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THE COMPUTATIONAL MODELLING OF ACOUSTIC SHIELDS BY THE BOUNDARY AND SHELL ELEMENT METHOD

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Abstract—In this paper a numerical method for the computation of the acoustic field surrounding a set of vibrating bodies and coupled, forced shells (or shields) is introduced. The method is derived through the reformulation of the Helmholtz equation, which governs the acoustic field, as an integral equation termed the boundary and shell integral equation. Collocation is applied to this equation and a method termed the boundary and shell element method (BSEM) results. Modal modelling of the shell is then included to complete the model of the full acoustic–structure system. The method is implemented for general three-dimensional problems in a Fortran subroutine ABSEMGEN. The subroutine is applied to test problems and results are demonstrated.

1. INTRODUCTION

Acoustic shields, screens or barriers are sometimes used as a means to reducing the noise from machines, engines or roads. When it comes to determining a mathematical model for an acoustic shield, the main property to take into account is that the shield is a thin structure and, because it is thin, its vibratory motion can become coupled to the acoustic field. Thus a shield is equivalent to a discontinuity in the sound pressure field and the shield contributes to the acoustic field through coupling. Because of its topology, the shield is termed a shell.

The traditional boundary element method (BEM) is only suitable for computing the acoustic field exterior to non-thin bodies [1–3]. One purpose of this paper is to extend the BEM for the solution of three-dimensional acoustic radiation problems, as considered in Refs [4–8], to acoustic–vibratory systems involving vibrating bodies and thin, acoustically coupled shells. This is brought about by first introducing an integral equation which unifies the direct boundary integral equation of Burton and Miller [9] —which reformulates the Helmholtz equation exterior to bodies only—and the integral equation given in Ben Mariem and Hamdi [10] and Warham [11]—which reformulates the Helmholtz equation exterior to shells only. The resulting equation is termed the *boundary and shell integral equation*, which is a reformulation of the Helmholtz equation exterior to a set of bodies and shells. On the application of an integral equation method, a numerical method termed the *boundary and shell element method* (BSEM) is derived.

The acoustic radiation model that is considered in this paper is that of a set of vibrating bodies and shells lying in an acoustic medium. The dynamic properties of the shells are governed by the linear equations of motion and they may be directly forced. In this paper the boundary and shell integral equation which governs the acoustic field is stated. It is shown how a particular BSEM can then be derived by approximating the surface of the bodies and shells by a set of triangles and by approximating the boundary functions by a constant on each triangle and then applying the classical technique of collocation. The *in vacuo* vibratory properties of each shield are assumed known. A Fortran library subroutine ABSEMGEN which is able to compute the properties of the three-dimensional acoustic field around a set of vibrating bodies and coupled shells is described. The results from applying ABSEMGEN to test problems having the general form of a cube with a vibrating face shielded by a square are given.

2. MODELLING

The acoustic radiation model consists of a set of vibrating bodies and forced shells, all of which are separate, surrounded by an infinite acoustic field, and the shells assume linear elastic behaviour. Let E be the infinite domain exterior to a set of closed surfaces S (enclosing regions D) and shells Γ. The shell(s) is assumed to have two surfaces: an upper surface Γ_+ and a lower surface Γ_-. The normal to the upper surface Γ_+ at the point $\mathbf{p}(\in \Gamma)$ is denoted $\mathbf{n}_p$. The model is illustrated in Fig. 1.

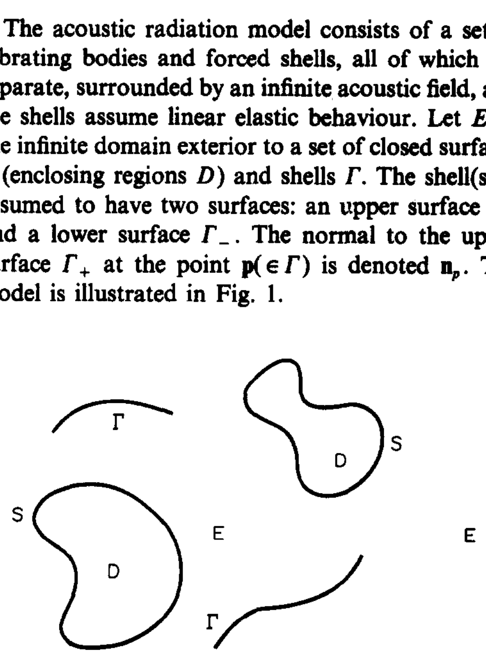

Fig. 1. The acoustic radiation model.

