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Performance approximation and design of pick-and-pass order picking systems

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An approximation method based on $G/G/m$ queuing network modeling using Whitt's queuing network analyzer to analyze pick-and-pass order picking systems is presented. The objective of this approximation method is to provide an instrument for obtaining rapid performance estimates (such as order lead time and station utilization) of an order picking system. The pick-and-pass system is decomposed into conveyor segments and pick stations. Conveyor segments have a constant processing time, whereas the service times at a pick station depend on the number of order lines in the order to be picked at the station, the storage policy at the station and the work practices. The proposed approximation method appears to be sufficiently accurate for practical purposes. It can be used to rapidly evaluate the effects of the storage methods in pick stations, the number of order pickers at stations, the size of pick stations, the arrival process of customer orders and the impact of batching and splitting orders on system performance.

Keywords: Warehousing, order picking, pick-and-pass, queuing, stochastic

1. Introduction

Order picking, the process of picking products to fill customer orders, is one of the most important activities in warehouses due to its high contribution (about 55%) to the total warehouse operating cost (Tompkins *et al.*, 2003). This paper considers a common type of pick-and-pass order picking system, which consists of a conveyor connecting all pick stations located along the conveyor line, as sketched in Fig. 1. Storage shelves are used to store products at each pick station. A customer order contains several order lines (an order line is a number of units of one article). A bin is assigned to a customer order together with a pick list when it arrives at the order picking system. To fill an order, the order bin is transported on the conveyor passing various pick stations. If an order line has to be picked at a station, the transportation system automatically diverts the bin to the station, so that the main flow of bins cannot become blocked by bins waiting for picking. After entering the pick station, the order bin moves to the pick position. Order pickers are assigned to pick stations to fill customer orders. An order bin is processed by one order picker at a station and an order picker works on one order at a time. This paper assumes that the order picker picks one order line per picking trip. The picker starts his/her trip from the pick position, reads the next article on the bin's pick list, walks to the storage shelves indicated, picks the required article, goes back to the pick position and deposits the picked article into the bin.

Although in some systems multiple lines may be picked in a picking trip, we model the case where only one article is picked per trip. Systems that we have observed that adhere to this constraint include a parts Distribution Center (DC) of an international motor production company (we use this example in our model validation in Section 4) where one article is picked per trip since articles are relatively heavy and need to be barcode scanned. In another warehouse we studied, even light articles were not batched to reduce pick errors. Having finished the pick list, the order picker pushes the bin back onto the main conveyor, which transports the bin to the next pick station. Such pick-and-pass systems are typically applicable in the case of a large daily number of multi-line orders. De Koster (1996) summarizes the advantages of such order picking systems.

Recent trends in warehouses operation mean that companies now need to accept late orders while still providing a rapid and timely delivery within tight time windows, which implies the time available for order picking is shorter (De Koster *et al.*, 2007). Hence, minimizing order throughput time is an important objective in many warehouses, and it is used commonly in the order picking literature (see Chew

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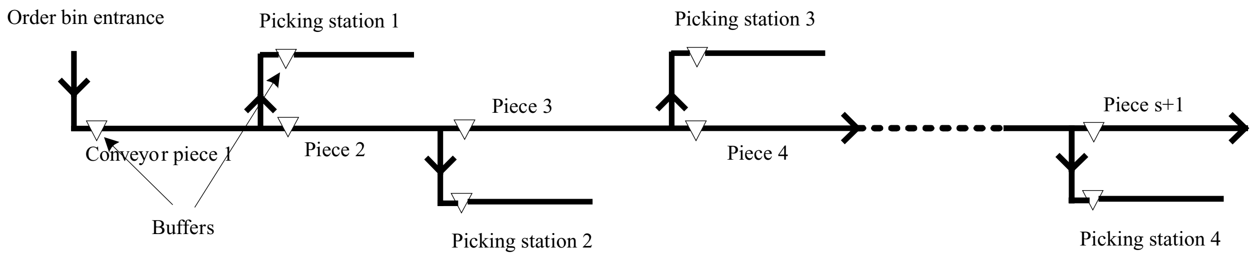

Fig. 1. Illustration of the pick-and-pass order picking system.

and Tang (1999), Roodbergen (2001) and Le-Duc and De Koster (2007)). Exact analysis of the pick-and-pass system described above is difficult due to the large state space required to model bin positions on the conveyor and difficulties in obtaining the exact distribution of service times at stations. This paper proposes an approximation-based modeling and analysis method to evaluate the mean order throughput time in such systems. The method provides a fast tool to evaluate alternatives in designing pick-and-pass systems. Our model relaxes the Jackson queuing network modeling of De Koster (1994) by allowing a general order arrival process and general service time distributions, which represent real-life warehouses more accurately and provide a deeper understanding of the pick-and-pass order picking system. The modeling and the analysis of the system is based on the analysis of a $G/G/m$ queuing network by Whitt (1983). We show the approximation method leads to acceptable results by comparing it with both simulation and with the real order picking process at a parts DC of an international motor production company.

The paper is organized as follows. In Section 2, we review the literature on pick-and-pass order picking. Section 3 describes the approximation model followed by model validation in Section 4. In Section 5, we analyze the impact of different warehousing activities on the system performance. We draw conclusions and discuss possible extensions of this paper in Section 6.

2. Related literature

The literature on order picking is extensive and mainly focuses on the following four issues that influence the order picking system performance: storage, batching, routing and zoning. A comprehensive literature review on order picking is given by De Koster *et al.* (2007). Research on pick-and-pass order picking systems is, however, not abundant. Jane (2000) considers a pick-and-pass order picking system, which he refers to as a *relay* picking system. He proposes several heuristic algorithms to address the problem of assigning n products into m storage zones (one picker per zone) with the objective of balancing the workload among order pickers in zones. Meller and Parikh (2006) propose a mathematical model, analogous to the classical dual bin-packing problem, to batch orders in both batch picking and zone picking systems with the objective of minimizing the workload imbalance among order pickers and zones. De Koster and Yu (2007) set up an integer programming model and design a heuristics to balance workload between progressive distribution zones for an internal distribution process at a Dutch flower auction. De Koster (1994) approximates pick-and-pass order picking systems by means of a Jackson-type network modeling and analysis approach. His model assumes the service time at each pick station is exponentially distributed and customer orders arrive according to a Poisson process. Jewkes *et al.* (2004) study the optimal pick position of an order bin at a pick station, the optimal product location at pick stations and the size of pick stations in a pick-and-pass order picking system with the objective of minimizing the order throughput time. Since they consider a static setting, only the travel time to pick orders is considered in their paper. Our paper considers a dynamic setting, where the waiting times of an order bin in front of pick stations is taken into account.

3. Approximation model

The pick-and-pass order picking system is represented by a sequence of pick stations connected by conveyor segments (see Fig. 1).

The service time for an order bin at a pick station consists of several components: *setup time* (time for starting and finishing the pick list, checking, weighing, labeling, etc.), *travel time* and the *picking time* for order lines. The travel time depends on the number of order lines to be picked at the station, the location of these order lines in the pick station and the travel speed of pickers. The picking time is proportional to the number of order lines to be picked in the station. We assume that the setup time and the pickers' travel speed are constants. We also assume that the picking time per order line, which may consist of multiple units, is constant, and is independent of the product type and the number of units picked. These assumptions are reasonable when the variance of the number of units picked per order line and the pick time itself are relatively small. We assume a pick-frequency class-based storage policy (see De Koster *et al.* (2007)) in each station. Similar to other research (see e.g., Petersen *et al.* (2004)), we assume that the demand is uniformly distributed over the products within a product class. The service time at a pick station is modeled as having a general distribution and is characterized only by its mean and Squared Coefficient of Variation (SCV). It is reasonable to use only two moments because in reality the service time is hard to fit to a theoretical distribution, whereas the information on the mean and the variance of the service time is relatively easy to obtain.

