Beliefs, Aggregate Risk, and the U.S. Housing Boom*

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Abstract

This paper develops a quantitative framework where unprecedented shifts in credit conditions are a source of optimistic beliefs about future house prices during the U.S. housing boom of the 2000s. In a general equilibrium life-cycle model with incomplete markets and aggregate risk, agents form beliefs about future house prices and learn the actual evolution. Because agents lack historical experience in an aggregate state characterized by high productivity and loose credit conditions, they are more likely to perceive forecast errors as permanent shifts in house prices. Consequently, agents underpredict the extent to which house prices mean revert. These learning dynamics account for 20% of the empirical house price volatility observed throughout the U.S. housing boom improving upon existing frameworks that only match 5%.

Keywords: housing boom; aggregate risk; heterogeneous agents; adaptive learning
JEL Codes: E20, E3, C68, R21

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1 Introduction

Loose credit conditions and optimistic beliefs about future house prices are the most common explanations for the record 40% increase in aggregate house prices building up to the 2007-2009 financial crisis.[1] Optimism is the newer of the two channels and its proponents mainly focus on *how* a shift in beliefs can account for house price dynamics during the 1998-2006 housing boom. *Why* there was a shift in beliefs about future house prices remains relatively unexplored. A better understanding of the source of optimistic beliefs and their relationship to credit conditions is important to guide policy efforts aimed at preventing a repeat of the 2007-2009 financial crisis.

This paper proposes that incomplete information about the evolution of house prices, coupled with looser credit conditions, gives rise to overly optimistic beliefs. As productivity increases, credit conditions loosen, and the economy enters a state with little historical precedent. The resulting predictions about future house prices lack a sufficient historical data sample and forecast errors are consequently perceived as permanent shifts in house prices. Beliefs about future house prices thus underpredict the extent to which house prices mean revert which helps account for 20% of the empirical house price volatility observed from 1998 to 2006. Existing frameworks match only 5% of this volatility suggesting that incomplete information may be key to understanding movements in beliefs and hence house prices.[2] By developing a framework that links optimistic beliefs and loose credit conditions, this paper also complements growing efforts to unify the most common explanations of the U.S. housing boom.

This paper embeds adaptive learning about the evolution of house prices into a general equilibrium life-cycle model with incomplete markets and aggregate risk. Allowing for aggregate shocks captures the economy-wide nature of shifts in productivity and mortgage finance in the late 1990s. The resulting heterogeneous households also face idiosyncratic income shocks as they make consumption, borrowing, and housing decisions subject to housing adjustment costs and borrowing constraints. These market frictions –along with additional

[1] See Favilukis et al. (2017), Greenwald (2018), Landvoigt (2016), Kermani (2012), Justiniano et al. (2019), Mian and Sufi (2017) and Di Maggio and Kermani (2017) for the role of loose credit conditions in the housing boom. See Kaplan et al. (2019), Burnside et al. (2016), Piazzesi and Schneider (2009), Gelain and Lansing (2014), Adelino et al. (2018), Nathanson and Zwick (2018), Glaeser and Nathanson (2017), and Foote et al. (2012) for the role of optimistic expectations. Low interest rates have also been studied as a source of the housing boom. Although Jordà et al. (2015) link loose monetary policy to high house prices in 140 years of data spanning 14 advanced countries, research is mixed on the specific role of low interest rates in the housing boom. Even though Adam et al. (2012) and Garriga et al. (2019) successfully link low interest rates to high house prices, Dokko et al. (2011) and Glaeser et al. (2013) struggle to find a similar connection.

[2] See Piazzesi and Schneider (2016) for a discussion of why full information rational expectations models give rise to a house price volatility puzzle in both the housing boom and other economic cycles.

model richness in income and taxes– help assure an empirically plausible distribution of wealth which is important to discipline the response of house prices to looser credit conditions. Without household heterogeneity, all households would either be constrained or unconstrained resulting in a less precise house price response to changes in credit conditions.

With incomplete markets and aggregate risk, agents must keep track of the potentially infinitely dimensional distribution over individual states and its law of motion to determine prices. Computational tractability can be achieved by solving for a Krusell and Smith (1998) approximate equilibrium where agents instead track house prices directly. As a result, agents form boundedly rational expectations with full knowledge of exogenous shocks but beliefs about endogenous variables. Underlying this well-established state space reduction technique is the assumption that agents maintain access to the entire history of endogenous variables when forming expectations.

Adaptive learning about house price forecasts relaxes the assumption that agents know the entire evolution of house prices in each aggregate state of the economy. Agents fix the form of the forecasting rule and update its parameter values in response to incoming data. How closely their beliefs about future house prices align with the actual evolution depends on the amount of historical data available for a particular aggregate state. The economy wide changes to mortgage finance observed throughout the 1990s/2000s had no historical precedent making it reasonable to assume that agents lacked a sufficient data sample to know the evolution of house prices. The resulting learning dynamics correspond to the U.S. housing boom and generate higher and more volatile house prices relative to frameworks where agents always know the evolution of houses prices.