2.1. *Acoustic modelling*

The equation governing the acoustic field is the wave equation

$$\nabla^2\Psi(\mathbf{p},t)=\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Psi(\mathbf{p},t), \qquad (1)$$

where $\Psi(\mathbf{p},t)$ is the scalar time-dependent velocity potential related to the time-dependent particle velocity by

$$\mathbf{V}(\mathbf{p},t)=\nabla\Psi(\mathbf{p},t) \qquad (2)$$

and c is the propagation velocity ($\mathbf{p}$ and t are the spatial and time variables). The time-dependent sound pressure $Q(\mathbf{p},t)$ is given in terms of the velocity potential by

$$Q(\mathbf{p},t)=-\rho\frac{\partial\Psi}{\partial t}(\mathbf{p},t), \qquad (3)$$

where ρ is the density of the acoustic medium.

Only periodic solutions to the wave equation are considered, thus it is sufficient to consider time-dependent velocity potentials of the form

$$\psi(\mathbf{p},t)=\varphi(\mathbf{p})\,\mathrm{e}^{-\mathrm{i}\omega t} \qquad (4)$$

where ω is the angular frequency ($\omega=2\pi\eta$, where η is the frequency in hertz) and $\varphi(\mathbf{p})$ is the (time-independent) velocity potential. The substitution of expression (4) into (1) reduces it to the Helmholtz (reduced wave) equation

$$\nabla^2\varphi(\mathbf{p})+k^2\varphi(\mathbf{p})=0,$$

where $k^2=\omega^2/c^2$ and k is the wavelength. The Sommerfeld radiation condition is also satisfied. Note that for the purpose of the acoustic modelling, the shells are assumed to be infinitesimally thin, so that if $\mathbf{p}\in\Gamma$ then $\mathbf{p}\in\Gamma_+$ and $\mathbf{p}\in\Gamma_-$.

2.2. *Notation*

Let the function $v(\mathbf{p})$ for $\mathbf{p}\in S$ be defined as follows:

$$v(\mathbf{p})=\lim_{\epsilon\to 0}\frac{\partial\varphi}{\partial\mathbf{n}_p}(\mathbf{p}+\epsilon\mathbf{n}_p). \qquad (5)$$

The potential φ and its derivatives are generally discontinuous at the shell. However, $\varphi(\mathbf{p})$ ($\mathbf{p}\in\Gamma$) and its derivatives take limiting values on Γ_+ and Γ_-. Let the functions $\varphi_+(\mathbf{p})$, $\varphi_-(\mathbf{p})$, $v_+(\mathbf{p})$ and $v_-(\mathbf{p})$ ($\mathbf{p}\in\Gamma$) be defined as follows:

$$\varphi_+(\mathbf{p})=\lim_{\epsilon\to 0}\varphi(\mathbf{p}+\epsilon\mathbf{n}_p),$$

$$\varphi_-(\mathbf{p})=\lim_{\epsilon\to 0}\varphi(\mathbf{p}-\epsilon\mathbf{n}_p),$$

$$v_+(\mathbf{p})=\lim_{\epsilon\to 0}\frac{\partial\varphi}{\partial\mathbf{n}_p}(\mathbf{p}+\epsilon\mathbf{n}_p).$$

$$v_-(\mathbf{p})=\lim_{\epsilon\to 0}\frac{\partial\varphi}{\partial\mathbf{n}_p}(\mathbf{p}-\epsilon\mathbf{n}_p).$$

Let the functions $\delta(\mathbf{p})$, $\nu(\mathbf{p})$, $\Phi(\mathbf{p})$ and $V(\mathbf{p})$ for $\mathbf{p}\in\Gamma$ be defined as follows:

$$\delta(\mathbf{p})=\varphi_+(\mathbf{p})-\varphi_-(\mathbf{p}),$$

$$\nu(\mathbf{p})=v_+(\mathbf{p})+v_-(\mathbf{p}),$$

$$\Phi(\mathbf{p})=\tfrac{1}{2}(\varphi_+(\mathbf{p})+\varphi_-(\mathbf{p})),$$

$$V(\mathbf{p})=\tfrac{1}{2}(v_+(\mathbf{p})-v_-(\mathbf{p})).$$

2.3. *Structural modelling of the shell*

Let $\mathbf{u}_1(\mathbf{p})$, $\mathbf{u}_2(\mathbf{p}),\ldots,(\mathbf{p}\in\Gamma)$ be the (orthogonal) *in vacuo* mode shapes of the shell(s) listed in order of most fundamental to least. Let a unit distribution of force $\mathbf{u}_j(\mathbf{p})$ ($\mathbf{p}\in\Gamma$) at wavenumber k produce a displacement of $\lambda_j(k)\mathbf{u}_j(\mathbf{p})$ ($\mathbf{p}\in\Gamma$) for $j=1,2,\ldots$ for the shell lying in a vacuum. The functions $\lambda_j(k)$ may include the effect of structural damping. The mode shapes $\mathbf{u}_j(\mathbf{p})$ and the functions $\lambda_j(k)$ ($j=1,2,\ldots$) can generally be computed straightforwardly via the finite element method up to some maximum value of j, depending on the discretization of the shell [12].