A conveyor segment j can contain k_j order bins and it is assumed to have a constant speed, vl_j. We approximate it as k_j servers in parallel, each of which has a constant service rate of vl_j/k_j. This means that the output rate of a conveyor segment j exactly equals vl_j if and only if it is completely full with bins. In the approximation, the output rate of a conveyor segment is proportional to the number of bins on it. At the end of a conveyor segment, a transition is made by the order bin to the subsequent conveyor segment, or it is pushed into a pick station. The transition probability for an order bin to enter a pick station depends on the bin's pick list and the storage assignment of products in that station. We approximate this behavior by a Markovian transition probability, which is justified in the case of a large bin throughput (the typical application area of these systems). The transition probabilities at the end of a conveyor segment and at leaving a pick station are calculated in Section 3.2. After finishing the picking at a station, the bin is pushed onto the next conveyor segment downstream of the pick station.

We assume that each pick station has an infinite storage capacity (buffer) for order bins. This assumption is reasonable because in reality order pickers at pick stations will ensure that the system will not be blocked when their stations become full. If a pick station tends to become full, the order pickers can temporarily put the bins on the floor. We also assume that there is a buffer with an infinite capacity in front of each conveyor segment which means that the arrivals will not be lost and pick stations and conveyor segments can not become blocked because of the lack of output capacity. This assumption is also realistic because the conveyor segments can normally contain a sufficiently large number of bins.

The whole pick-and-pass order picking system is approximately modeled as a $G/G/m$ queuing network consisting of $C + S$ nodes preceded by unlimited waiting space in front of them. Nodes $1, 2, \ldots, C$ represent the conveyor segments and nodes $C + 1, C + 2, \ldots, C + S$ represent the pick stations. The number of servers at each node is equal to the capacity of each conveyor segment or the number of order pickers working in the station.

The data used to analyze the queuing network are as follows.

S = the number of pick stations, with index j;
C = the number of conveyor segments, with index j;
J = the total number of nodes, which is equal to $S + C$, with index j;
I = the number of product classes stored in the pick stations, with index i;
N = the maximum number of order lines contained in a customer order, with index n;
O_n = probability that an order contains n order lines, $n = 1, 2, \ldots, N$;
f_i = order frequency of product class i, it is the probability that an order line belongs to the ith product class, $i = 1, 2, \ldots, I$;
vl_j = the velocity of conveyor segment j, expressed in bins per second, $j = 1, 2, \ldots, C$;
k_j = the capacity of conveyor segment j, expressed in bins, $j = 1, 2, \ldots, C$;
h_j = the number of order pickers at station j, $j = C + 1, C + 2, \ldots, C + S$;
m_j = the number of servers at node j, $m_j = k_j$ for $j = 1, 2, \ldots, C$, and $m_j = h_j$ for $j = C + 1, C + 2, \ldots, C + S$;
l_{ij} = the storage space (in meters) used to store products of the ith class in the racks at station j, $i = 1, 2, \ldots, I, j = C + I, C + 2, \ldots, C + S$;
sc = setup time per bin at a pick station, expressed in seconds;
tp = picking time per order line, expressed in seconds;
λ_{01} = external arrival rate of order bins to the system, entering node 1, expressed in bins/second;
c^2_{01} = SCV of inter-arrival time of order bins to the system.

The variables are as follows:

V_j = probability of visiting station j for an order bin, $j = C + 1, C + 2, \ldots, C + S$;
τ_j = total service time at station j if the order bin enters station $j, j = C + 1, C + 2, \ldots, C + S$;
wk_j = total travel time at station j if the order bin enters station $j, j = C + 1, C + 2, \ldots, C + S$;
pk_j = total picking time at station j if the order bin enters station $j, j = C + 1, C + 2, \ldots, C + S$;
c^2_{sj} = SCV of service time at node $j, j = 1, 2, \ldots, C + S$;
c^2_{aj} = SCV of inter-arrival time to node j, $j = 1, 2, \ldots, C + S$;
λ_j = internal arrival rate of order bins to node j, $j = 1, 2, \ldots, C + S$;
q_{kj} = transition probability from node k to node j, $k = 1, 2, \ldots, C + S, j = 1, 2, \ldots, C + S$;
vt_j = number of visits of an order bin to node j (either zero or one), $j = 1, 2, \ldots, C + S$;
W_j = waiting time of an order bin in front of node j, $j = 1, 2, \ldots, C + S$;
T_j = sojourn time of an order bin at node j, $j = 1, 2, \ldots, C + S$.

In the next two subsections, we will derive expressions for the mean and the SCV of the service time at each node and then calculate the mean throughput time of an order bin in the system.

3.1. *Mean and SCV of service times at pick stations and conveyor segments*

The mean service time at station j if the order bin enters station j, has three components, *the setup time sc, the travel time* wk_j and the *picking time* pk_j. The mean service time is calculated by

$$E[\tau_j] = sc + E[wk_j] + E[pk_j], \quad \forall j > C. \tag{1}$$

We next derive the expressions for the last two components in Equation (1).

The probability that an order line of class i is stored in station j depends on the order frequency of the ith class products and the space used to stored the ith class products in station j. It is given by

$$p_{ij} = f_i \times \frac{l_{ij}}{\sum_{j=1}^{S} l_{ij}}, \quad \forall i, \quad \forall j > C. \tag{2}$$

Therefore, the probability that an order line is picked in station j is the summation of p_{ij} over i:

$$P_j = \sum_{i=1}^{I} p_{ij}, \quad \forall j > C. \tag{3}$$

Thus, the conditional probability for an order bin to enter station j given that there are n order lines in the order is equal to the probability that there is at least one order line to be picked at station j:

$$V_{jn} = 1 - (1 - P_j)^n, \quad \forall j > C, \quad \forall n, \tag{4}$$

where $(1 - P_j)^n$ is the probability that none of the order lines in this order bin is to be picked in station j. The probability that an order bin enters station j now becomes:

$$V_j = \sum_{n=1}^{N} V_{jn} \times O_n, \quad \forall j > C. \tag{5}$$

The number of order lines to be picked in station j given that the order bin contains n order lines is a random variable with a binomial distribution, that is

$$P\{X_j = x_j | n \text{ order lines in an order}\} = \binom{n}{x_j} P_j^{x_j}(1 - P_j)^{n - x_j}, \quad \forall j > C. \tag{6}$$

Canceling out the condition, we have that:

$$P\{X_j = x_j\} = \sum_{n=1}^{N} P\{X_j = x_j | n \text{ order lines in an order}\} \times O_n = \sum_{n=1}^{N} \binom{n}{x_j} P_j^{x_j}(1 - P_j)^{n - x_j} O_n, \quad \forall j > C. \tag{7}$$

The expected number of lines to be picked at station j given the bin enters station j is

$$\begin{aligned} E[X_j | x_j > 0] &= \sum_{x_j=1}^{N} x_j \times P\{X_j = x_j | x_j > 0\} \\ &= \sum_{x_j=1}^{N} x_j \times \frac{P\{X_j = x_j, x_j > 0\}}{P\{X_j > 0\}} \\ &= \left(\sum_{x_j=1}^{N} x_j \sum_{n=1}^{N} \binom{n}{x_j} P_j^{x_j}(1 - P_j)^{n - x_j} O_n \right) \Big/ (1 - P\{X_j = 0\}), \quad \forall j > C. \end{aligned} \tag{8}$$

To obtain the expected travel time, $E[wk_j]$, for an order bin, we need information on the products' locations in a pick station. Under the pick-frequency class-based storage policy, the optimal locations of products and the *picker's home base* (pick position of order bins) in a pick station is illustrated in Fig. 2 (see Jewkes *et al.* (2004)), where class A refers to the class of those products with the highest demand frequency, class B the second highest class, and so on.

