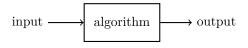
# Undergraduate Complexity Theory Lecture 2: Turing Machines

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July 8, 2022

#### 1 Lecture Notes

Today we're mainly going to talk about hwo to define an algorithm using Turing Machines.



**Definition 1.1.** A language L is a subset of strings  $L \subseteq \Sigma^*$ .

Fact 1.2. In general, decision problem  $f: \Sigma^* \to \{yes, no\}$  can convert language  $L = \{x \in \Sigma^* : f(x) = yes\}$ , and conversely, language L convert to decision problem  $f(x) = \begin{cases} yes, & x \in L \\ no, & x \notin L \end{cases}$ .

e.g. IsPrime:  $\{0,1\}^* \to \{\text{yes}, \text{no}\}\$ is equal to language  $\mathsf{PRIMES} = \{\langle x \rangle : x \in \mathbb{N}, x \text{ is prime}\}.$ 

Daily programming languages are fun to program, but hard to formalize.

Ways to formalize "algorithm":

- 1. Turing Machine ('36)
- 2. Lambda Calculus ('36)
- 3. Post Machine ('36)
- 4. Wang Machine ('50s)
- 5. P" ('64)

think they as programming language but "machines". Easy to formalize, but annoying to program.

Church-Turing Thesis: Any real-world algorithm can be simulated by (compiled to) Turing Machines. **Definition 1.3.** "computable" means computable by TMs.

Fact 1.4. An algo running in time T in C-like pssudocode can be compiled to a TM running in time  $\approx T^4$ . Extended Church-Turing Thesis: Any real-world algorithm can ...with at most polynomial slowdown.

Our official model: (1-tape, two-way infinite) Turing Machine.

input alphabet  $\Sigma$ , tape alphabet  $\Gamma = \Sigma \cup \{b\}$  where b is the blank symbol. ...many definitions about TM.

**Definition 1.5.** The *computation trace* of M on input X is a sequence of configurations  $C_0, C_1, \ldots, C_n$  where  $C_0$  is the initial configuration,  $C_i$  yields  $C_{i+1}$ , and  $C_n$  is an halting configuration.

**Definition 1.6.** TM M is a decider if M on X halt for all  $X \in \Sigma^*$ .

**Definition 1.7.** A TM M decides language L iff M is a decider and M accept X iff  $X \in L$ , otherwise reject.

## 2 Reading

### 2.1 Sipser 3.1 (Turing Machines)

- 1. configuration
- 2. yield
- 3. starting / accepting / rejecting / halting configuration
- 4. L(M): the language of / recognized by M
- 5. decider
- 6. Turing recognizable (recursively enumerable language)
- 7. Turing decidable (recursive language)

### 2.2 Sipser 3.3 (The Definition of Algorithm)

- 1. hilbert's 10th problem, history background
- 2. Church-Turing Thesis
- 3. 3-level Description of TM: formal description, impl description, high-level description