We may represent a general harmonic forcing distribution over the shell in the form

$$\mathbf{g}(\mathbf{p})=\sum_j G_j\mathbf{u}_j(\mathbf{p}),\quad \mathbf{p}\in\Gamma. \qquad (6)$$

The harmonic displacement $\mathbf{w}(\mathbf{p})$ and velocity $\mathbf{v}(\mathbf{p})$ of the shell may then be written in the form

$$\mathbf{w}(\mathbf{p})=\sum_j W_j\mathbf{u}_j(\mathbf{p}),\quad \mathbf{v}(\mathbf{p})=\sum_j V_j\mathbf{u}_j(p),\quad p\in\Gamma. \qquad (7)$$

Hence

$$G_j=\lambda_j(k)V_j,\quad j=1,2,\ldots. \qquad (8)$$

Note that since $\mathbf{v}(\mathbf{p})=-\mathrm{i}\omega\mathbf{w}(\mathbf{p})$ then

$$V_j=-\mathrm{i}\omega W_j,\quad j=1,2,\ldots. \qquad (9)$$

Let $\mathbf{f}(\mathbf{p})$ ($\mathbf{p}\in\Gamma$) represent the direct forcing on the shell. Thus the actual forcing distribution in the direction of $\mathbf{n}_p$ is

$$g(\mathbf{p})=f(\mathbf{p})-\mathrm{i}\rho\omega\delta(\mathbf{p}),\quad \mathbf{p}\in\Gamma, \qquad (10)$$

where

$$g(\mathbf{p})=\mathbf{g}(\mathbf{p})\cdot\mathbf{n}_p \text{ and } f(\mathbf{p})=\mathbf{f}(\mathbf{p})\cdot\mathbf{n}_p.$$

2.4. *Definition of some acoustic properties*

The sound pressure is related to the velocity potential as follows:

$$p(\mathbf{p}) = \mathrm{i}\rho\omega\varphi(\mathbf{p}), \quad \mathbf{p} \in E. \tag{11}$$

The sound power produced by the system is given by

$$\begin{aligned} P &= \tfrac{1}{2}\,\mathrm{Re}\left\{\int_S \mathrm{i}\rho\omega\varphi v^*(\mathbf{p})\,\mathrm{d}S_p \right. \\ &\quad + \int_\Gamma \mathrm{i}\rho\omega\varphi_+(\mathbf{p})\,v_+^*(\mathbf{p})\,\mathrm{d}S_p \\ &\quad \left. + \int_\Gamma \mathrm{i}\rho\omega\varphi_-(\mathbf{p})v^*{}_-(\mathbf{p})\,\mathrm{d}S_p\right\} \\ &= \tfrac{1}{2}\,\mathrm{Re}\left\{\int_S \mathrm{i}\rho\omega\varphi v^*\,\mathrm{d}S_p \right. \\ &\quad \left. + \int_\Gamma \mathrm{i}\rho\omega\delta_+(\mathbf{p})V^*(\mathbf{p})\,\mathrm{d}S_p\right\}. \end{aligned} \tag{12}$$

where the asterisk indicates the complex conjugate. The radiation ratio is given by

$$\begin{aligned} \sigma_{RAD} &= \frac{P}{\tfrac{1}{2}\rho c\,\mathrm{Re}\left(\int_S v(\mathbf{p})v^*(\mathbf{p})\,\mathrm{d}S_p + \int_\Gamma v_+(\mathbf{p})v_+^*(\mathbf{p})\,\mathrm{d}S_p + \int_\Gamma v_-(\mathbf{p})v_-^*(\mathbf{p})\,\mathrm{d}S_p\right)} \\ &= \frac{P}{\tfrac{1}{2}\rho c\,\mathrm{Re}\left(\int_S v(\mathbf{p})v^*(\mathbf{p})\,\mathrm{d}S_p + 2\int_\Gamma V(\mathbf{p})V^*(\mathbf{p})\,\mathrm{d}S_p\right)}. \end{aligned} \tag{13}$$

3. INTEGRAL EQUATION REFORMULATION OF THE ACOUSTIC FIELD

The problem of reformulating the Helmholtz equation exterior to a radiating or scattering body as an integral equation that forms a reliable basis for solution by a numerical method has interested researchers for several decades. For the background to this, see Refs [4, 8, 13, 14]. The similar problem, but exterior to shells, has also received some attention [10, 11, 15]. In this section the direct boundary integral equation of Burton and Miller [9] and the integral equation reformulation for the problem exterior to shells are unified to give the formulation of the full system illustrated in Fig. 1. A similar technique has been applied to find the boundary and shell integral equation for Laplace problems in Ref. [16].