The expected travel time at station j given that the order bin will enter station j is

$$E[wk_j] = \frac{1}{ws} E\left[2 \times \sum_{i}^{I} d_{ij} \times X_{ij} | X_j > 0 \right], \quad \forall j > C, \tag{9}$$

where ws is the travel speed of the order pickers expressed in meters/second, X_{ij} is the number of lines of the ith class to be picked at station j and d_{ij} is the travel distance from the *picker's home base* to the location of the ith class of products. X_{ij} is equal to $X_j \times (p_{ij}/P_j)$ in distribution. As mentioned before, we assume that within each class, products are stored randomly and the demands are uniformly distributed over products. Hence, the d_{ij} are uniformly distributed random variables with a probability density function of

$$f_{d_{ij}}(x) = \begin{cases} \frac{2}{l_{ij}} & \text{for} \frac{1}{2} \sum_{k=0}^{i-1} l_{kj} \le x \le \frac{1}{2} \sum_{k=0}^{i} l_{kj}, \forall i, \forall j > C, \\ 0 & \text{elsewhere.} \end{cases} \tag{10}$$

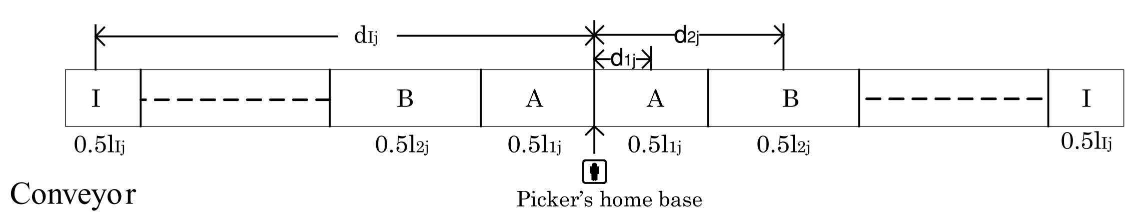

Fig. 2. Product locations in the storage rack at station j.

We define $l_{0j} = 0$ in the equation above. Because d_{ij} are independent from X_j, and the p_{ij}/P_j are not random variables, we obtain:

$$E[wk_j] = \frac{2}{ws} E[X_j | X_j > 0] \times \sum_{i}^{I} \frac{p_{ij}}{P_j} E[d_{ij}], \quad \forall j > C, \tag{11}$$

where $E[d_{ij}]$ is the expected value of d_{ij} given by

$$E[d_{ij}] = \int_{-\infty}^{\infty} x f_{d_{ij}}(x) \mathrm{d}x = \frac{1}{2} \sum_{k=0}^{i-1} l_{kj} + \frac{1}{4} l_{ij}, \quad \forall i, \quad \forall j > C. \tag{12}$$

Using Equation (8), we can calculate the expected picking time at station j given that the order bin will enter station j:

$$E[pk_j] = tp \times E[X_j | X_j > 0], \quad \forall j > C. \tag{13}$$

From Equations (1), (11) and (13), we can obtain the expected service time at station j given that the order bin will enter station j.

To obtain the SCV of the service time of an order bin at station j, we need to calculate the second moment of the service time, which is given by

$$\begin{aligned} E\left[\tau_j^2\right] &= E[(wk_j + pk_j + sc)^2], \quad \forall j > C \\ &= E\left[wk_j^2\right] + E\left[pk_j^2\right] + 2E[wk_j pk_j] + 2sc \\ &\quad \times E[wk_j] + 2sc \times E[pk_j] + sc^2. \end{aligned} \tag{14}$$

The second moment of wk_j is calculated as follows:

$$\begin{aligned} E\left[wk_j^2\right] &= \frac{4}{ws^2} E\left[X_j^2 | X_j > 0\right] \times E\left[\left(\sum_{i=1}^{I} D_{ij}\right)^2\right] \\ &= \frac{4}{ws^2} E\left[X_j^2 | X_j > 0\right] \\ &\quad \times E\left[2 \times \sum_{i=1}^{I} \sum_{k=i+1}^{I} (D_{ij} D_{kj}) + \sum_{i=1}^{I} D_{ij}^2\right] \\ &= \frac{4}{ws^2} E\left[X_j^2 | X_j > 0\right] \\ &\quad \times \left\{2 \times \sum_{i=1}^{I} \sum_{k=i+1}^{I} E[D_{ij}] E[D_{kj}] + E\left[\sum_{i=1}^{I} D_{ij}^2\right]\right\} \\ &\qquad \forall j > C, \end{aligned} \tag{15}$$

where

$$D_{ij} = \frac{p_{ij}}{P_j} \times d_{ij},$$

and

$$E[D_{ij}] = \frac{p_{ij}}{P_j} E[d_{ij}].$$

The last step of Equation (15) follows from the independence of D_{ij} and D_{kj} if $i \neq k$. The conditional second moment of X_j is given by

$$\begin{aligned} E\left[X_j^2 | X_j > 0\right] &= \sum_{x_j=1}^{N} x_j^2 \times \frac{P\{X_j = x_j, x_j > 0\}}{P\{X_j > 0\}} \\ &= \left(\sum_{x_j=1}^{N} x_j^2 \sum_{n=1}^{N} \binom{n}{x_j} P_j^{x_j} (1 - P_j)^{n - x_j} O_n\right) \Big/ \\ &\quad P\{X_j > 0\}, \quad \forall j > C. \end{aligned} \tag{16}$$

The second moment of D_{ij} is given below:

$$\begin{aligned} E\left[D_{ij}^2\right] &= \left(\frac{p_{ij}}{P_j}\right)^2 \times E\left[d_{ij}^2\right] \\ &= \left(\frac{2}{l_{ij}}\right) \left(\frac{p_{ij}}{P_j}\right)^2 \times \int_{0.5 \sum_{k=0}^{i-1} l_{kj}}^{0.5 \sum_{k=0}^{i} l_{kj}} x^2 \mathrm{d}x \\ &= \frac{1}{3} \left(\frac{2}{l_{ij}}\right) \left(\frac{p_{ij}}{P_j}\right)^2 \left[\left(\frac{1}{2} \sum_{k=0}^{i} l_{kj}\right)^3 - \left(\frac{1}{2} \sum_{k=0}^{i-1} l_{kj}\right)^3\right], \\ &\qquad \forall i, \quad \forall j > C. \end{aligned} \tag{17}$$

From Equations (15) to (17), we obtain $E[wk_j^2]$. The second moment of pk_j is obtained by

$$E\left[pk_k^2\right] = tp^2 \times E\left[X_j^2 | X_j > 0\right]. \tag{18}$$

The component $E[wk_j pk_j]$ is calculated as

$$\begin{aligned} &E[wk_j pk_j] \\ &= \frac{1}{ws} E\left[2 \times \left(\sum_{i=1}^{I} d_{ij} \times \frac{p_{ij}}{P_j} \times X_j\right) \times tp \times X_j | X_j > 0\right] \\ &= \frac{2 \times tp}{ws} \times E\left[X_j^2 | X_j > 0\right] \times \left\{\sum_{i=1}^{I} \frac{p_{ij}}{P_j} \times E[d_{ij}]\right\}, \ \forall j > C. \end{aligned} \tag{19}$$

From Equations (11) to (19), we can obtain the second moment of the service time at a pick station given that the order bin will enter that station. With the value of the first and second moments of the service time, we can calculate the SCV of the service time at station j:

$$c_{\mathrm{sj}}^2 = \frac{E\left[\tau_j^2\right] - E[\tau_j]^2}{E[\tau_j]^2}, \quad \forall j > C. \tag{20}$$

As mentioned at the beginning of this section, the service rate of each server of a conveyor segment is constant; therefore the values of the SCVs for conveyor segments are zero, that is

$$c_{\mathrm{sj}}^2 = 0, \quad \forall j \leq C. \tag{21}$$

The mean service time of a server on a conveyor segment is the reciprocal of its service rate:

$$E[\tau_j] = \frac{k_j}{v l_j}, \quad \forall j \leq C. \tag{22}$$

Using this information on the mean and the SCV of the service time at each node, we will calculate the order throughput time in the system in the next subsection.

