# Salad Model Parameters Study

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## Contents

# 1 Variables

**Dry weight**

- The main variable defining the plant growth.
- It is composed of 2 parts:
  - Structural dry weight
    - Weight of the plant's structural components, such as stems, leaves, and roots, after all water has been removed.
  - Non-structural dry weight
    - All other components of the plant biomass that are not considered structural, such as sugars, starches, oils, proteins, and other organic compounds.
- The difference in between these 2 weighs is that the structural components contribute to the physical structure and support of the plant, while non-structural components play roles in energy storage, metabolism, and other biochemical processes.

**Non-constant Variables**

$r_{\mathrm{gr}}$ – specific growth rate ($g \cdot m^{-2} \cdot s^{-1}$)
– represents the growth rate of the plant already considering the initial amount of the plant (the difference from the growth rate is that the growth rate considers only the absolute rate – from zero to a specific size)

$\Phi_{\mathrm{phot}}$ – gross canopy photosynthesis rate ($g \cdot m^{-2} \cdot s^{-1}$)
– total rate at which photosynthesis occurs in the canopy of a vegetation cover (it represents the combined photosynthetic activity of all the plants within the canopy)
it provides insights into the amount of CO2 uptake and carbon fixation occurring within a given vegetation cover
it is influenced by various factors, including light availability (the higher the better), temperature (photosynthesis rates typically increase with temperature up to an optimal point, beyond which they may decrease due to enzyme denaturation or other factors), CO2 concentration (higher concentrations of CO2 generally lead to higher rates of photosynthesis, up to a certain point – increased CO2 availability can enhance carbon fixation), leaf age (the older, the worse) and others

$\Phi_{\mathrm{phot,max}}$ – gross CO2 assimilation rate for a canopy with 1 $\mathrm{m}^{-2}$ of effective surface area ($g \cdot m^{-2} \cdot s^{-1}$)
– response of the canopy photosynthesis to the photosynthetically active radiation, the carbon dioxide concentration in the greenhouse air, and the temperature (photosynthesis response)

$\Phi_{\mathrm{resp}}$ – maintenance respiration $(g \cdot m^{-2})$
– energy expended by an organism to maintain basic cellular functions and processes necessary for its survival, even in the absence of growth or other metabolic activities
– maintenance respiration is distinct from growth respiration, which refers to the energy expended by an organism for growth and synthesis of new biomass (while growth respiration is proportional to the rate of growth and biomass accumulation, maintenance respiration occurs continuously and is independent of growth rate)

$\epsilon$ – current light use efficiency $(g \cdot J^{-1})$
– efficiency with which plants convert absorbed light energy into chemical energy through the process of photosynthesis (rate at which plants assimilate carbon dioxide and produce biomass per unit of absorbed photosynthetically active radiation)
– key determinant of plant productivity and is influenced by various factors, including the efficiency of light capture, the efficiency of photosynthetic electron transport, and the efficiency of carbon fixation

$\Gamma$ – quantity $(g \cdot m^{-3})$
– it depends on the temperature

$\sigma$ – leaf conductance $(m \cdot s^{-1})$
– refers to the overall conductance of gases through the leaf surface, including diffusion through the cuticle and epidermis
– represents the resistance encountered by gases as they move across the leaf surface before reaching the stomata

$c_{\mathrm{car}}$ – carboxylation conductance $(m \cdot s^{-1})$
– internal conductance which describes the movement of CO2 from sub-stomatal cavities to sites of carboxylation

$c_{\mathrm{gr,max}}$ – saturation growth rate $(s^{-1})$
– the maximum growth rate that a population can achieve under ideal conditions when all necessary resources are abundant and there are no limiting factors

$c_{\gamma}$ – growth rate coefficient (–)
– the rate at which a population increases in size over time (a measure of the intrinsic growth potential of the population under optimal conditions)