3.1. *Notation*

The Helmholtz integral operations L_k, M_k, M_k^t, and N_k are defined as follows:

$$\{L_k\mu\}_\Pi(\mathbf{p}) \equiv \int_\Pi G_k(\mathbf{p},\mathbf{q})\mu(\mathbf{q})\,\mathrm{d}S_q, \quad \mathbf{p} \in E\cup S\cup\Gamma,$$

$$\{M_k\mu\}_\Pi(\mathbf{p}) \equiv \int_\Pi \frac{\partial G_k}{\partial n_q}(\mathbf{p},\mathbf{q})\mu(\mathbf{q})\,\mathrm{d}S_q, \quad \mathbf{p} \in E\cup S\cup\Gamma,$$

$$\{M_k^t\mu\}_\Pi(\mathbf{p}) \equiv \frac{\partial}{\partial n_p}\int_\Pi G_k(\mathbf{p},\mathbf{q})\mu(q)\,\mathrm{d}S_q, \quad \mathbf{p} \in S\cup\Gamma,$$

$$\{N_k\mu\}_\Pi(\mathbf{p}) \equiv \frac{\partial}{\partial n_p}\int_\Pi \frac{\partial G_k}{\partial n_q}(\mathbf{p},\mathbf{q})\mu(\mathbf{q})\,\mathrm{d}S_q, \quad \mathbf{p} \in S\cup\Gamma,$$

where Π is a surface (not necessarily closed), $\mathbf{n}_q$, $\mathbf{n}_p$ are unit 'outward' normal to Π at $\mathbf{q}$, $\mathbf{p}$ and $\mu(\mathbf{q})$ is a bounded function defined for $\mathbf{q} \in \Pi$. $G_k(\mathbf{p},\mathbf{q})$ is the free-space Green's function for the Helmholtz equation

$$G_k(\mathbf{p},\mathbf{q}) = \frac{e^{ikr}}{4\pi r} \quad \text{in three dimensions,} \tag{14}$$

where $\mathbf{r} = \mathbf{p} - \mathbf{q}$ and $r = |\mathbf{r}|$.

3.2. *Integral equation formulation*

The equations that make up the boundary and shell integral equation formulation of the Helmholtz equation are given in this subsection. For points on the boundary the following equation holds

$$\begin{aligned} &\alpha\{M_k\varphi\}_S(\mathbf{p}) - \tfrac{1}{2}\alpha\varphi(\mathbf{p}) + \beta\{N_k\varphi\}_S(\mathbf{p}) \\ &\quad = \alpha\{L_k v\}_S(\mathbf{p}) + \beta\{M_k^t v\}_S(p) + \tfrac{1}{2}\beta v(\mathbf{p}) \\ &\quad\quad - \alpha\{M_k\delta\}_\Gamma(\mathbf{p}) - \beta\{N_k\delta\}_\Gamma(\mathbf{p}) \\ &\quad\quad + \alpha\{L_k\nu\}_\Gamma(\mathbf{p}) + \beta\{M_k^t\nu\}_\Gamma(\mathbf{p}), \quad \mathbf{p} \in S, \end{aligned}$$

where α and β are complex numbers and S is smooth at $\mathbf{p}$. This equation relates $\varphi(\mathbf{p})$ and $v(\mathbf{p})$ for points $\mathbf{p}$ on the boundary S. For points on the shell, we have the following equations

$$\begin{aligned} \Phi(\mathbf{p}) &= \{M_k\delta\}_\Gamma(\mathbf{p}) - \{L_k\nu\}_\Gamma(\mathbf{p}) \\ &\quad + \{M_k\varphi\}_S(\mathbf{p}) - \{L_k v\}_S(\mathbf{p}), \quad \mathbf{p} \in \Gamma, \\ V(\mathbf{p}) &= \{N_k\delta\}_\Gamma(\mathbf{p}) - \{M_k^t\nu\}_\Gamma(\mathbf{p}) \\ &\quad + \{N_k\varphi\}_S(\mathbf{p}) - \{M_k^t\varphi\}_S(\mathbf{p}), \quad \mathbf{p} \in \Gamma, \end{aligned}$$

where Γ is smooth at $\mathbf{p}$. The value of $\varphi(\mathbf{p})$ for points in the exterior domain are related to the solutions on S and Γ through the following equation

$$\begin{aligned} \varphi(\mathbf{p}) &= \{M_k\varphi\}_S(\mathbf{p}) - \{L_k v\}_S(\mathbf{p}) \\ &\quad + \{M_k\delta\}_\Gamma(\mathbf{p}) - \{L_k\nu\}_\Gamma(\mathbf{p}), \quad \mathbf{p} \in E. \end{aligned}$$