In the benchmark model without adaptive learning, higher productivity and looser credit conditions have a relatively small quantitative impact on house prices and struggle to generate any volatility. House prices only increase by 5.5% which is far below the 40% increase observed in the data. Productivity is the main driver of movements in house prices with an extra boost from a small fraction of constrained agents increasing housing expenditures in response to looser credit conditions. Because household heterogeneity limits the number of constrained agents, looser credit conditions have a relatively modest effect on house prices. House price volatility is negligible at only 5% of its empirical value because house prices are roughly constant in a particular state and only substantially move in response to the aggregate shocks.

Aggregate consumption and housing expenditures also rise along with house prices in response to positive aggregate shocks to productivity and credit constraints. Consumption increases by 4.5% which is slightly below the empirical rise observed in the early part of the boom but below the eventual peak before the bust. Leverage, which is defined as the ratio of borrowing to housing value, counterfactually falls by 15% while it remains flat in the data. Although a fraction of constrained agents increase borrowing in response to looser credit conditions, the unconstrained majority decreases borrowing in response to the increase in productivity resulting in a drop in aggregate leverage. A larger share of constrained agents as well as higher house prices more in line with their empirical counterparts could help the model generated leverage better match the data.

With adaptive learning, house prices instead rise by 10% which is roughly double the increase observed in the benchmark version of the model. Although this increase hardly matches the empirical boom, volatility jumps from almost nothing to 0.06 which is closer to the 0.28 annualized empirical value. Higher volatility arises from the relatively bigger jump in house prices as the economy transitions to the boom state as well as more variable house prices throughout the boom state. Consistent with a housing wealth effect, consumption and housing demand are slightly higher with adaptive learning compared to the benchmark model.[3] Because house prices are higher and more volatile with adaptive learning, my results suggest that incomplete information may be important for understanding movements in house prices throughout the housing boom. Although the direct effect of looser credit conditions on higher house prices is relatively muted, credit conditions maintain the indirect yet important role of triggering learning dynamics.

Results from both the benchmark and the adaptive learning versions of the model exhibit a lack of consumption smoothing in response to the aggregate boom shocks and understate the responsiveness of aggregate housing expenditures. Kaplan and Violante (2014) reconcile this responsiveness of consumption by noting that adjustments to goods consumption are less costly than adjustments to housing expenditures unless the underlying income shock is substantial.[4] The lack of savings market also contributes to the responsiveness of consumption and under responsiveness of housing expenditures. Absent a savings instrument, households insure against both income fluctuations and the retirement phase of the life-cycle by accumulating housing. They are thus more likely to adjust consumption in response to wealth changes and build a buffer stock of housing over the life-cycle. Introducing a savings market and developing a more detailed housing market may help discipline these mismatched responses to aggregate shocks.[5]

To understand how the parameters regulating adaptive learning affect the boom dynamics

[3]The size of the housing wealth effect remains debated. While Berger et al. (2018) find large consumption responses to house price movements, Guren et al. (2019) note that responses in the 2000s are smaller than those of the 1980s.

[4]In a series of experiments, I increased the elasticity of housing along with the magnitude of income shocks and generated a larger response in housing expenditures.

[5]Computation with a savings instrument is not entirely straightforward and is discussed in more detail in the result section (5).

of house prices, I run a series of sensitivity tests where I adjust the learning gain parameter to change the speed at which agents update their forecasts with incoming information. In this extension, I ask whether different speeds of learning can help explain some of the variation in the timing and magnitude of the housing boom across MSAs as documented by Ferreira and Gyourko (2017) and Charles et al. (2015). In areas experiencing persistent local economic headwinds such as Cleveland, OH, agents may be more likely to perceive positive forecast errors as transitory. Agents would then place less weight on new information and adjust their forecasts more gradually throughout the boom leading to relatively lower house prices. In contrast, agents in areas with local economic tailwinds such as Phoenix, AZ, may be more willing to interpret positive forecast errors as a structural shift and more aggressively update their forecasts generating higher prices.[6] Empirical estimates of adaptive learning coefficients as well as transaction level expected capital gains corroborate the hypothesis of slow information updating in Cleveland and quick information updating in Phoenix.

2 Related Literature

Methodologically, this paper is most similar to Favilukis et al. (2017), Kaplan et al. (2019), and Hoffman (2016) who also develop quantitative frameworks with incomplete markets and aggregate risk to study the U.S. housing boom. Favilukis et al. (2017) do not allow for shifts in beliefs and account for the housing boom with higher productivity, relaxed borrowing constraints, and an influx of foreign borrowers in bond markets. In contrast, Kaplan et al. (2019) develop an explicit role for beliefs and find that they are quantitatively more important than credit conditions in explaining the dynamics of both the boom *and* the bust. By modeling beliefs as exogenous shocks orthogonal to credit conditions, Kaplan et al. (2019) abstract away from interaction channels between the two sources of the boom. Moreover, exogenous beliefs can neither address *why* beliefs shifted in the late 1990s nor account for the volatility of house prices as is the goal of this paper.

