Beliefs, Aggregate Risk, and the U.S. Housing Boom*

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Abstract

Endogenously optimistic beliefs about future house prices can account for the increase, time-path, and volatility of house prices in the U.S. housing boom of the 2000s without shocks to housing preferences. In a general equilibrium model with incomplete markets and aggregate risk, heterogeneous agents form beliefs about future house prices in response to shocks to fundamentals. When fundamentals like credit conditions loosen, agents do not know the extent to which demand for housing services increases, leading them to revise up their beliefs, but only partially. Consistent with both novel and existing empirical evidence, beliefs are increasingly optimistic throughout the boom. How beliefs are formed in housing booms has direct implications for prudential policy as endogenous beliefs are sensitive to policy interventions.

Keywords: housing boom; aggregate risk; heterogeneous agents; incomplete information
JEL Codes: E20, E3, C68, R21

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1 Introduction

Record house prices in the 2000s preceded one of the of the worst recessions in modern U.S. history. Understanding what drove this housing boom has been a central question for researchers and policymakers as they seek to safeguard against a repeat of the economic damage. The two most commonly proposed drivers are looser credit conditions and optimistic beliefs about futures house prices.[1] This paper combines these two drivers in a general equilibrium model with incomplete markets and aggregate risk to address why beliefs became more optimistic in the 2000s and how belief formation matters for assessing the effectiveness of prudential policies.

Incomplete information about the evolution of house prices in an economic expansion with looser credit conditions can generate endogenously optimistic beliefs and hence higher house prices. Given today's ongoing debate about the degree to which credit conditions affect house prices, it is unlikely that everyone in the early 2000s had a clear understanding. And how could they have known if expectations about future house prices were in line with fundamentals when innovations in mortgage finance that allowed for looser credit conditions had never before been experienced on a nation-wide scale? For these reasons, this paper models loose credit conditions as an aggregate state where the exact effect of fundamentals on house prices is unknown. This is achieved by relaxing the complete information assumption that the evolution of house prices is time invariant and fully known in all aggregate states.

When credit conditions loosen, agents must form beliefs to forecast future house prices and do so via adaptive learning—for which this paper is one of the first to embed in a framework with incomplete markets and aggregate risk to study housing.[2] Under adaptive learning, the partial updating of beliefs prevents agents from fully internalizing the increase in demand for housing services as credit conditions loosen, which results in house prices rising by more than expected. Agents interpret this forecast miss as a signal that their beliefs were not optimistic enough, revise their beliefs upward, and will continue to do so absent a new

[1]See Favilukis et al. (2017), Greenwald (2018), Arslan et al. (2022), Lind (2021), Johnson (2019), Landvoigt (2016), Kermani (2012), Justiniano et al. (2019), Mian and Sufi (2017) and Di Maggio and Kermani (2017) for the role of loose credit conditions in the housing boom. See Kaplan et al. (2020), Burnside et al. (2016), Piazzesi and Schneider (2009), Gelain and Lansing (2014), Adelino et al. (2018), Nathanson and Zwick (2018), Glaeser and Nathanson (2017), Albanesi et al. (2022), Foote et al. (2012), and Foote et al. (2018) for the role of optimistic beliefs. See Duca et al. (2021) for an overview of the drivers of housing cycles in the U.S. and other countries.

[2]Moll (2024) highlights the benefits of an approach like that used in this paper: forecasting prices directly in models with incomplete markets and aggregate risk along, using learning to update prices, and disciplining parameters with survey evidence. Giusto (2014) and Hoffman (2016) embed adaptive learning with incomplete markets and aggregate risk, the former in the canonical Krusell and Smith (1998) model and the later to study the interactions of beliefs and income fluctuations. Broer et al. (2022, 2023), Porapakkram and Young (2007), and Kübler and Scheidegger (2021) explore aggregate risk under information frictions more generally.

signal that suggest their beliefs are too optimistic. Because agents pull forward demand for housing services when they expect house prices to go up in the future, optimistic beliefs bring to fruition higher house prices.

The model's key mechanism for generating optimistic beliefs—and hence higher house prices—is persistently positive house price forecast errors. To corroborate this mechanism with external empirical evidence, this paper develops a novel proxy for house price beliefs in the 2000s from the University of Michigan Survey of Consumers. Although Kuchler et al. (2023, Table 2) document limited availability of empirical house price expectations prior to 2007, this paper exploits the tight correlation of a question on beliefs that is only available starting in 2007 with a question going back to 1992 to construct the empirical proxy. Adam et al. (2024) and Kindermann et al. (2024) also provide supporting evidence of forecast error persistence in their own applications of learning to housing.

Because optimistic beliefs are formed endogenously from incomplete information in this paper, shocks to income and credit conditions can generate the U.S. housing boom of the 2000s without exogenous beliefs in the form housing preference shocks. Under endogenous beliefs, model generated house prices can match 75 percent of the empirical level increase observed from 1997 to 2007, which is similar to the fit under exogenous beliefs as in Kaplan et al. (2020). However, this paper's external empirical evidence developed from the Michigan Survey validates the model's persistently positive house price forecast errors generated under endogenous beliefs instead of the negative forecast errors generated under the exogenous beliefs of Kaplan et al. (2020). Finally, endogenous beliefs can match additional features of house prices including the time path, 96 percent of the autocorrelation, and 89 percent of the volatility.

In the model, looser credit conditions unaccompanied by optimistic beliefs have only a slight direct effect on house prices, but are key for matching the housing boom dynamics of homeownership, mortgage leverage, and foreclosures. The small effect of credit conditions on house prices is the result of 1) only a fraction of heterogeneous households—those who are younger and have lower income—being constrained from homeownership and 2) these households buying similar sized properties as those they were renting.[3] None-the-less, in the

[3]Constrained households are thus the closest parallel to subprime borrowers who have been shown by Mian and Sufi (2009, 2017), Griffin et al. (2021), and others to account for the dynamics of the housing boom. Additional channels could result in credit conditions having a larger effect on house prices, but may still fall short of matching house price statistics other than the level increase as is the goal of this paper. Favilukis et al. (2017) also include aggregate risk and incomplete markets and link credit conditions to house prices with a potentially counterfactually high risk aversion, a less simplified financial sector, and influx of foreign borrowers. Subsequent work explore the role of market segmentation of renters and homeowners [Greenwald and Guren (2024)], the type of credit conditions [Greenwald (2018) and Justiniano et al. (2019)], and the feedback between households and the financial sector [Arslan et al. (2022)].

framework of this paper, any increase in demand for housing services—like the slight increase from constrained households—can generate persistently positive house price forecast errors and hence endogenously optimistic beliefs.

In addition to explaining several features of house prices during the 2000s housing boom and unifying the explanations of beliefs and credit conditions, this paper contributes one of the first studies of how belief formation affects the management of housing booms. The prudential policies studied are similar to those of Allen and Greenwald (2022) and Graham and Sharma (2024) who assess the effects of borrowing limits and higher interest rates on housing dynamics, respectively. Under the endogenous beliefs proposed by this paper, these prudential policies dampen or eliminate an outsized increase in house prices. By contrast, the same prudential policies have little effect on house prices under exogenous beliefs.

House prices are sensitive to prudential policy under endogenous beliefs because beliefs are 1) time-varying and 2) depend on recent house price forecast errors. Changes in fundamentals like higher interest rates or lower borrowing limits can dampen demand for housing services which, in turn, pushes down forecast errors resulting in relatively less optimistic beliefs in current and subsequent periods. By contrast, under exogenous beliefs, optimism is the result of a news shock where agents expect higher demand for housing services in the future. Unless a change in fundamentals such as higher interest rates or lower borrowing constraints can counteract the increase in demand, optimism and the resulting housing boom will continue. Chodorow-Reich et al. (2023) similarly find that housing boom dynamics are sensitive to interest rates under incomplete information. Both Igan and Kang (2011) and Kuang et al. (2024) relatedly document that house price expectations respond to changes in borrowing constraints.

2 Related Literature

Why beliefs about future house prices became optimistic in the 2000s is also addressed by Howard and Liebersohn (2023) and Chodorow-Reich et al. (2023) who propose regional divergence and relocation to downtown neighborhoods, respectively, as fundamentals-based explanations relying on regional heterogeneity. Although regional heterogeneity is beyond the scope of this paper, it is complementary to household heterogeneity and incomplete information, the latter of which is also a key feature of Landvoigt (2016).

By allowing for an interaction of beliefs and fundamentals via incomplete information about the evolution of house prices, this paper contributes a new methodological frontier to study housing booms with incomplete markets and aggregate risk. This paper builds off of the state-of-the art quantitative frameworks of Favilukis et al. (2017) and Kaplan et al. (2020) which rely on complete information. Relatedly, work by Hoffman (2016) also forms endogenous beliefs via adaptive learning to study the interactions of beliefs and income fluctuations across several economic cycles rather than credit conditions in the 2000s. The interactions between beliefs and fundamentals in other frameworks is also featured in Johnson (2019), Cox and Ludvigson (2021), and Dong et al. (2022).

More specifically, this paper contributes a tractable computational method to incorporate adaptive learning into rich quantitative models to study house prices. Although other structural models with learning have successfully accounted for housing market dynamics, these studies typically rely on representative agents which may result in an overstated direct effect of looser credit conditions on higher house prices [Adam et al. (2012), Boz and Mendoza (2014), Kuang (2014), and Caines (2020)]. Chodorow-Reich et al. (2023) and Kindermann et al. (2024) are an exception by allowing for regional heterogeneity and learning about local fundamentals. While they, and most other adaptive learning applications, assume imperfect information about model quantities, Adam and Marcet (2011) provide microfoundations for learning about market clearing prices, as is the case in this paper.

Because this paper's contribution of endogenous belief formation adds several additional dimensions to already computationally intensive frameworks, assuming homogeneous beliefs allows for tractability and clean interpretation of mechanisms. In structural models with heterogeneous beliefs, Piazzesi and Schneider (2009), Burnside et al. (2016), Glaeser and Nathanson (2017), Dong et al. (2022) show that optimists push up house prices to generate the housing boom, which is also consistent with the survey evidence of Kindermann et al. (2024). This paper's homogeneous beliefs can thus be interpreted as a first step towards incorporating these richer forms of beliefs into computationally intensive quantitative models, as advocated by Moll (2024).

Even though explicit measures of house price expectations are only available post 2007, this paper's novel empirical proxy and other evidence supports optimistic beliefs during the 2000s. Surveys by Case and Shiller (1988, 2004), Case et al. (2012), and Shiller and Thompson (2022) document high expectations for house price growth during local booms. For the 2000s boom specifically, Piazzesi and Schneider (2009), Ben-David et al. (2019), and Soo (2018) all document optimism rising with house prices.

Optimistic beliefs and loose credit conditions are not the only explanations of the U.S. housing boom of the 2000s. Low interest rates have also been studied with mixed success. Even though Adam et al. (2017), Garriga et al. (2019), Jordà et al. (2015), Chodorow-Reich et al. (2023) and Diamond and Landvoigt (2022) successfully link low interest rates to high house prices, Dokko et al. (2011) and Glaeser et al. (2013) do not find a similar connection.

To that end, the counterfactual prudential policy simulations in this paper are in line with previous studies of interest rate policy under learning. Adam and Woodford (2021), Adam et al. (2024), and Caines and Winkler (2021) show that it may be optimal for monetary policy to lean against house prices and other assets when information is incomplete. While this paper abstracts away from the rich general equilibrium feedback from other sectors of the economy, like those also shown to be important by Kinnerud (2024), the prudential policy counterfactual simulations none-the-less provide a clean mechanism for showing how belief formation can matter in housing boom management.

3 Model

3.1 Environment

The model introduces incomplete information about future house prices into the framework of Kaplan et al. (2020) to allow for endogenously optimistic beliefs.[4] Time is discrete and the economy is populated by a continuum of measure one finitely lived households who are heterogeneous in ages and idiosyncratic income endowments. These households trade goods and services with a lending sector, a rental housing sector, housing construction firms, and final goods firms. Lowercase letters denote individual household quantities and uppercase letters denote aggregates.

Preferences: Households are active in economic life for $j = 1, \ldots, J$ periods where they work from $j = 1$ to J^{ret-1} and retire at J^{ret} until they exit the economy with certainty at J. All j subscripts have been dropped unless needed. Households have preferences over final goods consumption and housing expenditures $\{c_j, s_j\}_{j=1}^{J}$ with final goods consumption as the numeraire. The utility function is given as:

$$u_j(c, s) = e_j \frac{\left[(1-\phi)c^{1-\gamma} + \phi s^{1-\gamma}\right]^{\frac{1-\sigma}{1-\gamma}} - 1}{1-\sigma}$$

Where ϕ is the taste for housing relative to goods consumption, $1/\gamma$ is the elasticity of substitution between housing expenditures and goods consumption, and σ is the intertemporal elasticity of substitution. A deterministic equivalence scale $\{e_j\}_{j=1}^{J}$ adjusts consumption for changes in household size over the life-cycle. Expected life-time utility is given as:

$$\mathbb{E}_0 \left[\sum_{j=1}^{J} \beta^{j-1} u_j(c, s) + \beta^J v(\flat + \underline{\flat}) \right]$$

Where the warm-glow bequest motive at the end of life J follows the functional form of De

[4]Any discrepancies between the environment described in this section (3.1) and that of Kaplan et al. (2020) stem from their code sometimes differing slightly from their text. Although these discrepancies do not affect the main findings of beliefs as a driver of the U.S. housing boom of the 2000s, there are other corrections to their code that do have some effect on the calibration as shown in Appendix E.

Nardi (2004) which modifies that of Carroll (2002):

$$v(\flat) = \psi \frac{(\flat + \underline{\flat})^{1-\sigma} - 1}{1-\sigma} \tag{1}$$

The bequest motive prevents households from counterfactually ramping up debt and drawing down housing when they exit the economy. The strength of the bequest motive is regulated by ψ and the extent to which bequests are luxuries is given by $\underline{\flat}$.

Income endowments: While working, households receive an income endowment comprised of an aggregate stochastic endowment $\Theta(Z)$, a deterministic life-cycle profile that varies by age χ_j, and an idiosyncratic stochastic endowment $\epsilon_j(z)$ that follows a Markov process.

$$\log y = \log \Theta(Z) + \chi_j + \epsilon_j(z), \quad \text{when} \quad j < J^{ret}$$

The transition matrix for earnings $\epsilon_{j+1}(z') \sim \Upsilon_{j+1|j}(\epsilon_j(z))$ is age dependent which helps account for rising income volatility throughout working life.

In retirement, households receive a fraction ρ_{SS} of their last working period income with aggregate income averaged across states $\bar{\Theta}$. Heterogeneity in retirement income helps preserve the wealth distribution of retired agents.

$$y = \rho_{SS}(\log \bar{\Theta} + \chi_{J^{ret}-1} + \epsilon_{J^{ret}-1}), \quad \text{when} \quad J^{ret} \leq j \leq J \tag{2}$$

Income tax follows the functional form of Heathcote et al. (2017) where τ_y^0 sets the average level of taxation and τ_y^1 sets the degree of tax progressivity. To capture the tax benefits of homeowning, households can deduct ϱ fraction of the interest paid on their mortgage $r_m m$ for the first \$1,000,000 of mortgage debt, $\bar{m}$.

$$\mathcal{T}(y, m) = y - \tau_y^0(y - \varrho r_m min\{m, \bar{m}\})^{1-\tau_y^1} \tag{3}$$

If a household does not have a mortgage, the expression simplifies to, $\mathcal{T}(y) = y - \tau_y^0(y)^{1-\tau_y^1}$.

Housing: Households can either rent or own houses. To capture the lumpy dynamics of housing over the life-cycle—as described by Chambers et al. (2009)—housing units are indivisible and available in discrete fixed sizes $\mathcal{H} \in \{h^0, \dots, h^N\}$ for homeowners and $\tilde{\mathcal{H}} \in \{\tilde{h}^0, \dots, \tilde{h}^N\}$ for renters. If housing units were instead a continuum of sizes households would make a counterfactually large number of adjustments to square footage over the life-cycle as they could upsize or downsize by a fraction of a room.

Markets for rental and owner-occupied housing are both frictionless and competitive where the law of one price holds and selling is instantaneous.[5] Rental units cost $\rho(\mu, Z)$ and owner-occupied units cost $p(\mu, Z)$. Renting generates housing services equal to the size of the housing unit, $s = \tilde{h}$ while homeownership offers additional utility $s = \omega h$ for $\omega \geq 1$. Homeowners pay per-period taxes and maintenance costs $(\delta_h + \tau_h)p(\mu, Z)h$ where maintenance offsets the depreciation of the housing unit. When selling, homeowners pay a transaction cost $\kappa p(\mu, Z)h$ that is linear in housing value. Renters do not pay these costs and can adjust the size of their housing unit without transaction costs.

Liquid savings: Households can save in one-period bonds b at the exogenous risk-free rate q_b. Unsecured borrowing is prohibited and households cannot trade among themselves. Households transact bonds with risk-neutral non-modeled foreign agents with deep pockets.

Assuming an exogenous financial sector implies that the model is essentially a small open economy which is a fairly standard assumption in the literature [Arslan et al. (2022), Kaplan et al. (2020), Greenwald and Guren (2024), Chodorow-Reich et al. (2023)]. Arslan et al. (2022) explain that at the onset of the housing boom in the late 1990s, net exports decline and stay negative, which in turn implies that there is a net capital inflow to the U.S. consistent with the data. Although allowing for endogenous determination of the risk-free rate q_b would lend to more rigorous counterfactual simulations, it would also require zero net supply of financial instruments or capital rented to final goods firms. Not only would the dynamics and mechanisms be complicated by an additional endogenous price, but q_b would also need to be approximated via a Krusell and Smith (1998) equilibrium such that the coefficients on the evolution of $q_b(\mu, Z0)$ would need to be solved for in addition to house prices $p(\mu, Z)$.

Mortgages: Homeowners finance housing purchases with multi-period defaultable mortgages subject to a fixed origination cost $\kappa^m(Z)$. These mortgages are amortized over the life of the household at interest rate $r^m = (1 + \iota)r_b$ which is equal to the risk-free rate scaled by an intermediation wedge.[6] Default risk is thus priced by the mortgage pricing function $q_j(\mathbf{x}', y; Z, \mu)$ rather than the interest rate. This function depends on variables that predict future repayment which include the age j of borrowing households, individual state variables $\mathbf{x}' = \{b', h', m'\}$, individual income y, and aggregate states Z and μ.

When a household aged j obtains a mortgage with principal balance m' they receive $q_j(\mathbf{x}', y; Z, \mu)m'$ from the lender where $q_j(\mathbf{x}', y; Z, \mu) < 1$. The down payment at origination

[5] See Hedlund (2016) for the role of search frictions such as time delays in housing market dynamics.

[6] Allowing homeowners to choose the length of mortgages with borrower-specific interest rates would add more realism at the cost of more state variables. Tractability is assured by 1) mortgages being due at the deterministic end of economic life and by 2) amortization at the common mortgage interest rate r^m.

is thus $p(\mu, Z)h' - q_j(\mathbf{x}', y; Z, \mu)m'$. Households make $J - j$ mortgage payments until the mortgage is fully repaid, $\pi_j^{min}(m) \leq m(1 + r_m) - m'$. The minimum payment is determined by constant amortization:

$$\pi_j^{min}(m) = \left[\frac{r_m(1 + r_m)^{J-j+1}}{(1 + r_m)^{(J-j+1)} - 1}\right] m \tag{4}$$

A loan-to-value (LTV) constraint limits mortgage size to a fraction of the value of housing collateral $m' \leq \theta^{LTV}(Z)p(\mu, Z)h'$ and the payment-to-income constraint (PTI) caps the minimum mortgage payment at a fraction of household income $\pi_j^{min}(m') \leq \theta^{PTI}(Z)y$. Because mortgages are multi-period debt, households are only subject to these constraints in the period when mortgages are originated. If mortgages were instead one-period debt, households would be subject to these constraints in every period. One-period debt would make housing a riskier asset because a house price decline or negative income shock would tighten these constraints to the extent that some homeowners would de-lever or default. To avoid these potential negative outcomes, homeowners would take on less leverage than what is observed in the data. Multi-period debt is therefore important for both a more realistic household problem and matching the empirical cross section of household mortgage leverage.

HELOCs: Home equity lines of credit (HELOCs) allow homeowners to borrow one-period, non-defaultable debt up to a fraction of the value of housing collateral $-b' \leq \theta^{HELOC}p(\mu, Z)h$ with an interest rate equal to that on mortgages r_m. Although only a small fraction of homeowners have HELOCs, including these debt instruments is important for matching targeted net worth moments in the calibration of the cross-sectional distribution of households. Evidence is mixed as to whether or not HELOCS account for mortgage debt dynamics in the U.S. housing boom. Chen et al. (2020) find an important role while Kim (2021) argue that it is more negligible.

Shocks: Z indexes an aggregate state where income $\Theta(Z)$ fluctuates via a two state Markov chain with transition matrix $Z' \sim \Gamma_Z(Z)$ and credit conditions
$\mathcal{C}(Z) = \{\theta^{LTV}(Z), \theta^{PTI}(Z), \kappa^m(Z), \zeta^m(Z)\}$ loosen via a one-time unanticipated shock as in Favilukis et al. (2017).[7] Because agents expect permanently looser credit conditions in the housing boom, the tightening of credit conditions in the housing bust is also a one-time

[7] A two-state Markov chain for credit conditions as in Kaplan et al. (2020) would be ideal. However, solving under endogenous beliefs is more computational tractable when assuming a one-time loosening like other papers in the literature. Appendix E shows that the quantitative rise in house prices in the exogenous beliefs framework of Kaplan et al. (2020) is quite similar with credit conditions that loosen one-time as in this paper, or are Markov as in their original formulation. Given the negligible differences in these results, abstracting away from Markov credit conditions under endogenous beliefs allows for them to be compared.

unanticipated shock.

The two aggregate states of the economy are a high and low state ($Z = high, low$). In the housing boom, income and credit conditions simultaneously attain their high state values while the subsequent bust is thus a contraction back to their low state values.

Individual income $\epsilon_j(z)$ follows an AR(1) process with persistence ρ and an age-dependent standard deviation σ_{ε_j} resulting in an age dependent transition matrix $\epsilon_{j+1}(z') \sim \Upsilon_{j+1|j}(\epsilon_j(z))$

$$\epsilon_j(z) = \rho\epsilon_{j-1}(z_{-1}) + \varepsilon_j(z), \quad \varepsilon_j(z) \sim \mathcal{N}(0, \sigma^2_{\varepsilon_j}) \tag{5}$$

Aggregate state space: The distribution of individual household states μ is a necessary state variable for agents to correctly forecast next period prices under incomplete markets and aggregate risk. $\Gamma_\mu(\mu; Z, Z')$ is the equilibrium law of motion of the measure of agents such that $\mu' = \Gamma_\mu(\mu; Z, Z')$. The aggregate state space of the economy is thus the distribution over individual states and the aggregate shock $\{\mu, Z\}$. Let $\mathcal{X}^h = \mathcal{B} \times \mathcal{H} \times \mathcal{M} \times \mathcal{E} \times \mathcal{J}$ denote the set of individual states for homeowners and $\mathcal{X}^r = \mathcal{B} \times \mathcal{E} \times \mathcal{J}$ that of renters with measure $\int_{\mathcal{X}^h} \mu^h d\mu^h + \int_{\mathcal{X}^r} \mu^r d\mu^r = 1$.

Beliefs: This paper introduces a framework where agents form boundedly rational expectations with respect to fully known shocks $\{Z, \epsilon_j(z)\}$ and only form beliefs about house prices $p(\mu, Z)$ which are a subset of endogenous prices. I solve for a Krusell and Smith (1998) equilibrium by approximating the potentially infinitely dimensional distribution μ and its law of motion $\mu' = \Gamma_\mu(\mu; Z, Z')$ with lower dimensional vectors containing sufficient information to predict prices. Under my contribution of endogenous beliefs, agents know the form of the approximated law of motion but do not know its parameter values. Section 3.7 discusses in more detail the equilibrium properties of the model and Appendix B discusses the solution method with endogenous beliefs.

3.2 Households' Problem

See Appendix A.1 for the full recursive households' problems.

Renters are endowed with savings $b > 0$ and income y. They must choose to stay a renter or purchase a house and become a homeowner solving the problem:

$$V^r_j(b, y; \mu, Z) = \max\{V^{rent}_j(b, y; \mu, Z), V^{own}_j(b, y; \mu, Z)\} \tag{6}$$

Newly originated mortgages are subject to loan-to-value and payment to income constraints:

$$m' \leq \theta^{LTV}(Z) p_h(\mu, Z) h' \tag{7}$$

$$\pi_j^{min}(m') \leq \theta^{PTI}(Z) y \tag{8}$$

Relative to renting, homeowning offers households the benefits of extra utility, tax deductible mortgage interest payments, and collateral for borrowing liquid assets in the form of HELOCs. Renting has the advantage of shielding households from capital gains fluctuations due to movements in house prices. When model simulated house prices match the empirical standard deviation as is the case under endogenous beliefs, renting becomes relatively more advantageous due to more volatile house prices.