3.2. *Mean throughput time of an order*

We calculate the mean throughput time of an order bin in the pick-and-pass order picking system under consideration based on the $G/G/m$ queuing network approximation model of Whitt (1983, 1993) (see Appendix A). The mean order throughput time consists of the transportation times on the conveyor segments, the service times at pick stations and the waiting times in front of conveyor segments and pick stations. The approximation analysis uses two parameters to characterize the arrival process and the service time at each node, one to describe the rate, and the other to describe the variability. The two parameters for the service time are $E[\tau_j]$, and c_{sj}^2, as we derived in Section 3.1. For the arrival process, the parameters are λ_j, the arrival rate, which is the reciprocal of the mean inter-arrival time between two order bins to each node, and c_{aj}^2, the SCV of the inter-arrival time. Orders bins arrive at the system at conveyor segment 1 (see Fig. 1) with rate λ_{01}, and the SCV of the inter-arrival time is c_{01}^2. To calculate the internal arrival rate and the SCV of the inter-arrival time at each node, we need to know the transition probabilities between nodes. At the end of a conveyor segment, an order bin is either transferred to a subsequent conveyor segment for transportation or pushed into a pick station. In the case of a pick-and-pass system with the layout as sketched in Fig. 1, the transition probabilities between these nodes are given by

$$q_{jj+C} = V_{j+C}, \quad \forall j < C, \tag{23}$$

$$q_{jj+1} = 1 - V_{j+C}, \quad \forall j < C, \tag{24}$$

$$q_{jj-S} = 1, \quad \forall C < j \le C + S, \tag{25}$$

where the value of V_{j+C} is obtained from Equation (5). The transition probabilities between other nodes are zero. Because order bins leave the system from the last conveyor segment C, we have $q_{Cj} = 0$ for $\forall\, 1 \le j \le J$. The matrix of the transition probabilities is indicated by $\mathbf{Q}$. As an example, consider a network with three pick stations and four conveyor segments, i.e., $C = 4$ and $S = 3$. Assuming that at the end of each conveyor segment (except for segment 4, the last one), a bin has a probability of 0.6 of being pushed into the next pick station. Bins enter the system at node 1 and leave the system at node 4. The Markov transition matrix is then given by

$$\mathbf{Q} = \begin{pmatrix} 0 & 0.4 & 0 & 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0.4 & 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0 & 0.6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

With this probability transition matrix, we can obtain the internal traffic rates λ_j and the SCV of the inter-arrival time between two bins at each node (see Appendix A).

The utilization of a conveyor segment and a pick station is given by

$$\rho_j = \begin{cases} \lambda_j / vl_j, & \forall j \le C, \\ \lambda_j E[\tau_j]/h_j, & \forall j > C. \end{cases} \tag{26}$$

The expected sojourn time of a bin at node j is given by

$$E[T_j] = E[vt_j] \times (E[W_j] + E[\tau_j]), \quad \forall 1 \le j \le J, \tag{27}$$

where $E[W_j]$ is the expected waiting time in front of node j as calculated by Equation (A9), and $E[vt_j]$ is the expected number of visits to node j of an order bin. The probability mass function of vt_j is given by

$$vt_j = \begin{cases} 0 & \text{with probability } 1 - V_j \\ 1 & \text{with probability } V_j \end{cases} \quad \forall\, 1 \le j \le J, \tag{28}$$

where V_j is obtained from Equation (5) for $j > C$ and $V_j = 1$ for $j \le C$. Hence

$$E[vt_j] = 0 \times (1 - V_j) + V_j = V_j, \quad \forall 1 \le j \le J. \tag{29}$$

The total expected order throughput time is the summation of the expected sojourn time at each node.

4. Model validation

To validate the quality of the approximation method described in Section 3, we compare the results with both simulation and a real order picking process.

We built a simulation model in Automod® 10.0. For each scenario in the example, we used at least 20 000 orders, preceded by 2000 orders of initialization for the system to become stable, to guarantee that the 95%-confidence interval width of the Mean Order Throughput Time (MOTT) is below 1% of the mean value. The parameters used in the example are listed in Table 1.

Table 2 illustrates the storage assignments in stations and the probability that an order bin has to be handled at a station. We observe from Table 2 that stations have the same total storage space but use different storage spaces per product class (i.e., a non-uniform storage policy).

We varied the order arrival rates to the system to compare the performance of the approximation method to simulation under different workloads. The results are listed in Table 3. This table also illustrates the accuracy of $G/G/m$ modeling over the Jackson-type modeling used in De Koster (1994).

Table 3 shows that the relative errors between the approximation model and the simulation results are all below 6% under different workloads. It also shows that the larger the utilizations at stations, the more accurate is the $G/G/m$ modeling compared to Jackson-type modeling.

We also conducted other experiments with different parameters: the number of pick stations varied from four to 18, with a step size of two, and the utilization of pick stations varied from 0.2 to 0.9 with a step size of 0.1. In all

Table 1. Parameters used in the example

Parameter	*Value*
Order arrival process	Poisson distributed (we evaluate different arrival rates)
Number of stations	18
Number of order pickers	18
Product classes and order frequency per class	Class 1: $f_1 = 0.8$, Class 2: $f_2 = 0.15$, Class 3: $f_3 = 0.05$
Total fraction of storage space for product classes	Class 1: 0.2, Class 2: 0.3, Class 3: 0.5
Size of order bins	$60 \times 40 \times 35$ cm
Conveyor speed	0.7 bins per second (0.1 m minimum space between two bins)
Conveyor length	First segment 40 bins, 20 bins for others
Length of each pick station	28 m (40 bins)
Walk speed of order pickers	1 m/s
Picking time per line	18 s
Setup time	45 s
Maximum number of lines in an order bin	30
The number of order lines in an order	Empirical distribution (based on the data from a Dutch warehouse) with mean of 15.6 and standard deviation of 6.3

experimental settings, the relative error between the approximation model and the simulation results was below 7%.

To further validate our approximation method, we compared our results to the performance of a real order picking process in the bulky storage area at the parts DC of an international motor production company. The bulky storage area stores a total of 240 products divided into three classes. One class contains 48 heavy products and the other two classes are categorized according to their order frequencies, each containing 96 products. The whole area is divided into four pick stations connected by conveyor segments. Through analyzing the log files from the Warehouse Management System (WMS) for a picking day, which is chosen as a representative of its typical picking process, we obtained the data for the order arrival process to the system, the service times at pick stations and the routing probabilities of order bins to enter each station. The results are listed in Table 4. We also measured the capacities of conveyor pieces and their speeds. We input these data into our approximation model. The MOTT result is compared with the mean order throughput time obtained from the WMS.