The velocity at the upper surface of the shell is equal and opposite to that at the lower surface, that is $v(\mathbf{p}) \equiv 0$ ($\mathbf{p} \in \Gamma$). Hence the equations above can be simplified as follows:

$$\alpha\{M_k\varphi\}_S(\mathbf{p}) - \tfrac{1}{2}\alpha\varphi(\mathbf{p}) + \beta\{N_k\varphi\}_S(\mathbf{p}) = \alpha\{L_k v\}_S(\mathbf{p}) + \beta\{M_k^t v\}_S(\mathbf{p}) + \tfrac{1}{2}\beta v(\mathbf{p}) - \alpha\{M_k\delta\}_\Gamma(\mathbf{p}) - \beta\{N_k\delta\}_\Gamma(\mathbf{p}), \quad \mathbf{p} \in S, \quad (15)$$

$$\Phi(\mathbf{p}) = \{M_k\delta\}_\Gamma(\mathbf{p}) + \{M_k\varphi\}_S(\mathbf{p}) - \{L_k v\}_S(\mathbf{p}), \quad \mathbf{p} \in \Gamma, \quad (16)$$

$$V(\mathbf{p}) = \{N_k\delta\}_\Gamma(\mathbf{p}) + \{N_k\varphi\}_S(\mathbf{p}) - \{M_k^t v\}_S(\mathbf{p}), \quad \mathbf{p} \in \Gamma, \quad (17)$$

$$\varphi(\mathbf{p}) = \{M_k\varphi\}_S(\mathbf{p}) - \{L_k v\}_S(\mathbf{p}) + \{M_k\delta\}_\Gamma(\mathbf{p}), \quad \mathbf{p} \in E. \quad (18)$$

4. DERIVATION OF THE NUMERICAL METHOD

In this section collocation is applied to the integral equation formulation of the Helmholtz equation given in the previous section. The resulting discrete equations are then coupled with the equations that govern the motion of the structure. A linear system that models the coupled acoustic–structure interaction problem results. The Fortran subroutine ABSEMGEN is outlined.

4.1. *Application of collocation*

In this section it is shown how collocation is applied to derive the discrete form of the integral equations. The boundary and shell are divided into planar triangular elements. The boundary S is divided into n_S elements $\Delta S_1, \Delta S_2, \ldots, \Delta S_{n_S}$, the shell Γ is divided into n_Γ elements $\Delta\Gamma_1, \Delta\Gamma_2, \ldots, \Delta\Gamma_{n_\Gamma}$ and the boundary functions and shell functions are approximated by a constant on each element. Let $\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_{n_S}$ and $\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_{N_\Gamma}$ be the collocation point with $\mathbf{p}_i \in \Delta S_i$ for $i = 1, 2 \ldots, n_S$ and $\mathbf{q}_i \in \Delta\Gamma_i$ for $i = 1, 2, \ldots, n_\Gamma$ and each lying at the centroid of the respective element.

It is helpful to introduce the following notation. Let the matrices L_{SS}, $\mathrm{L}_{S\Gamma}$, $\mathrm{L}_{\Gamma S}$, and $\mathrm{L}_{\Gamma\Gamma}$ be defined as follows:

$$[\mathrm{L}_{SS}]_{ij} = [L_k h\}_{\Delta S_j}(\mathbf{p}_i), \quad i = 1, 2, \ldots, n_S, \; j = 1, 2, \ldots, n_S,$$

$$[\mathrm{L}_{S\Gamma}]_{ij} = \{L_k h\}_{\Delta\Gamma_j}(\mathbf{p}_i), \quad i = 1, 2, \ldots, n_S, \; j = 1, 2, \ldots, n_\Gamma,$$

$$[\mathrm{L}_{\Gamma S}]_{ij} = \{L_k h\}_{\Delta S_j}(\mathbf{q}_i), \quad i = 1, \ldots, n_\Gamma, \; j = 1, 2, \ldots, n_S,$$

$$[\mathrm{L}_{\Gamma\Gamma}]_{ij} = \{L_k h\}_{\Delta\Gamma_j}(\mathbf{q}_i), \quad i = 1, 2, \ldots, n_\Gamma, \; j = 1, 2, \ldots, n_\Gamma,$$

where h is the unit function. The other integral operators can be discretized in a similar way. This allows us to construct the following linear systems of approximations which are the discrete analogues of eqns (15), (16) and (17)

$$[\alpha(\mathrm{M}_{SS} - \tfrac{1}{2}\mathrm{I}_{SS}) + \beta\mathrm{N}_{SS}]\underline{\varphi}_S \approx [\alpha\mathrm{L}_{SS} + \beta(\mathrm{M}_{SS}^t + \tfrac{1}{2}\mathrm{I}_{SS})]\underline{v}_S - [\alpha\mathrm{M}_{S\Gamma} + \beta\mathrm{N}_{S\Gamma}]\underline{\delta}_\Gamma, \quad (19)$$

