput time of an arbitrary order in the system is an important measurement of the efficiency of an order picking system. The faster the order can be picked, the sooner it can be ready for shipment and the higher service level the warehouse can provide. Order batching and zoning of the picking area are two important factors that will influence the order picking efficiency. This paper uses a $G/G/m$ queuing network approximation model to analyze the impact of batching and zoning on order picking system performance with online order arrivals. The approximation model shows to have acceptable quality. Errors are in general small, but when the utilization or the number of zones becomes large, errors increase. Through the experiments carried out in Section 5 and other extensive experiments with different input parameters, such as setup time at pick zones, different order arrival rates to the systems, and the different order arrival distributions, we find an optimal batch size always exists and batch size has large impact on mean order throughput time as shown in Fig. 4. From Fig. 5, we find the mean order throughput time in the system is quite robust for a varying number of zones around the optimum number of zones. We also find, for given order arrival rates, the precise shape of the order arrival distribution has only slight impact on mean order throughput time. This is especially true when the utilizations at zones are small. In general, many factors influence the system performance. This phenomenon reflects the complexity of the pick-and-pass system. The preference of one operational strategy over the other depends on the settings of the system. The approximation model developed in this paper can therefore be used as a fast tool to analyze these alternatives.

Although the development of the approximation model is based on random storage, S-shape routing, and Poisson order arrivals, it is possible to extend our model to different storage strategies, routing policies, system layouts, and other order arrivals. The expression of service time at zones and the SCV of interarrival time of batches to the system may alter, but the $G/G/m$ queuing network approximation model still can be used to analyze these situations.

Appendix: *G/G/m* queuing network models

The approach described in Chapter 10.1.3 in Bolch et al. (2006) is to decompose the $G/G/m$ queuing network into separate service facilities that are analyzed in isolation. To estimate the mean pick bin throughput time in this $G/G/m$ queuing network system, the internal flow parameters need to be calculated at first. The internal flow rate to each node λ_j is obtained by solving the following linear equations:

$$\lambda_j = \lambda_{0j} + \sum_{i=1}^{J} \lambda_i t_{ij} \quad 1 \leqslant j \leqslant J, \tag{A.1}$$

where λ_{0j} is the external arrival rate of pick bins to node j, J is the total number of nodes (conveyor pieces, pick zones and the sorting station) in the system, and t_{ij} is the transition probability from node i to node j obtained from Eqs. (29)–(32).

The calculation of the SCVs of the inter-arrival times to each node is done iteratively using the following three phases.

Phases 1: Merging Several arrival processes to each node are merged into a single arrival process. The SCV of the inter-arrival time is obtained by

$$c_{ai}^2 = \frac{1}{\lambda_i} * \left(\sum_{j=1}^{J} c_{ji}^2 * \lambda_j * t_{ji} + c_{0i}^2 * \lambda_{0i} * t_{0i} \right) \quad \forall 1 \leqslant i \leqslant J, \tag{A.2}$$

where c_{0i}^2 is the SCV of the external inter-arrival time to node i, and

$$c_{0i}^2 = 0 \quad \forall i > 1, \tag{A.3}$$

since the pick bins enter the system from node 1, the first conveyor piece.

Phase 2: Flow The SCVs of the inter-departure times of pick bins from each node, c_{di}^2, depends on the SCV of the inter-arrival time c_{ai}^2 and the service time c_{si}^2:

$$c_{di}^2 = 1 + \frac{\rho_i^2 * (c_{si}^2 - 1)}{\sqrt{m_i}} + (1 - \rho_i^2) * (c_{ai}^2 - 1) \quad \forall 1 \leqslant i \leqslant J. \tag{A.4}$$

Phase 3: Splitting

$$c_{ij}^2 = 1 + t_{ij} * (c_{di}^2 - 1) \qquad \forall 1 \leqslant i \leqslant J, \quad \forall 1 \leqslant j \leqslant J. \tag{A.5}$$

The iteration starts with phase 1 and initial values of $c_{ij}^2 = 1$ and stops when c_{ai}^2 converge.

With the internal flow parameter λ_j, and c_{aj}^2, and the service time parameter $E[\mathrm{SE}_j]$, and c_{sj}^2, based on the method developed by Whitt (1993), the expected waiting time in front of each node can be approximated. Since the focus is on a single node, the subscript indexing the node is omitted in deriving the expected waiting time in front of a node.

For a multi-server node with m servers, the expected waiting time is given by

$$E[W]_{G/G/m} = \phi(\rho, c_a^2, c_s^2, m) \left(\frac{c_a^2 + c_s^2}{2} \right) E[W]_{M/M/m}, \tag{A.6}$$

where c_a^2 and c_s^2 are obtained from (A.2)–(A.5) and Eq. (17), respectively, ρ is given by Eq. (33), $E[W]_{M/M/m}$ is the waiting time in queue of a multi-server node with Poisson arrivals and exponential service distribution. The exact expression for $E[W]_{M/M/m}$ is given by

$$E[W]_{M/M/m} = \frac{P(N \geqslant m)}{\mu m (1 - \rho)}, \tag{A.7}$$

where μ is the reciprocal of mean service time at each node.

