# Undergraduate Complexity Theory
# Lecture 6: Problems in P

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## 1 Lecture Notes

Recap: Time Hierarchy Theorem $\implies$ $\exists$ languages decidable in exponential time but not in P.

**Definition 1.1.** $\mathsf{EXP} = \bigcup_{k\in\mathbb{N}} \mathsf{TIME}(2^{n^k})$ = languages decidable in $2^{\mathrm{poly}(n)}$ time.

**Fact 1.2.** $\mathsf{P} \subsetneq \mathsf{EXP}$.

1. (ST-)PATH $\in$ EXP, and with a cleverer algo, PATH $\in$ P.
2. 2-COL $\in$ EXP, with a cleverer algo, 2-COL $\in$ P too.
3. 3-COL $\in$ EXP, but believed not in P.

**Theorem 1.3** (BE '05)**.** 3-COL *is in* $O(1.33^n)$ *time.*

4. LCS $\in$ EXP, and with techniques like memorization and dynamic programming, LCS $\in$ P.
5. 3-CLIQUE $\in O(n^3)$. in $O(n^2)$? not sure.

**Fact 1.4.** *can do* 3-CLIQUE *in* $O(n^3/\log^2 n)$ *time, or* $O(n^{2.38})$ *time.*

6. 4-CLIQUE $\in \mathsf{TIME}(n^4)$. in $\mathsf{TIME}(n^3)$?
7. k-CLIQUE within brute force is time $O(\binom{n}{k}\binom{k}{2}) = O(n^k)$: k-CLIQUE $\in$ EXP. in P? don't know.

## 2 Reading

### 2.1 Sipser 7.2 (The Class P)