$$\underline{\Phi}_\Gamma \approx \mathrm{M}_{\Gamma\Gamma}\underline{\delta}_\Gamma + \mathrm{M}_{\Gamma S}\underline{\varphi}_S - \mathrm{L}_{\Gamma S}\underline{v}_S, \quad (20)$$

$$\underline{V}_\Gamma \approx \mathrm{N}_{\Gamma\Gamma}\underline{\delta}_\Gamma + \mathrm{N}_{\Gamma S}\underline{\varphi}_S - \mathrm{M}_{\Gamma S}^t\underline{v}_S, \quad (21)$$

where $\underline{\varphi}_S = [\varphi_1, \varphi_2, \ldots, \varphi_{n_S}]$, $\underline{\delta}_\Gamma = [\delta_1, \delta_2, \ldots, \delta_{n_\Gamma}]$, $\underline{v}_S = [v_1, v_2, \ldots, v_{n_S}]$.

4.2. *Discretizing the vibratory properties of the shell*

The following equation is immediate from (10)

$$\underline{g}_\Gamma = \underline{f}_\Gamma - \mathrm{i}\rho\omega\underline{\delta}_\Gamma, \quad (22)$$

where $\underline{g}_\Gamma = [g_1, g_2, \ldots, g_{n_\Gamma}]^T$ with $g_j = g(\mathbf{p}_j)$ and $\underline{f}_\Gamma = [f_1, f_2, \ldots, f_{n_\Gamma}]^T$ with $f_j = f(\mathbf{p}_j)$. Let the vibratory properties of the shell be modelled by the n_M most fundamental mode shapes. Let the $n_\Gamma \times n_M$ matrix of mode shapes $\mathrm{U}_{\Gamma M}$ be defined as follows:

$$[\mathrm{U}_{\Gamma M}]_{ij} = \mathbf{u}_j(\mathbf{p}_i) \cdot \mathbf{n}_p, \quad \text{for} \quad i = 1, 2, \ldots, n_\Gamma \quad \text{and} \quad j = 1, 2, \ldots, n_M. \quad (23)$$

We may then write $\underline{g}_\Gamma \approx U_{\Gamma M}\underline{G}_M$, $\underline{f}_\Gamma \approx U_{\Gamma M}\underline{F}_M$, and $\underline{v}_\Gamma \approx U_{\Gamma M}\underline{V}_M$ where $\underline{G}_M = [G_1, G_2, \ldots, G_{n_M}]$, $\underline{F}_M = [F_1, F_2, \ldots, F_{n_M}]$ and $\underline{V}_M = [V_1, V_2, \ldots, V_{n_M}]$.

4.3. *Discrete formulation of the coupled problem*

Equation (22) can now be written in the form

$$\mathrm{U}_{\Gamma M}\underline{G}_M \approx \mathrm{U}_{\Gamma M}\underline{F}_M - \mathrm{i}\rho\omega\underline{\delta}_\Gamma, \quad (24)$$

Let D_{MM} be the $n_M \times n_M$ diagonal matrix defined as follows:

$$\mathrm{D}_{MM} = \mathrm{diag}\{\lambda_1(k), \lambda_2(k), \ldots, \lambda_M(k)\}.$$

Thus we may write

$$\underline{G}_M = \mathrm{D}_{MM}\underline{V}_M. \quad (25)$$

Thus from (24), (25) we have

$$\mathrm{U}_{\Gamma M}\mathrm{D}_{MM}\underline{V}_M \approx \mathrm{U}_{\Gamma M}\underline{F}_M - \mathrm{i}\rho\omega\underline{\delta}_\Gamma. \quad (26)$$

In the physical system of interest, the velocity of the surface of the bodies, the modal forcing on the shells and the dynamic properties of the shell are assumed known. From approximation (19), (20) and (21) we may now construct the following linear system for computing approximations $\hat{\underline{\varphi}}_S$, $\hat{\underline{\delta}}_\Gamma$, and $\hat{\underline{V}}_M$ to $\underline{\varphi}_S$, $\underline{\delta}_S$, and $\underline{V}_M$

$$\begin{bmatrix} \alpha(\mathrm{M}_{SS}-\frac{1}{2}\mathrm{I}_{SS})+\beta\mathrm{N}_{SS} & \alpha\mathrm{M}_{S\Gamma}+\beta\mathrm{N}_{S\Gamma} & 0_{SM} \\ \mathrm{N}_{\Gamma S} & \mathrm{N}_{\Gamma\Gamma} & -\mathrm{U}_{\Gamma M} \\ 0_{MS} & i\rho\omega\mathrm{U}^T_{M\Gamma} & \mathrm{U}^T_{\Gamma M}D_{MM} \end{bmatrix} \begin{bmatrix} \hat{\underline{\varphi}}_S \\ \hat{\underline{\delta}}_\Gamma \\ \hat{\underline{V}}_M \end{bmatrix} = \begin{bmatrix} (\alpha\mathrm{L}_{SS}+\beta(\mathrm{M}_{SS}+\frac{1}{2}\mathrm{I}_{SS}))\underline{v}_S \\ \mathrm{M}^t_{\Gamma S}\underline{v}_S \\ \mathrm{U}^T_{M\Gamma}\mathrm{U}_{\Gamma M}\underline{F}_M \end{bmatrix}. \tag{27}$$