$P(N \geqslant m)$ is the probability that all servers are busy and is given by

$$P(N \geqslant m) = \left(\frac{(m\rho)^m}{m!(1-\rho)} \right) \zeta, \tag{A.8}$$

with

$$\zeta = \left(\frac{(m\rho)^m}{m!(1-\rho)} + \sum_{k=0}^{m-1} \frac{(m\rho)^k}{k!} \right)^{-1}. \tag{A.9}$$

The expression for ϕ in (A.6) is given by

$$\phi(\rho, c_a^2, c_s^2, m) = \begin{cases} \left(\frac{4(c_a^2 - c_s^2)}{4c_a^2 - 3c_s^2} \right) \phi_1(m, \rho) \\ \quad + \left(\frac{c_s^2}{4c_a^2 - 3c_s^2} \right) \psi(c^2, m, \rho) c_a^2 \geqslant c_s^2, \\ \left(\frac{c_s^2 - c_a^2}{2(c_a^2 + c_s^2)} \right) \phi_3(m, \rho) \\ \quad + \left(\frac{c_s^2 + 3c_a^2}{2(c_a^2 + c_s^2)} \right) \psi(c^2, m, \rho) c_a^2 \leqslant c_s^2, \end{cases} \tag{A.10}$$

with

$$\phi_1(m, \rho) = 1 + \gamma(m, \rho), \tag{A.11}$$

$$\phi_3(m, \rho) = (1 - 4\gamma(m, \rho)) e^{\frac{2(1-\rho)}{3\rho}}, \tag{A.12}$$

$$\gamma(m, \rho) = \min \left\{ 0.24, \frac{(1-\rho)(m-1)[(4+5m)^{0.5} - 2]}{16 m \rho} \right\}, \tag{A.13}$$

and

$$\psi(c^2, m, \rho) = \begin{cases} 1 c^2 > 1 \\ \phi_4(m, \rho)^{2(1-c^2)} 0 \leqslant c^2 \leqslant 1, \end{cases} \tag{A.14}$$

with

$$c^2 = \frac{c_a^2 + c_s^2}{2}, \tag{A.15}$$

and

$$\phi_4(m, \rho) = \min \left\{ 1, \frac{\phi_1(m, \rho) + \phi_3(m, \rho)}{2} \right\}. \tag{A.16}$$

References

Bolch, G., Greiner, S., De Meer, H., Trivedi, K.S., 2006. Queuing Networks and Markov Chains. John Wiley & Sons, Hoboken, NJ.

Chen, C.-M., Gong, Y., De Koster, R., Van Nunen, J., 2008. A DEA-based evaluative framework for order picking systems. Working Paper, Erasmus University.

Chew, E., Tang, L.C., 1999. Travel time analysis for general item location assignment in a rectangular warehouse. European Journal of Operational Research 112 (3), 582–597.

Choe, K., Sharp, G.P., Serfozo, R.S., 1992. Aisle-based Order Pick Systems with Batching, Zoning and Sorting. Progress in Material Handling Research. The Material Handling Industry of America, Charlotte, NC. pp. 245–276.

De Koster, R., 1994. Performance approximation of pick-to-belt orderpicking systems. European Journal of Operational Research 72 (3), 558–573.

De Koster, R., Le-Duc, T., Roodbergen, K.J., 2007. Design and control of warehouse order picking: A literature review. European Journal of Operational Research 182 (2), 481–501.

Gibson, D.R., Sharp, G.P., 1992. Order batching procedures. European Journal of Operational Research 58 (1), 57–67.

Gong, Y., De Koster, R., 2008. A polling-based dynamic order picking system for online retailers. IIE Transactions 40 (11), 1070–1082.

Le-Duc, T., De Koster, M.B.M., 2005. On determine the optimal number of zones in a pick-and-pack order picking system. Working Paper, RSM, Erasmus University Rotterdam.

Le-Duc, T., De Koster, M.B.M., 2007. Travel time estimation and order batching in a 2-block warehouse. European Journal of Operational Research. 176 (1), 374–388.

Manzini, R., Gamberi, M., Regattieri, A., 2005. Design and control of a flexible order picking system (FOPS) a new integrated approach to the implementation of an expert system. Journal of Manufacturing Technology Management 16 (1), 18–35.

Manzini, R., Gamberi, M., Persona, A., Regattieri, A., 2007. Design of a class based storage picker to product order picking system. International Journal of Advanced Manufacturing Technology 32 (17), 811–821.

Meller, R.D., Parikh, P.J., 2006. The Effect of Batching Orders on Workload-imbalance in Order Picking Systems. Progress in Material Handling Research. Material Handling Institute, Charlotte. pp. 271–286.

Petersen, C.G., 2002. Considerations in order picking zone configuration. International Journal of Operations and Production Management 27 (7), 793–805.

Petersen, C.G., Aase, G., Heiser, D.R., 2004. Improving order-picking performance through the implementation of class-based storage. International Journal of Physical Distribution and Logistics Management 34 (7), 534–544.

Roodbergen, K.J., 2001. Layout and routing methods for warehouses, Rotterdam School of Management Erasmus University, Ph.D. Dissertation.

Roodbergen, K.J., Vis, I.F.A., 2006. A model for warehouse layout. IIE Transactions 38, 799–811.

Rosenwein, M.B., 1996. A comparison of heuristics for the problem of batching orders for warehouse selection. International Journal of Production Research 34 (3), 657–664.

Tompkins, J.A., White, J.A., Bozer, Y.A., Frazelle, E.H., Tanchoco, J.M.A., 2003. Facilities Planning. John Wiley & Sons, Hoboken, NJ.

Van Nieuwenhuyse, I., De Koster, R., Colpaert, J., 2007. Order batching in multi-server pick-and-sort warehouses. FETEW Research Report KBI_0731, K.U. Leuven, pp. 29–39.

Whitt, W., 1993. Approximations for the GI/G/m queue. Production and Operations Management 2 (2), 114–159.

Yu, M., De Koster, R., 2008. Performance approximation and design of pick-and-pass order picking systems. IIE Transactions 40 (11), 1054–1069.