From Table 4, we find that the relative error is around 6%. We conclude that the quality of the approximation method is acceptable for practical purposes. In the next section, we use this approximation method as a tool to estimate the pick-and-pass order picking system performance under various warehousing policies.

5. Scenario analyses

In this section, we use the approximation method to analyze the impact of different warehousing policies on the order picking system's performance. These policies include the storage assignments in pick stations, the size of pick stations, the number of order pickers in stations, and order batching and splitting decisions in the order release process. The parameters used for these scenario analyses are illustrated in Table 1.

5.1. *The effects of storage policies on system performance*

Storage policies affect the order throughput time in the pick-and-pass system, as they impact the workload balance between stations. In this subsection, we will compare the impact of uniform (stations use identical storage spaces to store a certain class of products) and non-uniform (stations use different storage spaces to store a certain class of products) storage policies on the MOTT. We expect that the uniform storage policy leads to a shorter order throughput time, as it leads to workload balance between stations.

The storage space for each class of products in stations and the probability for a bin to enter a pick station under the uniform storage policy are shown in Table 5.

Table 6 illustrates a comparison with the non-uniform storage policy (refer to Tables 2 and 3). As the stations are now balanced on average, we find from Table 6 that the

Table 2. Storage space and the bin visit probabilities to stations under the non-uniform storage policy

$l_{ij}(m)$	*St. 1*	*St. 2*	*St. 3*	*St. 4*	*St. 5*	*St. 6*	*St. 7*	*St. 8*	*St. 9*	*St. 10*	*St. 11*	*St. 12*	*St. 13*	*St. 14*	*St. 15*	*St. 16*	*St. 17*	*St. 18*
Class 1	4.9	5.6	6.3	4.9	5.6	6.3	4.9	5.6	4.9	6.3	4.9	5.6	6.3	4.9	5.6	6.3	4.9	5.6
Class 2	7.7	8.4	9.1	7.7	8.4	9.1	7.7	8.4	7.7	9.1	7.7	8.4	9.1	7.7	8.4	9.1	7.7	8.4
Class 3	15.4	14	12.6	15.4	14	12.6	15.4	14	15.4	12.6	15.4	14	12.6	15.4	14	12.6	15.4	14
Bin visit probability	0.36	0.39	0.43	0.36	0.39	0.43	0.36	0.39	0.43	0.36	0.39	0.43	0.36	0.39	0.43	0.36	0.39	0.43

Table 3. Validation results for the example and comparisons to Jackson modeling

Input rate (bin/s)	*MOTT (s)(G/G/m)*				*MOTT(s) (Jackson)*
	Numerical	*Simulation*	*Rel. error (%)*	*Station utilization (max)*	
0.008	1615.5	1556.2 ± 4.6	3.81	0.409	1867.5
0.011	1725.0	1647.3 ± 5.2	4.72	0.517	2119.9
0.013	1889.8	1789.5 ± 6.1	5.60	0.630	2518.6
0.016	2290.8	2171.5 ± 8.3	5.49	0.780	3559.0
0.018	3116.0	3023.4 ± 15.7	3.06	0.893	5792.4
0.019	4312.8	4247.4 ± 24.4	1.54	0.944	9078.5

MOTTs are shorter under the uniform storage policy than under the non-uniform storage policy. The improvement is substantial when the workload of the system increases.

Because of the advantage of the uniform storage policy, we will focus our analysis on this storage policy in the following discussions.

5.2. *The effects of station sizes and the number of pickers on system performance*

The size of the pick stations and the number of order pickers in stations impact the MOTT. With a fixed length of the whole order picking system (i.e., a fixed storage capacity of the system) and a fixed number of order pickers, the larger the size of the pick stations, the fewer number of stations we have in the system, and the more order pickers are available at each pick station. Pick stations of larger size will increase the service time due to longer travel times, and a fewer number of stations tends to increase the utilization of pick stations due to higher bin arrival rates. Therefore, they lead to an increase in the MOTT. However, on the other hand, a lower number of stations leads to fewer station visits for an order bin (hence fewer queues and a smaller setup time); more order pickers per station implies decreasing utilization at pick stations, which reduces the MOTT. In pick-and-pass order picking system design, a main question therefore is to find the right trade-off between these opposite effects by selecting the right number of stations. Table 7 shows the system performance for various combinations of station sizes and order pickers per station. It shows that under the current settings, the scenario of six stations with three order pickers per station has the best performance for all used arrival rates.

5.3. *The effect of batching orders on system performance*

As we have seen from the analysis above, the input rate of order bins to the system has a significant impact on system performance. A large arrival rate results in a higher workload for the system, and it will subsequently increase the MOTT. One way to reduce the input rate to the system is to batch orders. We consider the following batching rules. We batch two successive order bins each containing at most L lines into one bin, and then send it to the system. The order bins with more than L lines are sent directly to the system. The batching threshold, L can take any value between one and $\lfloor N/2 \rfloor$, where $\lfloor * \rfloor$ means rounding down to the nearest integer. Otherwise, the number of lines in a batched bin

Table 4. Data and comparison with results of the real order picking system

Parameter	*Value*
Number of stations	4
Number of order pickers per station	1
Number of order lines to pick per order	Empirical distribution (mean, 2.5, stdv, 1.9)
Order inter-arrival time to the system (s)	Empirical distribution (mean, 28.9, stdv, 52.4)
Service time at station A (s)	Empirical distribution (mean, 40.1, stdv, 41.6)
Service time at station B (s)	Empirical distribution (mean, 51.0, stdv, 51.1)
Service time at station C (s)	Empirical distribution (mean, 54.1, stdv, 48.0)
Service time at station D (s)	Empirical distribution (mean, 38.8, stdv, 35.0)
Prob. To enter station A	0.385
Prob. To enter station B	0.254
Prob. To enter station C	0.271
Prob. To enter station D	0.435
MOTT from $G/G/m$ approximation model (s)	302.1
MOTT from WMS (s)	321.7
Relative error (%)	6.1

Table 5. Storage space and the bin visit probabilities to stations under the uniform storage policy

$l_{ij}(m)$	St. 1	St. 2	St. 3	St. 4	St. 5	St. 6	St. 7	St. 8	St. 9	St. 10	St. 11	St. 12	St. 13	St. 14	St. 15	St. 16	St. 17	St. 18
Class 1	5.6	5.6	5.6	5.6	5.6	5.6	5.6	5.6	5.6	5.6	5.6	5.6	5.6	5.6	5.6	5.6	5.6	5.6
Class 2	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4
Class 3	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
Bin visit probability	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39

may exceed the bin's capacity. We assume that N (the maximum number of lines in an order) is also the capacity of an order bin. By batching small orders, we can decrease the input rate to the system, leading to a decrease in the MOTT. On the other hand, the service time at each station and the probability of entering a pick station will increase because more order lines have to be picked. These factors imply an increase of the MOTT. When we batch two successive bins with fewer than L lines, the first bin has to wait for several inter-arrival time periods to be processed. However, since the mean order inter-arrival time is normally very small compared to the total MOTT, and only those bins containing less than L lines are batched, this effect is small and can be neglected. The impact of order batching on system performance depends on the trade-off between the above-mentioned factors. We can analyze this impact with a slight modification of the approximation method discussed earlier.