In subroutine ABSEMGEN the integrals corresponding to the discrete integral operators were computed using Gaussian quadrature on a triangle [15] when regular. In the case when the integrals are singular, techniques similar to those given in [6] and [8] are used. See Ref. [8] for more details. In this work, the complex numbers α and β were chosen so that $\alpha = \|\mathrm{N}_{SS}\|$ and $\beta = i\|\mathrm{M}_{SS}-\frac{1}{2}\mathrm{I}_{SS}\|$ which is consistent with the method advocated in Ref. [18].

For the computation of the solution in the exterior domain and the sound power and radiation ratio, the corresponding discrete analogues of eqns (18), (11), (12) and (13) are implemented.

In outline, subroutine ABSEMGEN accepts the frequencies of interest, a description of the shape and position of the bodies and shells, the dynamic properties of the shells, the distribution of vibration on the bodies, the distribution of the direct forcing on the shells and the points in the exterior where the sound pressure is sought. As output, the subroutine gives, for each chosen frequency, the surface intensity pattern, the sound power, the radiation efficiency and the sound pressure at the selected exterior points.

5. TEST PROBLEMS AND RESULTS

In this section some results from the application of subroutine ABSEMGEN to a shielded cube problem are considered. The test problems each consist of a 10 cm cube (S) of which one face is vibrating uniformly, the other faces of the cube are rigid. A square plate (Γ) with sides of length 10 cm is placed 10 cm from the vibrating face. The acoustic medium is air (density = 1.29 kg/m^3, speed of sound = 331 m/sec). In the initial tests the square plate is assumed rigid. In the final test the square plate assumes the properties of 1 mm thick steel (density = 7800 kg/m^3, Young's modulus = 209×10^9 Pa, Poisson's ratio = 0.3). In each test the acoustic medium is air.

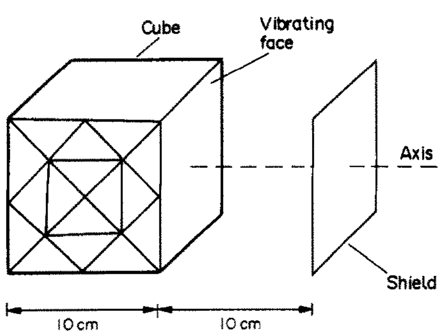

Fig. 2. The test problem.

The test problem is illustrated in Fig. 2. In the implementation of the BSEM via subroutine ABSEMGEN, the cube is divided into 96 boundary elements and the shell is divided into 16 shell elements, so that the elements are all of uniform size.

The radiation ratios of the system were computed at 10, 20, . . . , 5000 Hz. The results from this test were compared with the results obtained when the square plate is removed. For these tests the plate is assumed rigid. The results are illustrated in Fig. 3.

Figure 3 demonstrates that the shield has a major effect on the acoustic properties of the system. Below *circa* 1300 Hz, the unshielded and shielded cube have approximately the same radiation ratio. However, the shield has the effect of significantly reducing the

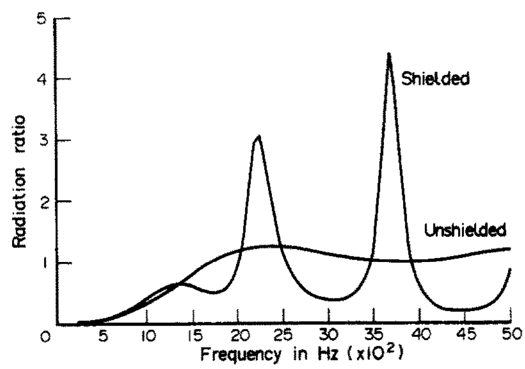

Fig. 3. Radiation ratio curves for both a shielded and unshielded cube.

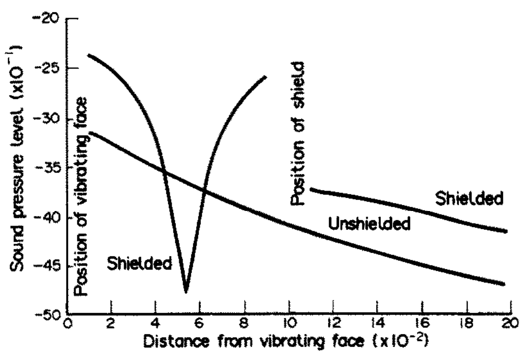

Fig. 4. The computed effect on the axial sound pressure level in air at 2300 Hz of placing a rigid 10 cm square shield in front of a uniformly vibrating face of a 10 cm cube.