Hoffman (2016) matches the volatility of aggregate house prices across a wider range of economic cycles by also embedding an adaptive learning forecasting rule into the Krusell and Smith (1998) solution method. He abstracts away from the interaction of beliefs with credit conditions by focusing instead on the response of house prices to detailed income fluctuations. Other adaptive learning models succeed in accounting for high house prices during the housing boom by requiring agents to learn about the persistence of shifts in fundamentals. Adam et al. (2012), Boz and Mendoza (2014), Kuang (2014), and Caines

[6] This extension does not allow for multiple geographic regions. It instead compares the solution of the aggregate model with different learning calibrations.

(2015) all rely on a representative agent which may overstate the direct impact of looser borrowing constraints on higher house prices.

By assuming that fluctuations in credit conditions and productivity trigger adaptive learning about house price forecasts, this framework takes a stance that beliefs respond to credit conditions. Although some evidence suggests a reverse ordering where beliefs instead move credit conditions, the direction of causality is difficult to identify and remains debated.[7] Moreover, with a detailed household sector and a simplified financial sector, the framework in this paper is better suited to focus on the responses of households to exogenous changes in the supply of credit.[8]

In line with the results of Kiyotaki et al. (2011) and Kaplan et al. (2019), I find that looser borrowing constraints have a muted quantitative impact on higher house prices. This contrasts Favilukis et al. (2017) who generate a large quantitative impact from loose credit conditions in a similar framework because of a potentially counterfactually high risk aversion and a less simplified financial sector. Alternative representations of credit conditions suggest that looser payment-to-income constraints [Greenwald (2018)] or looser *lending* constraints [Justiniano et al. (2019)] have a larger quantitative impact on house prices than borrowing constraints. Adding additional constraints to a model with household heterogeneity and aggregate risk complicates both the computation and the interpretation of dynamics. To thus allow for more straightforward model mechanisms, I maintain the standard and simple representation of credit conditions by a loan-to-value constraint on borrowing.

In my framework, constrained households who increase borrowing when credit conditions loosen are the closest parallel to subprime borrowers. A more explicit role for households with blemished credit histories is beyond the scope of this paper. While the pioneering work of Mian and Sufi (2009, 2017) finds that the expansion of mortgage debt to subprime borrowers is key for understanding the dynamics of the housing boom and bust, the subsequent findings of Adelino et al. (2018), Albanesi et al. (2017), Foote et al. (2016) suggest that wealthier prime households played a more central role. To reconcile the borrowing and housing expenditure patterns of wealthier households, Adelino et al. (2018) and Foote et al. (2012) propose an optimistic shift in beliefs as an alternative driver of the U.S. housing boom.

A myriad of empirical evidence supports the view that beliefs became more optimistic at the onset and throughout the housing boom even though explicit measures of house price expectations are few in number. Case and Shiller (1988, 2004) and Case et al. (2012) conduct

[7] While work by Mian and Sufi (2009, 2017) finds that changes in credit conditions increased optimism about future house prices, work by Adelino et al. (2018) and Foote et al. (2012) suggests the opposite direction of causality.

[8] See Bordalo et al. (2018) for a study of how credit conditions change in response optimistic beliefs.

surveys across cities and find that home buyers have higher expectations for future house price increases during local boom episodes. Analyzing the Michigan Survey of Consumers, Piazzesi and Schneider (2009) note that a large and increasing share of households perceived that it was good time to buy a house in the early phase of the housing boom. In the later phase of the boom, households perceived housing as too expensive but were still optimistic that house prices would continue to rise.[9]

Soo (2018) develops a housing sentiment index measuring the tone of local housing news. She finds that beliefs peaked before house prices during the housing boom and argues that beliefs predict movements in house prices. The VAR evidence of Cox and Ludvigson (2019) contrasts this finding and instead demonstrates that beliefs cannot predict house prices once credit conditions and other fundamentals are controlled for. Even though Cox and Ludvigson (2019) find that credit conditions are the most important driver of movements in house prices, they also uncover contemporaneous correlation between beliefs and credit conditions corroborating the premise of this paper.[10]

Structural models exploring the role of beliefs in the housing boom typically rely on heterogeneous beliefs where relative optimists push up house prices. Although incomplete information similarly generates swings in house prices, these frameworks abstract away from the link between belief formation and aggregate fluctuations as is the focus of this paper. Piazzesi and Schneider (2009) use a search model to highlight the importance of optimists and Burnside et al. (2016) build upon this framework by introducing social dynamics where agents may change beliefs after interacting with other agents.[11] Glaeser and Nathanson (2017) also rely on the interactions of rational and optimistic agents to match momentum, mean reversion, and house price volatility.

[9]Analyzing the formation of house price expectations more generally, the information experiment of Armona et al. (2019) concludes that households actually update beliefs with incoming information. Niu and van Soest (2014) infer house price expectations from the Rand American Life Panel longitudinal survey and also find that household adjust beliefs.

[10]The VAR estimates of Ben-David et al. (2019) instead suggest that expectation shocks are the main driver of the housing boom. By focusing on innovations to expectations, they abstract away from expectation formation as is the focus of this paper.