Assuming the original input process to the system is Poisson distributed with rate λ_{01}, an order bin has a probability O_n to contain n order lines. The flow of order bins with n order lines is also a Poisson process with rate $\lambda_{01} \times O_n$. After batching, the original process is split into two subprocesses. The first subprocess refers to the batched bins, and the second subprocess is the unbatched bins. According to the properties of a Poisson process, the inter-arrival time of the first subprocess is Gamma distributed with parameters (2, $\lambda_{01} \times \sum_{n=1}^{L} O_n$). The input rate of this type of order flow is $\tilde{\lambda}_{011} = 0.5\lambda_{01} \times \sum_{n=1}^{L} O_n$, and the SCV of the order inter-arrival time is $c_{011}^2 = 0.5$. The second subprocess is Poisson distributed with rate $\tilde{\lambda}_{012} = \lambda_{01} \times \sum_{n=L+1}^{N} O_n$, where N is the maximum number of lines in a bin. The SCV of the order inter-arrival time is $c_{012}^2 = 1$.

The basic idea to calculate the MOTT with two input flows is derived from Whitt (1983). The procedure is first to calculate the mean and the SCV of the service time at each pick station, the transition probabilities between nodes and the internal traffic flows to each node separately for each input flow, and then convert these two types of flows into one (see Appendix B). The method of Appendix A is then used to obtain MOTT. Following the example at the beginning of this section, we assume that L is equal to 15. Table 8 compares the system performance between batching and non-batching scenarios.

Table 8 shows that the input rates decrease, and the service times at pick stations increase when orders are batched. Batching orders can slightly reduce the utilizations of pick stations. The impact of pick station utilizations on waiting times in front of the stations is marginal when the utilizations are low, but becomes substantial when the utilizations get higher. We observe that when the system is not heavily loaded, order batching increases the MOTT. This is mainly due to the longer service time at pick stations, and the increased probability of entering pick stations. However, when the system is heavily loaded, the MOTT decreases when we batch orders. Under a heavy load, the waiting time is the major component of the order throughput time; reducing pick stations utilizations by batching orders can significantly reduce the waiting time in front of the pick stations, and therefore reduces the MOTT.

5.4. *The effects of splitting orders on system performance*

As an alternative to batching orders, splitting an order into two small orders will reduce the order bin service times in pick stations and the probabilities of entering pick stations.

Table 6. Comparison of system performance between uniform and non-uniform storage policies in pick stations

	MOTT(s)			Utilization	
Input rate (bin/s)	Uniform	Non-uniform	Improvement (%)	Uniform	Non-uniform
0.008	1613.0	1615.5	0.15	0.376	0.409
0.011	1720.3	1725.0	0.27	0.475	0.517
0.013	1876.4	1889.8	0.71	0.579	0.630
0.016	2236.6	2290.8	2.37	0.716	0.780
0.018	2849.1	3116.0	8.57	0.821	0.893
0.019	3436.9	4312.8	20.31	0.868	0.944

Table 7. System performances under various station sizes and the number of order pickers per station

Input rate (bin/s)	*Number of stations (number of pickers per station) (station size in meters)*											
	18 (1)(28)		*9 (2)(56)*		*6 (3)(84)*		*3 (6)(168)*		*2 (9)(252)*		*1 (18)(514)*	
	MOTT(s)	*Utilization*	*MOTT(s)*	*Utilization*	*MOTT(s)*	*Utilization*	*MOTT(s)*	*Utilization*	*MOTT(s)*	*Utilization*	*MOTT(s)*	*Utilization*
0.008	1613.0	0.376	1370.7	0.348	1330.2	0.345	1426.6	0.398	1607.1	0.482	2304.4	0.771
0.011	1720.3	0.475	1407.9	0.439	1351.7	0.436	1441.2	0.503	1630.3	0.608	4867.1	0.974
0.013	1876.4	0.579	1463.7	0.535	1386.6	0.531	1474.0	0.612	1706.1	0.741	inf	>1
0.016	2236.6	0.716	1586.9	0.663	1468.9	0.657	1587.6	0.758	2332.0	0.917	inf	>1
0.018	2849.1	0.821	1765.7	0.759	1591.7	0.753	1870.8	0.868	inf	>1	inf	>1
0.019	3436.9	0.868	1904.3	0.803	1687.0	0.796	2263.9	0.918	inf	>1	inf	>1
0.020	4226.5	0.903	2052.3	0.835	1788.2	0.828	3113.5	0.955	inf	>1	inf	>1
0.021	6110.2	0.940	2294.8	0.870	1951.6	0.863	16 765.0	0.994	inf	>1	inf	>1

Table 8. Comparison of system performances between batching and non-batching scenarios

	$L = 15$											
Order arrival rate (bins/s)	*0.0083*	*0.0105*	*0.0128*	*0.0159*	*0.0182*	*0.0185*	*0.0186*	*0.0189*	*0.0192*	*0.0200*	*0.0204*	*0.0208*
Rate after batching (bins/s)	*0.0063*	*0.0079*	*0.0096*	*0.0119*	*0.0136*	*0.0138*	*0.0139*	*0.0142*	*0.0144*	*0.0150*	*0.0153*	*0.0156*
MOTPT (s)												
Batching	1864.2	1973.9	2123.4	2435.8	2891.8	2971.6	3004.5	3114.5	3162.0	3679.9	3991.1	4424.2
Non-batching	1613.0	1720.3	1876.4	2236.6	2849.1	2968.2	3018.4	3191.7	3436.9	4226.5	4925.5	6110.2
Utilization												
Batching	0.358	0.452	0.551	0.682	0.781	0.792	0.797	0.810	0.826	0.859	0.876	0.895
Non-batching	0.376	0.475	0.579	0.716	0.821	0.833	0.837	0.852	0.868	0.903	0.921	0.940
Mean waiting time (s)												
Batching	22.1	31.0	43.1	68.4	105.3	111.7	114.4	123.3	135.2	169.0	194.2	229.2
Non-batching	23.7	34.3	49.7	85.2	145.5	157.3	162.2	179.3	203.5	281.3	350.1	466.9
Mean service time (s)												
Batching	83.4	83.4	83.4	83.4	83.4	83.4	83.4	83.4	83.4	83.4	83.4	83.4
Non-batching	80.1	80.1	80.1	80.1	80.1	80.1	80.1	80.1	80.1	80.1	80.1	80.1
Bin visit probability												
Batching	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69
Non-batching	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56

Table 9. Comparison of system performances between splitting and non-splitting scenario

	R = *15*											
Order arrival rate (bins/s)	*0.0083*	*0.0105*	*0.0128*	*0.0159*	*0.0164*	*0.0166*	*0.0167*	*0.0169*	*0.0172*	*0.0175*	*0.0182*	*0.0192*
Rate after splitting (bins/s)	*0.0130*	*0.0164*	*0.0200*	*0.0248*	*0.0256*	*0.0259*	*0.0260*	*0.0264*	*0.0269*	*0.0274*	*0.0284*	*0.0300*
MOTPT (s)												
Splitting	1344.6	1451.1	1624.1	2128.2	2295.8	2369.6	2404.3	2536.4	2700.9	2911.8	3581.9	7023.3
Non-splitting	1613.0	1720.3	1876.4	2236.6	2333.2	2372.8	2390.9	2456.6	2532.3	2620.4	2849.1	3436.9
Utilization												
Splitting	0.416	0.526	0.640	0.793	0.819	0.828	0.833	0.847	0.861	0.876	0.908	0.961
Non-splitting	0.376	0.475	0.579	0.716	0.740	0.749	0.752	0.765	0.778	0.792	0.821	0.868
Mean waiting time (s)												
Splitting	26.9	40.5	63.0	128.2	149.8	159.3	163.8	180.9	202.2	229.4	316.0	760.6
Non-splitting	23.7	34.3	49.7	85.2	94.7	98.6	100.4	106.8	114.3	123.0	145.5	203.5
Mean service time (s)												
Splitting	74.5	74.5	74.5	74.5	74.5	74.5	74.5	74.5	74.5	74.5	74.5	74.5
Non-splitting	80.1	80.1	80.1	80.1	80.1	80.1	80.1	80.1	80.1	80.1	80.1	80.1
Bin visit probability												
Splitting	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43
Non-splitting	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56