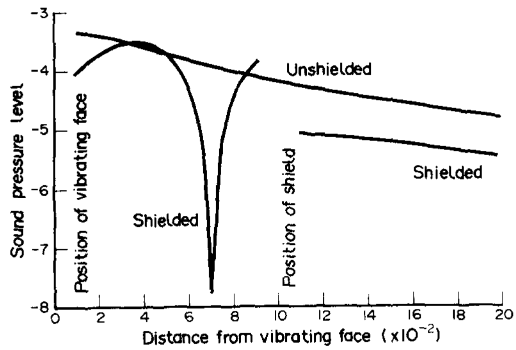

Fig. 5. The computed effect on the axial sound pressure level in air at 3050 Hz of placing a rigid 10 cm square shield in front of a uniformly vibrating face of a 10 cm cube.

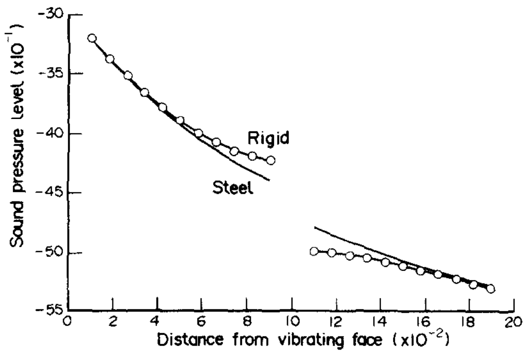

Fig. 7. A comparison of the axial sound pressure levels in air at 490 Hz of shielding a cube with a uniformly vibrating face with (I) a rigid 10 cm square and (II) A 1 mm thick 10 cm square shield of steel.

radiation ratio at around 1800, 3000 and 4500 Hz and significantly increasing it at around 2300 and 3700 Hz.

Let us now go on to study the effect of the shield on the pressure field at a set of chosen frequencies. Figures 4–6 show the sound pressure level (in this case simply the logarithm of the sound pressure) at points along the line from the centre of the vibrating face to the centre of the plate and beyond at frequencies of 2300, 3050 and 3700 Hz.

Finally, a case where the plate is non-rigid is considered. It is assumed that the plate is a 1 mm thick piece of steel that is hinged on all four sides and it is assumed to have no structural damping. The dynamic properties of the square plate can be found in many textbooks on the analytic vibratory analysis of simple structures, for example Ref. [19]. The first resonant frequency of the plate is near 490 Hz. The shield is not directly forced. Figure 5 compares the sound pressure level for the steel shield with that of the rigid shield at 490 Hz.

6. CONCLUDING DISCUSSION

As verification of the subroutine ABSEMGEN, the results in Fig. 3 compare well with the results in [20], wherein the radiation ratio curve for a cube shielded by another cube is given. Figure 3 shows the occurrence of near-resonances and near-antiresonances in the acoustic system.

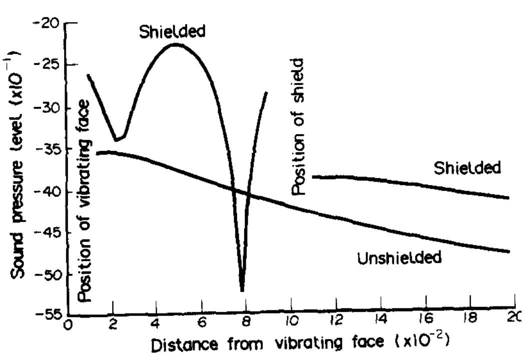

Fig. 6. The computed effect on the axial sound pressure level in air at 3700 Hz of placing a rigid 10 cm square shield in front of a uniformly vibrating face of a 10 cm cube.

At the low frequencies, where the shield is much smaller than the acoustic wave-length, the shield does not appear to have a significant effect on the global properties of the acoustic field. The sound pressure plots in Figs 4–6 show the influence of the shield on the sound pressure field near the acoustic resonant frequencies. Figures 4 and 6 show the shield has the effect of increasing the farfield sound pressure at the acoustic resonances and Fig. 5 illustrates the opposite effect at the anti-resonant frequency.

Figure 7 shows the effect of the shield near its first structural resonant frequency. The coupling between the acoustic field and the structural vibration acts to reduce the effect of the shield on the sound pressure field.

The boundary and shell element method is thus a useful generalization of the tradition BEM: greatly enhancing the range of applications. The subroutine ABSEMGEN, which implements the BSEM, is described in greater detail in reference [20]. The subroutine clearly has a wide range of applications—its application to the problem of predicting the effect of a shield on the noise from an engine is considered in Ref. [21].

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