$c_{\mathrm{q10,gr}}$ – measure of growth rate sensitivity to the canopy temperature (–)
– how changes in canopy temperature affect the growth rate of plants
– canopy temperature is an integrative trait that reflects the plant water status or the resultant equilibrium between the root water uptake and shoot transpiration
– is related to a temperature change of 10°C (therefore q10)

**Constant Variables**

$c_{\text{ch2o}}$ – conversion rate of CO2 into CH2O (–)
– CO2 is converted into CH2O through a series of enzymatic reactions during photosynthesis
– CH2O, along with other intermediate compounds, contributes to the synthesis of organic molecules necessary for plant growth and development

$c_{\text{k}}$ – light extinction coefficient (–)
– attenuation of light as it passes through a medium, such as a canopy of vegetation (how quickly light is absorbed, scattered, or reflected by leaves and other canopy elements as it penetrates through the canopy)

$c_{\text{lar}}$ – structure leaf are ratio ($g^{-1} \cdot m^{-2}$)
– proportion of a plant's biomass that is allocated to leaf tissue relative to other structural components, such as stems and roots (investment of the plant in leaf area as a fraction of its total structural biomass)

$c_{\tau}$ – root dry mass ratio to the overall dry mass of the crop (–)
– measure used to describe the proportion of a plant's total dry biomass that is allocated to root tissue (what is the proportion of the roots mass to the total mass)

$c_{\text{resp,sht}}$ – shoot maintenance respiration coefficients ($s^{-1}$)
– rate at which shoot tissues respire for maintenance purposes (energy expended by plants to sustain basic cellular functions and processes necessary for their survival, even in the absence of growth or other metabolic activities – protein turnover, ion transport, and maintenance of cellular integrity)

$c_{\text{resp,rt}}$ – root maintenance respiration coefficients ($s^{-1}$)
– rate at which root tissues respire for maintenance purposes (similar to shoot maintenance respiration, but for roots)

$c_{\text{q10,resp}}$ – measure of respiration rate sensitivity to the canopy temperature (–)
– how changes in canopy temperature affect the rate of respiration in plants (similar to measure of growth rate sensitivity to the canopy temperature, but for respiration)

$c_{\epsilon}$ – light use efficiency ($g \cdot J^{-1}$)
– light use efficiency at very high CO2 concentrations and the carbon dioxide compensation point

$c_{\Gamma}$ – CO2 compensation point at 20°C ($g \cdot m^{-3}$)
– rate of photosynthetic CO2 fixation exactly balances the rate of CO2 release through respiration in the plant (net uptake of CO2 by the plant is zero)
– it is influenced by temperature, light intensity, and the efficiency of photosynthetic enzymes

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| $c_{\mathrm{q10},\Gamma}$ | – measure of CO2 compensation rate sensitivity to the canopy temperature (–)<br>– how changes in canopy temperature affect the rate of CO2 compensation in plants (similar to measure of growth/respiration rate sensitivity to the canopy temperature, but for CO2 compensation) |
| $c_{\mathrm{bnd}}$ | – boundary layer conductance ($m \cdot s^{-1}$)<br>– ease with which gases can move across the boundary layer between the plant surface and the surrounding air (it quantifies the resistance encountered by gases as they diffuse through the boundary layer)<br>– boundary layer refers to the thin layer of still air that surrounds the surfaces of leaves, stems, and other plant parts |
| $c_{\mathrm{stm}}$ | – stomatal conductance ($m \cdot s^{-1}$)<br>– refers to the ease with which gases can move through the stomata, which are small pores located on the leaf surface and surrounded by guard cells (primarily controls the rate of gas exchange between the leaf interior and the atmosphere) |
| $c_{\mathrm{car},1}$, $c_{\mathrm{car},2}$, $c_{\mathrm{car},3}$ | – carboxylation conductance parameters ($m \cdot s^{-1} \cdot {}^{\circ}C^{-2}$, $m \cdot s^{-1} \cdot {}^{\circ}C^{-1}$, $m \cdot s^{-1}$) |
| $c_{\mathrm{y}}$ | – yield factor (–)<br>– represents the ratio of the actual biomass or yield produced by the plant to the amount of resources consumed or utilized during growth (refers to the efficiency with which plants convert available resources, such as light, water, or crop yield) |