On the other hand, splitting orders increases the arrival flow rate because more order bins enter the system. To analyze the impact of order splitting on system performance, we split an order bin containing R or more than R lines into two bins, one containing $\lfloor R/2 \rfloor$ lines and the other containing $R - \lfloor R/2 \rfloor$ lines. Again, assuming the original arrival process is Poisson distributed, the input process is divided into two Poisson processes: the input flow of non-split bins with rate $\tilde{\lambda}_{011} = \lambda_{01} \times \sum_{n=1}^{R-1} O_n$, and the input flow of bins to be split with rate $\tilde{\lambda}_{012} = \lambda_{01} \times \sum_{n=R}^{N} O_n$. Before arriving at the first conveyor segment, we suppose that the input flow of bins to be split will first pass through an artificial node with a very small constant service time. A new order bin is created following the completion of service at the artificial node. According to the approximation method given in Sections 2.2 and 4.6 of Whitt (1983), the departure process, i.e., the arrival process to the first conveyor segment of this flow of split bins, has a rate of $2\tilde{\lambda}_{012}$ and an approximated SCV of the inter-arrival time of two.

The total arrival process to the first conveyor segment is therefore the combination of a Poisson process, with rate $\tilde{\lambda}_{011} = \lambda_{01} \times \sum_{n=1}^{R-1} O_n$, and a process with a rate of $2\tilde{\lambda}_{012}$ and an SCV of the inter-arrival time of two. Similar to the approaches used to analyze batching orders, we can obtain the system performance for the order splitting scenario. The results in comparison with the non-splitting scenario, as illustrated in Table 9, show that splitting orders increases the input rate to the system and reduces the service times at pick stations and the probabilities of entering pick stations. Splitting orders increases the utilizations of pick stations. MOTT shortens when the station utilizations are low. This is mainly due to the reduction in service times and the probabilities of entering pick stations. When station utilization becomes high ($\rho > 0.75$ approximately for R equal to 15), order splitting increases the MOTT because the waiting time in front of a station becomes longer due to higher utilization.

We note that the approximation model underestimates the MOTT when we consider each split as a separate order. However, in reality, orders are only split when the number of order lines is large, and the impact of this error on the MOTT will be slight. The approximation model will give a reasonable estimation for the MOTT from a practical point of view.

6. Conclusions and extensions

In this paper, we propose an approximation method based on $G/G/m$ queuing network modeling to analyze performance of a pick-and-pass order picking system. The method can be used as a fast tool to estimate the impact of design alternatives on the MOTT of the order picking system. These alternatives include the storage policies, the size of pick stations, the number of order pickers in stations and the arrival process of customer orders. In general, the preference of one alternative over others is subject to a detailed specification of the order picking system. The quality of the approximation method is acceptable for practical purposes. Therefore, it enables planners to evaluate various system alternatives, which is essential at the design phase of the order picking system. Additionally, the approximation method can also be used to evaluate various operational policies such as order batching and order splitting on system performance.

The model lends itself to several modifications and extensions that are left for future research. Although we assumed in this paper that pickers pick only one order line in their picking trip, it is possible to relax this assumption and derive the first and second moments of the service time for picking multiple lines in a pick trip. We may also take the number of units to pick in an order line into consideration and differentiate the picking time for different articles. In such cases, the number of units to pick in an order line and the picking time per article are both stochastic variables. The $G/G/m$ queuing network approximation model still can be used to analyze these situations, but characterizing the service distribution is less straightforward. The layout of pick stations can be altered (we here assumed a line layout) to, for example, a parallel-aisle layout.

We also estimated the standard deviation of the order throughput time using the method described in Whitt (1983). However, the method did not provide good estimation results. It would also be interesting to find a more accurate approach to estimate the standard deviation of the order throughput time, which together with the MOTT provides a better description of the order picking system performance. Another interesting extension of the paper is to consider the situation when an order picker is responsible for picking at multiple pick stations. Furthermore, in reality, the buffer capacity in front of each pick station is finite, which influences the performance in high-utilization situations. It might be possible to derive estimates for the mean throughput time using approximation methods for finite-buffer queuing networks.

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Appendices

Appendix A

According to Whitt (1983), to estimate the MOTT in this $G/G/m$ queuing network system, we need to calculate the internal flow parameters. The internal flow rate to each node, λ_j, is obtained by solving the following linear equation:

$$\lambda_j = \lambda_{0j} + \sum_{i=1}^{J} \lambda_i q_{ij}, \quad 1 \le j \le J, \tag{A1}$$

where λ_{0j} is the external arrival rate to node j, J is the total number of nodes (conveyor segments and pick stations) in the system and q_{ij} is the transition probability from node i to node j.

The arrival rate to node j from node i is given by

$$\lambda_{ij} = \lambda_i q_{ij}, \quad \forall 1 \le i \le J, \quad \forall 1 \le j \le J. \tag{A2}$$

The proportion of arrivals to j that come from i, is calculated by

$$pr_{ij} = \lambda_{ij}/\lambda_j, \quad \forall 0 \le i \le J, \quad \forall 1 \le j \le J. \tag{A3}$$

The variability of the parameters of the internal flow, i.e., the SCVs of the inter-arrival time of the arrival processes to nodes, are calculated by solving the following linear equation:

$$c_{aj}^2 = a_j + \sum_{i=1}^{J} c_{ai}^2 b_{ij}, \quad 1 \le j \le J, \tag{A4}$$

where

$$a_j = 1 + \omega_j \left\{ \left(pr_{0j} c_{0j}^2 - 1\right) + \sum_{i=1}^{J} pr_{ij}\left[(1 - q_{ij}) + q_{ij}\rho_i^2 x_i\right]\right\}, \tag{A5}$$

$$b_{ij} = \omega_j pr_{ij} q_{ij}\left(1 - \rho_i^2\right), \tag{A6}$$

and c_{0j}^2 is the SCV of the external inter-arrival time to node j, and $c_{0j}^2 = 0$ for $\forall j > 1$, since the order bins enter the system from the first conveyor segment. ρ_i is the utilization of node i obtained from Equation (26), and

$$x_i = 1 + m_i^{-0.5}\left(\max\left\{c_{si}^2 .0.2\right\} - 1\right), \tag{A7}$$

with m_i being the number of servers at node i, and c_{si}^2 the SCV of service time at node i obtained from Equations (20) and (21):

$$\omega_j = [1 + 4(1 - \rho_j)^2(\upsilon_j - 1)]^{-1}, \tag{A8}$$

with

$$\upsilon_j = \left[\sum_{i=0}^{J} pr_{ij}^2\right]^{-1}.$$

With the internal flow parameters, λ_j and c_{aj}^2, and the service time parameters, $E[\tau_j]$, and c_{sj}^2, Whitt (1983) decomposes the network into separate service facilities that are analyzed in isolation. Each service facility is a $G/G/m$ queue. Whitt (1993) provides the following approximation for the expected waiting time in queues. Since we are focusing on a single node, we omit the subscript indexing the node in deriving the expected waiting time in front of a node.