Homeowners have four options: sell their house to purchase a new house or become a renter, default and become a renter, stay in their house and pay their existing mortgage, or stay in their house and refinance a new mortgage.

$$V_j^h(\mathbf{x}, y; \mu, Z) = \max\{V_j^r(b^n, y; \mu, Z), V_j^{default}(b, y; \mu, Z), V_j^{stay,pay}(\mathbf{x}, y; \mu, Z), V_j^{stay,refi}(\mathbf{x}, y; \mu, Z)\} \tag{9}$$

3.3 Lending Sector

The lending sector originates mortgages m' to households aged j at price $q_j(\mathbf{x}', y; \mu, Z) \leq 1$. Although lenders are competitive, uninsurable aggregate risk may induce profits and losses along the equilibrium path. Lenders are owned by non-modeled foreign agents with deep pockets who receive these profits and losses as net exports. The mortgages market clears loan-by-loan with the pricing of mortgage points depending on individual future default probabilities and collateral of foreclosed homes.[8]

$$q_j(\mathbf{x}', y; \mu, Z) = -\zeta(Z) + \mathbb{E}_{Z',\epsilon'} \begin{cases} 1 & \text{sell/refi} \\ \dfrac{(1 - \delta_h^d - \tau_h - \kappa_h) p'(\mu', Z') h'}{(1 + r_m) m'} & \text{default} \\ \dfrac{(1 + r_m) m' - m'' + q_{j+1}(\mathbf{x}'', y'; \mu', Z') m''}{(1 + r_m) m'} & \text{pay} \end{cases} \tag{10}$$

If a homeowner sells or refinances, they repay the full balance of their mortgage so that the lender receives the principal plus interest less the intermediation cost $\zeta(Z)$. If the household defaults, the lender forecloses and sells the house to recover the market value of the house as a fraction of the original mortgage. If the homeowner pays their existing mortgage, the lender values the contract as the next period mortgage payment $(1 + r_m)m' - m''$ plus the continuation value of the contract $q_{j+1}(\mathbf{x}'', y'; \mu', Z')m''$.

[8] In their text Kaplan et al. (2020) have the intermediation cost $\zeta(Z)$ as a proportional cost while in their code it is a fixed cost as in this paper.

3.4 Final Goods and Construction Firms

See Appendix A.2 for the full recursive problems of final goods and housing construction firms. The competitive final goods sector has a linear constant returns to scale technology $Y = \Theta(Z)N_c$ with inelastic labor supply N_c. Profit maximization delivers an aggregate wage equal to aggregate productivity:

$$w(\mu, Z) = \Theta(Z) \tag{11}$$

A competitive construction sector produces houses with technology $H_h = (\Theta N_h)^{\alpha}\bar{L}^{1-\alpha}$ where N_h is labor services and $\bar{L}$ is the amount of newly available buildable land. Using the equilibrium wage in equation (11) housing supply is:

$$H_h = [\alpha p(\mu, Z)]^{\frac{\alpha}{1-\alpha}}\bar{L} \tag{12}$$

The profit maximization of the construction sector thus pins down a single price for housing $p(\mu, Z)$ via aggregate housing supply H_h and the aggregate productivity shock $\Theta(Z)$.

3.5 Rental Sector

The rental rate is pinned down by a user-cost formula that is a function of current and future house prices along with a per-period operating cost Ξ as shown in Appendix A.2. The competitive rental sector frictionlessly buys and sells housing units to convert them into rental housing which incurs the same depreciation and taxes as owner-occupied housing.

$$\rho(\mu, Z) = \Xi + p(\mu, Z) - (1 - \delta_h - \tau_h)\mathbb{E}_{Z',\epsilon'|Z,\epsilon}[q_b p'(\mu', Z')] \tag{13}$$

Greenwald and Guren (2024) show that looser credit conditions can quantitatively account for the drop in the rent-price ratio observed throughout the housing boom when landlords are heterogeneous with disperse ownership costs. Although Kaplan et al. (2020, Appendix D) and Arslan et al. (2022) introduce these important frictions in their frameworks, doing so in this paper is complicated by aggregate risk. With costs on the conversion of newly purchased housing units into rentals, the aggregate stock of rental housing $\tilde{H}'$ becomes an additional state variable with an unknown evolution that must be solved as a fixed point along with house prices $p(\mu, Z)$. In this setting, it would also require the formation of endogenous beliefs about the future stock of rental housing in addition to house prices. Although frictions on the stock of rental housing would give credit conditions a more important direct role and beliefs potentially less, this conclusion is not inconsistent with the goal of this paper to unify these two explanations of the housing boom and relate the shift in beliefs to fundamentals.

3.6 Government

The sum of income tax revenues $\mathcal{T}(y, m)$ less mortgage interest rate deductions, property tax revenues $p(\mu, Z)\tau_h \int_{\mathcal{X}^h} h d\mu^h$, and land permit revenues $p(\mu, Z)H_h - w(\mu, Z)N_h$ net of pension outlays $\int_{\mathcal{X}} y_{ret} d\mu_{\mathcal{J}^{ret}}$ are always positive and spent on government services G that are not valued by households and thus discarded.

3.7 Computation of Equilibrium

See Appendix A.3 for the definition of the recursive competitive equilibrium and Appendix B for the computational algorithm and its goodness of fit. The solution method developed in this paper is a variation of the Krusell and Smith (1998) algorithm where aggregate risk and incomplete markets rule out an equilibrium distribution over individual states μ corresponding to a steady state. To determine prices, agents must keep track of a potentially infinitely dimensional object to compute the distribution's equilibrium law of motion $\mu' = \Gamma_\mu(\mu; Z, Z')$.[9] An approximate equilibrium thus achieves computational tractability by tracking house prices directly and updating them with a forecasting rule for each combination of current and future aggregate states, $\mathcal{Z} = \{Z, Z'\}$.[10]

$$\log p'(p(Z); \mathcal{Z}) = a^0_{\mathcal{Z}} + a^1_{\mathcal{Z}} \log p(Z) \quad \iff \quad \mu' = \Gamma_\mu(\mu; \mathcal{Z}) \tag{14}$$

The log-linear AR(1) forecasting rule for house prices in equation (14) is standard in macro housing applications and assumes that the conditional sample log means of house prices $\mu^p_{\mathcal{Z}} = a^0_{\mathcal{Z}}/(1 - a^1_{\mathcal{Z}})$ are sufficient statistics to accurately predict future prices. While the aggregate capital stock in Krusell and Smith (1998) is predetermined, aggregate housing is not and requires explicit market clearing, which makes the computational algorithm in this paper closer to that of Krusell and Smith (2006). Although additional statistics, lags, or forms of a forecasting rule may be used, Krusell and Smith (1998) find that more complexity only incrementally improves accuracy. Moreover, Pancrazia and Pietrunti (2019) find that models relying on simple forecasting rules are best at fitting the path house prices throughout the housing boom.

The standard Krusell and Smith (1998) solution method assumes that agents have full knowledge of the forecasting coefficients in equation (14) and solves for these coefficients

[9] Ahn et al.'s (2018) alternative to the Krusell and Smith (1998) method first solves for a stationary equilibrium with only idiosyncratic shocks and then linearizes around the steady state of aggregate shocks. Linearization may only be suitable if the wealth distribution remains close to the aggregate steady state and this may not be the case when allowing for time-varying beliefs and hence booming house prices. The Krusell and Smith (1998) method may thus be more robust than linearization in the setting of this paper.

[10] A single housing price $p(\mu, Z)$ is the only price determined via market clearing. The lending sector, rental sector, final goods firms, and construction firms are all perfectly competitive with linear objective functions. The number of prices is thus reduced from four to one.

as fixed point. To compute an approximate equilibrium, one first guesses values for these coefficients $\boldsymbol{a}_{\mathcal{Z}} = (a^0_{\mathcal{Z}}, a^1_{\mathcal{Z}})'$ and then solves and simulates the model. Using the time series of simulated market clearing prices p_t, one estimates new coefficients via an ordinary least squares regression shown in equation (15). These steps are repeated until the new coefficients are close to the originals, $\boldsymbol{a}_{\mathcal{Z}} \approx \boldsymbol{a}^{new}_{\mathcal{Z}}$. Let $\mathcal{Z}_t = \{Z, Z'\}$ index the partition of current and future aggregate states at time t.

$$\log p_{\mathcal{Z}_{t+1}} = \underbrace{a^0_{\mathcal{Z}} + a^1_{\mathcal{Z}} \log p_{\mathcal{Z}_t}}_{\boldsymbol{x}'_{\mathcal{Z}_t}\boldsymbol{a}_{\mathcal{Z}}} + e_{\mathcal{Z}_{t+1}}, \qquad \boldsymbol{a}^{new}_{\mathcal{Z}} = \left(\sum_{t=1}^{T} \boldsymbol{x}_{\mathcal{Z}_t}\boldsymbol{x}'_{\mathcal{Z}_t}\right)^{-1} \sum_{t=1}^{T} \boldsymbol{x}_{\mathcal{Z}_t} \log p_{\mathcal{Z}_{t+1}} \tag{15}$$

Embedding adaptive learning into the Krusell and Smith (1998) solution method relaxes the assumption that agents know the true values of the forecasting coefficients in states of the economy lacking a historical precedent. Rather than solving for converged forecasting coefficients as a fixed point, agents instead update the coefficients each period $\boldsymbol{a}_{\mathcal{Z}_t}$ with a combination of past coefficients and weighted lagged forecast errors e_{t-1} as shown by the recursive stochastic gradient mechanism for adaptive learning in equation (16).[11] Let $\boldsymbol{x}_{t-2} = (1, \log p_{t-2})'$.

$$\boldsymbol{a}_{\mathcal{Z}_t} = \boldsymbol{a}_{\mathcal{Z}_{t-1}} + g_t \boldsymbol{x}_{t-2} \underbrace{(\log p_{t-1} - \boldsymbol{x}'_{t-2}\boldsymbol{a}_{\mathcal{Z}_{t-1}})}_{e_{t-1}} \tag{16}$$

Following the convention of Evans and Honkapohja (1999), agents form beliefs at time t using information available at $t-1$.[12] Because the next period aggregate state Z' is unknown at time t, agents will need to calculate as many vectors of coefficients as there are next period aggregate states Z'. With four possible aggregate states, agents will compute four different coefficients each period. The time t expected future house prices is $\mathbb{E}_t[\log p_{t+1}] = \sum_{Z'} \pi_{Z,Z'} \exp\left\{a^0_{\mathcal{Z}_t} + a^1_{\mathcal{Z}_t} \log p_t\right\}$.[13]

As agents learn the true values of the forecasting coefficients, beliefs about future house prices may not yet correspond to the actual evolution of prices. The resulting temporary equilibria may be self-referential where optimism leads to relatively higher market clearing

[11] Evans et al. (2010) show that stochastic gradient learning does not weight the forecast error by the inverse of the matrix of second moments, $\boldsymbol{R}_{\mathcal{Z}_t} = \boldsymbol{R}_{\mathcal{Z}_{t-1}} - g_t(\boldsymbol{x}'_{t-2}\boldsymbol{x}_{t-2} - \boldsymbol{R}_{\mathcal{Z}_{t-1}})$. Stochastic gradient learning has the advantages of simplicity and ease of interpretation when embedding in a general equilibrium framework. Furthermore, Mele et al. (2020) argue that stochastic gradient learning keeps the state space small by abstracting from the evolution of the estimated second moments. However, stochastic gradient learning has different convergence criteria than least squares learning as shown by Barucci and Landi (1997).

[12] A forecasting rule like that in Malmendier and Nagel (2016) where time t information is included in time t beliefs via the forecast error, $e_t = \log p_t - \boldsymbol{x}'_{t-1}\boldsymbol{a}_{\mathcal{Z}_{t-1}}$, would induce a simultaneity problem in this general equilibrium setting as agents would be determining house prices p_t while also using them to form beliefs.

[13] There are several alternative forms of equation (16) and Appendix F.2 shows that these have little material difference on the main housing boom simulation results.

realizations or pessimism leads to relatively lower. Following the terminology of Adam and Marcet (2011), agents are internally rational because they form dynamically consistent beliefs about the future absent full information. How the values of time-varying coefficients $\boldsymbol{a}_{\mathcal{Z}_t}$ differ from their near-rational known counterparts $\boldsymbol{a}_{\mathcal{Z}}$ depends on the evolution of the house price forecast error, which, in turn, depends on two key parameters: the gain parameter g_t governing the speed of updating and the initial coefficients $\boldsymbol{a}_{\mathcal{Z}_0}$.

As shown in figure (1), the convergence of coefficients to an ergodic distribution is used for equilibrium selection under either the standard known coefficients $\boldsymbol{a}_{\mathcal{Z}}$ or adaptive learning with unknown time-varying coefficients $\boldsymbol{a}_{\mathcal{Z}_t}$. How coefficients converge differs between the two solution methods—the standard known coefficients converge via a fixed point while the time-varying coefficients converge sequentially. Under the standard known coefficients, the fixed-point solution method solves and simulates the model many times at many different coefficient values. These coefficients remain fixed throughout simulation and are updated at each iteration until the original guess is close to the simulated value as shown by the dashed lines in figure (1). By contrast, under adaptive learning, the model is solved on a grid of future prices so that coefficients can vary throughout simulation. Although the resulting sequence of coefficients may diverge from the near-rational fixed-point counterparts temporarily—as shown by the solid line in Figure (1)—the sequence does eventually converge to a nearby ergodic distribution.

Given that there are the many forms of expectations that allow for incomplete information, this paper does not claim that the adaptive learning rule given by equation (16) is uniquely suitable for endogenizing beliefs to quantitatively account for booming house prices. In fact, the solution method in Appendix B is generalizable to many forms of time-varying beliefs. Nonetheless, adaptive learning has the advantages of tractability when embedded in a detailed quantitative model and well-studied equilibrium convergence properties.[14] Alternatives such as extrapolative, diagnostic, or natural expectations may be equally suitable for explaining the housing boom, their equilibrium properties in macroeconomic models are less well developed relative to those for adaptive learning. Furthermore, Gelain et al. (2016), Glaeser and Nathanson (2017), Case and Shiller (1989), Pancrazia and Pietrunti (2019), and Granziera and Kozicki (2015) all find that house price forecasts with a backward looking component, like that found in adaptive learning, best fit house price data.

[14] See Evans and Honkapohja (2001) for adaptive learning convergence to rational expectations equilibria.

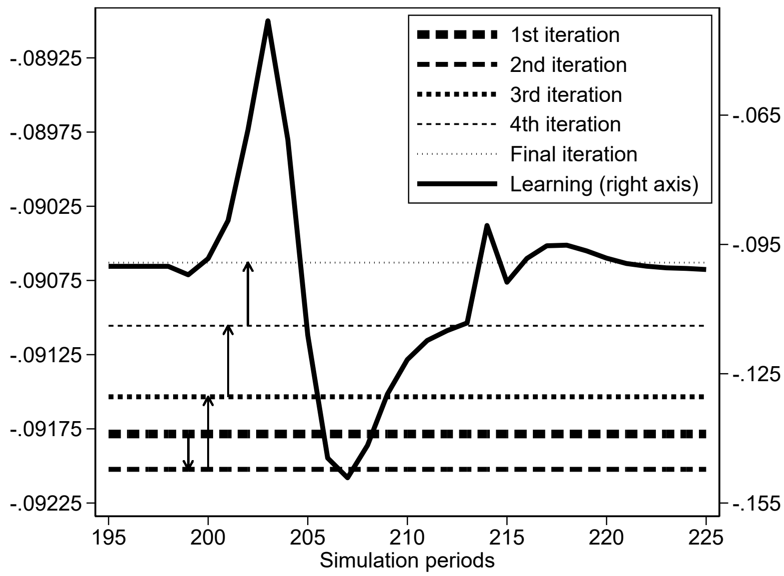


Figure 1: House price forecast coefficient $a^0_{Z_{low_t},Z'_{low}}$ solved for as a fixed point or sequentially or under learning, which starts in period 200. The spikes in the coefficients under learning (solid thick line) are periods where the economy changes aggregate states. The coefficients eventually return to near their ergodic distribution (pre 200 values) by about period 220.

4 Parameterization and Calibration

4.1 Parameters

The model's parameters are set to resemble the U.S. economy in the late 1990s with the cross-sectional moments from the 1998 Survey of Consumer Finances via Kaplan et al. (2020) and a novel proxy on house price beliefs constructed from the University of Michigan Survey of Consumers. Table (1) lists targeted moments from the model's stochastic steady state which is characterized by tight credit conditions and fluctuations in income. These parameters follow Kaplan et al. (2020) and are discussed in detail in Appendix C.1. Parameters that differ from Kaplan et al. (2020) are discussed below along with this paper's contribution of external empirical evidence on belief formation.

Housing Preferences: The preference for housing relative to goods consumption ϕ pins down housing demand so that the average share of housing expenditure to total expenditures from the model is 0.16, as in the data. When beliefs are exogenous, this parameter is not fixed and instead follows a stochastic process where it can take on a low value $\phi_{low} = 0.12$ and a high value $\phi_{high} = 0.20$. However, the stochastic process follows a new shock such that ϕ cannot transition directly from the low to high value and must first enter an intermediate state where the transition has not yet occurred, but is highly likely. This intermediate state is what generates exogenously optimistic beliefs and thus the boom in house prices.

Moment	Parameter	Empirical Value	Model Value
Agg. net worth/annual agg. labor income	β	5.5	4.9
Median ratio of net worth to labor income	β	1.2	1.2
Median net worth: age 75/age 50	ψ	1.55	1.48
% of bequests in bottom 1/2 of wealth dist.	$\underline{\flat}$	0	0
Housing/total cons. expenditures	ϕ	0.16	0.16
Aggregate home-ownership rate	ω	0.66	0.68
Foreclosure rate	ξ	0.005	0.0002
P10 housing/total net worth of owners	$\min \mathcal{H}$	0.11	0.13
P50 housing/total net worth of owners	$\#\mathcal{H}$	0.5	0.33
P90 housing/total net worth of owners	gap $\mathcal{H}$	0.95	0.74
Average sized owned house/rented house	$\min \tilde{\mathcal{H}}$	1.5	2
Average earnings of owners to renters	$\#\tilde{\mathcal{H}}$	2.1	2.8
Annual fraction of houses sold	κ_h	0.1	0.09
Homeownership rate of < 35 y.o.	Ξ	0.39	0.34
Employment in construction sector	$\bar{L}$	0.05	0.04

Table 1: Targeted moments in calibration corresponding to model parameters.

Aggregate shocks: The economy has aggregate shocks to income $\Theta(Z)$ and credit conditions $\mathcal{C}(Z) = \{\theta^{LTV}(Z), \theta^{PTI}(Z), \kappa^m(Z), \zeta^m(Z)\}$. Aggregate income follows a two-state Markov chain with values $\{\Theta(Z_{low}), \Theta(Z_{high})\}$ following a discrete approximation of an AR(1) process estimated from a linearly de-trended series of total U.S. labor productivity. The accompanying transition probabilities $\pi^{\Theta}_{Z,Z'}$ are similarly obtained from the Markov chain approximation.

Aggregate credit conditions loosen via a one-time unanticipated shift to characterize the onset of the housing boom in the 1990s. They are represented by perfectly correlated movements in the loan-to-value constraint $\theta^{LTV}(Z)$, payment-to-income-constraint $\theta^{PTI}(Z)$, the fixed mortgage origination cost $\kappa^m(Z)$, and the mortgage intermediation wedge $\zeta(Z)$. Although higher loan-to-value constraints are the most common representation of looser credit conditions in macro housing models, other innovations in mortgage finance also played a role in extending relatively more credit to households and are hence included.[15] Dokko et al. (2019) find that 60 percent of all mortgages contained at least one non-traditional feature by 2005 and Greenwald (2018) and Ma and Zubairy (2020) show that payment-to-

[15]Loan-to-value constraints also make the households' problems well defined as explained by Engelhardt (1996). Along with Kiyotaki et al. (2011), Engelhardt (1996) notes that loan-to-value constraints are a reduced-form representation of contract enforcement frictions in credit markets. Lenders require both a down payment and the house to be pledged as collateral because they fear that households might not repay mortgages. By forcing households to hold equity in their homes, loan-to-value constraints distribute some of the down-side risk of house price declines from lenders to borrowers.

income constraints $\theta^{PTI}(Z)$ are quantitatively more important than loan-to-value constraints $\theta^{LTV}(Z)$ in accounting for mortgage debt throughout the 2000s.

The loan-to-value constraint is equal to 0.95 in the low state which is slightly higher than Duca et al.'s (2011) estimates for cumulative loan-to-value ratios of first-time home buyers in the 1990s. Following Kaplan et al. (2020), the value 0.95 replicates the 90th percentile of the loan-to-value distribution observed in the pre-boom data. The high state value of 1.1 targets a 15 percentage point rise in combined loan-to-value ratios from the late 1990s to 2006. The payment-to-income constraint values are set at $\theta^{PTI}(Z_{high}) = 0.5$ and $\theta^{PTI}(Z_{low}) = 0.25$ which are slightly lower than the values of Greenwald (2018) to account for the life time amortization of mortgages being longer than the typical 30 years. Falling financial intermediation and funding costs are represented by a decrease in the fixed mortgage origination costs $\kappa^m(Z)$ from \$2,000 to \$1,200 as described by Favra and Imbs (2015) and a decrease in the mortgage intermediation wedge $\zeta^m(Z)$ from 100 basis points to 60 capture as described by Arslan et al. (2022).

By assuming that shifts in aggregate credit conditions along with fluctuations in aggregate income trigger adaptive learning about house price forecasts, this paper takes the stance that beliefs respond to credit conditions. Although some evidence suggests the opposite direction of causality, identification remains debated and difficult to disentangle.[16]

Beliefs: This paper's contribution of endogenous beliefs via adaptive learning introduces two parameters that affect the formation of house price expectations.

First, the learning gain g_t assigns the speed at which agents update their beliefs with incoming information. Following Marcet and Nicolini (2003) and Milani (2014), a switching gain specifies a constant gain $g_t = g$ in boom states so that beliefs are updated more aggressively to more recent house price observations and a decreasing gain $g_t = g_{t-1}/(g_{t-1}+1)$ in non-boom states so that beliefs more closely track the sample mean of house price observations. The aggregate state characterized by high income and loose credit conditions is the boom state with constant gain and all other states are the non-boom states with a decreasing gain.

A switching gain has the benefit of rapid adjustments in boom states and dampened oscillations in non-boom states. Updating beliefs aggressively with incoming information under a constant gain is useful in the boom state when agents do not know the evolution of house prices, which can be interpreted as them not knowing the sample mean as $\mu^p_{\mathcal{Z}} =$

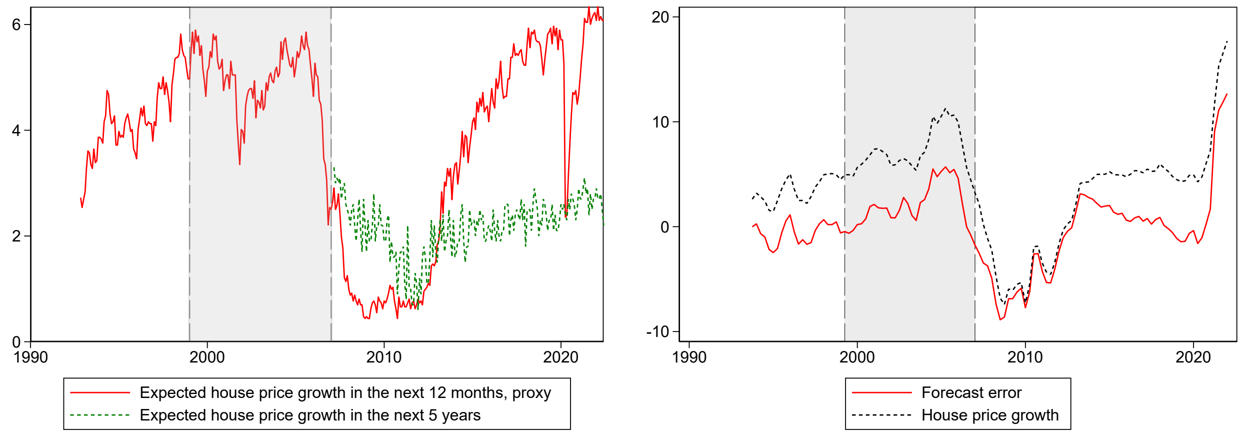
[16]While Mian and Sufi (2009, 2017) find that changes in credit conditions increased expected future house prices, Adelino et al. (2018) and Foote et al. (2012) suggest the opposite direction of causality. Cox and Ludvigson (2021) find contemporaneous correlation between beliefs and credit conditions.