## 2 Growth-influencing Variables

$c_{\mathrm{gr,max}}$ – saturation growth rate
– it depends on intrinsic growth rate (the maximum growth rate of the population in the absence of limiting factors – it is dependent on the biomass size and the rate of its change), biomass size and carrying capacity of the environment (the maximum population size or biomass that the environment can sustain indefinitely – the equilibrium point at which the birth rate equals the death rate, resulting in a stable population size)
– if its value is high, the plant needs to reproduce quickly (which is good for a large yield)

$c_{\mathrm{k}}$ – light extinction coefficient
– it depends on factors such as leaf angle distribution, leaf optical properties, leaf area index, and the presence of other elements such as branches and stems
– it is related to the initial light intensity (the higher the light intensity, the faster the growth) and the distance in the medium it needs to pass through ( indirect ratio)
– if its value is high, the light penetrates deeper into the medium before being significantly attenuated (more light availability for the plant growth), but it may lead to increased competition for light among plants, thus potentially result in shading and reduced growth for certain species, particularly those occupying lower layers of the canopy or understory

$c_{\mathrm{lar}}$ – structural leaf area ratio
– higher structural leaf area ratio indicates that a larger proportion of the plant's biomass is invested in leaf tissue, which can enhance the plant's photosynthetic capacity and potential for growth

$c_{\epsilon}$ – light use efficiency ($g \cdot J^{-1}$)
– if its value is high, it is generally advantageous, because it indicates that plants are effectively converting absorbed light energy into biomass through photosynthesis (a high light use efficiency implies that plants are maximizing their productivity and resource use efficiency, which can lead to increased crop yields, improved plant growth, and enhanced ecosystem productivity)
– improving light use efficiency is a key goal in plant breeding, crop management, and agricultural practices aimed at increasing crop yields and resource use efficiency

$c_{\mathrm{y}}$ – yield factor
– it quantifies the efficiency of resource use by plants and provides insights into factors affecting plant productivity and crop yields (a higher yield factor indicates greater efficiency in resource use and higher productivity per unit of resource input)
– high yield factor is considered desirable in plant growth and agricultural production

## 3 Equations

A pair of differential equations of the growth model based on plant dry weight, determining the evolution of structural dry weight $x_1(t)$ and non-structural dry weight $x_2(t)$:

$$\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} = r_{\mathrm{gr}}(t)x_1(t), \tag{1}$$
$$\frac{\mathrm{d}x_2(t)}{\mathrm{d}t} = c_{\mathrm{ch2o}}\Phi_{\mathrm{phot}}(t) - r_{\mathrm{gr}}(t)x_1(t) - \Phi_{\mathrm{resp}}(t) - \frac{1-c_{\mathrm{y}}}{c_{\mathrm{y}}}r_{\mathrm{gr}}(t)x_1(t). \tag{2}$$

For the substitution of the time-dependent parameters of the differential equations, the specific growth rate $r_{\mathrm{gr}}(t)$ is defined by

$$r_{\mathrm{gr}}(t) = c_{\mathrm{gr,max}}\frac{x_2(t)}{c_\gamma x_1(t) + x_2(t)}c_{\mathrm{q10,gr}}^{\frac{u(t)}{10}-2}, \tag{3}$$

the gross canopy photosynthesis growth $\Phi_{\mathrm{phot}}(t)$ is defined by

$$\Phi_{\mathrm{phot}}(t) = \left(1 - e^{-c_{\mathrm{k}}c_{\mathrm{lar}}(1-c_\tau)x_1(t)}\right)\Phi_{\mathrm{phot,max}}(t), \tag{4}$$

and the maintenance respiration $\Phi_{\mathrm{resp}}(t)$ is defined by

$$\Phi_{\mathrm{resp}}(t) = \left(c_{\mathrm{resp,sht}}\left(1-c_\tau\right) + c_{\mathrm{resp,ct}}c_\tau\right)x_1(t)c_{\mathrm{q10,resp}}^{\frac{u(t)}{10}-\frac{5}{2}}. \tag{5}$$

