MR-HIFU

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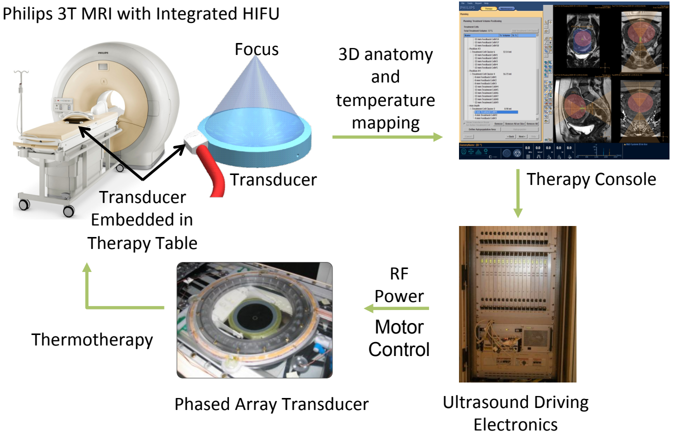
The MR-HIFU Process

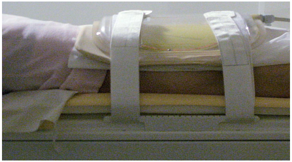
HIFU is a process which may be used in destroying cancerous tissue using hyperthermia

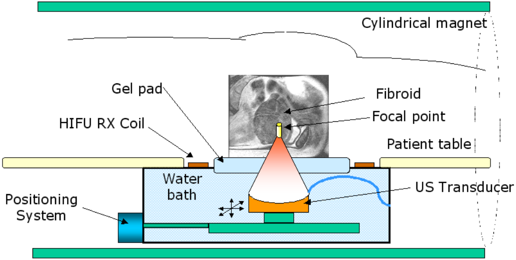
- Transducer produces 40W Ultrasound signal at 1.2 MHz.
- Ultrasound signal is focused into a region $8mm \times 2mm \times 2mm$.
- Pressure variations due to the ultrasound lead to a temperature source $Q(\mathbf{x}, t)$.
- Temperature $T(x, t)$ changes due to the action of the source, diffusion and perfusion due to blood flow.
- High temperatures over a sustained time lead to tissue damage.

How Does MR-HIFU Work?



System Setup





Questions to address

- Find a semi-analytic expression for the spatially extended temperature field $T(x,t)$
- Obtain numerical computations of a simplified model which allows us to look at the effects of parameter variation and compare these with calculations from a more sophisticated model
- Determine useful and usable models for the spatial tissue damage which depend on material parameters which takes into account the thermal dose, thermal conductivity, thermal diffusion, specific heat, and perfusion of the issue of interest and surrounding structures.

The source term Q

- Transducer produces a pressure field P computable using a Rayleigh-Sommerfeld integral method.
- This gives a heat source Q with

$$Q = \frac{\alpha P^2}{2\rho c}.$$

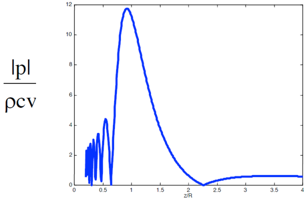
Orthogonal components of pressure

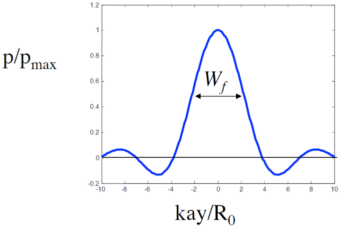
P along axis of propagation

$$|P| = Ae^{-\alpha z}\left|1 - e^{ika^2/2z}\right|$$

P orthogonal to axis of propagation

$$|P| = \frac{J_1(ky)}{ky}$$





Pennes Equation

The temperature $T(x,t)$ obeys the Pennes equation: a linear heat equation.
Heat generation due to source Q, Heat loss due to conduction and perfusion by blood

$$\rho c T_t = \nabla(k \nabla T) - \gamma(T - T_b) + Q.$$

k is thermal conductivity , γ is the perfusion constant (material dependent).Some values:

- $\rho c \approx 3 \times 10^6$
- $\gamma \approx 2000$
- $Q \approx 4\rho c - 10\rho c$
- Length scale of $L \approx 10^{-3}$ at the focus.
- $k \approx 1/2$

Some scalings

- At the focus

$$\nabla(k\nabla T) \approx 10^6, \gamma(T - T_b) \approx 10^3, Q \approx 10^6$$

  Perfusion is unimportant (unless we are close to a major blood vessel).