[11]Heterogeneous expectations may require additional state variables complicating computation. Porapakkram and Young (2007) solve a standard Krusell and Smith (1998) economy where households have heterogeneous expectations resulting from different information sets. Kübler and Scheidegger (2018) also develop a computational method to allow types of agents to have different forecasts and expectations.

3 Model

3.1 Environment

This framework is similar to the model developed by Kaplan et al. (2019). Time is discrete where next period quantities are denoted by primes $'$. Individual household quantities are lowercase and aggregate quantities are upper case with a subscript denoting the type of agent. The economy is populated by a lending sector (l), housing construction firms (h), and final goods firms (c) as well as a continuum of measure one finitely lived households (j). Households are heterogeneous in ages and idiosyncratic income endowments.

Preferences

Households are active in economic life for $j = 1, \ldots, J$ periods where they work from $j = 1$ to J^{ret-1} and retire at J^{ret} until they exit the economy with certainty at J. All j subscripts have been dropped unless completely necessary to clarify ambiguity. Households have preferences over final goods consumption and housing expenditures $\{c_j, h'_j\}_{j=1}^{J}$ with final goods consumption as the numeraire. The utility function is given as:

$$u_j(h', c) = e_j \frac{[(1-\phi)c^{1-\gamma} + \phi h'^{1-\gamma}]^{\frac{1-\sigma}{1-\gamma}} - 1}{1-\sigma} \tag{1}$$

Where ϕ is the taste for housing relative to goods consumption, $1/\gamma$ is the elasticity of substitution between housing and goods consumption, and σ is the intertemporal elasticity of substitution. A deterministic equivalence scale $\{e_j\}_{j=1}^{J}$ adjusts consumption for changes in household size over the life-cycle. Expected life-time utility is given as:

$$\mathbb{E}_0 \left[\sum_{j=1}^{J} \beta^{j-1} u_j(h', c) + \beta^J v(\flat) \right] \tag{2}$$

Where the warm-glow bequest motive at the end of life J follows the functional form of De Nardi (2004) which modifies that of Carroll (2002):

$$v(\flat) = \psi \frac{(\flat + \underline{\flat})^{1-\sigma} - 1}{1-\sigma} \tag{3}$$

The bequest motive prevents households from counterfactually ramping up their borrowing and drawing down their housing as they exit the economy. The strength of the bequest motive is regulated by ψ and the extent to which bequests are luxuries is determined by $\underline{\flat}$.

Income Endowments

While working, households receive an endowment of income comprised of three components: an aggregate stochastic shock $\Theta(Z)$, a deterministic life-cycle profile that varies by age χ_j, and an idiosyncratic stochastic shock ϵ_j that follows a first-order Markov process. The transition matrix for earnings $\epsilon_{j+1} \sim \Upsilon_{j+1|j}(\epsilon_j)$ helps account for rising income volatility throughout working life.

$$\log y = \log \Theta(Z) + \chi_j + \epsilon_j, \quad \text{when} \quad j < J^{ret} \tag{4}$$

In retirement, households receive an aggregate income endowment that is the average aggregate income across states $\bar{\Theta}$ and a fraction ρ_{SS} of their last working period income which helps preserve the wealth distribution of retired agents.

$$y = \rho_{SS} y_{Ret-1}(\bar{\Theta}), \quad \text{when} \quad J^{ret} \leq j \leq J \tag{5}$$

Income tax follows the functional form of Heathcote et al. (2017) where τ_y^0 sets the average level of taxation and τ_y^1 sets the degree of tax progressivity.

$$\mathcal{T}(y, b) = y - \tau_y^0 (y)^{1-\tau_y^1} \tag{6}$$

Markets and Individual State Space

Incomplete markets prevent households from insuring against income risk. Households cannot trade among themselves and can only obtain one period loans b' from lenders at price $q = 1/(1 + r_b)$ where $r_b = r(1 + \iota)$ is equal to the risk-free interest rate r plus an intermediation wedge ι.[12] There is no short selling of loans $(0 \leq b')$ and borrowing is limited to a fraction of the value of housing collateral $(b' \leq \theta^{LTV}(Z)ph')$. Households can only purchase houses h' from the housing construction sector at price p which also prevents households from using housing as a mechanism to risk share among themselves. Housing has per-period maintenance and tax costs $(\delta_h + \tau_h)ph$ where maintenance offsets the depreciation of the structure. Adjustments to housing are also subject to a linear cost $\mathbb{1}_{h \neq h'} \kappa_h ph$.

Households receive an initial endowment of loans b and an initial endowment of housing h. They begin economic life with no debt and inherit the housing stock from the agent they are replacing. The state space of individual households consists of loans, housing, idiosyncratic income, and age $\{b, h, \epsilon, j\}$.

[12] Including multi-period mortgage loans adds realism but introduces additional computational complexity by requiring agents to make explicit choices when they adjust mortgages.