The gross canopy photosynthesis growth $\Phi_{\mathrm{resp}}(t)$ is related to gross CO2 assimilation rate for a canopy with 1 $\mathrm{m}^{-1}$ of effective surface area $\Phi_{\mathrm{phot,max}}(t)$, which is defined as

$$\Phi_{\mathrm{phot,max}}(t) = \frac{\epsilon(t)u_{\mathrm{par}}(t)\sigma(t)\left(u_{\mathrm{CO2}}(t) - \Gamma(t)\right)}{\epsilon(t)u_{\mathrm{par}}(t) + \sigma(t)\left(u_{\mathrm{CO2}}(t) - \Gamma(t)\right)}. \tag{6}$$

The gross canopy photosynthesis growth $\Phi_{\mathrm{phot,max}}$ is dependent on the value of the current light use efficiency $\epsilon(t)$, which can be calculated as

$$\epsilon(t) = c_\epsilon\frac{u_{\mathrm{CO2}}(t) - \Gamma(t)}{u_{\mathrm{C02}}(t) + 2\Gamma(t)}, \tag{7}$$

the value of the quantity $\Gamma(t)$, which can be calculated as

$$\Gamma(t) = c_\Gamma c_{\mathrm{q10},\Gamma}^{\frac{u(t)-20}{10}}, \tag{8}$$

and the value of the leaf conductance $\sigma(t)$, which can be calculated as

$$\frac{1}{\sigma(t)} = \frac{1}{c_{\mathrm{bnd}}} + \frac{1}{c_{\mathrm{stm}}} + \frac{1}{c_{\mathrm{car}}(t)}, \tag{9}$$

where the carboxylation conductance $c_{\mathrm{car}}(t)$ is a quadratically related to the temperature as

$$c_{\mathrm{car}}(t) = c_{\mathrm{car},1}u^2(t) + c_{\mathrm{car},2}u(t) + c_{\mathrm{car},3}. \tag{10}$$

Putting all together, the extended form of differential equations is

$$\frac{\mathrm{d}\boldsymbol{x_1(t)}}{\mathrm{d}t} = \boldsymbol{c}_{\mathrm{gr,max}} \frac{\boldsymbol{x_2(t)}}{c_\gamma \boldsymbol{x_1(t)} + \boldsymbol{x_2(t)}} c_{\mathrm{q10,gr}}^{\frac{\boldsymbol{u(t)}}{10}-2} \boldsymbol{x_1(t)}, \tag{11}$$

