15-455: Midterm

March 3, 2020

Name:

Andrew ID:

Instructions

- Fill in the box above with your name and your Andrew ID. **Do it, now!**
- No notes, electronic devices, binoculars, ...
- Clearly mark your answers in the allocated space. If need be, use the back of a page for scratch space. If you have made a mess, cross out the invalid parts of your solution, and circle the ones that should be graded.
- Scan the test first to make sure that none of the 11 pages are missing. Some parts may be on the back of the page.
- The problems are of varying difficulty and are not necessarily sorted in order of increasing difficulty. You might wish to pick off the easy ones first.
- You have 80 minutes. Good luck.

1	10	
2	10	
3	20	
4	20	
5	20	
6	20	
Total	100	

Problem 1: Boolean Formulae (10 pts.)

Determine the complexity of the following decision problems (brief explanation, no full proof).

A. Tautology testing for formulae in conjunctive normal form.

B. Satisfiability testing for formulae in disjunctive normal form.

Problem 2: Evaluation (10 pts.)

in the discussion of computability, it is perfectly reasonable to insist that there should be an evaluation function $\mathsf{eval}(e, x)$ that returns the result of running program e on input x, and that this evaluation function should itself be computable.

A. Explain why this simple requirement forces us to deal with partial functions.

B. What is the fundamental connection between having to deal with partial functions and undecidability?

Problem 3: Infinite Sets (20 pts.)

Recall the collection of all semidecidable sets with finite complement:

$$\mathsf{COF} = \{\, e \mid W_e \text{ is infinite} \,\}$$

A. Why does Rice's theorem apply to COF?

B. What does Rice's theorem tell us about COF?

Problem 4: Wurzelbrunft and Ochsenfiesl vs. Collatz (20 pts.)

Wurzelbrunft and Ochsenfiesl are two grad students at a little known university in a galaxy far, far away. They both are fascinated by the infamous Collatz function $C(x) = x/2$ for x even, and $C(x) = 3x + 1$ otherwise. The Collatz Conjecture says that, starting at any positive integer x, repeated application of C will ultimately lead to 1. The conjecture appears to be true, but it is wide open and seems brutally difficult to prove.

Having taken 15-455, Wurzelbrunft and Ochsenfiesl decided to study the computational complexity of the Collatz set

$$S = \{ x \in \mathbb{N}_+ \mid \exists k \, C^k(x) = 1 \}$$

Thus, S is the set of positive integers for which the Collatz conjecture holds. Wurzelbrunft thinks he has a proof that S is in $\mathbb{NP}$, based on ideas similar to Pratt's primality test. He also claims that his result, together with the well-known fact that S is infinite, immediately implies the Collatz conjecture. His friend Ochsenfiesl, on the other hand, thinks he has a proof that S is undecidable. He claims his result implies that the Collatz conjecture is false.

If you were their PhD adviser, what professional, well-reasoned advice would you give to them?

A. Wurzelbrunft:

B. Ochsenfiesl:

Problem 5: Dominating Sets (20 pts.)

A dominating set in a ugraph $G = \langle V, E \rangle$ is a set $D \subseteq V$ such that every vertex not in D is adjacent to a vertex in D.

Problem: **Dominating Set (DS)**

Instance: A ugraph G, a bound k.

Question: Does G have a dominating set of size k?

Note that a dominating set is rather similar to a vertex cover. Sure enough, there is a polynomial time reduction from VC to DS. In fact, we can keep the same bound k, we just need to change G to a new graph H.

A. Define the graph H. Hint: H has $|V| + |E|$ vertices, so $V(H)$ is clear. Then figure out what $E(H)$ should be.

vertices of H:

edges of H:

B. Show how a vertex cover C of size k in G produces a dominating set D of size k in H.

C. Show how a dominating set D of size k in H produces a vertex cover C of size k in G.

Problem 6: ROBDD (20 pts.)

Assume the standard variable ordering $x_1 < x_2 < \ldots < x_n < 0, 1$.

A. Draw a picture of the ROBDD $x_1 \vee x_2 \vee \ldots \vee x_n$. What is its size (count internal nodes plus terminals)?

B. What would happen to the ROBDD if we changed the variable ordering to

$$x_{\pi(1)} < x_{\pi(2)} < \ldots < x_{\pi(n)} < 0, 1$$

for some permutation π of $[n]$.

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