Shocks

The aggregate state Z evolves according to a two state Markov chain with transition matrix $Z' \sim \Gamma_Z(Z)$. Shocks to aggregate productivity $\Theta(Z)$ and borrowing constraints $\theta^{LTV}(Z)$ are assumed to be perfectly correlated to maintain computational tractability when solving the model with adaptive learning.[13] Perfect correlation counterfactually assumes that every economic expansion is accompanied by looser credit conditions which either overstates the frequency of shifts in credit conditions or understates the frequency of economic expansions. There is, however, no definitive convention on the timing and relationship of these aggregate shocks.[14] Moreover, some correlation may be justified given the link between technological improvements in automated underwriting and relaxed borrowing constraints established by Guler (2015). With credit conditions following a Markov process, I am imposing that agents placed some probability on looser credit conditions accompanying higher productivity even though the economy had never experienced this state before the late 1990s. This assumption can be interpreted as agents expecting changes in productivity to diffuse to the financial sector which then results in more accommodating credit conditions.

There are two states of the economy, a high state ($Z = high$) and a low state ($Z = low$) where productivity and loan-to-value constraints are higher in the high state relative to the low state.

$$\begin{aligned} \Theta(Z_{high}) >& \Theta(Z_{low}) \\ \theta^{LTV}(Z_{high}) >& \theta^{LTV}(Z_{low}) \end{aligned}$$

Individual income ϵ_j follows an AR(1) process with persistence ρ and an age-dependent standard deviation σ_{ε_j} resulting in an age dependent transition matrix $\epsilon_{j+1} \sim \Upsilon_{j+1|j}(\epsilon_j)$

$$\epsilon_j = \rho\epsilon_{j-1} + \varepsilon_j, \quad \varepsilon_j \sim \mathcal{N}(0, \sigma^2_{\varepsilon_j}) \tag{7}$$

Aggregate State Space

Incomplete markets and aggregate risk make the distribution of agents across individual household states μ a necessary state variable for agents to correctly forecast next period prices.[15] $\Gamma_\mu(\mu; Z, Z')$ is the equilibrium law of motion of the measure of agents such that

[13] Adaptive learning requires as many additional grids as there are aggregate states. Section (3.6) explains in more detail why increasing the number of aggregate states becomes cumbersome with time varying beliefs.

[14] Kaplan et al. (2019) specify independent shocks to income and credit conditions. Favilukis et al. (2017) model income as a Markov process and the shock to credit conditions as unanticipated.

[15] Although there are some instances where the distribution over individual states μ does not completely describe the aggregate state, the conditions for the existence of a recursive equilibrium described by Duffie et al. (1994) are satisfied in this economy.

$\mu' = \Gamma_\mu(\mu; Z, Z')$. The aggregate state space of the economy is thus the aggregate shock and the distribution over individual states $\{Z, \mu\}$. Let $\mathcal{X} = \mathcal{B} \times \mathcal{H} \times \mathcal{E} \times \mathcal{J}$ denote the set of individual states with measure $\int_{\mathcal{X}} \mu d\mu = 1$.

Beliefs

Agents form boundedly rational expectations with fully known shocks $\{Z, \epsilon\}$ but beliefs about endogenous variables. I will solve for a Krusell and Smith (1998) equilibrium by approximating the potentially infinitely dimensional distribution μ and its law of motion $\mu' = \Gamma_\mu(\mu; Z, Z')$ with lower dimensional vectors containing sufficient information to accurately predict prices. Under adaptive learning, agents know the form of the approximated law of motion but do not know its parameter values. How aggressively agents update the values of these parameters depends on their experience in a particular aggregate state. Section 3.6 discusses in more detail the equilibrium properties in both the standard near-rational benchmark version of the model and its extension that allows for adaptive learning.

Government

The sum of income tax revenues $\mathcal{T}(y)$, property tax revenues $p\tau_h \int_{\mathcal{X}} h d\mu$, and land permit revenues (see section 3.4) net of pension outlays $\int_{\mathcal{X}} y_{ret} d\mu_{\mathcal{J}^{ret}}$ is always positive and spent on government services G that are not valued by households and thus discarded.

3.2 Households' Problem

See Appendix A.1 for the full recursive households' problem. Households aged j receive an endowment of housing h and choose housing expenditures h' valued at price $p(\mu, Z)$ subject to a fixed adjustment cost κ. They borrow loans b' costing $q_j(\mu, Z)$ and repay lenders the full amount of the loan b.[16] The households' budget constraint is thus:

$$c = y - \mathcal{T}(y) + p(\mu, Z)[(1 - \delta_h - \tau_h - \mathbb{1}_{h' \neq h}\kappa)h - h'] + q_j(\mu, Z)b' - b$$

Borrowing b' is limited to a fraction $\theta^{LTV}(Z)$ of the value of the housing stock:

$$b' \leq \theta^{LTV}(Z)p(\mu, Z)h'$$

Individual households aged j will be able increase the amount they borrow b' if there is an increase in either house prices $p(\mu, Z)$ or the loan-to-value limit $\theta^{LTV}(Z)$.

[16] An earlier version of this paper included default which was removed to more cleanly explore the role of beliefs about future house prices.