Interpretation	Parameter	Value
Aggregate income $\Theta(\mathbf{Z})$		
Aggregate income	$\{\Theta(high), \Theta(low)\}$	$\{1.035, 0.965\}$
Transition probability	$\pi^{\Theta}_{h,h} = \pi^{\Theta}_{l,l}$	0.9
Aggregate credit conditions $\mathcal{C}(Z)$		
Loan-to-value ratio	$\{\theta^{LTV}(loose), \theta^{LTV}(tight))\}$	$\{1.1, 0.95\}$
Payment-to-income ratio	$\{\theta^{PTI}(loose), \theta^{PTI}(tight)\}$	$\{0.5, 0.25\}$
Fixed origination cost	$\{\kappa^{m}(loose), \kappa^{m}(tight)\}$	$\{\$1,200, \$2,000\}$
Proportional origination cost	$\{\zeta(loose), \zeta(tight)\}$	$\{0.006, 0.010\}$
Beliefs/learning		
Constant gain	$g_t = g$	0.508
Initial learning coefficients	$\boldsymbol{a}_0 = \boldsymbol{a}_{Z_{high}, Z'_{high}}$	{-0.102,0.838}

Table 2: Parameters for aggregate shocks to income and credit conditions along with those for endogenous beliefs characterized via adaptive learning. The model period is two years and values are not annualized.

$a^0_{\mathcal{Z}}/(1-a^1_{\mathcal{Z}})$. If agents interpret large forecast errors as a signal that their beliefs are far from the sample mean, it may be advantageous to adjust their beliefs as quickly as possible in the direction of the forecast error, as prescribed by the constant gain. By contrast, aggressive belief updating in a non-boom state may assign too much signal to noisy observations and pull beliefs away from their average values. Consequently, because the decreasing gain puts successively less weight on recent observations, beliefs are less likely to assign much signal to new information. Although decreasing gain learning can potentially guarantee convergence to a rational expectations equilibrium, the Krusell-Smith approximate equilibrium solved for in this paper may not necessarily be a true rational expectations equilibrium, and convergence may only be to an ergodic distribution.

Following Caines (2020), this paper calibrates the housing boom's constant gain $g_t = g$ for the boom state by minimizing the difference between mean squared house price forecast errors from the model and an empirical proxy constructed from the University of Michigan Survey of Consumers detailed in Appendix D and shown in Figure (2). Even though series of expected house price growth are only available at the end of the housing boom in 2007, the "next 12 months" expectations series is tightly correlated with a series on selling conditions that dates back to 1992. Backcasting the shorter series from the longer can deliver a proxy of house price expectations throughout the 2000s. This proxy and its counterpart of expected house price growth for the next 5 years were elevated in the 2000s housing boom as shown in Panel (2a). Because the series for the next 5 years was at an all-time high even after house price growth peaked, it is reasonable to conclude that households were optimistic about housing. The resulting forecast errors used to calibrate the constant learning gain $g_t = g$ are large and have a steady upward path following that of house prices as shown in Panel (2b).



(a) Expectations of house price growth (b) House price growth and forecast errors

Figure 2: 12-month percent change in expected house price growth and 4-quarter percent change in house price growth with forecast errors calculated as house price growth less the proxy. Shaded bar between the dashed vertical lines marks the U.S. housing boom (1999 to 2007).

With an annualized value of about $g = 0.3$ (0.508 non-annualized), this paper's constant gain is on the larger end of values typically found elsewhere in the literature. Milani (2014) notes that when forecast errors are large, agents may be concerned that the economy is experiencing a structural break and assign a large weight to incoming information and hence a high value for the constant gain. Caines (2020), Adam et al. (2012), Adam et al. (2024) use annualized gains of about 0.03 to 0.06 in their applications of adaptive learning to housing. These smaller values may arise from 1) agents learning about house price growth instead of house price forecasting coefficients and 2) differences in house price expectations used in calibration. For example, both Caines (2020) and Adam et al. (2024) use post-boom house price expectations from the Case-Shiller house price futures or the Michigan Survey. On the other hand, Marcet and Nicolini (2003) estimate an annualized constant gain ranging from 0 to 0.5 in boom-like settings, Milani (2014) estimates constant gains ranging from 0.21 to 0.32, and Carvalho et al. (2023) close to 0.45 at an annualized frequency. Taken together, this evidence suggests that an annualized constant gain of about 0.3 is high but still within an established range of values.

The decreasing gain $g_t = g_{t-1}/(g_{t-1}+1)$ only affects the non-boom states and is initialized at $g_t = g/(1 + t \times g)$ which follows the convention of Marcet and Nicolini (2003) and Milani (2014) plus a decay parameter $t = 5$ that takes into account the number of periods since the start of the housing boom and thus incomplete information about the evolution of house prices. Appendix F.3 discusses simulations without the decay parameter.

The second learning parameter, initial beliefs $\boldsymbol{a}_0$, determines the initial forecast error and there is no definitive convention, as noted by Lubik and Matthes (2016). This paper targets the initial boom period forecast error of 1.36 percent and attain a value of 1 percent using the known pre-boom coefficients for tight aggregate credit conditions and the high income state $\boldsymbol{a}^{tight}_{Z_{high},Z'_{high}}$ as initial beliefs for the boom state. Appendix F.2 shows that quantitative results are similar if the initial coefficient is set at values such that the initial forecast error is exactly 1.36 percent. The advantage of the former approach of setting the coefficients to their known values $\boldsymbol{a}^{tight}_{Z_{high},Z_{high}}$ is that agents start the housing boom with a forecasting rule they have previously used.

4.2 Calibration

See Appendix C.2 for an in depth discussion of the cross-sectional distribution of households in the model's stochastic steady state which assumes known forecasting coefficients, tight aggregate credit conditions, and fluctuations in aggregate income.

Parameter	Empirical Value	Model Value
Fraction of homeowners w/mortgage	0.66	0.65
Fraction of Homeowners w/HELOC	0.06	0.02
Aggr. mortgage debt/housing value	0.42	0.48
P10 LTV ratio for mortgages	0.15	0.02
P50 LTV ratio for mortgages	0.57	0.5
P90 LTV ratio for mortgages	0.92	0.85
Share of NW held by bottom quintile	0	0
Share of NW held by middle quintile	0.05	0.09
Share of NW held by top quintile	0.81	0.66
Share of NW held by top 10 percent	0.7	0.43
Share of NW held by top 1 percent	0.46	0.06
P10 house value/earnings	0.9	0.94
P50 house value/earnings	2.1	1.8
P90 house value/earnings	5.5	4.1

Table 3: Untargeted moments in calibration.

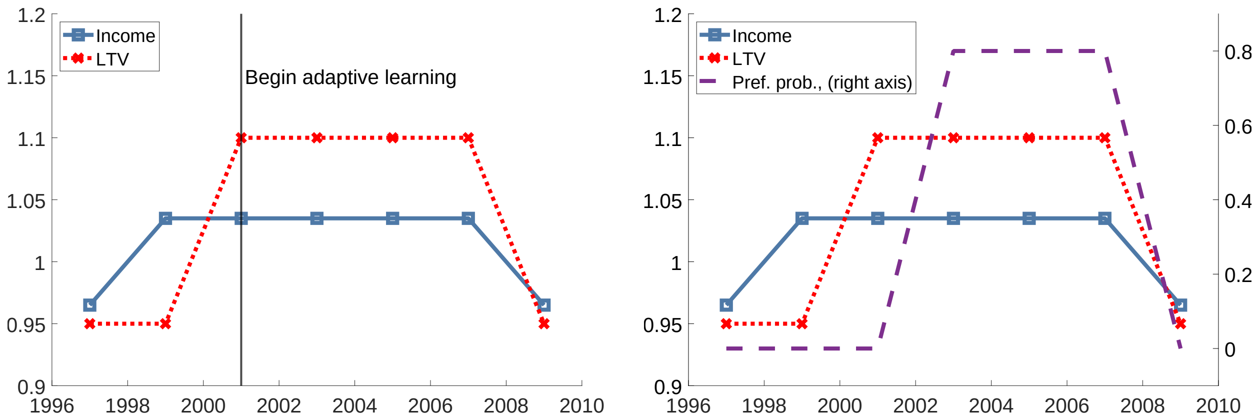
Table (3) shows a comparison of untargeted moments from the model to their empirical counterparts. The model matches the distributions of homeownership and mortgage debt shown in the top panel and the bottom 90th percentile of the wealth distribution shown in the bottom panel. As is common in models with heterogeneous households, the top 10th percentile of the model's wealth distribution is far below that of the data because wealthy households can only hold housing or liquid financial instruments and thus accumulate a counterfactually high share.[17] Appendix C.3 discusses the assumption on the partial seg-

[17] Although allowing for risky assets with heterogeneous returns is a solution proposed by Cioffi (2021) and Xavier (2021), it is beyond the scope of this paper as it would complicate how a shift in beliefs affects house prices.

mentation of housing unit sizes by renters and homeowners.

5 Housing Boom Simulations

This section shows the responses of prices and quantities to a sequence of aggregate shocks. The results under endogenous beliefs formed via time-varying house price forecasting coefficients are compared to counterparts 1) from the data and 2) the exogenous belief specification of Kaplan et al. (2020).



(a) Aggregate shocks under endogenous beliefs (b) Aggregate shocks under exogenous beliefs

Figure 3: Aggregate shocks corresponding to values in table (2) for 1997-2009.

The housing boom is initiated by the same sequence of aggregate shocks to income and credit conditions under either belief formation, as shown in figure (3). In 1999, aggregate income (blue lines with squares) transitions from the low state to the high state and aggregate credit conditions (red dashed lines with Xs) unexpectedly loosen in 2001. After remaining in this boom state for 4 periods (8 years), aggregate income transitions back to the low state and aggregate credit conditions unexpectedly tighten back to their pre-boom values. The housing boom is thus a period where agents do not anticipate looser credit conditions, but then expect them to stay in place permanently.

Under endogenous beliefs, there are no additional aggregate shocks. As shown in panel (3a), adaptive learning about house price forecasting coefficient begins in 2001 when aggregate credit conditions unexpectedly loosen as this is an economic state that had little historical precedent. Appendix F.3 shows that the results are similar if adaptive learning instead begins at the start of the model simulation instead of in 2001.

Under exogenous beliefs, a third shock to housing preferences shown in panel (3b) generates the U.S. housing boom. This shock is a news shock that occurs in 2003, one period following the shift in credit conditions in 2001. While the housing preference parameter ϕ is fixed under endogenous beliefs it evolves according to a three state Markov process under exogenous beliefs. In the first state, it is at its low value and has zero probability of shifting to its high value. In the second state, it also has its low value but has a probability of 0.85 to shift to its high value.

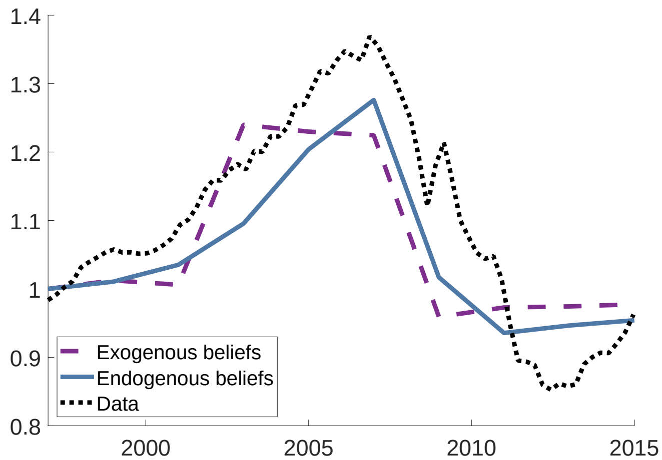
As explained by Kaplan et al. (2020) and shown in Appendix F.1, the shocks to income and credit conditions cannot generate a boom in house prices without optimistic beliefs. Although demand for housing services—and hence house prices—is generally determined by income, the 7 percentage point increase in aggregate income shown in figure (3) is not large enough to generate the nearly 35 percentage point increase in house prices. Relatedly, the increase in housing demand from looser credit conditions is limited to only a fraction of households due to heterogeneous incomes and ages. Because these constrained households tend to be younger and buy the same size of house they were renting when credit conditions loosen in this framework, overall demand for housing services only increases slightly, which results in almost no increase in house prices.[18] As will be discussed in this section and shown by Ma and Zubairy (2020), looser credit conditions are important for accounting for homeownership dynamics across the age distribution.

Although shifts in income and credit conditions fall short of generating a boom in house prices in this framework, shifts in beliefs can succeed because of the self-fulling dynamics of beliefs. Essentially, households demand more housing when they expect prices to go up in the future and this increase in demand brings to fruition higher house prices. Whether beliefs are exogenous or endogenous affects the responses of prices and quantities as well as the effectiveness of policy interventions as discussed later in section 5.1.

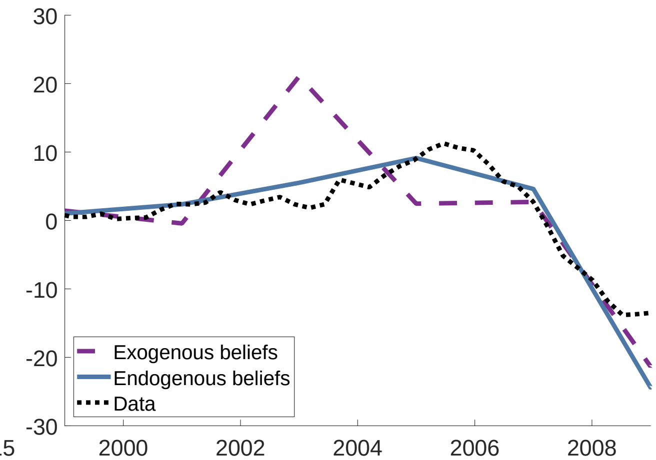
Figure (4a) shows that optimistic beliefs can match the boom in U.S. house prices in the 2000s. Empirical aggregate house prices peak about 35 percent above their pre-boom values and model generated house prices under endogenous beliefs can generate peak house prices about 30 percent above their pre-boom values. Although the model generated peak in house prices is similar under either belief formation, endogenous beliefs can also match additional moments of aggregate house prices because endogenous beliefs generate a persistent rise house prices rather than a counterfactual jump in a single period under exogenous beliefs.

More specifically, model generated house prices under endogenous beliefs can match the standard deviation and autocorrelation of empirical house prices as shown in table. The autocorrelation of 0.440 under endogenous beliefs nearly perfectly matches that of the data and the standard deviation of 0.113 is 89 percent of the data. By contrast, exogenous beliefs either severely undershoot or overshoot the data's autocorrelation and standard deviation.

[18] Greenwald and Guren (2024) show that this is not a general result and frameworks that incorporate endogenous investment in the housing stock along with landlord heterogeneity can generate higher house prices from looser credit conditions.



(a) House prices, 1997=1



(b) House price forecast error, percentage points

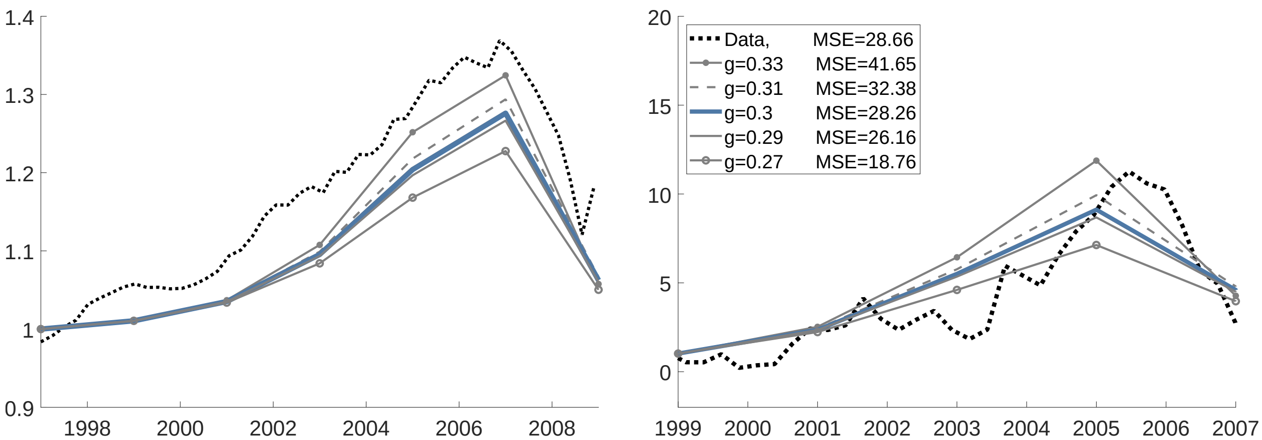
Figure 4: House prices and forecast errors from the data and the model solved under endogenous beliefs and exogenous beliefs. The model's forecast error is $\log p_t - \mathbb{E}_{Z_t}[a^0_{\mathcal{Z}_{t-1}} + a^1_{\mathcal{Z}_{t-1}} \log p_{t-1}]$. See Appendix G for sources and definitions of the data series and Appendix D for details on the construction of the proxy for empirical forecast errors from University of Michigan Survey of Consumers.

	St. dev., σ_{p_t}	$\frac{\sigma_{p_t}}{\sigma_{data}}$	Autocorr., $\rho(p_t, p_{t-1})$	$\frac{\rho(p_t,p_{t-1})}{\rho_{data}}$
Exogenous beliefs, 2003-2007	0.020	16%	0.325	71%
Exogenous beliefs, 1999-2007	0.152	115%	0.225	39%
Endogenous beliefs, 1999-2007	0.113	89%	0.440	96%
Data, 1999-2007	0.127	100%	0.458	100%

Table 4: Biannual house prices statistics from the data and model for the U.S. housing boom.

The house price forecast errors shown in panel (4b) provide external empirical validation of belief formation in the model and shed light on why endogenous beliefs can successfully replicate the path of empirical aggregate house prices. The empirical proxy form the University of Michigan Survey of Consumers (dotted black lines) shows that forecast errors throughout the 2001 to 2007 period are positive and persistent. The partial updating of endogenous beliefs via adaptive learning allows the model to successfully replicate these features of the data, as shown by the solid blue lines. Agents can only partially internalize the increase in aggregate demand when credit conditions loosen, which results in house prices coming in higher than expected. This positive forecast error is interpreted as signal that beliefs were too low and agents then form even more optimistic beliefs. If updating were full instead of partial, agents could form beliefs with more weight on recent forecast misses resulting in forecast errors that alternative between positive and negative values rather than ones that are persistently positive as observed in the data. Under exogenous beliefs, the forecast errors display this counterfactual sign change as they are negative in all periods except for 2003.

How closely model generated house prices and forecast errors match what is observed in the data under endogenous depends on the constant gain parameter g as shown in Figure (5). Under all constant gain values shown, agents underpredict house prices because their initial beliefs do not fully internalize the increase in housing demand from looser credit conditions. Since beliefs are combination of past values and recent forecast errors, partial updates of large positive forecast errors can result in persistently underpredicted house prices, as observed in this paper's novel empirical proxy.



(a) House prices, 1997=1

(b) House price forecast errors, percent

Figure 5: House prices and forecast errors from the data and model solved under endogenous beliefs. The model's forecast error is $\log p_t - \mathbb{E}_{Z_t}[a^0_{\mathcal{Z}_{t-1}} + a^1_{\mathcal{Z}_{t-1}} \log p_{t-1}]$. The parameter g is the learning gain and is annualized in the legend.

Panel (5a) shows that although house prices steadily rise under all values of the constant gain g shown, a smaller constant gain results in a more muted housing boom while a larger constant gain results in one that is more pronounced. There is a positive correlation between the constant gain and house price growth because the constant gain determines the speed at which agents incorporate new information into their beliefs—a larger value of the constant gain g thus updates beliefs more aggressively.

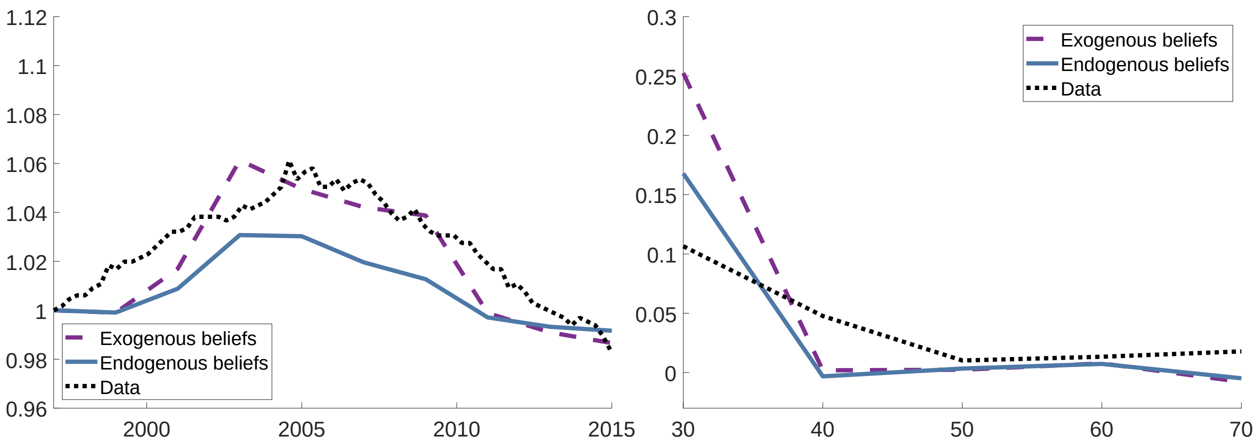
Self-referential dynamics and lack of historical experience in the boom state account for larger values of the constant gain g resulting in larger forecast errors as shown in Panel (5b). Because agents do not know the evolution of house prices in the boom, they revise up their beliefs absent incoming information suggesting too much optimism. Under all values of the constant gain g, forecast errors match the upward path observed in the data, but with less volatility due to their observation at a lower two year frequency.[19] At 28.66, the mean

[19]Because it is unclear if it is better to aggregate via end of period observations or averages, I do not transform the quarterly data to a 2-year frequency.

squared error for the annualized constant gain $g = 0.2986$ (g=0.508 at a two-year frequency) is the closest fit to the data and hence used in the main results.

Appendix F.1 shows that if the economy did not revert back to the non-boom state in 2007, agents would form pessimistic beliefs about future house prices as they realize that their beliefs were too optimistic given fundamentals. However, in the main simulations shown, the boom does not last long enough for agents to self-correct their over optimism. Furthermore Appendix F.4 shows that beliefs do eventually converge to an ergodic distribution in a longer simulation with a counterfactual recurrence of the housing boom state.

Given that endogenous beliefs generate house prices that better match the path, autocorrelation, and standard deviation of the data, this paper next investigates their effect on the homeownership rate which is a key determinant of housing demand and hence house prices.



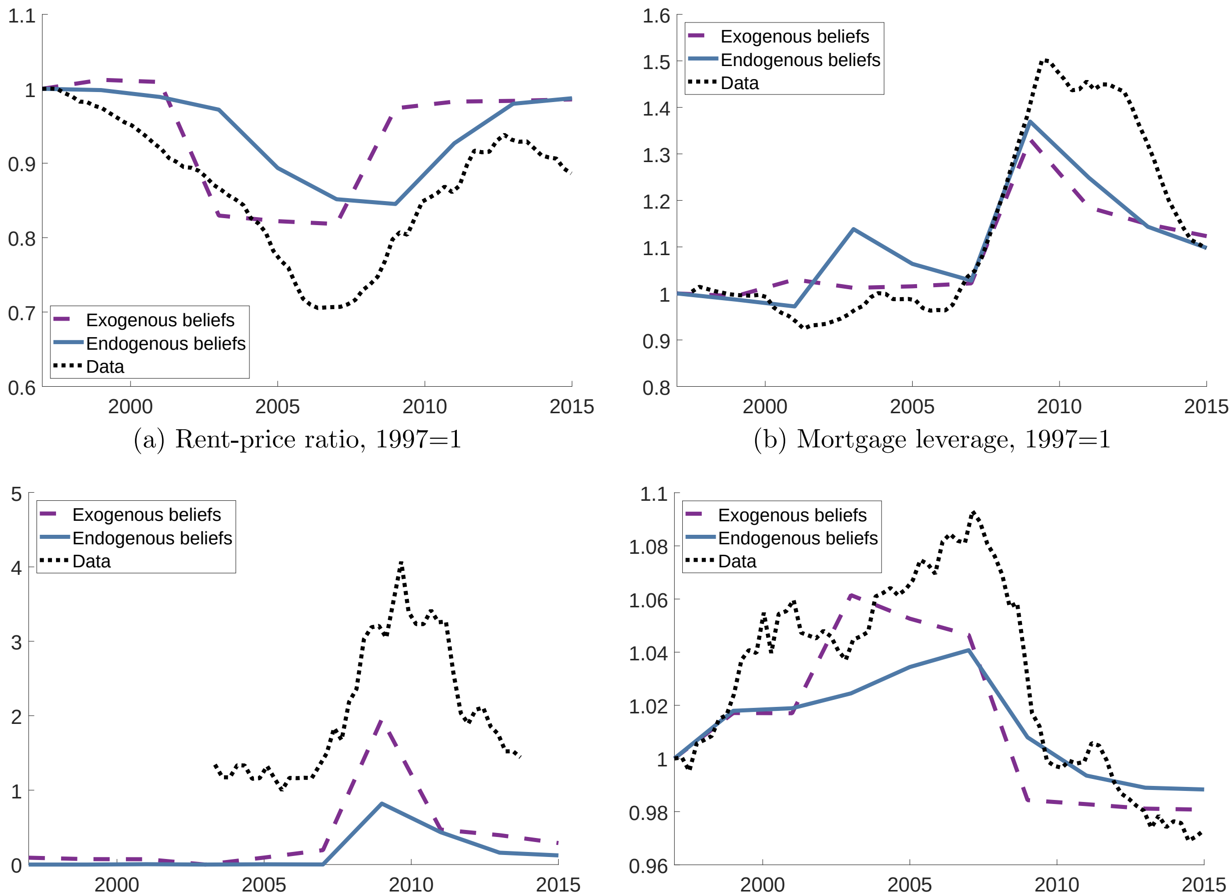
(a) Homeownership rate, 1997=1

(b) Log change in homeownership by age

Figure 6: The homeownership rate from the data and the model solved under endogenous and exogenous beliefs.

