

# Undergraduate Complexity Theory

## Lecture 6: Problems in P

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### 1 Lecture Notes

Recap: Time Hierarchy Theorem  $\implies \exists$  languages decidable in exponential time but not in P.

**Definition 1.1.**  $\text{EXP} = \bigcup_{k \in \mathbb{N}} \text{TIME}(2^{n^k}) =$  languages decidable in  $2^{\text{poly}(n)}$  time.

**Fact 1.2.**  $\text{P} \subsetneq \text{EXP}$ .

1. (ST-)PATH  $\in \text{EXP}$ , and with a cleverer algo, PATH  $\in \text{P}$ .
2. 2-COL  $\in \text{EXP}$ , with a cleverer algo, 2-COL  $\in \text{P}$  too.
3. 3-COL  $\in \text{EXP}$ , but believed not in P.

**Theorem 1.3** (BE '05). 3-COL *is in*  $O(1.33^n)$  *time*.

4. LCS  $\in \text{EXP}$ , and with techniques like memorization and dynamic programming, LCS  $\in \text{P}$ .
5. 3-CLIQUE  $\in O(n^3)$ . in  $O(n^2)$ ? not sure.

**Fact 1.4.** *can do* 3-CLIQUE *in*  $O(n^3 / \log^2 n)$  *time, or*  $O(n^{2.38})$  *time*.

6. 4-CLIQUE  $\in \text{TIME}(n^4)$ . in  $\text{TIME}(n^3)$ ?
7. k-CLIQUE within brute force is time  $O(\binom{n}{k} \binom{k}{2}) = O(n^k)$ : k-CLIQUE  $\in \text{EXP}$ . in P? don't know.

### 2 Reading

#### 2.1 Sipser 7.2 (The Class P)