3.3 Lending Sector

The lending sector makes loans b' to households aged j at price $q_j(\mu, Z)$ and receives b in repayment. Although lenders are competitive, uninsurable aggregate risk may induce profits and losses along the equilibrium path. Lenders are owned by non-modeled foreign agents with deep pockets who receive these profits and losses as net exports (NX). The loan market clears loan-by-loan with the same risk premium for all borrowers given that there is no default.[17] Lenders charge a premium relative to the risk-free rate $r_b = (1 + \iota)r$. The loan pricing function is given as:

$$q_j(\mu, Z) = 1/(1 + r_b) \tag{8}$$

3.4 Final Goods and Construction Firms

See Appendix A.2 for the full recursive problems of final goods and housing construction firms. The competitive final goods sector has a linear constant returns to scale technology:

$$Y = \Theta(Z)N_c$$

Where N_c is the unit of labor services. With inelastic labor supply, profit maximization delivers the wage equal to productivity:

$$w(\mu, Z) = \Theta(Z)$$

A competitive construction sector produces houses with technology $H_h = (\Theta(Z)N_h)^{\alpha}\bar{L}^{1-\alpha}$ where N_h is labor services and $\bar{L}$ is the amount of newly available buildable land. Using the equilibrium condition $w(\mu, Z) = \Theta(Z)$ from the final goods firms, the supply of housing is given as:

$$H_h \equiv [\alpha p(\mu, Z)]^{\frac{\alpha}{1-\alpha}}\bar{L} \tag{9}$$

The profit maximization of the construction sector pins down one house price $p(\mu, Z)$ via aggregate housing supply H_h and the aggregate productivity shock $\Theta(Z)$.

3.5 Recursive Competitive Equilibrium

- A *recursive competitive equilibrium* consists of:
  - A sequence of income endowments y to households
  - Prices for houses, wages, and loans: $\{p(\mu, Z), w(\mu, Z), q_j(\mu, Z)\}$

[17] Otherwise lenders would adjust the risk premium based on individual future default probabilities and the collateral value of foreclosed homes.

- Government parameters for the loan-to-value constraint, land permits, taxes, and social security payments: $\{\theta^{LTV}(Z), \bar{L}, \mathcal{T}(y), \tau_h, \rho_{SS}\}$
- Perceived laws of motion for the state space $\mu = \Gamma_\mu(\mu; Z, Z')$ where μ is the measure over the set of individual states $\mathcal{X} = (\mathcal{B} \times \mathcal{H} \times \mathcal{E} \times \mathcal{J})$

- Value function V_j and policy functions for consumption, loan demand, and housing demand $\{c, b', h'\}$ solve the individual households' problem.
- The lending sector maximizes profits with the loan market clearing loan-by-loan with pricing function $q_j(\mu, Z)$.

$$\int_{\mathcal{X}} b'(b, h, \epsilon, j; \mu, Z) d\mu = B_l'$$

- Firms in the construction sector maximize profits with policy functions $\{N_h, H_h\}$. There is a single housing price $p(\mu, Z)$ that clears the housing market.

$$\int_{\mathcal{X}} [h'(b, h, \epsilon, j; \mu, Z) - (1 - \delta_h)h] d\mu = H_h$$

- Final goods firms maximize profits so that the labor market clears at $\Theta(Z) = w(\mu, Z)$ with the total labor supply $N_h + N_c$ normalized to 1.

$$\int_{\mathcal{X}} \exp(\chi_j + \epsilon) d\mu_{\mathcal{J}_{work}} = \underbrace{N_h + N_c}_{1}$$

- The government collects revenue from income taxes $\mathcal{T}(y)$, property taxes τ_h, and land permits $\bar{L}$ to finance expenditures on social security benefits and non-valued government spending:

$$\mathcal{T}(y) + \tau_h p(\mu, Z) \int_{\mathcal{X}} h d\mu + [p(\mu, Z) H_h - w(\mu, Z) N_h] = \rho_{ss} \int_{\mathcal{X}} y_{ret} d\mu_{\mathcal{J}_{ret}} + G$$

- The aggregate resource constraint is satisfied where household consumption, government spending, net exports of profits or losses on lending equal output less housing adjustment costs.

$$\int_{\mathcal{X}} c d\mu + G + NX = Y - \kappa p(\mu, Z) \int_{\mathcal{X}} h d\mu$$

- Consistency is satisfied and perceived laws of motion of the state space $\mu' = \Gamma_\mu(\mu, Z, Z')$ is consistent with individual behavior

3.6 Computation of Equilibrium

The solution method is adapted from Kaplan et al. (2019), Favilukis et al. (2017), and Hoffman (2016) who use a variation of the Krusell and Smith (1998) algorithm. See Appendix B for computational details. Aggregate risk and incomplete markets rule out an equilibrium distribution over individual states μ corresponding to a steady state. To determine prices, agents must then keep track of this potentially infinitely dimensional object to compute its equilibrium law of motion $\Gamma_\mu(\mu; Z, Z')$.[18]