Figure (6) shows that the homeownership rate largely matches its empirical counterpart over time and in the cross section. Panel (6a) shows that the aggregate homeownership rate under endogenous beliefs matches the gradual rise and decline observed in the data throughout the boom. The increase in the homeownership rate in both the model and the data is mostly driven by households under the age of 40 purchasing homes as shown in Panel (6b). The size of this increase in homeownership among these young households in panel (6b) is reflected in the aggregate homeownership in (6a). Looser credit conditions allow a larger fraction of young households to overcome payment-to-income and loan-to-value constraints earlier in life than under tighter credit conditions. New homeowners tend to purchase the same sized housing unit they were previously renting which results in slightly more aggregate demand for housing services, but almost no increase in house prices. Higher house prices can mostly be attributed to unconstrained household upsizing their housing units in response to wealth effects from higher expected future house prices.

Figure (7) shows that other prices and quantities also match the data under either exogenous or endogenous beliefs. Matching the volatility of house prices under endogenous beliefs leads to the rent-price ratio and aggregate consumption better matching the path observed in the data which is useful for understanding the consumption response to house prices.



(a) Rent-price ratio, 1997=1

(b) Mortgage leverage, 1997=1

(c) Foreclosure rate, percent

(d) Goods consumption, 1997=1

Figure 7: Housing boom prices and quantities from the data and the model solved under endogenous and exogenous beliefs.

Panel (7a) shows that the decline in the rent-price ratio under either exogenous or endogenous beliefs matches about half of the decline observed in the data. Although macro housing models succeed at generating an increase in prices and homeownership, it is often at the expense of counterfactually rising rents. The rental rate in equation (13) shows that high expected future house prices—like those from optimistic beliefs—are key to rents decreasing and thus a drop in the rent-price ratio. When model generated house prices more closely track the gradual increase throughout the boom, as under endogenous beliefs, the model generated rent-price ratio also better tracks its empirical counterpart.

Panel (7b) shows that modeled generated mortgage leverage remains near its pre-boom values throughout most of the boom and then rises in the bust, similar to what is observed in the data. The only shortcoming of endogenous beliefs is a counterfactual one-time jump during the boom that is not present in their exogenous beliefs counterparts.

The model's foreclosure rate shown in Panel (7c) rises in 2009 when income contracts and credit conditions tighten in the bust. The rise in the foreclosure rate is smoother under endogenous beliefs relative to exogenous beliefs reflecting a smoother path of house prices rather than a one-time jump under exogenous beliefs.

Finally, panel (7d) shows that optimistic beliefs and higher house prices lead to an empirically consistent increase in goods consumption throughout the housing boom. In line with a housing wealth effect, the paths of consumption under either exogenous or endogenous beliefs show a strong correlation with the respective paths of house prices.[20] The model under endogenous beliefs can thus better match the data's steady rise in the path of goods consumption due to its corresponding steady rise in house prices.

5.1 Policy counterfactuals

Although model generated house prices under endogenous beliefs are validated by external empirical evidence and match many features of house prices in addition to the level increase, other prices and quantities from the model are quite similar under either type of belief formation which raises questions about the implications of belief formation. This section shows that belief formation affects the sensitivity of house prices to prudential policy interventions such that endogenous beliefs are more sensitive than their exogenous counterparts. To address the question of whether or not prudential policy can manage housing booms, this paper studies two counterfactual scenarios: higher interest rates and statutory tighter credit conditions.

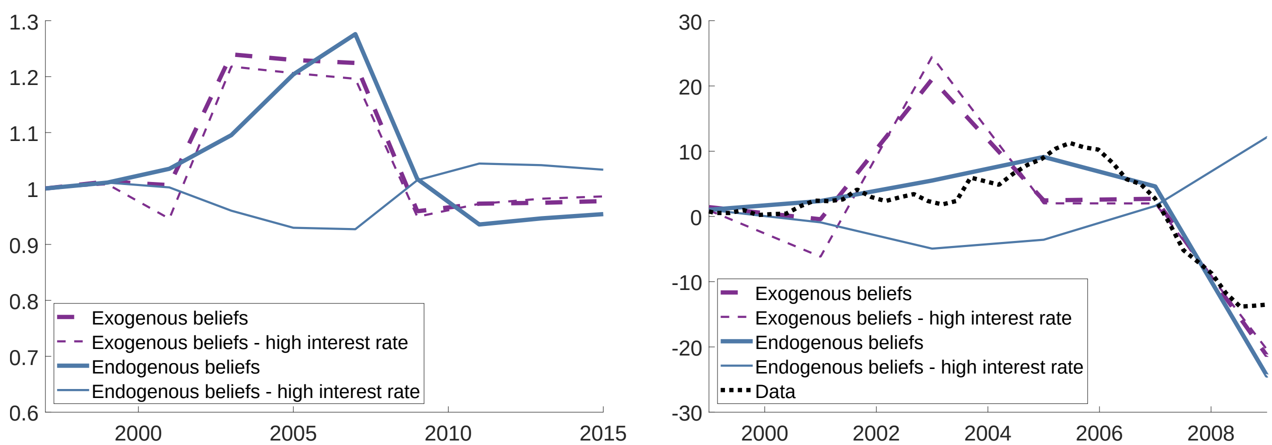
Using monetary policy to "lean against" rising house prices has been studied as a prudential policy for housing boom management because higher policy rates push up mortgage rates, which, in turn, dampens demand for housing services. However, because higher interest rates also 1) dampen output while dampening house prices and 2) may make homeownership less affordable, researchers have cautioned that lowering house prices may be too costly [Benati (2021), Lambertini et al. (2013), Ehrenbergerova et al. (2021), Svensson (2017)]. On the other hand, researchers have also shown that leaning against house prices maybe optimal under imperfect information [Caines and Winkler (2021) and Adam and Woodford (2021)].

The higher interest rate scenario relies on the same aggregate shocks as the main housing boom simulation shown previously along with an additional unexpected shock to the risk-

[20]The size of the housing wealth effect remains debated. While Berger et al. (2018) find large consumption responses to house price movements, Guren et al. (2020) note that responses in the 2000s are smaller than those of the 1980s.

free rate that occurs in 2001 when credit conditions loosen. More specifically, the risk-free rate r_b rises 50 basis points from 3 percent to 3.5 percent on an annual basis resulting in the mortgage rate $r^m = (1+\iota)r_b$ rising from 4 percent to 4.5 percent on an annual basis.

Figure (8) shows that house prices under endogenous beliefs are much more sensitive to a 50 basis point increase in the risk-free rate than their exogenous counterparts. In fact, there is a bust instead of a boom as shown in panel (8a) via the thin solid blue line. House prices drop because the decrease in demand from higher interest rates leads to a negative forecast error that persists due to the slow updating of beliefs under adaptive learning, as shown in panel (8b). The lower expected future house prices—or pessimistic beliefs—results in lower house prices and thus no boom. By contrast, under exogenous beliefs house prices and forecast errors are little different in the scenario with higher interest rates (thin dashed purple lines) relative to the main simulation (thick dashed purple lines). Because exogenous beliefs rely on households pulling forward higher expected demand that never materializes, house prices will only be substantially lower if the dampening effects of higher interest rates can offset these amplifying effects of beliefs. The counterfactual shows that this unlikely with only a 50 basis point increase in.



(a) House prices, 1997=1

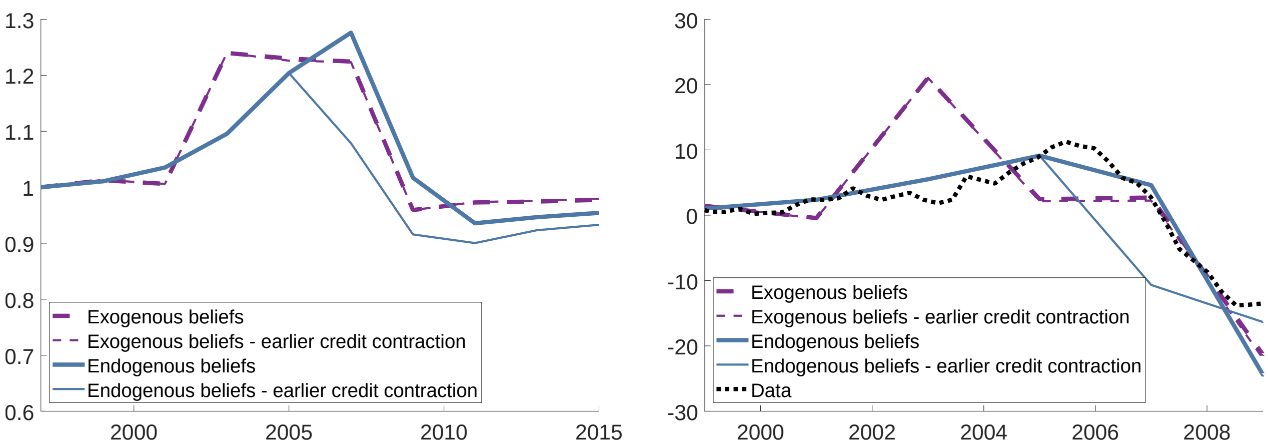
(b) House price forecast errors, percent

Figure 8: House prices and forecast errors with counterfactually higher interest rates. The model's forecast error is $\log p_t - \mathbb{E}_{Z_t}[a^0_{\mathcal{Z}_{t-1}} + a^1_{\mathcal{Z}_{t-1}} \log p_{t-1}]$.

Raising interest rates to lean against high house prices is one of many prudential policies in housing boom management. Statutory limits on loan-to-value ratios and payment-to-income ratios are more widely studied as these policies directly limit leverage, which, in turn, can affect housing demand [see Kelly et al. (2018), Fuster and Zafar (2016), and Bolliger et al. (2024)]. However, these borrowing limits can make homeownership unaffordable for younger and less well-off borrowers as house prices rise [Gete and Reher (2016), Carozzi (2019), and others].

The statutory tighter credit conditions scenario is the same as the main housing boom simulation shown previously, but with credit conditions tightening in 2005 instead of 2009. This scenario can be interpreted as one where innovations in mortgage finance allow for looser credit conditions, but the government reacts by imposing statutory limits at pre-boom values.

Figure (9), like figure (8), shows that house prices are more sensitive to macroprudential policy under endogenous belief than under exogenous beliefs. Under endogenous beliefs, house prices begin to contract as credit conditions tighten midway through the boom, as shown via the thin solid blue lines in panel (9a). When house prices are high and credit conditions tighten, households that are younger and with relatively lower income can no longer afford to become homeowners and this dampens demand for housing. The decrease in house prices pushes down the house price forecast error, as shown in panel (9b), which then pushes subsequent prices down even more. By contrast, under exogenous beliefs there is little difference between the simulations when credit conditions contract earlier in the boom.



(a) House prices, 1997=1

(b) House price forecast errors, percent

Figure 9: House prices and forecast errors with counterfactually tighter credit conditions midway through the boom. The model's forecast error is $\log p_t - \mathbb{E}_{Z_t}[a^0_{\mathcal{Z}_{t-1}} + a^1_{\mathcal{Z}_{t-1}} \log p_{t-1}]$.

The two counterfactual scenarios studied have several caveats. First, the model is assumed to be a small open economy where interest rates are taken as given and fixed rather than endogenously responding to macroeconomic conditions as they would in other settings. Second, the rich household heterogeneity along with incomplete markets and aggregate risk necessitates simplifying some of the general equilibrium feedback from other sectors of the economy found elsewhere in the literature. None-the-less, these scenarios show that the sensitivity of house prices to macroprudential policy can depend on belief formation.

6 Conclusion

This paper first addresses *why* beliefs about future house prices shifted in the early 2000s to push up house prices throughout the 1999-2007 U.S. housing boom. When the evolution of house prices is unknown in an economic expansion accompanied by looser credit conditions, agents are more likely to underpredict future house prices and update their time-varying beliefs in the direction of these positive forecast errors. The resulting endogenously optimistic beliefs are externally validated with empirical evidence obtained via a novel proxy developed from the University of Michigan Survey of Consumers. Furthermore, endogenous optimistic beliefs can match the time path, autocorrelation, and standard deviation of empirical aggregate house prices in addition to the level increase.

Next, this paper shows that the effectiveness of prudential policies can depend on belief formation. Prudential policies like higher interest rates or tighter credit conditions can dampen endogenously formed optimistic beliefs such that the boom in house prices is shorter or non-existent. By contrast, these policies have little material effect on house prices under exogenously optimistic beliefs. Because raising interest rates may lower output, and tightening credit conditions decrease homeownership, studying the effectiveness of these policies is important for understanding when benefits outweigh costs.

Finally, this paper's contribution of endogenous belief formation in a model with incomplete markets and aggregate risk can generalize to settings beyond adaptive learning and the 2000s housing boom. An unexpected shift to an economic state without precedent could describe events such as the onset of the COVID-19 pandemic in 2020 where house prices also boomed.[21]

[21] Loewenstein and Willen (2024) find that beliefs explain the 2000s boom, but a combination of beliefs and preference for larger house account for that of the 2020s.

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A Appendix: Recursive Problems

A.1 Recursive Households' Problem

Households are renters r or homeowners h with distribution $\mu^r + \mu^h = \mu = 1$. In the final period, ($j = J$), all solve the bequest problem. Let $\Upsilon_{j+1|j}(y)$ denote the distribution of $y'|y$ which embeds the Markov transition for the individual shocks $\epsilon_j(z)$ and deterministic χ_j.

Renters' Problem ($\mathbf{j < J}$)

Renters have two choices: stay renters or purchase a house and become homeowners.

$$V_j^r(b, y; \mu, Z) = \max\{V_j^{rent}(b, y; \mu, Z), V_j^{own}(b, y; \mu, Z)\} \tag{6}$$

Rent to rent: if renters choose to stay renters, they solve

$$V_j^{rent}(b, y; \mu, Z) = \max_{\{b', \tilde{h}', c\}} \left\{U_j(c, s) + \beta \mathbb{E}_{Z', \epsilon'|Z, \epsilon}[V_{j+1}^r(b', y'; \mu', Z')]\right\} \tag{6.a}$$

$$\begin{aligned} s.to. \quad c + q_b b' + \rho(\mu, Z)\tilde{h}' =& y - \mathcal{T}(y) + b \\ 0 \leq& b' \\ s =& \tilde{h}' \in \tilde{\mathcal{H}} \\ \mu' =& \Gamma_\mu(\mu; Z, Z') \\ Z' \sim& \Gamma_Z(Z) \\ y' \sim& \Upsilon_{j+1|j}(y) \end{aligned}$$

Liquid financial instruments b, income y, and age j are individual state variables. Rental housing $\tilde{h}'$ costs $\rho(\mu, Z)$ and enters the budget constraint as a cost of foregone consumption. **Rent to own**: if renters choose to purchase a house and become homeowners, they solve

$$V_j^{own}(b, y; \mu, Z) = \max_{\{b', h', c\}} \left\{U_j(c, s) + \beta \mathbb{E}_{Z', \epsilon'|Z, \epsilon}[V_{j+1}^h(b', h', m', y'; \mu', Z')]\right\} \tag{6.b}$$

$$\begin{aligned} s.to. \quad c + p(\mu, Z)h' + q_b b' =& y - \mathcal{T}(y) + b + q_j(b', h', m', y; \mu, Z)m' - \kappa^m(Z) \\ 0 \leq& b' \\ 0 \leq m' \leq& \theta^{LTV}(Z)p_h(\mu, Z)h' \\ \pi_j^{min}(m') \leq& \theta^{PTI}(Z)y \\ s =& \omega h', \quad h' \in \mathcal{H} \\ \mu' =& \Gamma_\mu(\mu; Z, Z') \\ Z' \sim& \Gamma_Z(Z) \\ y' \sim& \Upsilon_{j+1|j}(y) \end{aligned}$$

Where the minimum mortgage payment is defined in equation (4) as:

$$\pi_j^{min}(m) = \left[\frac{r_m(1+r_m)^{J-j+1}}{(1+r_m)^{(J-j+1)}-1}\right] m \tag{4}$$

Renters who purchase a house have the homeowners' continuation value $V_{j+1}^h(b', h', m', y'; \mu, Z)$ which has a state space that also includes the newly originated mortgage m' and the purchased house h'.

Homeowners' Problem $(j < J)$

Homeowners have five options: sell their house and purchase a new house, sell their house and become a renter, default and become a renter, stay in their house and pay their existing mortgage, or stay in their house and refinance a new mortgage. Let $\mathbf{x} \equiv \{b, h, m\}$.

$$V_j^h(\mathbf{x}, y; \mu, Z) = \max \begin{cases} V_j^{sell,buy}(b^n, y; \mu, Z) \\ V_j^{sell,rent}(b^n, y; \mu, Z) \\ V_j^{default}(b, y; \mu, Z) \\ V_j^{stay,pay}(\mathbf{x}, y; \mu, Z) \\ V_j^{stay,refi}(\mathbf{x}, y; \mu, Z) \end{cases} \tag{9}$$

Sell to rent or buy: homeowners who sell their house solve the renters' problem (6) with financial assets b^n equal to one-period liquid financial instruments b plus the equity from the sale of the house—the sale price less housing costs and the outstanding mortgage balance.

$$b^n = b + (1 - \delta_h - \tau_h - \kappa_h)p(\mu, Z)h - (1 + r_m)m$$

Default: homeowners who default become renters, incur a utility penalty ξ, and live in the smallest rental housing unit $\tilde{h}^0$. Their property is foreclosed and the lender receives the proceeds from the sale of the housing unit less property taxes and depreciation. In default, depreciation is higher $\delta_h^\delta > \delta_h$.[22] Defaulters are not subject to recourse meaning that the lender cannot lay claim to other assets should the housing collateral not cover all of the defaulted mortgage balance.

[22] As studied by Gabriel et al. (2021), not all defaults lead to foreclosure and there is sometimes a time lag between the two events. A default is failure to meet the terms of mortgage contract and a defaulted mortgage is foreclosed when the homeowner's rights are to the property are eliminated. See Fannie Mae's glossary: https://www.knowyouroptions.com/find-resources/information-and-tools/glossary.

$$V_j^{default}(b, y; \mu, Z) = \max_{\{b',c\}} \left\{ U_j(c, \tilde{h}^0) - \xi + \beta \mathbb{E}_{Z',\epsilon'|Z,\epsilon}[V_{j+1}^r(b', y'; \mu', Z')] \right\} \tag{9.a}$$

$$\begin{aligned} s.to. \quad c + q_b b' + \rho(\mu, Z)\tilde{h}^0 =& y - \mathcal{T}(y) + b \\ 0 \leq& b' \\ \tilde{h}^0 =& \min \tilde{\mathcal{H}} \\ \mu' =& \Gamma_\mu(\mu; Z, Z') \\ Z' \sim& \Gamma_Z(Z) \\ y' \sim& \Upsilon_{j+1|j}(y) \end{aligned}$$

Stay and pay: homeowners who stay in their house and pay their existing mortgage solve

$$V_j^{stay,pay}(\mathbf{x}, y; \mu, Z) = \max_{\{b',m',c\}} \left\{ U_j(c, s) + \beta \mathbb{E}_{Z',\epsilon'|Z,\epsilon}[V_{j+1}^h(\mathbf{x}', y'; \mu', Z')] \right\} \tag{9.b}$$

$$\begin{aligned} s.to. \quad c + q_b b' + (\delta_h + \tau_h)p(\mu, Z)h + m(1 + r_m) =& y - \mathcal{T}(y, m) + b + m' \\ \pi_j^{min}(m) \leq& (1 + r_m)m - m' \\ 0 \leq& m' \\ -b' \leq& \theta^{HELOC} p(\mu, Z)h \\ s =& \omega h, \quad h' = h \\ \mu' =& \Gamma_\mu(\mu; Z, Z') \\ Z' \sim& \Gamma_Z(Z) \\ y' \sim& \Upsilon_{j+1|j}(y) \end{aligned}$$

Because these homeowners stay in the same house $h' = h$, there is no housing choice. Instead, these homeowners pay the housing depreciation costs and tax $(\delta_h + \tau_h)p(\mu, Z)h$ and a mortgage payment greater than the minimum payment $\pi_j^{min}(m)$ described in equation (4). These homeowners can also borrow against the value of their house in the form of HELOCs.

Stay and refi: homeowners who stay in their house and refinance a new mortgage solve:

$$V_j^{stay,refi}(\mathbf{x}, y; \mu, Z) = \max_{\{b', m', c\}} \left\{ U_j(c, s) + \beta \mathbb{E}_{Z', \epsilon' | Z, \epsilon}[V_{j+1}^h(\mathbf{x}', y'; \mu', Z')] \right\} \tag{9.c}$$

$$\begin{aligned}
s.to. \quad c + q_b b' + (\delta_h + \tau_h) p(\mu, Z) h + m(1 + r_m) =& y - \mathcal{T}(y, m) + q_j(\mathbf{x}', y; \mu, Z) m' - \kappa^m(Z) \\
0 \leq m' \leq& \theta^{LTV}(Z) p_h(\mu, Z) h' \\
\pi_j^{min}(m') \leq& \theta^{PTI}(Z) y \\
-b' \leq& \theta^{HELOC} p(\mu, Z) h \\
s =& \omega h, \quad h' = h \\
\mu' =& \Gamma_\mu(\mu; Z, Z') \\
Z' \sim& \Gamma_Z(Z) \\
y' \sim& \Upsilon_{j+1|j}(y)
\end{aligned}$$

These homeowners stay in their house h and pay housing maintenance costs $(\delta_h + \tau_h) p(\mu, Z)$. They pay off their existing mortgage m with the proceeds of a new mortgage m' that is subject to points $q_j(\mathbf{x}', y; \mu, Z) \leq 1$, the fixed mortgage origination cost $\kappa^m(Z)$, and loan-to-value and payment-to-income constraints described in equations (7)-(8).

Homeowners can thus extract equity from HELOCS or mortgage refinancing which is essentially a cash-out equity extraction because all mortgages amortize at the same rate r_m.[23] Because mortgage refinancing requires the payment of fixed origination cost $\kappa^m(Z)$ and mortgage points $q_j(\mathbf{x}', y; \mu, Z)$, it is preferred to HELOCS when homeowners are extracting relatively large amounts of equity or have a higher income such that $q_j(\mathbf{x}', y; \mu, Z)$ is closer to 1. Otherwise, HELOCs may be preferred.

Bequest Problems ($\mathbf{j = J}$)

When households exit economic life, they solve the following problems with the bequest motive $V_{J+1} = v(\flat + \underline{\flat})$ described in equation (1).

Renters' Bequest Problem

Renters must stay renters and cannot become homeowners in the final period of economic life. Their value function is thus $V_J^r(b, y; \mu, Z) = V_J^{rent}(b, y; \mu, Z)$.

[23] Wong (2019) relaxes the fixed interest rate assumption when modeling mortgage refinancing and finds that younger households or those with larger mortgages are more likely to refinance.

$$V_J^{rent}(b, y; \mu, Z) = \max_{\{b', \tilde{h}', c\}} \{U_j(c, s) + \beta v(\flat + \underline{\flat})\} \tag{17}$$

$$\begin{aligned} s.to. \quad c + q_b b' + \rho(\mu, Z)\tilde{h}' =& y - \mathcal{T}(y) + b \\ \flat =& b' \\ 0 \leq & b' \\ s =& \tilde{h}' \in \tilde{\mathcal{H}} \end{aligned}$$

Homeowners' Bequest Problem

Homeowners have three choices in the final period of economic life. They can sell their houses and become renters, default and become renters, or stay in their house and leave it as a bequest after paying off the residual mortgage.[24] Homeowners can neither purchase a new house nor refinance a new mortgage in the final period of economic life.

$$V_J^h(\mathbf{x}, y; \mu, Z) = \max \begin{cases} V_J^{sell,rent}(b^n, y; \mu, Z) \\ V_J^{default}(b, y; \mu, Z) \\ V_J^{stay,pay}(\mathbf{x}, y; \mu, Z) \end{cases} \tag{18}$$

Sell to rent: when homeowners sell their house and choose to become renters, they solve the renter's bequest problem (17) with liquid financial instruments b^n.

$$b^n = b + (1 - \delta_h - \tau_h - \kappa_h)p(\mu, Z)h - (1 + r_m)m$$

Default: homeowners who default in the last period of economic life solve:

$$V_J^{rent}(b, y; \mu, Z) = \max_{\{b', c\}} \left\{U_j(c, \tilde{h}^0) - \xi + \beta v(\flat + \underline{\flat})\right\} \tag{18.a}$$

$$\begin{aligned} s.to. \quad c + q_b b' + \rho(\mu, Z)\tilde{h}^0 =& y - \mathcal{T}(y) + b \\ 0 \leq & b' \\ \flat =& b' \end{aligned}$$

[24] Strategic bequests and dynasties are beyond the scope of this paper, see Brandsaas (2024) for a study.