$$\begin{aligned}\frac{\mathrm{d}\boldsymbol{x_2(t)}}{\mathrm{d}t} = {} & c_{\mathrm{ch2o}} \left(1 - e^{-\boldsymbol{c}_{\mathrm{k}} \boldsymbol{c}_{\mathrm{lar}} (1-c_\tau) \boldsymbol{x_1(t)}}\right) \cdot \\ & \cdot \frac{\boldsymbol{c_\epsilon} \frac{\boldsymbol{u}_{\mathrm{CO2}}\boldsymbol{(t)} - c_\Gamma c_{\mathrm{q10},\Gamma}^{\frac{\boldsymbol{u(t)}-20}{10}}}{\boldsymbol{u}_{\mathrm{C02}}\boldsymbol{(t)} + 2 c_\Gamma c_{\mathrm{q10},\Gamma}^{\frac{\boldsymbol{u(t)}-20}{10}}} \boldsymbol{u}_{\mathrm{par}}\boldsymbol{(t)} \frac{c_{\mathrm{bnd}} c_{\mathrm{stm}} \left(c_{\mathrm{car},1} \boldsymbol{u^2(t)} + c_{\mathrm{car},2} \boldsymbol{u(t)} + c_{\mathrm{car},3}\right)}{c_{\mathrm{stm}} \left(c_{\mathrm{car},1} \boldsymbol{u^2(t)} + c_{\mathrm{car},2} \boldsymbol{u(t)} + c_{\mathrm{car},3}\right) + c_{\mathrm{bnd}} \left(c_{\mathrm{car},1} \boldsymbol{u^2(t)} + c_{\mathrm{car},2} \boldsymbol{u(t)} + c_{\mathrm{car},3}\right) + c_{\mathrm{bnd}} c_{\mathrm{stm}}} \left(\boldsymbol{u}_{\mathrm{CO2}}\boldsymbol{(t)} - c_\Gamma c_{\mathrm{q10},\Gamma}^{\frac{\boldsymbol{u(t)}-20}{10}}\right)}{\boldsymbol{c_\epsilon} \frac{\boldsymbol{u}_{\mathrm{CO2}}\boldsymbol{(t)} - c_\Gamma c_{\mathrm{q10},\Gamma}^{\frac{\boldsymbol{u(t)}-20}{10}}}{\boldsymbol{u}_{\mathrm{C02}}\boldsymbol{(t)} + 2 c_\Gamma c_{\mathrm{q10},\Gamma}^{\frac{\boldsymbol{u(t)}-20}{10}}} \boldsymbol{u}_{\mathrm{par}}\boldsymbol{(t)} + \frac{c_{\mathrm{bnd}} c_{\mathrm{stm}} \left(c_{\mathrm{car},1} \boldsymbol{u^2(t)} + c_{\mathrm{car},2} \boldsymbol{u(t)} + c_{\mathrm{car},3}\right)}{c_{\mathrm{stm}} \left(c_{\mathrm{car},1} \boldsymbol{u^2(t)} + c_{\mathrm{car},2} \boldsymbol{u(t)} + c_{\mathrm{car},3}\right) + c_{\mathrm{bnd}} \left(c_{\mathrm{car},1} \boldsymbol{u^2(t)} + c_{\mathrm{car},2} \boldsymbol{u(t)} + c_{\mathrm{car},3}\right) + c_{\mathrm{bnd}} c_{\mathrm{stm}}} \left(\boldsymbol{u}_{\mathrm{CO2}}\boldsymbol{(t)} - c_\Gamma c_{\mathrm{q10},\Gamma}^{\frac{\boldsymbol{u(t)}-20}{10}}\right)} - \\ & - \boldsymbol{c}_{\mathrm{gr,max}} \frac{\boldsymbol{x_2(t)}}{c_\gamma \boldsymbol{x_1(t)} + \boldsymbol{x_2(t)}} c_{\mathrm{q10,gr}}^{\frac{\boldsymbol{u(t)}}{10}-2} \boldsymbol{x_1(t)} - \left(c_{\mathrm{resp,sht}} \left(1 - c_\tau\right) + c_{\mathrm{resp,ct}} c_\tau\right) \boldsymbol{x_1(t)} c_{\mathrm{q10,resp}}^{\frac{\boldsymbol{u(t)}}{10}-\frac{5}{2}} - \frac{1 - \boldsymbol{c}_{\mathrm{y}}}{\boldsymbol{c}_{\mathrm{y}}} \boldsymbol{c}_{\mathrm{gr,max}} \frac{\boldsymbol{x_2(t)}}{c_\gamma \boldsymbol{x_1(t)} + \boldsymbol{x_2(t)}} c_{\mathrm{q10,gr}}^{\frac{\boldsymbol{u(t)}}{10}-2} \boldsymbol{x_1(t)}.\end{aligned} \tag{12}$$

All state variables are marked in cyan in bold and input variables in magenta in bold. Growth-influencing variables are depicted in orange.