Computational tractability is achieved by solving for an *approximate* equilibrium where agents keep track of house prices $p(\mu, Z)$ directly and update them with a forecasting rule for each combination of current and future aggregate states, $\mathcal{Z} = \{Z, Z'\}$.[19]

$$\mu' = \Gamma_\mu(\mu; \mathcal{Z})$$
$$\iff$$
$$\log p'(p(Z); \mathcal{Z}) = a^0_{\mathcal{Z}} + a^1_{\mathcal{Z}} \log p(Z) \tag{10}$$

The log-linear AR(1) forecasting rule for house prices in equation (10) is standard in the literature across several different types of macro housing models.[20] This specification assumes that the conditional sample means of house prices $\mu^p_{\mathcal{Z}} = \exp\{a^0_{\mathcal{Z}}/(1 - a^1_{\mathcal{Z}})\}$ are sufficient statistics to accurately predict future prices.[21] Although additional statistics, lags, or forms of a forecasting rule may be used, Krusell and Smith (1998) find that more complexity only incrementally improves accuracy. Moreover, Pancrazi and Pietrunti (2019) estimate house price dynamics and find that models relying on simple forecasting rules better fit the data throughout the housing boom.

The standard Krusell and Smith (1998) solution method solves for a fixed point of coefficient values in equation (10). To compute an approximate equilibrium, one guesses values for the vectors of coefficients $\boldsymbol{a}_{\mathcal{Z}}$ and then solves and simulates the individual households' problems. Using the time series of simulated market clearing prices, new coefficients are estimated via an ordinary least squares regression shown in equation (11).[22] The model is

[18]Ahn et al. (2018) develop an alternative to the Krusell and Smith (1998) method for solving models with incomplete markets and aggregate risk. After solving for the stationary equilibrium with only idiosyncratic shocks, the solution is then linearized around the steady state of aggregate shocks. The authors mention on page 13 that this approach is unsuitable for asset pricing applications where aggregate uncertainty directly affects individual decision rules. Because movements in the loan-to-value constraint directly affect housing expenditures and loan choices of constrained agents, the Krusell and Smith (1998) method may be more robust in the setting of this paper.

[19]A single house price $p(\mu, Z)$ is the only price determined via market clearing. The lending sector, final goods firms, and construction firms are all perfectly competitive with linear objective functions. The number of prices is thus reduced from three to one because the profit maximization of the final goods firm pins down wages $w(\mu, Z)$ and the lending sector's pricing function $q_j(\mu, Z)$ pins down loan prices.

[20]See Favilukis et al. (2017), Kaplan et al. (2019), and Hoffman (2016).

[21]While the aggregate capital stock in Krusell and Smith (1998) is predetermined, aggregate housing is not and requires explicit market clearing. The computational strategy more closely resembles that of an incomplete markets model with risk-free bonds or endogenous labor supply described in Krusell and Smith (2006).

[22]Ergodicity permits the calculation of moments from data produced by simulations of the model. Cao

solved and simulated again but now with the new coefficients. These steps are repeated until the coefficients used to solve the households' problem are nearly equal to the coefficients estimated from the time series of market clearing prices, $\boldsymbol{a}_{\mathcal{Z}} \approx \boldsymbol{a}_{\mathcal{Z}}^{new}$.

$$\underbrace{\log p'(p(Z);\mathcal{Z})}_{Y_{\mathcal{Z}_t}} = \underbrace{a_{\mathcal{Z}}^0 + a_{\mathcal{Z}}^1 \log p(Z)}_{\boldsymbol{x}'_{\mathcal{Z}_t}\boldsymbol{a}_{\mathcal{Z}}} + e_{\mathcal{Z}_{t+1}}$$

$$\boldsymbol{a}_{\mathcal{Z}}^{new} = \left(\sum_{t=1}^{T} \boldsymbol{x}_{\mathcal{Z}_t}\boldsymbol{x}'_{\mathcal{Z}_t}\right)^{-1} \sum_{t=1}^{T} \boldsymbol{x}_{\mathcal{Z}_t} y_{\mathcal{Z}_t} \tag{11}$$

Embedding adaptive learning into the Krusell and Smith (1998) solution method creates a convenient framework to introduce an explicit role for beliefs about future house prices. Ljungqvist and Sargent (2018, p. 228-229) explain that solving for a Krusell and Smith (1998) approximate equilibrium is conceptually similar to the convergence of adaptive learning to a rational expectations equilibrium.[23] In the context of this paper, both equilibrium concepts rely on beliefs about future house prices converging to the actual evolution of model simulated house prices.

Rather than solving for converged forecasting coefficients as a fixed point, adaptive learning instead solves for them during simulation which allows the evolution of beliefs to be explicitly tracked.[24] As agents learn the true values of the forecasting coefficients, beliefs about future house prices may not yet correspond to the actual evolution of prices. The resulting temporary equilibria may be self-referential where optimism or pessimism impacts the market clearing realizations of house prices. Following the terminology of Adam and Marcet (2011b), agents are internally rational in the sense that they form dynamically consistent beliefs about the future absent full information about the true stochastic processes.[25]

The learning mechanism detailed in equations (12)-(13) relaxes the assumption that agents know the true values of the forecasting coefficients. Agents instead update coefficients each period. Current period coefficients $\boldsymbol{a}_{\mathcal{Z}_t}$ thus become a combination of past coefficients and weighted forecast errors $e_{\mathcal{Z}_t}$. How the coefficient values differ from their benchmark counterparts depends on three key parameters governing the learning mechanism detailed in equations (12)-(13): the gain parameter g_t, the initial coefficients $\boldsymbol{a}_{\mathcal{Z}_{t-1}}$, and the length of the training sample $t_{learn} - t_{burn}$. The calibration of these parameters is discussed in more

(2019) details how to apply the results of Duffie et al. (1994) to guarantee ergodicity.