Stay and pay: homeowners who pay off their mortgage and leave their house in bequest solve:

$$V_J^{pay,stay}(\mathbf{x}, y; \mu, Z) = \max_{\{b',c\}} \left\{U_j(c, s) + \beta \mathbb{E}_{Z'|Z}[v(\flat + \underline{b})]\right\} \tag{18.b}$$

$$\begin{aligned} s.to. \quad c + q_b b' + (\delta_h + \tau_h)p(\mu, Z)h + (1 + r_m)m' =& y - \mathcal{T}(y, m) + b \\ 0 \leq& b' \\ \flat =& b' + (1 - \kappa_h)p'(\mu'; Z, Z')h \\ s =& h\omega, \quad h \in \mathcal{H} \\ \mu' =& \Gamma_\mu(\mu; Z, Z') \\ Z' \sim& \Gamma_Z(Z) \end{aligned}$$

A.2 Final Goods, Construction, and Rental Firms' Problems

Final goods firms: V_c denotes the value function of final goods producing firms.

$$\begin{aligned} V_c(N_c; \mu, Z) =& \max_{N_c} \{Y - w(\mu, Z)N_c\} \\ s.to. \qquad Y =& \Theta(Z)N_c \\ 0 \leq& N_c \\ \mu' =& \Gamma_\mu(\mu; Z, Z') \end{aligned}$$

Because final goods consumption C is the numeraire, the price for final goods has been normalized to one. Taking the first order condition with respect to labor N_c pins down the aggregate equilibrium wage:

$$w(\mu, Z) = \Theta(Z) \tag{11}$$

Housing construction firms: V_h denotes the value function of housing construction firms.

$$\begin{aligned} V_h(N_h; \mu, Z) =& \max_{N_h} \{p(\mu, Z)H_h - w(\mu, Z)N_h\} \\ s.to. \qquad H_h =& [\Theta(Z)N_h]^\alpha \bar{L}^{1-\alpha} \\ 0 \leq& N_h \\ \mu' =& \Gamma_\mu(\mu; Z, Z') \end{aligned}$$

Taking the first order condition with respect to labor N_h and using the above equilibrium expression for aggregate wage $w(\mu, Z)$ yields:

$$\alpha\Theta(Z)p(\mu, Z)[\Theta(Z)N_h]^{\alpha-1}\bar{L}^{1-\alpha} = \Theta(Z)$$

Re-arranging delivers the expression for housing supply:

$$\underbrace{[\Theta(Z)N_h]^\alpha \bar{L}^{1-\alpha}}_{\equiv H_h} = [\alpha p(\mu, Z)]^{\frac{\alpha}{1-\alpha}} \bar{L} \tag{12}$$

Rental housing sector: V_r denotes the value function of the rental housing sector with the stock of housing units owned by the rental company given as $\tilde{H}'$ and the rental ready housing units given as X'.

$$\begin{aligned}
V_r(\tilde{H}, X; \mu, Z) =& \max_{\tilde{H}', X'} \big\{ [\rho(\mu, Z) - \Xi] X' - p(\mu, Z)[\tilde{H}' - (1 - \delta_h - \tau_h)\tilde{H}] \dots \\
& \dots + q_b \mathbb{E}_{Z', \epsilon' | Z, \epsilon}[V_r(\tilde{H}', X'; \mu', Z')] \big\} \\
s.to. \qquad X' \leq& \tilde{H}' \\
0 \leq& \tilde{H}', X' \\
\mu' =& \Gamma_\mu(\mu; Z, Z')
\end{aligned}$$

First order and envelope conditions yield the rental housing pricing equation:

$$\rho(\mu, Z) = \Xi + p_h(\mu, Z) - (1 - \delta_h - \tau_h) q_b \mathbb{E}_{Z', \epsilon' | Z, \epsilon}[p'(\mu', Z')] \tag{13}$$

See Appendix D of Kaplan et al. (2020) for a more general version that allows for financial and convertibility frictions.

A.3 Recursive Competitive Equilibrium

A *recursive competitive equilibrium* consists of:

- A sequence of income endowments y to households.
- Prices for owner-occupied housing $p(\mu, Z)$, rental housing $\rho(\mu, Z)$, wages $w(\mu, Z)$, and mortgages $q_j(\mathbf{x}', y; \mu, Z)$ where $\mathbf{x} = \{b, h, m\}$.
- Government parameters for constraints on loan-to-value $\theta^{LTV}(Z)$ and payment-to-income $\theta^{PTI}(Z)$, land permits $\bar{L}$, taxes $\mathcal{T}(y, m)$, and social security payments ρ_{SS}.
- Perceived laws of motion for the state space $\mu = \Gamma_\mu(\mu; Z, Z')$ where $\mu = \mu^r + \mu^h$ is the measure over the set of individual states for renters r and homeowners h. Let $\mu = 1$ and the respective state spaces be defined as $\mathcal{X}^h = (\mathcal{B} \times \mathcal{H} \times \mathcal{M} \times \mathcal{E} \times \mathcal{J})$ and $\mathcal{X}^r = (\mathcal{B} \times \mathcal{E} \times \mathcal{J})$.
- Value functions for renters $V_j^r(b, y; \mu, Z)$ with policy functions for consumption, liquid financial instruments, owner-occupied housing, and rental housing $\{c, b', h', \tilde{h}'\}$ solve the renters' problem. Value functions for homeowners $V_j^h(\mathbf{x}, y; \mu, Z)$ with policy functions for consumption, liquid financial instruments, owner-occupied housing, rental housing, and mortgages $\{c, b', h', \tilde{h}', m'\}$ solve the homeowners' problem.

Markets clear:

- The lending sector maximizes profits and the mortgage market clears loan-by-loan with homeowner specific pricing functions $q_j(\mathbf{x}', y; \mu, Z)$.

$$\int_{\mathcal{X}^h} m' d\mu^h = M'$$

- The rental sector maximizes profits with policy function $\tilde{H}'$ at price $\rho(\mu, Z)$ to supply rental housing.

$$\underbrace{\int_{\mathcal{X}^r} \tilde{h}' d\mu^r}_{\text{renters}} + \underbrace{\int_{\mathcal{X}^h} \tilde{h}' d\mu^h}_{\substack{\text{sellers} \\ \& \\ \text{defaulters}}} = \tilde{H}'$$

- Housing construction firms maximize profits with policy functions for labor demand N_h and the supply of new housing units H_h. There is a single housing price $p(\mu, Z)$ that clears the housing market so that housing outflows (LHS) equal inflows (RHS).

$$\underbrace{\tilde{H}' - (1-\delta_h)\tilde{H}}_{\substack{\text{rental sector} \\ \text{housing purchases}}} + \underbrace{\int_{\mathcal{X}^r} h' d\mu_r}_{\substack{\text{purchases} \\ \text{by renters}}} + \underbrace{\int_{\mathcal{X}^h} h' d\mu_h}_{\substack{\text{purchases} \\ \text{by owners}}}$$

$$= \underbrace{H_h - \delta_h \int_{\mathcal{X}^h} h d\mu^h}_{\substack{\text{new construction} \\ \text{less depreciation}}} + \underbrace{\int_{\mathcal{X}^h} h[\mathbb{1}_{sell} + \mathbb{1}_{default}(1 - \delta_h^d + \delta_h)] d\mu^h}_{\text{sales by owners \& sales of foreclosures}} + \underbrace{\int_{\mathcal{X}^h} \mathbb{1}_{bequest} h' d\mu^h}_{\text{estate sales}}$$

- Final goods firms maximize profits so that the labor market clears at $\Theta(Z) = w(\mu, Z)$ with the total labor supply from housing construction and final goods firms $N_h + N_c$ normalized to 1.

$$\int_{\mathcal{X}} \exp(\chi_j + \epsilon_j(z)) d\mu_{\mathcal{J}_{work}} = \underbrace{N_h + N_c}_{1}$$

- The government collects revenue from income taxes $\mathcal{T}(y, m)$, property taxes τ_h, and the sale of land permits $[p(\mu, Z)H_h - w(\mu, Z)N_h]$ to finance expenditures on social security income for retirees y_{ret} and non-valued government spending G:

$$\mathcal{T}(y, m) + \tau_h p(\mu, Z) \int_{\mathcal{X}^h} h d\mu^h + [p(\mu, Z)H_h - w(\mu, Z)N_h] = \rho_{ss} \int_{\mathcal{X}} y_{ret} d\mu_{\mathcal{J}_{ret}} + G$$

- Profits or losses from the financial and rental sectors are expressed as net exports:

$$\begin{aligned} NX = & \underbrace{\int_{\mathcal{X}^r} [b - q_b b'] d\mu^r + \int_{\mathcal{X}^h} [b - q_b b' \mathbb{1}_{[b'>0]} - \big(r_b(1+\iota)\big)^{-1} b' \mathbb{1}_{[b'<0]}] d\mu^h}_{\text{lenders' net expenses from liquid financial instruments}} \\ & \ldots + \underbrace{\int_{\mathcal{X}^h} [(1+r_m)m + q_j(\mathbf{x}', y; \mu, Z)m'] d\mu^h}_{\text{lenders' net revenue from mortgages}} \\ & \ldots + \underbrace{(\rho(\mu, Z) - \Xi)\tilde{H}'}_{\substack{\text{rental sector revenue} \\ \text{less operating costs}}} - \underbrace{p(\mu, Z)[\tilde{H}' - (1 - \delta_h - \tau_h)\tilde{H}]}_{\substack{\text{rental sector housing} \\ \text{purchases}}} \end{aligned}$$

- The aggregate resource constraint is satisfied where household consumption C, government spending G, net exports NX equal output Y less housing and mortgage adjustment costs.

$$\begin{gathered} \int_{\mathcal{X}^h} c d\mu^h + \int_{\mathcal{X}^r} c d\mu^r + G + NX + \Xi \tilde{H}' \\ = Y - \kappa p(\mu, Z) \int_{\mathcal{X}^h} h(\mathbb{1}_{sell} + \mathbb{1}_{default}) d\mu^h - \iota r_b \int_{\mathcal{X}^h} (m + b\mathbb{1}_{\{b<0\}}) d\mu^h \\ -(\zeta + \kappa^m) \int_{\mathcal{X}^h} m'(\mathbb{1}_{buy} + \mathbb{1}_{refi}) d\mu^h \end{gathered}$$

- Consistency is satisfied and perceived laws of motion of the state space $\mu' = \Gamma_\mu(\mu, Z, Z')$ is consistent with individual behavior

B Appendix: Computational Algorithm

The algorithms with either fixed known or time-varying unknown forecasting coefficients rely on the assumption that agents keep track of house prices via a log-linear forecasting rule for each combination of current and future aggregate states $\mathcal{Z} = \{Z, Z'\}$ as an approximation of the entire distribution over individual states μ and its law of motion $\Gamma_\mu(\mu; \mathcal{Z})$.

$$\log p'(p(Z); \mathcal{Z}) = a^0_{\mathcal{Z}} + a^1_{\mathcal{Z}} \log p(Z) \quad \Longleftrightarrow \quad \mu' = \Gamma_\mu(\mu; \mathcal{Z})$$

The model is first solved under known coefficients that are obtained via a fixed point. Next, the model is solved with beliefs where time-varying coefficients are tracked throughout the simulation. Because beliefs require as many additional grids as there are aggregate states, the households' problem must be solved $3^2 = 9$ times for two belief grids with 3 points each.[25] Fortunately, the increase in dimensionality is not too much of a computational burden because the belief grids can be parallelized.[26] The model has been solved on the high-throughput computing clusters at Indiana University (Karst and Carbonate), the University of Texas at Dallas (BigTex), and the Federal Reserve Board.

1. Define grids over ages $j = 1, \ldots, J$, liquid financial instruments b, liquid financial instrument choices b', rental housing choices $\tilde{h}'$, owner-occupied housing h, owner-occupied housing choices h', mortgage balances m, mortgage choices m', aggregate income $\Theta(Z) \in \{\Theta(Z_{high}), \Theta(Z_{low})\}$, aggregate credit conditions $\mathcal{C}(Z) \in \{\mathcal{C}(Z_{high}), \mathcal{C}(Z_{low})\}$, and house prices p.
2. Define coefficients:
   - Fixed: to pin down next period house prices, guess coefficients $\boldsymbol{a}_{\mathcal{Z}}$ for each $\mathcal{Z}$ for a total of $\#Z^2 = 4$ vectors of coefficients.[27]

     $$\log p'(p(Z); \mathcal{Z}) = a^0_{\mathcal{Z}} + a^1_{\mathcal{Z}} \log p(Z)$$

   - Beliefs: define additional grids over future house prices in each future state $\{p'(Z'_{low}), p'(Z'_{high})\}$ for a total of $\#Z' = 2$ additional grids.

With a risk-neutral lending sector, the mortgage price $q_j(\mathbf{x}', y; p(Z), Z)$ is pinned down

[25] Experiments with 5 grid points instead of 3 show that although individual aggregate housing demand may vary, the demand schedule for housing is roughly similar.

[26] Allowing independent aggregate shocks to credit conditions and income would require defining four belief grids instead of two. With 3 points for each grid, the households' problem would then need to be solved $3^4 = 81$ times instead of $3^2 = 9$ times. Computation times increase drastically as a result of increasing the dimensions of the arrays and interpolation in two additional dimensions.

[27] The forecasting equation is $\log p_{\mathcal{Z}_{t+1}} = \mathbb{1}_{\mathcal{Z}} a^0_{\mathcal{Z}} + a^1_{\mathcal{Z}} \log p_{Z_t}$ for $\mathcal{Z} \equiv \{Z, Z'\}$ which is slightly different, but practically equivalent to equation (14), $\log p_{\mathcal{Z}_{t+1}} = a^0_{\mathcal{Z}} + a^1_{\mathcal{Z}} \log p_{\mathcal{Z}_t}$.

by the lenders' mortgage pricing function in equation (10). The price for financial assets q_b is taken as given. The price of rental housing $\rho(\mu, Z)$ is pinned down by the rental sector's maximization problem (13) and aggregate wages $w(\mu, Z)$ are pinned down by the final goods firms' maximization problem (11).

3. Solve the households' problem at each point on the house price grid p (and at each point on the belief grids if time-varying). I use value function iteration with grid search for housing and mortgages and a golden section solver for liquid financial instruments.

4. Simulate a long time series of aggregate states Z_t for $t = 1, \ldots, T$ where T =5000 with a burn in period of 100.

5. Fix an initial distribution of liquid financial instruments, idiosyncratic incomes, and ages assuming that all households are initially renters $\mu_t \in \mathcal{B} \times \mathcal{E} \times \mathcal{J}$ for $t = 1$. Beginning realizations $\{\mathcal{B}, \mathcal{E}, \mathcal{J}\}$ denote liquid financial instruments, income realizations, and ages, respectively. There are N =125000 households.

   Some notes on initialization:
   - endowments of liquid financial instruments equal zero for all households at $t = 1$.
   - because all agents are renters at $t = 1$, there is no initial housing endowment
   - initial household ages are drawn from a uniform distribution[28] $j \sim [1, \ldots, J]$
   - when household i reaches the final period of economic life, $j_{i,t} = J$, a new household replaces them with $j_{i,t+1} = 1$ as renters with no mortgage debt $m_{i,t} = 0$. These new households inherit a random draw of liquid financial instruments $b_{i,t}$ that are correlated with individual income so that household with a higher income are more likely to inherit larger quantities.
   - if solving with beliefs, set calibrated initial coefficients $\boldsymbol{a}_0$.

6. Simulate
   - Fixed: coefficients are already set at $\mathbf{a}_{\mathcal{Z}}$ at the beginning of the problem.
   - Beliefs: compute period t coefficients to pin down $p'(p(Z); \mathcal{Z})$. Coefficients for each future aggregate state $\boldsymbol{a}_{Z_t, Z'_{low}}$ and $\boldsymbol{a}_{Z_t, Z'_{high}}$ are necessary because the value of Z' is not yet known. For this reason, time-varying beliefs require as many additional grids as there are aggregate states.

7. Compute the aggregate demand schedule for housing at each point on the house price grid p from the housing policy functions for renters $\tilde{h}'$ and homeowners h'. Interpolate over $b_{i,t}$ for renters and $\{b_{i,t}, h_{i,t}, m_{i,t}\}$ for homeowners. Average by housing type:

[28] This assumption follows Kaplan and Violante (2014). Matching the distribution of ages would introduce generational demographic changes into equilibrium dynamics which is beyond the scope of this paper.

$$
\begin{aligned}
\text{Renters:} \quad \tilde{H}_{t+1}(p, \boldsymbol{X}_t) =& \frac{1}{N} \sum\nolimits_{i=1}^{N^r} \tilde{h}'(b_{i,t}, y_{i,t}, j_{i,t}; p, \boldsymbol{X}_t) \\
\text{Homeowners:} \quad H_{t+1}(p, \boldsymbol{X}_t) =& \frac{1}{N} \sum\nolimits_{i=1}^{N^h} h'(b_{i,t}, h_{i,t}, m_{i,t}, y_{i,t}, j_{i,t}; p, \boldsymbol{X}_t)
\end{aligned}
$$

The number of aggregate states in the vector $\boldsymbol{X}_t$ depends on the algorithm:

- Fixed: $\boldsymbol{X}_t = \{Z_t\}$
- Beliefs: $\boldsymbol{X}_t = \{Z_t, p'(Z'_{low}), p'(Z'_{high})\}$ which requires additional interpolation over $p'(Z'_{low})$ and $p'(Z'_{high})$ to incorporate the effects of time-varying beliefs.

8. Compute excess demand for aggregate housing at each point on the house price grid p and recover the equilibrium house price $p_t^*(Z_t)$ from the market clearing condition:

$$
H_{t+1}(p_t^*(Z_t), \boldsymbol{X}_t) + \tilde{H}_{t+1}(p_t^*(Z_t), \boldsymbol{X}_t) = H_h + (1 - \delta_h)[H_t + \tilde{H}_t]
$$

9. Interpolate the individual policy functions b', h', $\tilde{h}'$, and m' at the equilibrium house price $p_t^*(Z_t)$ and then average to calculate the aggregate equilibrium quantities.

$$
\begin{aligned}
B_{t+1}^*(p_t^*(Z_t), \boldsymbol{X}_t) =& \frac{1}{N} \sum\nolimits_{i=1}^{N} b'(b_{i,t}, h_{i,t}, m_{i,t}, y_{i,t}, j_{i,t}; p_t^*(Z_t), \boldsymbol{X}_t) \\
\tilde{H}_{t+1}^*(p_t^*(Z_t), \boldsymbol{X}_t) =& \frac{1}{N} \sum\nolimits_{i=1}^{N^r} \tilde{h}'(b_{i,t}, y_{i,t}, j_{i,t}; p_t^*(Z_t), \boldsymbol{X}_t) \\
H_{t+1}^*(p_t^*(Z_t), \boldsymbol{X}_t) =& \frac{1}{N} \sum\nolimits_{i=1}^{N^h} h'(b_{i,t}, h_{i,t}, m_{i,t}, y_{i,t}, j_{i,t}; p_t^*(Z_t), \boldsymbol{X}_t) \\
M_{t+1}^*(p_t^*(Z_t), \boldsymbol{X}_t) =& \frac{1}{N} \sum\nolimits_{i=1}^{N} m'(b_{i,t}, h_{i,t}, m_{i,t}, y_{i,t}, j_{i,t}; p_t^*(Z_t), \boldsymbol{X}_t)
\end{aligned}
$$

Again, the number of aggregate states $\boldsymbol{X}_t$ depends on the algorithm:

- Fixed: $\boldsymbol{X}_t = \{Z_t\}$
- Beliefs: $\boldsymbol{X}_t = \{Z_t, p'(Z'_{low}), p'(Z'_{high})\}$

$$
\boldsymbol{p'}(p_t^*(Z_t)) = \begin{pmatrix} \exp\{a^0_{Z_t, Z'_{low}} + a^1_{Z_t, Z'_{low}} \log(p_t^*(Z_t))\} \\ \exp\{a^0_{Z_t, Z'_{high}} + a^1_{Z_t, Z'_{high}} \log(p_t^*(Z_t))\} \end{pmatrix}
$$

10. Simulate for $t = 1, \ldots, T$ periods by repeating steps (6)-(9).

11. Compare coefficients:
    - Fixed: partition the time series of market clearing house prices $\{p_t^*(Z_t)\}_{t=burn}^{T}$ by $\mathcal{Z} = \{Z, Z'\}$ to generate $\#Z^2 = 4$ sub-samples. Estimate new forecasting coefficients for each sub-sample $\mathcal{Z}$ via ordinary least squares regression:

$$
\boldsymbol{a}_{\mathcal{Z}}^{new} = \left(\sum_{t=burn}^{T} \boldsymbol{x}_{\mathcal{Z}_t} \boldsymbol{x}'_{\mathcal{Z}_t} \right)^{-1} \sum_{t=burn}^{T} \boldsymbol{x}_{\mathcal{Z}_t} \log p_{\mathcal{Z}_{t+1}}
$$

Repeat steps (2) - (11) until coefficients converge, $\boldsymbol{a}_{\mathcal{Z}}^{new} \approx \boldsymbol{a}_{\mathcal{Z}}$

- Beliefs: adjust belief parameters to verify that the time-varying coefficients $\{\boldsymbol{a}_{\mathcal{Z}_t}\}_{t=burn}^{T}$ from step (9) converge to their near-rational counterparts $\boldsymbol{a}_{\mathcal{Z}}$.

Chipeniuk et al. (2022) propose a solution check called auctioneer iteration to avoid relying on potentially misleading R^2 statistics as measures of accuracy. Their method adds the following steps after coefficients are obtained via a fixed point:

12. Use converged $\boldsymbol{a}_{\mathcal{Z}}$ and a price grid that is a subset of simulated prices, $p =\subset \{p_t^*(Z_t)\}_{t=1}^{T}$.
13. Repeat steps (1)-(11) to obtain new converged coefficients and new prices $\{p_t^{new}(Z_t)\}_{t=1}^{T}$.
14. Stop if $\{p_t^{new}(Z_t)\}_{t=1}^{T} \approx \{p_t^*(Z_t)\}_{t=1}^{T}$, otherwise go to step (1).

B.1 Computational Details

The known forecasting coefficients $\boldsymbol{a}_{\mathcal{Z}}$ shown in Table (5) are solved as a fixed point in the model's stochastic steady state characterized by tight aggregate credit conditions and fluctuations in aggregate income. The coefficients have similar values in all combinations of current and future income states which is consistent with Kaplan et al. (2020) and suggests that shocks to aggregate income do not have much effect on house price forecasts. Approximating the sample mean of house prices $\bar{p}_{\mathcal{Z}}$ via the coefficients points to slightly more variation in the highest and lowest values attained in the transitions between income states.

Income states	$a_{\mathcal{Z}}^0$	a_Z^1	$\bar{p}_{\mathcal{Z}} \approx \frac{a_{\mathcal{Z}}^0}{1-a_Z^1}$	R^2	Den Haan	Chipeniuk et. al
High, High	-0.102	0.838	0.53	0.9999	0.01	0.01
High, Low	-0.111	0.838	0.50			
Low, High	-0.091	0.847	0.55			
Low, Low	-0.100	0.847	0.52			

Table 5: Converged forecasting coefficients under tight aggregate credit conditions.

Like Kaplan et al. (2020) and Favilukis et al. (2017), the model's R^2 statistic is close to 1 which suggests adequate goodness of fit standards. Because R^2 statistics can be misleading gauges of accuracy, Den Haan (2010) and Chipeniuk et al. (2022) propose alternative tests which both come in sufficiently low at 0.01.

C Appendix: Calibration

C.1 Parameter Values

The model's parameters are set to resemble the U.S. economy in the late 1990s with the cross-sectional moments from the 1998 Survey of Consumer Finances via Kaplan et al. (2020).

Demographics: Each model period is equal to two years. Households begin economic life at age 21 ($j = 1$), retire at age 65 ($J^{ret} = 23$), and exit economic life at age 79 ($J = 30$).[29]

Preferences: The elasticity of substitution between goods consumption and housing expenditures $1/\gamma$ is set to 1.25 based on the estimates of Piazzesi et al. (2007). The elasticity of intertemporal substitution equals 0.5 by setting $\sigma = 2$. A McClements (1977) scale sets the consumption equivalence scale $\{e_j\}$ to match the OECD average number of children across different age groups. The discount factor β is set to replicate the 1998 ratio of aggregate net worth to annual labor income of 5.5 for which the model comes in slightly below at 4.9.

