Undergraduate Complexity Theory
Lecture 20: The Immerman-Szelepcsényi Theorem

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1 Lecture Notes

complete the proof of TQBF's PSPACE-hardness.

Fact 1.1. NPSPACE = coNPSPACE = PSPACE

a "scaled down" version: NL = coNL?

Theorem 1.2 (Immerman-Szelepcsényi '88)**.** NL = coNL.

Proof. Show exists log-space verifier V that $\langle G, s, t\rangle \notin$ ST-PATH.

Let R_l be {vertices reachable from s in $\leq l$ steps}, $r_l = R_l$, the certificate is like concatenation of cert for r_1, cert for r_2, ..., cert for r_n, and cert that t not reachable from s.

keypoint: After V processed cert for r_l, only retains l, r_l on its work tapes.

Suppose V now convinced of r_n, what cert would convince V that $t \notin R_n$?

cert of $t \notin R_n$ consists of: $s \to v_1, s \to v_2, \ldots, s \to v_{r_n}$.

V checks:

1. each path is in G ($O(\log n)$)
2. exactly r_n paths presented (check in $O(\log n)$ space by presenting endpoints in strictly increasing order)
3. and t is not among the endpoints ($O(\log n)$)

cert of r_{l+1}: $v_1 \in R_{l+1}, \ldots, v_{r_{l+1}} \in R_{l+1}, v_{r_{l+1}+1} \notin R_{l+1}, \ldots, v_n \notin R_{l+1}$.

cert of $v_x \notin R_{l+1}$: list all vertices in R_l (in incr order), check no edge connects them them to v_x. □

2 Reading

2.1 sipser 8.6 (NL Equals coNL)