- Away from the focus, Q diminishes rapidy (inverse square) and both perfusion and diffusion act together to reduce the temperature.

Simple models

Semi-analytic model of a radially symmetric system close to the focus.

Let r be the distance from the focus. Length scale $L = 10^{-3}$

Set $s = r/L$. Use approximation

$$Q = Q_0 \rho c \left(\frac{J_1(r/L)}{r/L} \right)^2 = Q_0 \rho c \left(\frac{J_1(s)}{s} \right)^2 .$$

Rescale system with known parameter values to give

$$T_t = \frac{1}{6s}(sT)_{ss} - \frac{2}{3} \times 10^{-3}(T - T_b) + Q_0 \left(\frac{J_1(s)}{s} \right)^2$$

$$T_s(0) = 0, \quad T(\infty) = T_b.$$

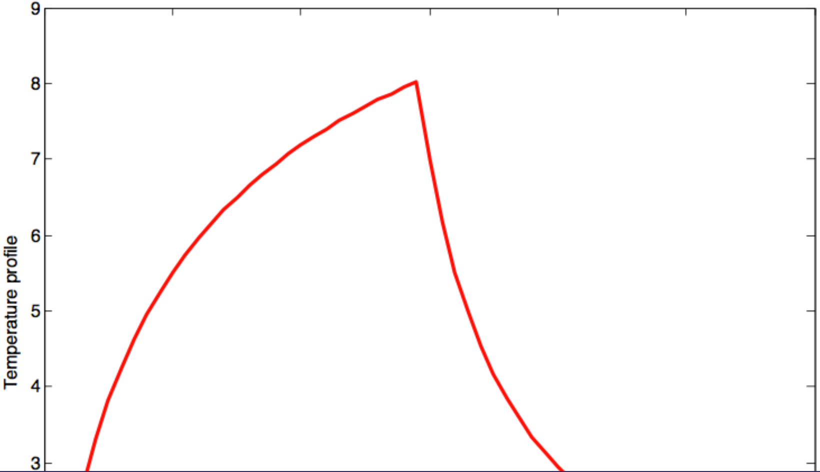
Numerical calculation

- Solve this numerically with a Crank-Nicolson method

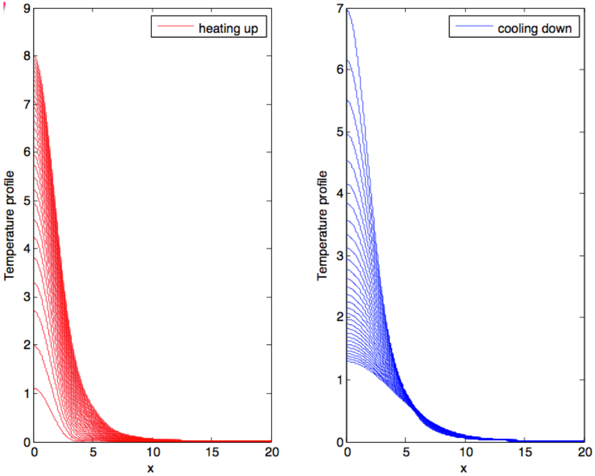
$$\Delta t = 1, \Delta x = 0.04, x \in [0, 20].$$

- Test by using $Q = \sin(r)/r$ which gives an analytic solution.
- Method converges rapidly using sparse matrix methods
- Run with Q as before with $Q_0 = 5$ for $0 < t < 30s$.
- Then take $Q_0 = 0$ for $t > 30$.