Two parameters, the strength of the bequest motive ψ and the extent to which bequests are luxuries $\underline{b}$, pin down the warm-glow bequest motive given in equation (1). The strength of bequests ψ is chosen to replicate the ratio of net worth at age 75 to age 50 of 1.55 which indicates the importance of bequests as a saving motive. The model comes in slightly below at 1.48. The luxuriousness of bequests $\underline{b}$ is chosen so that households in the bottom half of the model's wealth distribution do not leave bequests, as observed in the data.

Homeowners' additional utility ω sets the average homeownership rate which was 66 percent in the late 1990s and is slightly higher at 68 percent in the model. Defaulters' disutility ξ is set to match the average foreclosure rate in the late 1990s of 0.5 percent and is slightly lower at 0.02 percent in the model.

Income endowments: The deterministic life-cycle component of earning $\{\chi_j\}$ is from Kaplan and Violante (2014). Stochastic individual earnings $\epsilon_j(z)$ in equation (5) follow an AR(1) process in logs with an annual persistence of 0.97, annual standard deviation of 0.2, and an initial standard deviation of 0.42. The variance of log earnings rises by 2.5 between ages 21 and 64 which follows Heathcote et al. (2010). Bequested inheritances are correlated with individual income so that higher income households are more likely to inhert more.

Housing: The following three parameters discipline the size of owner-occupied housing

[29] Krueger and Kübler (2004) find that computational accuracy decays rapidly if the length of economic life extends beyond 30 periods when using aggregate risk in a life-cycle economy.

units $\mathcal{H}$: the size of the smallest unit h^0, the number of house sizes available $\#\mathcal{H}$, and the gap between housing sizes. The values of these parameters are obtained from targeting the 10th, 50th, and 90th percentiles of the distribution of the ratio of housing net worth to total net worth. The model matches the 10th percentile but comes in a bit below the 50th and 90th percentiles, as is common this class of model. The size of rental housing units $\tilde{\mathcal{H}}$ is chosen to target a ratio of median owner-occupied to rental housing of 1.5 square feet per person as detailed in Chatterjee and Eyigungor (2015) and a ratio of the average earnings of homeowners to renters equal to 2.1. The model generates moments that are mostly in line with these empirical targets.

The maintenance cost of housing that offsets depreciation δ_h equals 0.015 and replicates the empirical depreciation rate of the housing stock of 1.5 percent per year.[30] Defaulted housing has a higher depreciation rate δ_h^d equal to 0.22 to account for the loss of value from foreclosure. The linear transaction cost of housing adjustments κ_h equals 7 percent which falls within the 6 to 12 percent range estimated by Quigley (2002).[31] In line with the data, 9 percent of homeowners in the model buy or sell houses each year. The relative cost of renting versus owning is determined in part by the operating cost of the rental company Ξ and is particularly salient for young households. Ξ is chosen to match the 39 percent homeownership rate of households younger than 35 and comes in slightly lower at 34 percent.

Housing construction technology α is set so that the price elasticity of housing supply $\alpha/(1-\alpha)$ equals 1.5, the median among MSAs in Saiz (2010). Land permits $\bar{L}$ pin down employment in the construction sector at 5% of total employment which is consistent with the 1998 employment share of construction measured by the Bureau of Labor Statistics.

Financial instruments: The risk-free rate r is 2.5 percent per year and the lending wedge ι is 0.33 so that the interest rate on loans r_b is equal to 3 percent per year to replicate the gap between the average rate on 30-year fixed-term mortgage and the 10-year Treasury rate in the late 1990s. The maximum HELOC value θ^{HELOC} is 0.2.

Government: The property tax τ_h is set to 1% per year which is the median tax rate across U.S. states according to the Tax Policy Center. The income tax function in equation (3) follows the functional form of Heathcote et al. (2017). The parameter τ_y^0 indicates the average level of taxation and is set so that aggregate income tax revenues are 20% of income. The parameter τ_y^1 measures the degree of progressivity of the tax and transfer system and is

[30] Kaplan et al. (2020, p. 18) use the Bureau of Economic Analysis' Table 7.4.5 which details the consumption of fixed capital of the housing sector divided by the stock of residential housing at market value.

[31] Ghent (2012) finds a value of 13 percent and Ngai and Sheedy (2020) settle on 10 percent suggesting that 7 percent may be on the lower end of the established range.

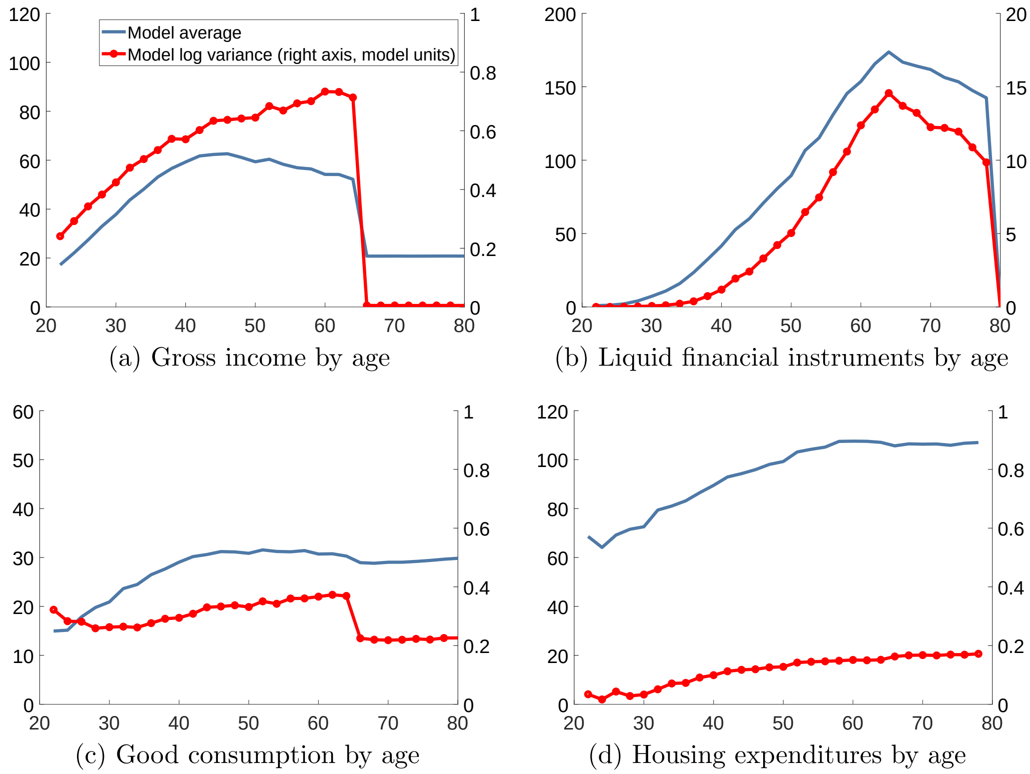
set at 0.15 based on the estimates of Heathcote et al. (2017). The share of mortgage interest deducted ϱ is set at 0.75 to so that only the first \$1,000,000 of mortgage debt is deductible. Social security payments in equation (2) maintain income heterogeneity by scaling the last realization of earnings y^{w}_{Jret-1} by the replacement rate ρ_{ss}. Kaplan et al. (2020) compute the ratio of average benefits to average lifetime earnings and find a replacement rate equal to 0.4.

Interpretation	Parameter	Value	Annualized
Demographics			
Maximum age	J	30	N
Retirement age	J^{ret}	23	N
Preferences			
Inverse elasticity of substitution	γ	0.8	N
Risk aversion	σ	2	N
Discount factor	β	0.97	Y
Strength of bequest motive	ψ	100	N
Extent of bequests as a luxuries	$\underline{b}$	7.7	N
Taste for housing	ϕ	0.13	N
Additional utility from owning	ω	1.015	N
Utility cost of foreclosure	ξ	0.8	N
Individual income			
Deterministic income	$\{\chi_j\}$	Kaplan & Violante (2014)	N
Annual persistence, ind. income	ρ_ϵ	0.97	Y
Annual st. dev., ind. income	σ_ϵ	0.20	Y
Initial st. dev., ind. income	σ_{ϵ_0}	0.42	Y
Distribution of bequest to new hhs	$b_{j=1} = b'_{j=J}$	Kaplan & Violante (2014)	N
Housing			
Owner-occupied housing unit sizes	$\mathcal{H}$	$\{1.5, 1.92, 2.46, \ldots$ $\ldots 3.15, 4.03, 5.15\}$	N
Rental housing unit sizes	$\tilde{\mathcal{H}}$	$\{1.125, 1.5, 1.92\}$	N
Depreciation rate of housing	δ_h	0.015	Y
Housing loss in foreclosure	δ_h^d	0.22	Y
Housing transaction cost	κ_h	0.07	N
Operating cost of rental company	Ξ	0.002	N
Housing supply elasticity	$\alpha/(1-\alpha)$	1.5	N
New land permits	$\bar{L}$	0.311	N
Financial instruments			
Risk-free interest rate	r	0.025	Y
Interest rate wedge on borrowing	ι	0.33	N
Maximum HELOC	θ^{HELOC}	0.2	N
Government			
Property tax on housing	τ_h	0.01	Y
Income tax function	τ_y^0, τ_y^1	0.75,0.151	N
Mortgage interest deduction fraction	ϱ	0.75	N
Mortgage interest deduction limit	$\bar{m}$	\$1 mil.	N
Social Security replacement rate	ρ_{SS}	0.42	N

Table 6: Parameter values. A period is two years and annualized values are noted in the final column with a Y. 1=\$52,000 which is the average value of income in the 1998 SCF.

C.2 Cross-sectional Distribution across the Life-Cycle

Before simulating the U.S. housing boom under this paper's contribution of endogenous beliefs, it is important to first discuss the cross-sectional distribution of households in the model's stochastic steady state which assumes known forecasting coefficients, tight aggregate credit conditions, and fluctuations in aggregate income.[32] Figure (10) shows that the life-cycle profiles of the means and variances of model quantities are largely consistent with a typical incomplete markets model where households succeed at some smoothing of goods consumption and housing expenditures by accumulating a buffer stock of liquid financial instruments.



(a) Gross income by age

(b) Liquid financial instruments by age

(c) Good consumption by age

(d) Housing expenditures by age

Figure 10: Life-cycle statistics from the model in thousands of 1998 dollars.

Panel (10a) shows that the average and log variance of income rise over the life-cycle consistent with the data from the 1998 Survey of Consumer Finances. The non-stationary age-dependent variance for idiosyncratic income helps match the more volatile earnings in the pre-retirement phase as is observed in the data.

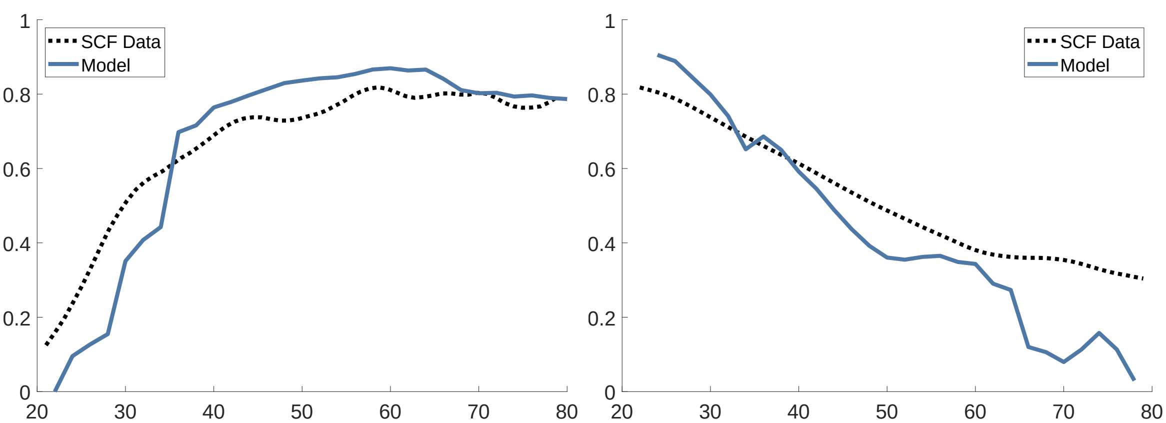
Panel (10b) shows that households accumulate liquid financial instruments, on average, as a buffer against both idiosyncratic shocks and the decrease in earnings at retirement.

[32] Appendix C.4 shows that the calibration is similar assuming the credit conditions can fluctuate between their loose and tight values via a Markov chain. Appendices E and F.1 confirm for impulse response functions are similar under credit conditions that shift one time or are Markov.

The pronounced hump-shape profile in both the average and variance (in levels, not logs) arises from a desire to smooth both consumption and housing expenditures in retirement. Average liquid financial instruments drop at the end of economic life because of bequest heterogeneity—only households in the top half of the wealth distribution leave inheritances.

The relatively flat profiles of both goods consumption and housing expenditures over the life-cycle shown in Panels (10c)-(10d) suggest that households partially self-insure against idiosyncratic shocks and retirement. Housing illiquidity and the availability of liquid financial assets are also both key for generating the flat profile of average housing expenditures, which is, in turn, key for matching the data and solving for a Krusell and Smith (1998) approximate equilibrium. Iacoviello and Pavan (2007) explain that illiquid housing results in households accumulating a buffer stock of wealth in liquid financial instruments instead of housing since households cannot adjust housing without paying additional transaction costs. If liquid financial instruments were not available, the average housing expenditure profile would look like that of liquid financial instruments with the hump-shape indicating more variation in marginal propensities to consume within and across households. The simple AR(1) forecast given by equation (14) would then be insufficient to approximate the evolution of house prices and would generate worse goodness of fit measures than those in Appendix B.

An empirically consistent distribution of housing is key for pinning down aggregate house prices throughout the housing boom and Figure (11) shows the model also matches the homeownership rate and mortgage leverage ratio over the life-cycle. Young households are particularly important because they are less likely to own homes and more likely to have maximum leverage which makes them the most sensitive to looser credit conditions.



(a) Homeownership rate by age, % (b) Mortgage leverage by age, % loan to value

Figure 11: Housing statistics over the life-cycle, 1998 dollars.

Panel (11a) shows that households remain renters, on average, in the first part of economic life in order to accumulate enough wealth and earnings to overcome the loan-to-value and payment-to-income constraints. The size of inheritances and initial income endowments determine the age in which households can afford to purchase housing.[33] Consistent with the data, the worse off households can never afford housing and remain life-long renters.

Panel (11b) shows that mortgage leverage—homeowners' ratio of mortgage debt to housing value—is highest at age 22 and declines thereafter as household amortize mortgage balances. Model simulated leverage is slightly below but mostly in line with its empirical counterpart until age 65. Thereafter, it takes on counterfactual oscillations and low values due to assumptions on mortgages. In the model, homeowners can take out a mortgage that lasts for only a few years while in practice mortgages may only be available longer tenures.

[33]See Brandsaas (2024) for an overview of how parental transfers help young households overcome mortgage constraints. Including intervivos transfers is beyond the scope of this paper.

C.3 Segmentation of Housing Unit Sizes

This Appendix evaluates the assumptions on the partial segmentation of housing unit sizes and shows that the model's distributions match those of the data minus a few discrepancies. Because housing size enters the households' utility function, empirically consistent partial segmentation can be represented by certain sizes of housing units only being available to a subset of households.[34] To best fit the pre-boom homeownership rate of 66 percent, I follow Kaplan et al. (2020) and assume that the smallest housing unit cannot be owned and the larger housing units cannot be rented. Table (7) shows that in the data only 9 percent of homeowners occupy the smallest sized unit and only 10 percent of renters occupy the largest sized unit which suggests that the partial segmentation assumption only restricts the model from matching a fraction of the overall size distribution. The model's distributions of owner-occupied and rental units overstate the demand for the smallest available units but match the rest of the distributions relatively well. Alternative segmentation assumptions shown in Table (7) provide small fixes to these shortcomings at the expense of distorting other targeted moments such as the homeownership rate or the housing expenditure share as shown in Table (8).

House Size	Data Owners	Benchmark Model	No seg.	Full seg.	Partial seg.	Smaller size 1	Larger size 1
1	9		19		19		
2	24	55	23		22	46	49
3	25	9	23		24	17	14
4	18	13	13	89	12	13	13
5	10	17	16	2	16	18	18
6	9	4	5	9	5	5	4
7	6	2	1	0	1	1	1
House Size	**Data Renters**	**Benchmark Model**	**No seg.**	**Full seg.**	**Partial seg.**	**Smaller size 1**	**Larger size 1**
1	51	79	76	75	76	73	77
2	28	14	14	12	14	18	14
3	11	7	6	13	10	9	9
4	5		2				
5	2		2				
6	1		1				
7	2		0				

Table 7: Distribution of housing unit sizes in percentage points by segmentation assumptions. Source: American Housing Survey, Kaplan et al. (2020).

[34] Landvoigt et al.'s (2015) empirical evidence points to partial segmentation of housing markets by quality of units rather than size which is beyond the scope of most macro housing models. Absent a standard segmentation convention, Arslan et al. (2022) rely on rental sector convertibility frictions and Greenwald and Guren (2024) assume separate housing stocks similar to the full segmentation shown in Table (7).

Moment	Empirical Value	Benchmark Model	No seg.	Full seg.	Smaller size 1	Larger size 1
Housing/total cons. expenditures	0.16	0.16	0.15	0.18	0.15	0.16
Aggregate home-ownership rate	0.66	0.68	0.7	0.54	0.71	0.68
Av. sized owned/rented house	1.5	2	1.6	2	1.6	1.7
Av. earnings of owners/renters	2.1	2.8	2.6	2.6	2.6	2.7
Homeownership rate of $<$ 35 y.o.	0.39	0.34	0.38	0.26	0.38	0.36

Table 8: Moments by segmentation assumptions. Source: American Housing Survey, Kaplan et al. (2020).

The segmentation assumptions are:

- **Benchmark**: renters can only choose the smallest three housing unit sizes and owners cannot choose the smallest size.
- **No segmentation**: renters and owners can choose all available housing unit sizes.
- **Full segmentation**: homeowners can only choose the largest four housing unit sizes so that there is no overlap between the units available to renters and homeowners.
- **Smaller unit 1**: size 1 is decreased to 1.07 (1.125 in the benchmark).
- **Larger unit 1**: size 1 is increased to 1.17 (1.125 in the benchmark).

Data	Rent-to-own	Rent-to-rent	Own-to-rent	Own-to-own
Data	0.25 (0.01)	0.04 (0.01)	-0.29 (0.01)	0.4 (0.01)
Model	0.29	0.15	-0.34	0.03

Table 9: Average log-changes in house size by type of transition. House size is the number of rooms of a house in the data which is the PSID form 1968 to 1996 via Kaplan et al. (2020). Standard errors are in parentheses.

C.4 Cross-sectional Calibration under Markov Credit Conditions

Moment	Parameter	Empirical Value	Model Value
Agg. net worth/annual agg. labor income	β	5.5	4.9
Median ratio of net worth to labor income	β	1.2	1.2
Median net worth: age 75/age 50	ψ	1.55	1.47
% of bequests in bottom 1/2 of wealth dist.	$\underline{b}$	0	0
Housing/total cons. expenditures	ϕ	0.16	0.16
Aggregate home-ownership rate	ω	0.66	0.7
Foreclosure rate	ξ	0.005	0.0014
P10 housing/total net worth of owners	$\min \mathcal{H}$	0.11	0.08
P50 housing/total net worth of owners	$\#\mathcal{H}$	0.5	0.29
P90 housing/total net worth of owners	gap $\mathcal{H}$	0.95	0.76
Average sized owned house/rented house	$\min \tilde{\mathcal{H}}$	1.5	2
Average earnings of owners to renters	$\#\tilde{\mathcal{H}}$	2.1	2.7
Annual fraction of houses sold	κ_h	0.1	0.1
Homeownership rate of < 35 y.o.	Ξ	0.39	0.42
Employment in construction sector	$\bar{L}$	0.05	0.04

Table 10: Targeted moments in calibration corresponding to model parameters. The table is similar to table (1) in the main text but with credit conditions that evolve according to a Markov process rather than a one-time shift.

Parameter	Empirical Value	Model Value
Fraction of homeowners w/mortgage	0.66	0.72
Fraction of Homeowners w/HELOC	0.06	0.01
Aggr. mortgage debt/housing value	0.42	0.51
P10 LTV ratio for mortgages	0.15	0.02
P50 LTV ratio for mortgages	0.57	0.52
P90 LTV ratio for mortgages	0.92	0.92
Share of NW held by bottom quintile	0	0
Share of NW held by middle quintile	0.05	0.09
Share of NW held by top quintile	0.81	0.66
Share of NW held by top 10 percent	0.7	0.43
Share of NW held by top 1 percent	0.46	0.06
P10 house value/earnings	0.9	0.94
P50 house value/earnings	2.1	1.8
P90 house value/earnings	5.5	4.1

Table 11: Untargeted moments in calibration. The table is similar to table (3) in the main text but with credit conditions that evolve according to a Markov process rather than a one-time shift.

D Appendix: Learning Calibration

When constant, the learning gain g is set to minimize the mean squared errors of house price forecasts from the model relative to an empirical proxy constructed from the University of Michigan Surveys of Consumers. Because the Michigan Survey's questions about future house prices are only available starting in 2007, there are no direct measures of house price expectations from 1999 to 2007 as shown by Kuchler et al. (2023, Table 2). To construct a house price expectations proxy, I exploit the 0.92 correlation between expected house price growth in the next 12 months (2007 start) and whether or not it is a relatively good time to sell a house (1992 start).[35]

Figure 12: Responses from the University of Michigan Surveys of Consumers, percent of respondents who say it is a good time to sell a house less those who say it is a bad time plus 100 and the median expected house price growth in the next 12 months expressed as 12-month percentage change. The shaded bar between the dashed vertical lines denotes the U.S. housing boom (1999 to 2007).

Given the tight correlation between the two series, I create a backcast of the expected change in house prices going back to 1992 by projecting expectations y_t onto selling conditions x_t for $t = \{\text{Jan. } 2007, \ldots, \text{Jun. } 2022\}$.

$$y_t = \beta_0 + \beta_1 x_t + e_t$$

[35]The `Relative` column of Table 43 is the `Good time to sell` column minus the `Bad time to sell` column plus 100. The survey asks, "Generally speaking, do you think now is a good time or a bad time to sell a house?" The `Median` column of Table 46 is the median response to the question, "By about what percent do you expect prices of homes like yours in your community to go (up/down), on average, over the next 12 months?"

Using the estimated coefficients $\hat{\boldsymbol{\beta}}$, I then backcast the expectations series $\hat{y}_t$ from the selling conditions series x_t when the former is not yet available, $t = \{\text{Jan. } 1992, \ldots, \text{Dec. } 2006\}$.

$$\hat{y}_t = \hat{\beta}_0 + \hat{\beta}_1 x_t$$

Figure (13) shows that the backcasted empirical proxy has a good in sample fit with an r^2 near 1. Although the higher frequency movements of the two series differ, the backcast captures the path of expectations which is key for the study of the U.S. housing boom.

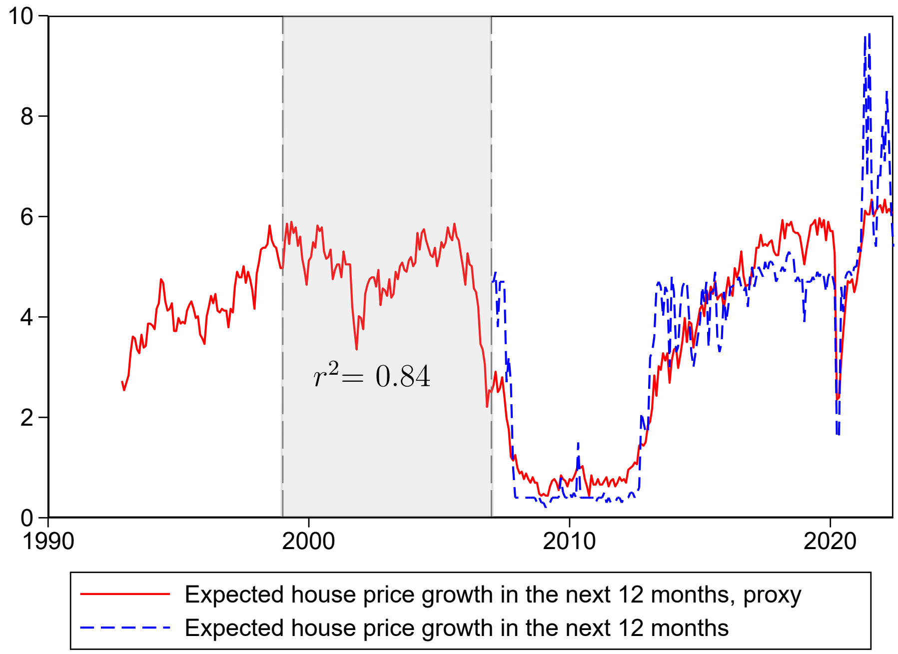


Figure 13: Median expected house price growth in the next 12 months from the University of Michigan Survey of Consumers, 12-month percentage change. Shaded bar between the dashed vertical lines marks the U.S. housing boom (1999 to 2007).

