

# Undergraduate Complexity Theory

## Lecture 2: Turing Machines

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### 1 Lecture Notes

Today we're mainly going to talk about how to define an algorithm using Turing Machines.



**Definition 1.1.** A language  $L$  is a subset of strings  $L \subseteq \Sigma^*$ .

**Fact 1.2.** In general, decision problem  $f : \Sigma^* \rightarrow \{\text{yes}, \text{no}\}$  can convert language  $L = \{x \in \Sigma^* : f(x) = \text{yes}\}$ , and conversely, language  $L$  convert to decision problem  $f(x) = \begin{cases} \text{yes}, & x \in L \\ \text{no}, & x \notin L \end{cases}$ .

e.g. `isPrime`:  $\{0, 1\}^* \rightarrow \{\text{yes}, \text{no}\}$  is equal to language `PRIMES` =  $\{\langle x \rangle : x \in \mathbb{N}, x \text{ is prime}\}$ .

Daily programming languages are fun to program, but hard to formalize.

Ways to formalize “algorithm”:

1. Turing Machine ('36)
2. Lambda Calculus ('36)
3. Post Machine ('36)
4. Wang Machine ('50s)
5. P ('64)

think they as programming language but “machines”. Easy to formalize, but annoying to program.

**Church-Turing Thesis:** Any real-world algorithm can be simulated by (compiled to) Turing Machines.

**Definition 1.3.** “computable” means computable by TMs.

**Fact 1.4.** An algo running in time  $T$  in C-like pseudocode can be compiled to a TM running in time  $\approx T^4$ .

**Extended Church-Turing Thesis:** Any real-world algorithm can ...with at most polynomial slowdown.

Our official model: (1-tape, two-way infinite) Turing Machine.

input alphabet  $\Sigma$ , tape alphabet  $\Gamma = \Sigma \cup \{b\}$  where  $b$  is the blank symbol. ...many definitions about TM.

**Definition 1.5.** The *computation trace* of  $M$  on input  $X$  is a sequence of configurations  $C_0, C_1, \dots, C_n$  where  $C_0$  is the initial configuration,  $C_i$  yields  $C_{i+1}$ , and  $C_n$  is an halting configuration.

**Definition 1.6.** TM  $M$  is a *decider* if  $M$  on  $X$  halt for all  $X \in \Sigma^*$ .

**Definition 1.7.** A TM  $M$  decides language  $L$  iff  $M$  is a decider and  $M$  accept  $X$  iff  $X \in L$ , otherwise reject.

## 2 Reading

### 2.1 Sipser 3.1 (Turing Machines)

1. configuration
2. yield
3. starting / accepting / rejecting / halting configuration
4.  $L(M)$ : the language of / recognized by  $M$
5. decider
6. Turing recognizable (recursively enumerable language)
7. Turing decidable (recursive language)

### 2.2 Sipser 3.3 (The Definition of Algorithm)

1. hilbert's 10th problem, history background
2. Church-Turing Thesis
3. 3-level Description of TM: formal description, impl description, high-level description