Results

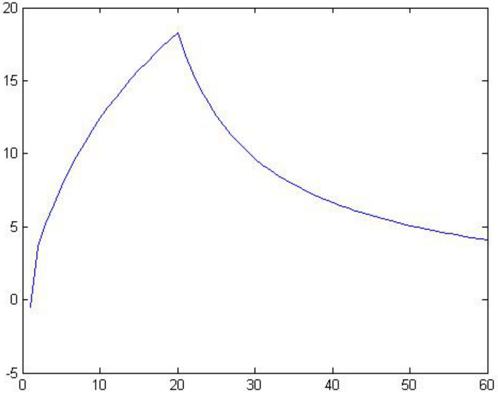


Results

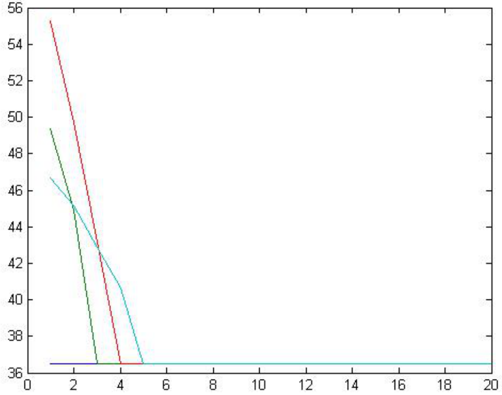


heating up
Temperature profile
x
cooling down
Temperature profile
x

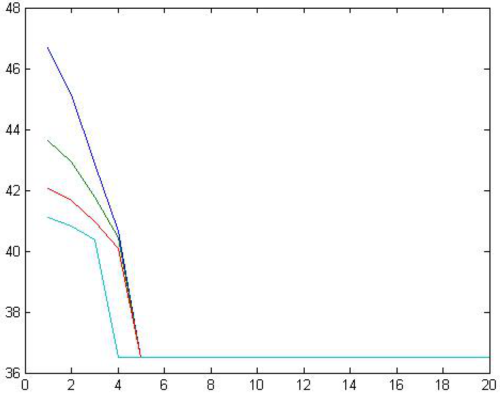
Comparison with results from more complex (3D finite difference) models



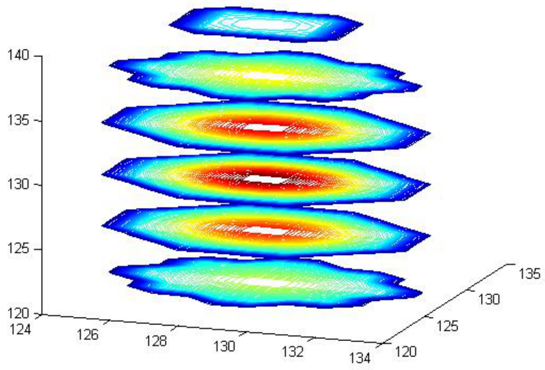
Comparison with results from more complex (3D finite difference) models



Comparison with results from more complex (3D finite difference) models



Comparison with results from more complex (3D finite difference) models



Classical Damage Theory

Classical theory (widely accepted, Saparto and Dewey, 1984) is based on the assumption that damage is measured by

$$TD(t) = \int_0^t r^{(43-T(t))} dt \qquad r = \begin{cases} 0.25 & T \leq 43^o C \\ 0.50 & T > 43^o C \end{cases}$$

This formula

- is entirely phenomenological/heuristic (has no mechanistic basis);
- Damage threshold varies widely among different tissues;
- No explanation of the significance/origin of the threshold at 43^oC.

Another Idea

Zhou, Chen, and Zhang, 2007 suggested that damage is the result of irreversible, protein denaturization, governed by the chemical reaction

$$P \to D,$$

($P =$ folded protein, $D =$ denatured protein) at an Arrhenius reaction rate

$$r(T) = A \exp\left(-\frac{\Delta G}{RT}\right)$$

where ΔG is activation energy, R is universal gas constant. This leads to damage fraction

$$\Omega(t) = \log\left(\frac{P_0}{P(t)}\right) = \int_0^t A \exp\left(-\frac{\Delta G}{RT(t)}\right)\, dt$$

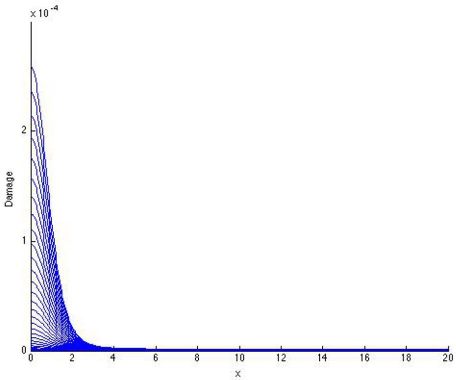
($P_0 =$ initial folded protein).

Comments

- While this formula was used for damage from a laser, we believe it is also applicable to damage from ultrasound (HIFU);
- The parameters A, ΔG and Ω_θ (damage threshold) can be chosen to match different tissue types.
- $\Omega(t)$ can be readily computed using the Pennes model.
- Irreparable damage if $\Omega > \Omega_\theta = 0.63$

Results

Using the previous computations of T we can estimate the damage using the above formula. Note values are small as we are not using a laser!



Conclusions

- The simplified model gives results very comparable to those of the more complex model and allows direct analysis
- Agreement between the two models gives us confidence in both
- New damage model has firmed theoretical foundation and is easy to implement
- We have confidence that a more complete analysis is now possible given more time

Thanks for your attention!