House price expectations from the model and the empirical proxy have four key differences that affect their comparability. Those from the model are for two-year real detrended house prices in levels while those from the data are for nominal expected 12-month growth rates. Calculations below discuss which differences affect the mean squared forecast error used to calibrate the constant learning gain.

Growth rates vs. levels: Forecast errors from growth rates and levels are roughly equivalent as shown by equations (19) and (20). Assuming the current month's house prices p_τ are known to survey participants, the h period ahead forecast error e_τ^{survey} for month τ can be defined as:

$$e_\tau^{survey} \equiv \log p_{\tau+h} - \log F_\tau p_{\tau+h}$$

Noting that the Michigan survey provides an expected 12-month growth rate $F_\tau g_{\tau+h}$ for $h = 12$, the above definition becomes:

$$e_\tau^{survey} \equiv \log p_{\tau+h} - \log\big(p_\tau(1 + F_\tau g_{\tau+h})\big)$$

Re-arranging and letting $\log(1 + F_\tau g_{\tau+h}) \approx F_\tau g_{\tau+h}$, the above expression becomes:

$$e_\tau^{survey} \equiv \log\big(p_{\tau+h}/p_t\big) - F_\tau g_{\tau+h} \tag{19}$$

In the model, the f period ahead forecast for period t in aggregate state Z_t with $\mathcal{Z}_t = \{Z_t, Z_{t+1}\}$ is given as:

$$e_t^{model} \equiv \log p_{t+f} - \mathbb{E}_{Z_{t+f}}[a^0_{\mathcal{Z}_t} + a^1_{\mathcal{Z}_t} \log p_t] \tag{20}$$

Where the period t is two years and $f = 1$ so that the forecast is one period ahead.

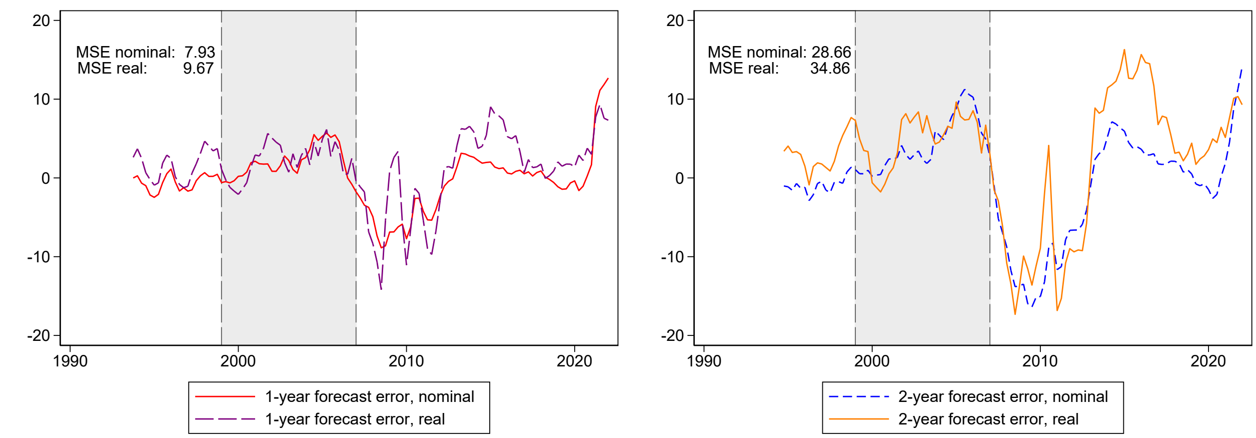
Forecast horizon: To compare the empirical proxy's 12-month forecast horizon to the two-year forecast horizon from the model, I assume that survey respondents expect house price growth for the next 12 months to remain for the following 12 months. The imputed forecast for the next $2 \times h = 24$ can be written as:

$$\begin{aligned} e_\tau^{survey} \equiv & \log p_{\tau+2\times h} - \log F_\tau p_{\tau+2\times h} \\ & \log p_{\tau+2\times h} - \log\big(p_\tau(1 + F_\tau g_{\tau+h})^2\big) \\ & \log(p_{\tau+2\times h}/p_\tau) - 2 \times F_\tau g_{\tau+h} \end{aligned} \tag{21}$$

Imputing the forecast for the next two years via compounding may overstate expected house price growth in the next two years, $2 \times F_\tau g_{\tau+h} > F_\tau g_{\tau+2\times h}$. Although Figure (14) shows that the forecast errors of house price growth at the 12-month and two-year horizons (equations 19 and 21) are quite similar throughout the housing boom, they differ in the later part when the two-year series is higher resulting in a larger mean squared error. However, because expected house prices for the next five years are at their all-time high at the end of the housing boom as shown in Panel (2a) in the main text, house price expectations for the next two years may have indeed been higher than those for the next 12 months.[36]

[36] The expectations for the next give years are the `median` column of Table 47, "Expected Change in Home Values During the Next Five Years." The survey asks, "By about what percent per year do you expect prices of homes like yours in your community to go (up/down), on average, over the next five years or so?" Two problems prevent interpolation of the two-year expectations from the one- and give-year expectations. First, the one- and five-year expectations do not have a monotonic ordering. Survey respondents expected higher house price growth in the next five years than in the next one year from early 2007 to mid-2013, but the opposite ordering from mid-2013 throughout the remainder of the period shown in Panel (2a). Second, there is no tight correlation of five-year series with other series in the Michigan survey thereby precluding a backcasting exercise to obtain proxies for the pre-2007 period.

Given the concerns about imputing the house price expectations for the next two years, one could instead calibrate the learning gain to the one-year forecast error. The resulting housing boom simulations would be like those shown in Figure (5) with a gain lower than g=0.508. With the lower value of the learning gain, the mean squared forecast error is closer to the 7.93 value of the one-year horizon rather than the 28.66 value at the two-year horizon. House prices still boom, but to a peak that is lower than the 75 percent match to the data observed in the main specification.



(a) 1-year nominal and real forecast errors

(b) 2-year nominal and real forecast errors

Figure 14: Panel (a): Real house price growth is the FHFA house price index scaled by the price index for non-durable consumption. Expected house price growth for the next 12 months is the empirical proxy constructed from the University of Michigan Survey of Consumers and is scaled by responses for expected inflation from the survey. Panel (b) is same as Panel (a), but with house price and inflation expectations both imputed to a two-year forecast horizon. The panels are given in the quarterly average of the 12-month percentage change. Shaded bar between the dashed vertical lines marks the U.S. housing boom (1999 to 2007).

Nominal vs. real: The real house price forecast error can be calculated by dividing both realized and expected house prices by actual and expected inflation. Let $\pi_{\tau+h}$ be one-year realized inflation and $F_\tau \pi_{\tau+h}$ be its expected value for month τ. The real forecast error is:

$$\begin{aligned} e_\tau^{survey,real} &\equiv \log\left(\frac{p_{\tau+h}}{1+\pi_{\tau+h}}\right) - \log\left(\frac{p_\tau(1+F_\tau g_{\tau+h})}{(1+F_\tau \pi_{\tau+h})}\right) \\ &\underbrace{\log(p_{\tau+h}/p_\tau) - F_\tau g_{\tau+h}}_{e_\tau^{survey}} - \left(\pi_{\tau+h} - F_\tau \pi_{\tau+h}\right) \end{aligned} \tag{22}$$

The real house price forecast error is the forecast error of nominal house price growth less the forecast for the inflation rate.[37] Figure (14) shows that the real and nominal house price

[37]The realized inflation series is the PCE price index for durables less consumption which is what is used elsewhere in the paper. Results are similar if the headline PCE price index is used instead.

forecasts are roughly similar for both the one-year expectations and the two-year imputed expectations throughout the housing boom.[38] Within each forecast horizon, there is little difference between the real or nominal mean squared errors at the one-year horizon. At the two-year horizon, the real error is larger than the nominal error and corresponds most closely to the value of 37.88 obtained under a larger constant shown in Figure (5).

De-trending: The empirical real house price forecast errors are not affected by subtracting out a time trend like the one used to construct house prices in the model. The predicted linear time trend $\hat{\Gamma} p_{\tau+h}$ cancels from both sides of equation (22):

$$\log\left(p_{\tau+h}(1-\Gamma)/(1+\pi_\tau)\right) - \log\left(p_\tau(1-\Gamma)(1+F_\tau g_{\tau+h})/(1+F_\tau \pi_{\tau+h})\right)$$

Additional caveats: Evidence from other surveys suggest that responses from the University of Michigan Survey of Consumers may actually underestimate expected house price growth which could lead to lower forecast errors than those presented above.[39] For this reason, I use the median house price expectations for the next 12 months instead of the mean as the former tends to run higher thus resulting in lower forecast errors. Moreover, figure (15) shows that the median point prediction for house price growth in the next 12 months from the NY Fed Survey of Consumer Expectations is not always higher than the empirical proxy used in this paper for the period in which they are both available.

[38]The 2-year real forecast error is: $\log(p_{\tau+2\times h}/p_\tau) - 2\times F_\tau g_{\tau+2\times h} - \left(\pi_{\tau+2\times h} - 2\times F_\tau \pi_{\tau+2\times h}\right)$.

[39]Kuchler et al. (2023) note in footnote 5 that house price expectations from the NY Fed Survey of Consumer Expectations (2013 start) show patterns similar to those of the University of Michigan Survey of Consumers with average one-year expectations usually about 2 percentage points higher. De Stefani (2021) notes that house prices expectations from the University of Michigan Survey of Consumers are only available for homeowners which may be problematic because Kindermann et al. (2024) find that renters looking to buy houses are the most informed about house price expectations.

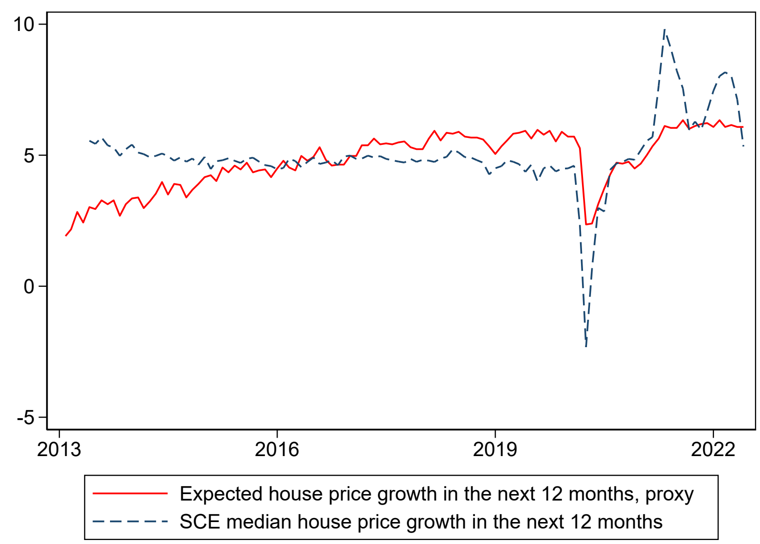


Figure 15: Expected house price growth for the next 12 months is the empirical proxy constructed from the University of Michigan Survey of Consumers and the median point prediction of expected house price growth for the next 12 months from the Federal Reserve Bank of New York Survey of Consumer Expectations, 12-month percentage change.

E Appendix: Comparison to Kaplan et al. (2020)

As noted in section 3.1, there are some small discrepancies between the model and code of this paper relative to that of Kaplan et al. (2020). These key discrepancies require a slightly lower operating cost of the rental company Ξ to calibrate to the correct homeownership rate for households under 35 years of age. They are listed below:

- The McClement's scale on utility e_j should be set so that $\{e_j\}_{j=21}^{30} = 1$ while in their code it is instead set at $\{e_j\}_{j=21}^{30} = \{e_j\}_{j=1}^{8} > 1$.
- At age 30, agents who choose to stay in their house should not borrow HELOCs.
- The mortgage pricing function was updating before it finished the age loop. This was fixed by closing off several loops before going into the value function iteration step.
- Retired agents are missing $-\zeta(Z)$ in their mortgage pricing function.

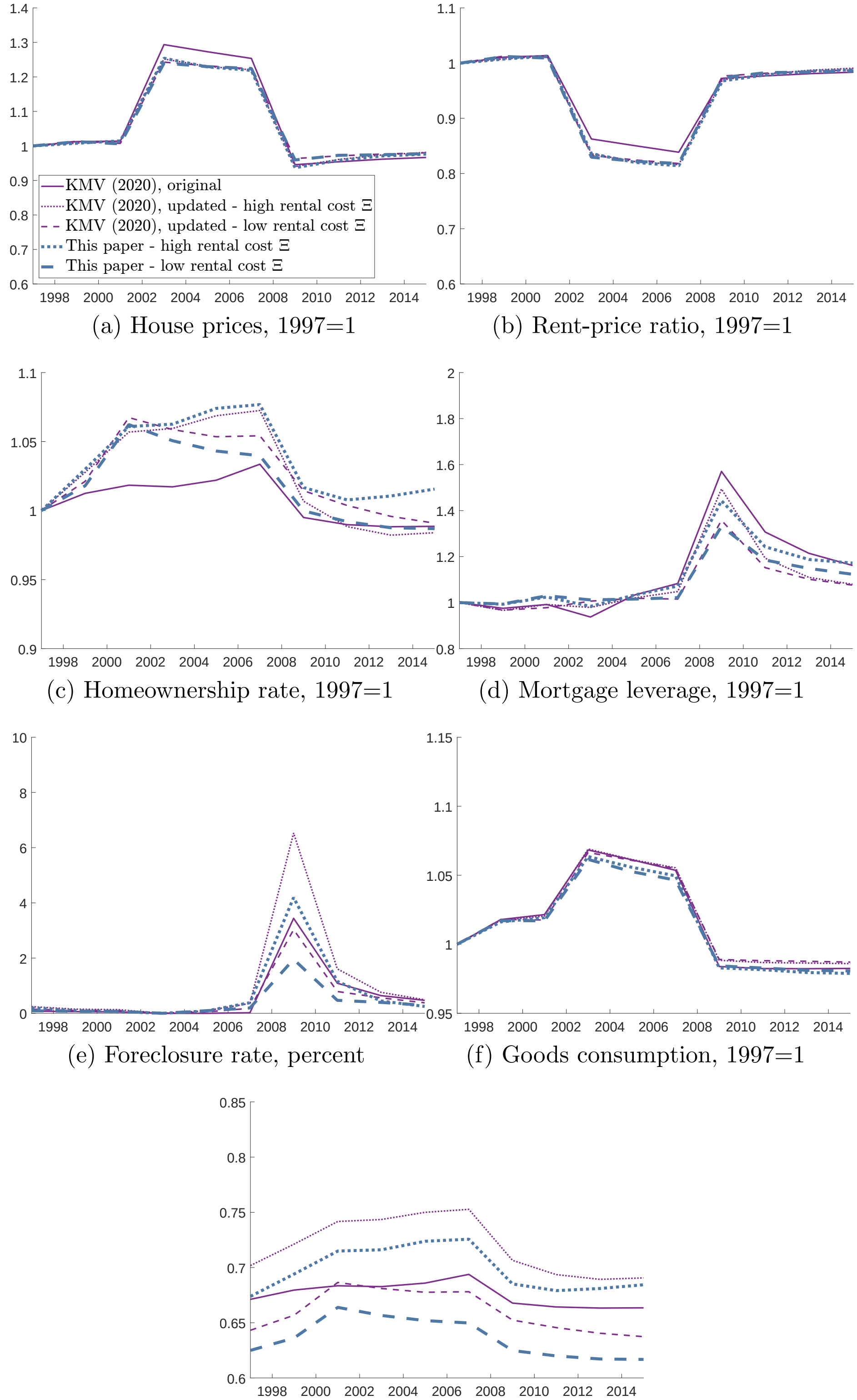
Figure (16) shows that these differences have almost no implications on the main findings of Kaplan et al. (2020) and only affect magnitudes and time paths of impulse responses. All purple lines shown in Figure (16) are results from Kaplan et al.'s (2020) code downloaded from their publication repository. All blue lines are from the code in this paper solved under the latter's forecasting coefficients for house prices.

The discrepancies noted above result in a slightly smaller boom in house prices, slightly larger drop in the rent-to-price ratio, and smaller rise in consumption, as shown in panels (16a), (16b), and (16f), respectively.

The effects are more substantial for the impulse responses for the normalized and un-normalized homeownership rate, leverage, and foreclosure rate as shown in panels (16c), (16g), (16d), and (16e), respectively. First, I note that leverage and the foreclosure rate (panels (16d) and (16e), respectively) both increase sharply in the bust with the degree of the increases depending on the operating cost of the rental company Ξ. The increases in the bust are relatively higher under the original high value of Ξ as shown by the dotted purple and blue lines than under the re-calibrated low value of Ξ as shown by the dashed purple and blue lines. Although the increase in both leverage and the foreclosure rate are lowest under the code of this paper with the re-calibrated low value of Ξ, panels (16d) and (16e) show that this can be accounted for, in part, due to the re-calibration.

Furthermore, the homeownership rate can also account for differences in the increase in leverage and foreclosures in the bust. The un-normalized homeownership rate shown in panel (16g) is relatively higher under the original high value of Ξ and lower under the re-calibrated lower value, as seen by the differences between the dotted and dashed lines. The purple lines from the code of Kaplan et al. (2020) generally start from a higher level of the homeownership rate than the blue lines of this paper's code. Finally, I note that for both the dashed lines and the dotted lines, the shape of the homeownership rate is quite similar, suggesting that the differences in the code can be explained.

Figure (17) compares the exogenous beliefs version of Kaplan et al. (2020) to a version with credit conditions that loosen one time instead of credit conditions that evolve according to a Markov process. The figure shows that the impulse response of housing boom simulations are nearly identical for all prices and quantities. This is the case because the probability of Markov credit conditions shifting is incredibly low at 1% so shutting it to 0% and having the shift be a surprise is quantitatively little different. This figure suggest that the model solved under endogenous beliefs and a one-time shift in credit conditions is readily comparable to its endogenous beliefs counterpart with Markov credit conditions.



(a) House prices, 1997=1

(b) Rent-price ratio, 1997=1

(c) Homeownership rate, 1997=1

(d) Mortgage leverage, 1997=1

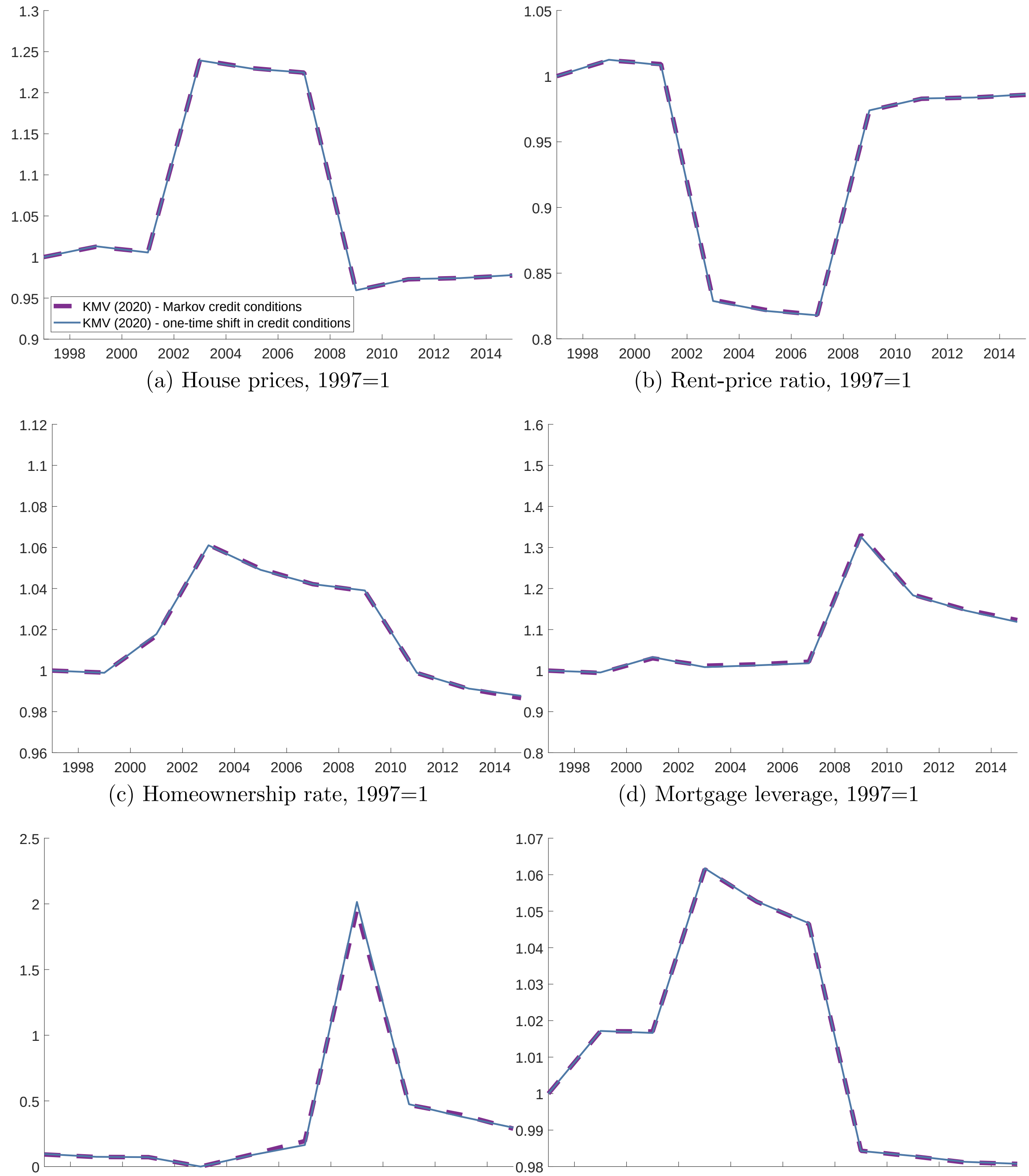
(e) Foreclosure rate, percent

(f) Goods consumption, 1997=1

(g) Homeownership level

Figure 16: Housing boom impulse responses under exogenous beliefs, 1997=1.

Note: purple lines are results from Kaplan et al.'s (2020) code repository and blue lines are from the code of this paper, set to the same values of forecasting rule coefficients. The solid purple line is the original impulse responses. The dashed and dotted lines correct from some small discrepancies with the former solved under the original value of the operating cost of the rental company Ξ which is higher than the re-calibrated lower value.



(a) House prices, 1997=1

(b) Rent-price ratio, 1997=1

(c) Homeownership rate, 1997=1

(d) Mortgage leverage, 1997=1

(e) Foreclosure rate, percent

(f) Goods consumption, 1997=1

Figure 17: Housing boom impulse responses under exogenous beliefs, 1997=1.

Note: simulations are from this paper's code solved under Markov credit conditions as in (2020) and under a one-time shift in credit conditions as in this paper.

F Appendix: Alternative Housing Boom Simulations

F.1 Alternative fundamentals

Figure (18) shows counterfactual housing boom simulations that parse the contributions of beliefs and credit conditions. Panel (18a) shows that house prices are highest and thus closest to matching the data in the main specification when there is a shift in income and credit conditions accompanied by endogenously optimistic beliefs.