For a multi-server node with m servers, the expected waiting time is given by

$$E[W]_{G/G/m} = \phi\left(\rho, c_a^2, c_s^2, m\right)\left(\frac{c_a^2 + c_s^2}{2}\right) E[W]_{M/M/m}, \tag{A9}$$

where c_a^2 and c_s^2 are obtained from Equation (A4), and Equations (20) and (21) respectively, ρ is given by Equation (26), and $E[W]_{M/M/m}$ is the waiting time in queue of a multi-server node with Poisson arrivals and exponential service distribution. The exact expression for $E[W]_{M/M/m}$ is given by

$$E[W]_{M/M/m} = \frac{P(N \ge m)}{\mu m(1 - \rho)}, \tag{A10}$$

where μ is the reciprocal of the mean service time at each node. $P(N \ge m)$ is the probability that all servers are busy and is given by

$$P(N \ge m) = \left(\frac{(m\rho)^m}{m!(1 - \rho)}\right)\zeta, \tag{A11}$$

with

$$\zeta = \left(\frac{(m\rho)^m}{m!(1 - \rho)} + \sum_{k=0}^{m-1} \frac{(m\rho)^k}{k!}\right)^{-1}.$$

The expression for ϕ in Equation (A9) is given by

$$\phi\left(\rho, c_a^2, c_s^2, m\right)$$

$$= \begin{cases} \left(\frac{4(c_a^2 - c_s^2)}{4c_a^2 - 3c_s^2}\right)\phi_1(m,\rho) + \left(\frac{c_s^2}{4c_a^2 - 3c_s^2}\right) \\ \quad \psi(c^2, m, \rho),\ c_a^2 \geq c_s^2, \\ \left(\frac{c_s^2 - c_a^2}{2(c_a^2 + c_s^2)}\right)\phi_3(m,\rho) + \left(\frac{c_s^2 + 3c_a^2}{2(c_a^2 + c_s^2)}\right) \\ \quad \psi(c^2, m, \rho),\ c_a^2 \leq c_s^2, \end{cases} \tag{A12}$$

with

$$\phi_1(m,\rho) = 1 + \gamma(m,\rho), \tag{A13}$$

$$\phi_3(m,\rho) = (1 - 4\gamma(m,\rho))e^{\frac{-2(1-\rho)}{3\rho}}, \tag{A14}$$

$$\gamma(m,\rho) = \min\left\{0.24, \frac{(1-\rho)(m-1)[(4+5m)^{0.5} - 2]}{16m\rho}\right\}, \tag{A15}$$

and

$$\psi(c^2, m, \rho) = \begin{cases} 1, & c^2 > 1, \\ \phi_4(m,\rho)^{2(1-c^2)}, & 0 \leq c^2 \leq 1, \end{cases} \tag{A16}$$

with $c^2 = (c_a^2 + c_s^2)/2$ and $\phi_4(m,\rho) = \min\{1, (\phi_1(m,\rho) + \phi_3(m,\rho))/2\}$.

Appendix B

Based on the work of Whitt (1983), we convert the two input flows into one. The external arrival rate to the system is given by

$$\lambda_{01} = \tilde{\lambda}_{011} + \tilde{\lambda}_{012} \tag{A17}$$

where λ_{01} is the combined external arrival rate to the system and $\tilde{\lambda}_{011}$ and $\tilde{\lambda}_{012}$ are the two separate external arrival rates to the system. The internal traffic rate to node j is given by

$$\lambda_j = \tilde{\lambda}_{j1} + \tilde{\lambda}_{j2}, \quad \forall 1 \leq j \leq J, \tag{A18}$$

where $\tilde{\lambda}_{j1}$ and $\tilde{\lambda}_{j2}$ are the internal traffic rates to node j from each input flow solved by the linear equations of Equation (A1).

The mean service time at pick station j is the weighted combination of the service times for two separate input flows:

$$E[\tau_j] = \frac{\tilde{\lambda}_{j1}E[\tau_{j1}] + \tilde{\lambda}_{j2}E[\tau_{j2}]}{\tilde{\lambda}_{j1} + \tilde{\lambda}_{j2}}, \forall j > C, \tag{A19}$$

where $E[\tau_{j1}]$ and $E[\tau_{j2}]$ are the mean service time for each separate input flow derived from Equation (1).

The second moment of service time at pick station j is derived by

$$E\left[\tau_j^2\right] = \frac{\tilde{\lambda}_{j1}E\left[\tau_{j1}^2\right] + \tilde{\lambda}_{j2}E\left[\tau_{j2}^2\right]}{\tilde{\lambda}_{j1} + \tilde{\lambda}_{j2}}, \quad \forall j > C, \tag{A20}$$

where $E[\tau_{j1}^2]$ and $E[\tau_{j2}^2]$ are the second moments of service time at pick station j for each input flow given by Equation (14).

The SCV of service time at pick station j, c_{sj}^2, can then be calculated from Equations (20), (A19) and (A20). Because the service time is constant at conveyor segments, the SCV and the mean of service time are obtained from Equations (21) and (22).

The SCV of inter-arrival time to each node, c_{aj}^2, is again obtained from Equation (A4). The required parameters are calculated as follows.

The transition probabilities from node i to node j are calculated as

$$q_{ij} = \begin{cases} \lambda_{ij}/\lambda_{01}, \ \forall 1 \leq i \leq C, j = i+1, \text{and } j = i + C, \\ 1, \qquad \forall C+1 \leq i \leq J, j = i - C. \end{cases} \tag{A21}$$

λ_{ij}, the arrival rate from node i to node j, is given by

$$\lambda_{ij} = \tilde{\lambda}_{ij1} + \tilde{\lambda}_{ij2}, \quad \forall 1 \leq i \leq J, \forall 1 \leq j \leq J, \tag{A22}$$

where $\tilde{\lambda}_{ij1}$ and $\tilde{\lambda}_{ij2}$ are the arrival rates from node i to node j for each separate input flow derived from Equation (A2).

The utilizations ρ_j at each node j are calculated from Equation (26). pr_{ij}, the proportion of arrivals to j that come from $i (i \geq 0)$, is obtained from Equation (A3).

The SCV for the inter-arrival time of orders to the system is given by

$$c_{01}^2 = (1 - \tilde{\omega}_1) + \tilde{\omega}_1\left[c_{011}^2\left(\frac{\tilde{\lambda}_{011}}{\tilde{\lambda}_{011} + \tilde{\lambda}_{012}}\right) + c_{012}^2\left(\frac{\tilde{\lambda}_{012}}{\tilde{\lambda}_{011} + \tilde{\lambda}_{012}}\right)\right], \tag{A23}$$

where $\tilde{\omega}_1 = [1 + 4(1-\rho_1)^2(\tilde{v}_1 - 1)]^{-1}$ with $\rho_1 = \lambda_{01}E[\tau_1] / m_1$ and

$$\tilde{v}_1 = \left[\left(\frac{\tilde{\lambda}_{011}}{\tilde{\lambda}_{011} + \tilde{\lambda}_{012}}\right)^2 + \left(\frac{\tilde{\lambda}_{012}}{\tilde{\lambda}_{011} + \tilde{\lambda}_{012}}\right)^2\right],$$

where c_{011}^2 and c_{012}^2 are the SCV for the inter-arrival time of orders to the system of each separate input flow.

At this point, we have converted the two input flows into one. We can apply the procedures in Appendix A to calculate the expected waiting time in front of each node and subsequently use Equation (27) to obtain the expected sojourn time of a bin at a node.

Biographies

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