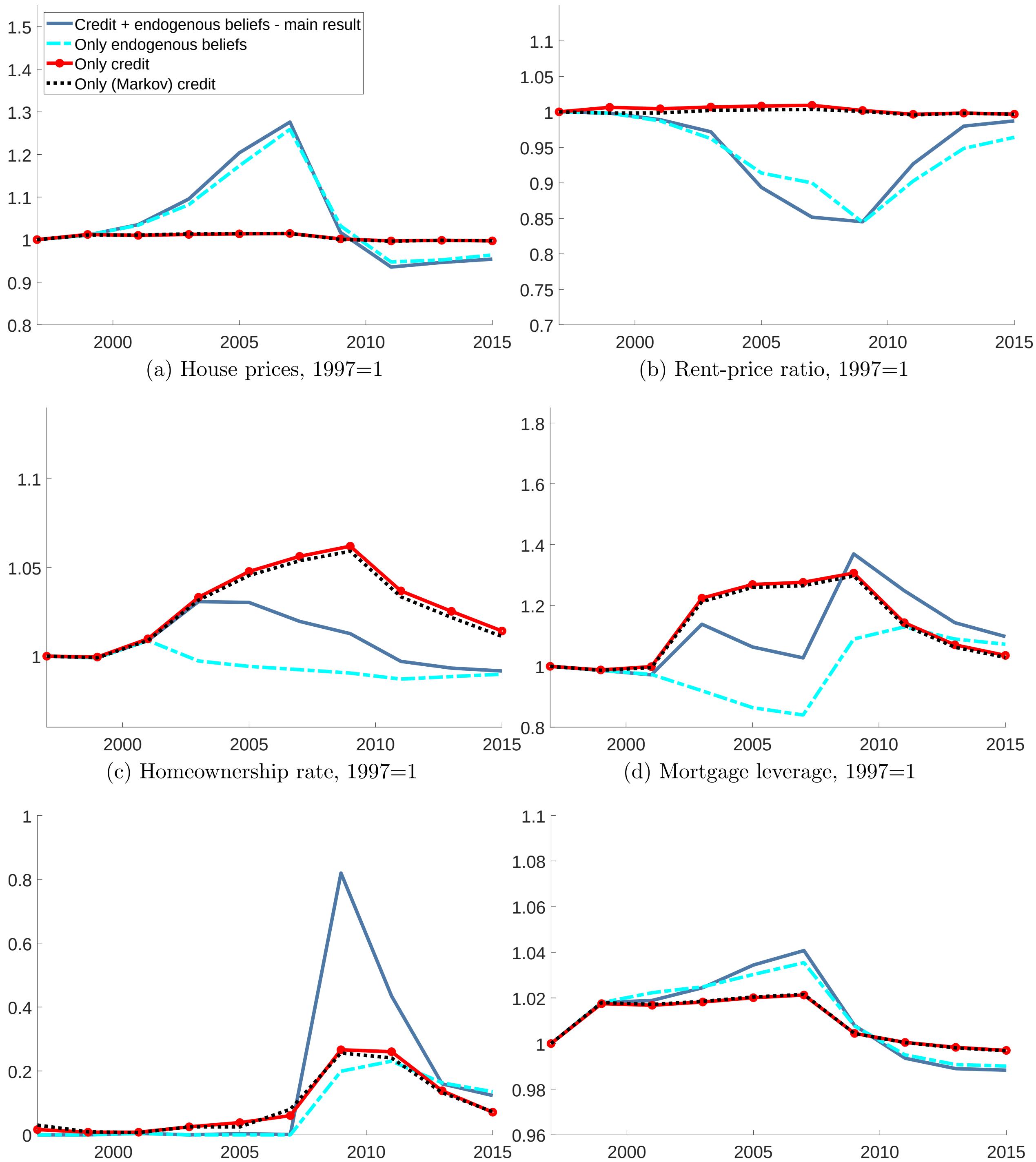
Panel (18a) shows via the dashed light blue lines that shutting down the shift in credit conditions, but still allowing for endogenous beliefs and the shock to income results in house prices that boom, albeit less than the main specification . This suggests that incomplete information plays an important role in generating an empirically consistent housing boom. Panels (18c)-(18e) show that without the loosening of credit conditions there are counterfactually low responses in the homeownership rate, mortgage leverage, and the foreclosure rate, respectively.

Shutting down endogenous beliefs, but allowing for shocks to income and credit conditions (red lines with circles) results in no boom in house prices as shown in Panel (18a).[40] Although there is an increase in homeownership as shown in Panel (18c), new homeowners are mostly purchasing the same sized house as they previously rented rather than upsizing. Homeowners must upsize housing units to push up house prices and this does not occur without optimistic expectations about future house prices. The rent-price ratio (Panel 18b) remains flat, mortgage leverage counter factually rises (Panel 18d) and consumption and foreclosures remain counterfactually low (Panels 18f-18e).

Together, these counterfactual simulations suggest that beliefs are quantitatively most important for determining booming house prices. Credit conditions, however, must loosen to account for other model housing dynamics.

Finally, figure (18) shows that simulations are similar whether credit conditions loosen via a one-time shock (red lines with circles) or are Markov (dotted black lines).

[40] For the housing boom simulation, one must first obtain the forecasting coefficients $\boldsymbol{a}_{\mathcal{Z}}^{loose}$ so that agents know how house prices evolve when credit conditions loosen. Assuming that coefficients are fixed and known, these coefficients are solved as a fixed point with fluctuations in aggregate income only and credit conditions set at their loose values. As in the main specification, the housing boom simulation then starts with the economy in the state with low aggregate income and tight aggregate credit conditions $\{\Theta(Z_{low}), \mathcal{C}(Z_{low})\}$ along with the corresponding coefficients $\boldsymbol{a}_{Z_{low},Z'}^{tight}$. As the economy transitions to the state with high aggregate income and loose credit conditions $\{\Theta(Z_{high}), \mathcal{C}(Z_{high})\}$ coefficients take on the values $\boldsymbol{a}_{Z_{high},Z'}^{loose}$.



(a) House prices, 1997=1

(b) Rent-price ratio, 1997=1

(c) Homeownership rate, 1997=1

(d) Mortgage leverage, 1997=1

(e) Foreclosure rate, percent

(f) Goods consumption, 1997=1

Figure 18: Counterfactual housing boom simulations from the model decomposition the effects of beliefs and credit conditions.

F.2 Alternative Forecast Error Formations

In equation (16) the lagged forecast error is defined as:

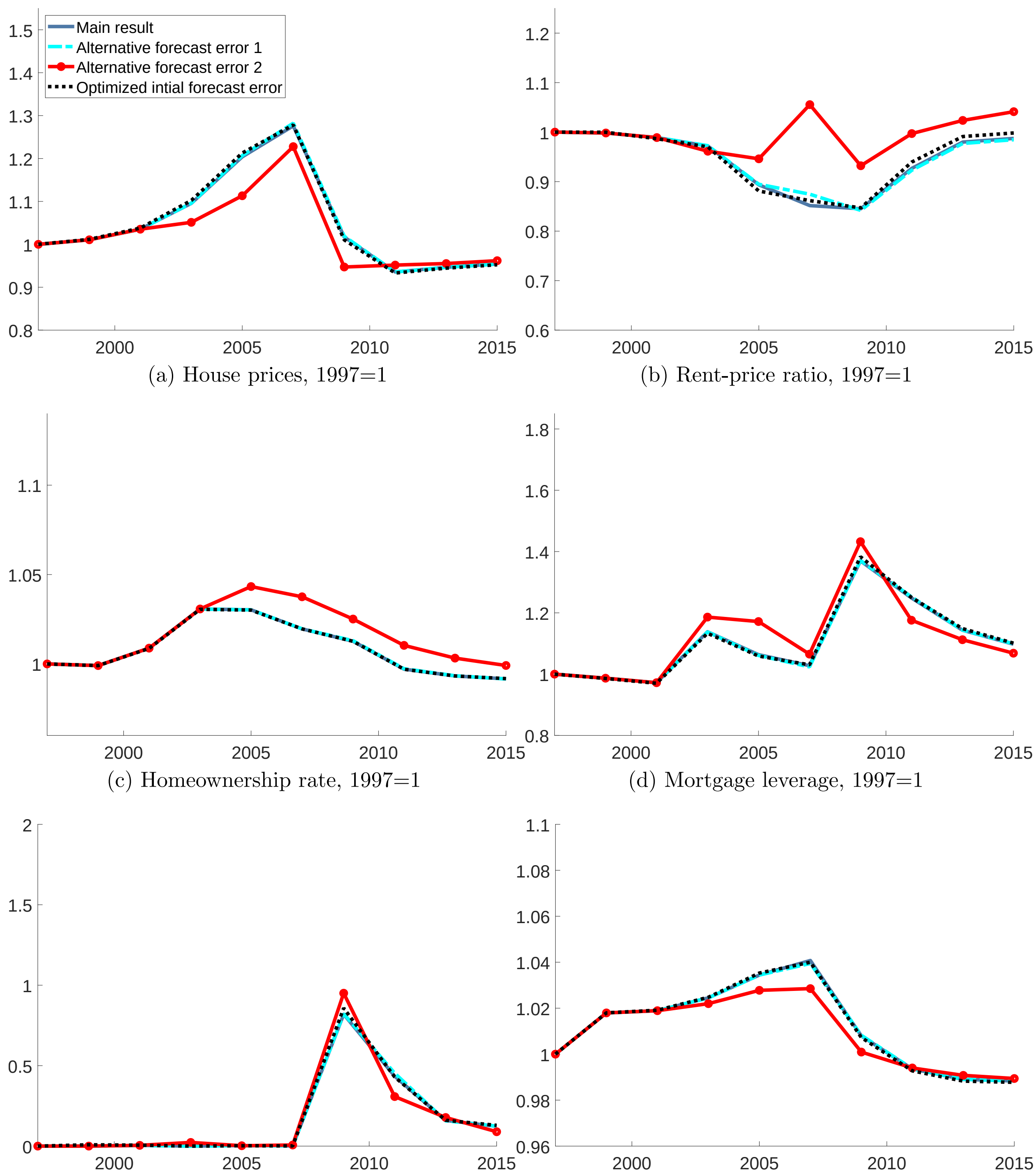
$$\boldsymbol{a}_{\mathcal{Z}_t} = \boldsymbol{a}_{\mathcal{Z}_{t-1}} + g_t \boldsymbol{x}_{t-2} \underbrace{\left(\log p_{t-1} - \boldsymbol{x}'_{t-2} \boldsymbol{a}_{\mathcal{Z}_{t-1}}\right)}_{e_{t-1}} \tag{16}$$

Because there is no definitive convention, this forecast error could take on other forms such as the lagged forecast error from a particular aggregate state or the realized forecast error shown below in equation (23) and (24), respectively.

$$e_{t-1} = \log p_{t-1} - \boldsymbol{x}'_{t-2} \boldsymbol{a}_{\mathcal{Z}_{t-2}} \tag{23}$$

$$e_{\mathcal{Z}_{t-1}} = \log p_{\mathcal{Z}_{t-1}} - \boldsymbol{x}'_{\mathcal{Z}_{t-2}} \boldsymbol{a}_{\mathcal{Z}_{t-1}} \tag{24}$$

Figure (19) shows that these forecast error formations, "Alternative forecast error 1" and "Alternative forecast error 2" shown by the light blue dashed and solid red lines with a circle, respectively, have little material effect on housing boom simulations in most cases. They are quite close to the main results shown by the solid blue line. The simulation "Optimized initial forecast error" has the initial forecast error set at its empirical value of 1.24 percent instead of the value of 1 percent that arises under the main calibration when assuming initial using boom beliefs are $\boldsymbol{a}^{tight}_{Z_{high}, Z'_{high}}$.



(a) House prices, 1997=1

(b) Rent-price ratio, 1997=1

(c) Homeownership rate, 1997=1

(d) Mortgage leverage, 1997=1

(e) Foreclosure rate, percent

(f) Goods consumption, 1997=1

Figure 19: housing boom simulations from the model under alternative forecast error formations.

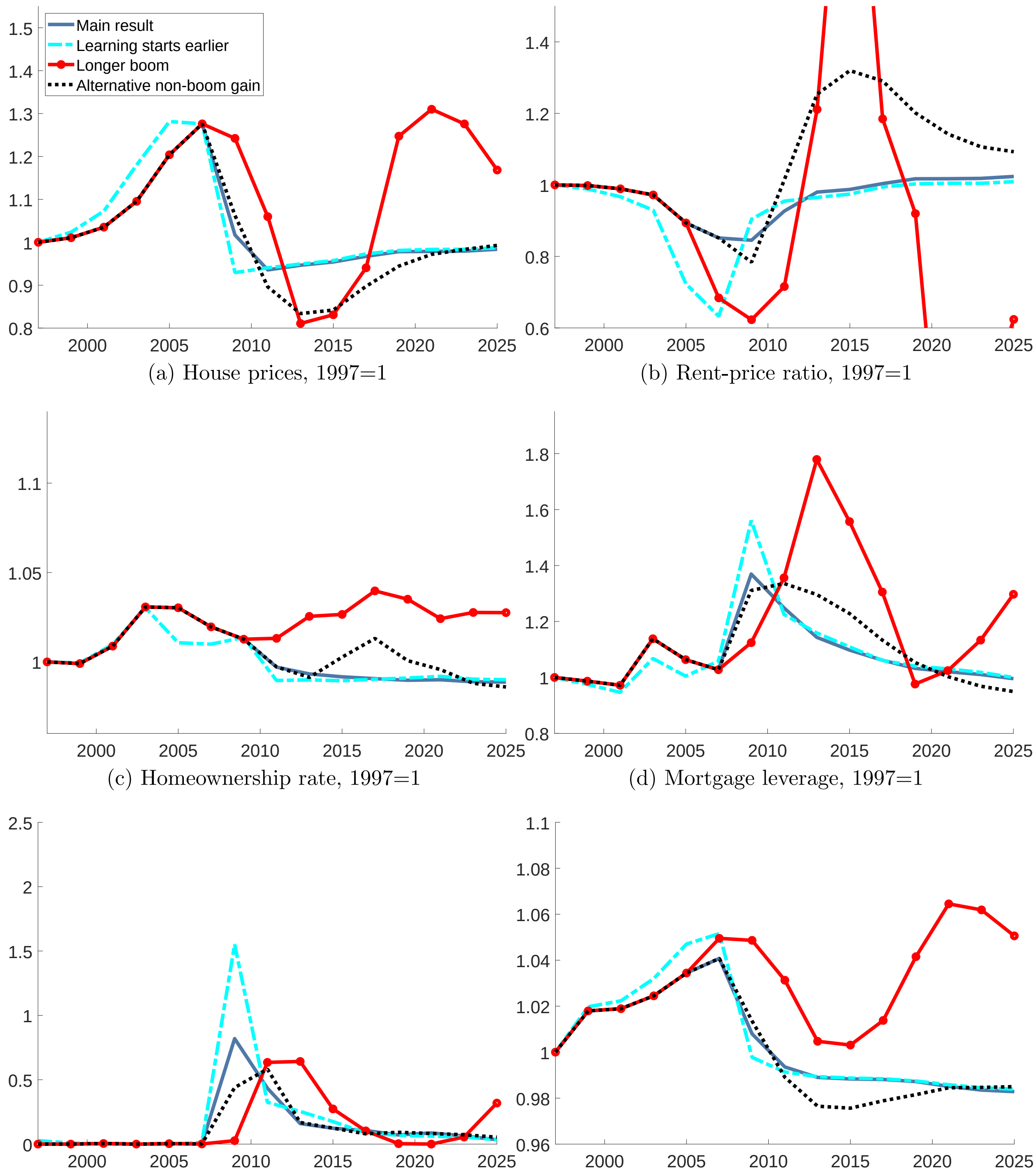
F.3 Exploring learning mechanisms

The results shown in figure (20) explore the properties of adaptive learning.

The simulation titled "Learning starts earlier" (dashed light blue lines), starts adaptive learning at the beginning of the simulation rather than when the economy transitions to the aggregate state characterized by looser credit conditions and high income in 2001 as under the main result (solid blue lines). The results are quite similar with notable differences being a steeper spike in the rent-price ratio, leverage, and foreclosures, shown in panels (20b), (20d), and (20e), respectively.

In the simulation titled "Longer boom" (solid red lines with circles), the housing boom state characterized by high income and looser credit conditions counterfactually lasts for another 10 years rather than ending in 2009. The simulation paths are exactly the same as those of the main result (solid blue lines) until 2009. Thereafter, house prices under the counterfactual, as shown in panel (20a), contract less suddenly and reach a deeper trough by 2013 than under the main result. Prices rebound above their pre-boom values around 2018 which actually results in a second housing boom that peaks in late 2022. Although this path tracks the actual rebound in aggregate house prices in the data quite closely, it also results in counterfactual movements in other model prices and quantities shown. For example, the counterfactually large rebound in the price-to-rent ratio is the result of large oscillations in the house price forecast error pushing up rents counterfactually high.

The simulation titled "Alternative non-boom gain" (dotted black lines) initializes the post-boom learning gain at $g_t = g/(1+g)$ as in Marcet and Nicolini (2003) rather than $g_t = g/(1+t \times g)$ as in the main simulation. The simulations shown are exactly the same in the housing boom, but the slower decaying of g_t post-boom results in a deeper decline in house prices and other model quantities. Like in the "Learning starts earlier" simulation, the protracted bust in house prices is at the expensive of a counterfactually high rent-price ratio.



(a) House prices, 1997=1

(b) Rent-price ratio, 1997=1

(c) Homeownership rate, 1997=1

(d) Mortgage leverage, 1997=1

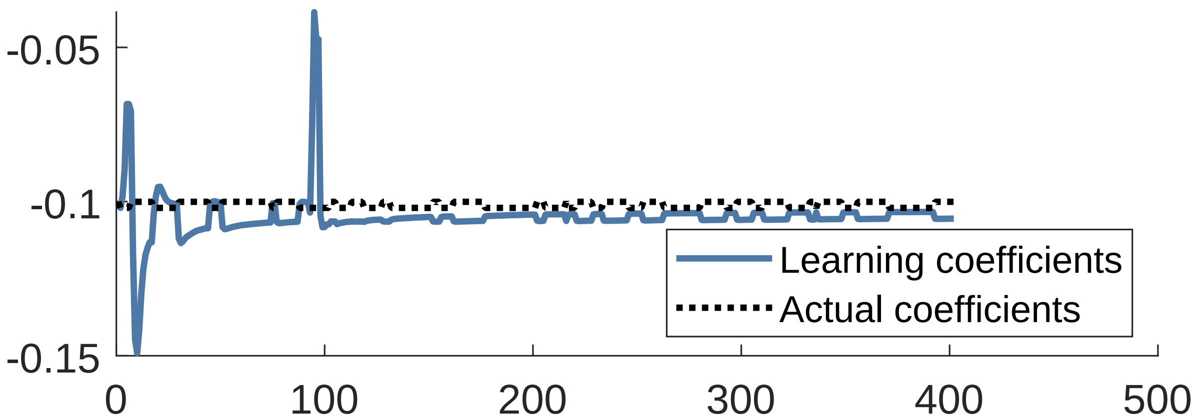
(e) Foreclosure rate, percent

(f) Goods consumption, 1997=1

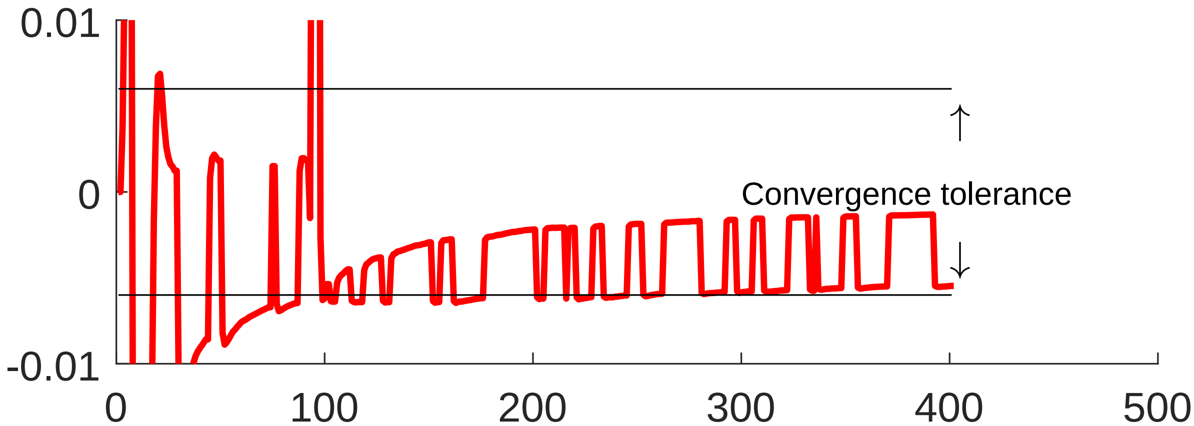
Figure 20: housing boom simulations from the model that explore the properties of adaptive learning.

F.4 Convergence of Learning Coefficients

Figures (21) and (22) show that coefficients under adaptive learning settle at an ergodic distribution near their known counterparts. A counterfactual return to the housing boom state 100 periods (200 years) after the 2000s boom shows that coefficients adjust in response to forecast errors but revert back to an ergodic distribution in the non-boom state. Although there are observable differences between the learning coefficients and their known counterparts in panels (21a) and (22a), panels (21b) and (22b) show that these differences are well within the convergence tolerance used to solve for the fixed coefficients.

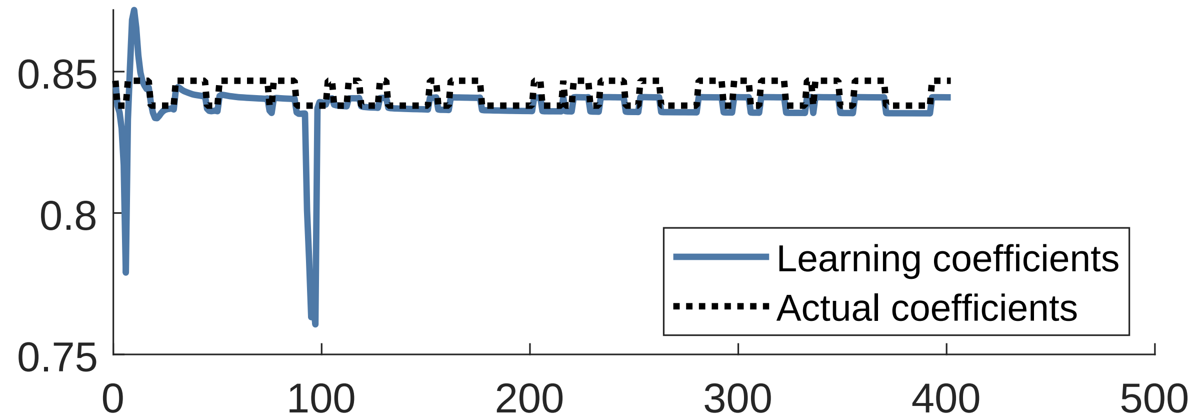


(a) Evolution of a_t^0 coefficient

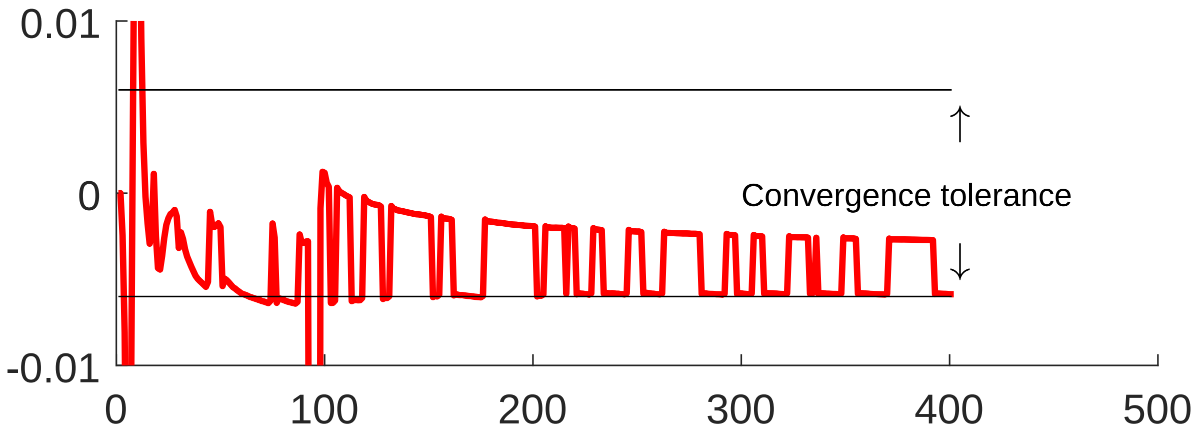


(b) Difference between learning and known coefficients

Figure 21: Time path of beliefs for 400 periods. Period 0 is the start of the housing boom near period 100 there is another counterfactual unexpected loosening of credit conditions.



(a) Evolution of a_t^1 coefficient



(b) Difference between learning and known coefficients

Figure 22: Time path of beliefs for 400 periods. Period 0 is the start of the housing boom and near period 100 there is another counterfactual unexpected loosening of credit conditions.

G Appendix: Data Definitions

The aggregate data definitions follow those in Appendix E.1 of Kaplan et al. (2020). They have been detrended using a linear time trend estimated from 1975 to 1997.

- **Consumption**: Quarterly nominal nondurable expenditures (line 8 of NIPA Table 2.3.5 Personal Consumption Expenditures by Major Type of Product) divided by the price index for nondurable consumption (line 8 of NIPA Table 2.3.4. Price Indexes for Personal Consumption Expenditures by Major Type of Product).
- **Homeownership**: Census Bureau U.S homeownership rate (Table 14) and by the age of household head (Table 19).
- **House Prices**: House price index for the entire United States (Federal Housing Finance Agency (`FRED: USSTHPI`)) divided by the price index for nondurable consumption (line 8 of NIPA Table 2.3.4. Price Indexes for Personal Consumption Expenditures by Major Type of Product).
- **Rent-Price Ratio**: Rents (Bureau of Labor Statistics Price Index for Rent of Primary Residences) divided by the FHFA house price index described above.
- **Foreclosures**: Number of consumers with new foreclosures and bankruptcies (Federal Reserve Bank of New York Quarterly Report on Household Debt and Credit) divided by the civilian noninstitutional population (`FRED: CNP16OV`).
- **Leverage**: Flow of Funds Table B.101 Balance Sheet of Households and Nonprofit Organizations. Home mortgage liabilities (FL163165505) divided by the sum of household owner occupied housing at market value (LM155035015) and nonprofit organization real estate at Market Value (LM165035005).
- **Labor Productivity**: U.S. Total Labor Productivity (`FRED: ULQELP01USQ661S`)
- **Mortgage and Treasury Interest Rates**: 30-year fixed rate mortgage (`FRED: GAGE30US`) and 10-year Treasury at a constant maturity (`FRED: GS10`)
- **House price expectations**: University of Michigan Surveys of Consumers. The construction of the empirical proxy and data definitions are discussed in detail in Appendix D. New York Fed Survey of Consumer Expectations.