[23]See Evans and Honkapohja (2001) and Marcet and Sargent (1989) for adaptive learning convergence criteria.

[24]See Kübler and Scheidegger (2018) for generalization of the existence and computation of an approximate equilibrium with self-referential beliefs.

[25]Adam and Marcet (2011b) also provide microfoundations for learning about prices instead of model fundamentals.

detail in the following section.

$$\boldsymbol{a}_{\mathcal{Z}_t} = \boldsymbol{a}_{\mathcal{Z}_{t-1}} + g_t R_{\mathcal{Z}_t}^{-1} \boldsymbol{x}_{\mathcal{Z}_{t-1}} \underbrace{(y_{\mathcal{Z}_{t-1}} - \boldsymbol{x}'_{\mathcal{Z}_{t-1}} \boldsymbol{a}_{Z_{t-1}})}_{e_{\mathcal{Z}_t}} \tag{12}$$

$$R_{\mathcal{Z}_t} = R_{\mathcal{Z}_{t-1}} + g_t(\boldsymbol{x}_{\mathcal{Z}_{t-1}} \boldsymbol{x}'_{\mathcal{Z}_{t-1}} - R_{\mathcal{Z}_{t-1}}), \quad R_{\mathcal{Z}_{t-1}} = (t_{learn} - t_{burn})^{-1} \sum_{i=t_{burn}}^{t_{learn}} \boldsymbol{x}_{\mathcal{Z}_i} \boldsymbol{x}'_{\mathcal{Z}_i} \tag{13}$$

When solving the model with adaptive learning, additional grids for state dependent future house prices $p'(Z'_{high})$ and $p'(Z'_{low})$ are defined so that the coefficients can vary over the course of the simulation. I first solve the model with the benchmark algorithm to obtain converged coefficients $\boldsymbol{a}_{\mathcal{Z}} = (a^0_{\mathcal{Z}}, a^1_{\mathcal{Z}})'$. I then re-solve the households' problem over the additional grids. I check that the time-series of learning coefficients $\boldsymbol{a}_{\mathcal{Z}_t}$ converges to an ergodic distribution close to their benchmark counterparts $\boldsymbol{a}_{\mathcal{Z}}$.[26] The extension in section (6) solves the model with a range of values for the learning gain g_t and finds that convergence to an ergodic distribution is robust.

4 Parameterization and Calibration

4.1 Parameters

The parameterization of the model largely follows that of Kaplan et al. (2019). The parameters are chosen to resemble the U.S. economy in the late 1990s with the cross-sectional moments from the 1998 Survey of Consumer Finances.

Demographics

Each model period is equal to two years. Households begin economic life at age 21 ($j = 1$), retire at age 65 ($J^{ret} = 23$), and exit economic life with certainty at age 79-80 ($J = 30$).[27]

Preferences

The elasticity of substitution between goods consumption and housing consumption $1/\gamma$ is set to 1.25 based on the estimates of Piazzesi et al. (2007). The elasticity of intertemporal substitution equals 0.5 by setting $\sigma = 2$. A McClements (1977) scale from Chang (2019) based on that of Pizzinelli (2018) is used to set the consumption equivalence scale $\{e_j\}$ to

[26] The following section discusses why convergence to the benchmark coefficients may not be guaranteed.

[27] Krueger and Kübler (2004) find that computational accuracy decays rapidly if the length of economic life extends beyond 30 periods.

match the OECD average number of children across different age groups from 1995 Census Data. The discount factor β is set to replicate the 1998 ratio of aggregate net worth to annual labor income.[28]

Two parameters, the strength of the bequest motive ψ, and the extent to which bequests are luxuries $\underline{b}$, pin down the warm-glow bequest motive given in equation (3). The strength of bequests ψ is chosen to replicate the ratio of net worth at age 75 to age 50 which indicates the importance of bequests as a saving motive. The luxuriousness of bequests $\underline{b}$ is chosen so that households in the bottom half of the wealth distribution do not leave a positive bequest.

Income Endowments

The deterministic life-cycle component of earning $\{\chi_j\}$ is from Kaplan and Violante (2014).[29] Stochastic individual earnings ϵ_j in equation (7) follow an AR(1) process in logs with an annual persistence of 0.97, annual standard deviation of 0.2, and an initial standard deviation of 0.42. The variance of log earnings rises by 2.5 between the ages of 21 and 64 which follows Heathcote et al. (2010).

Housing

The maintenance cost of housing that offsets depreciation δ_h replicates the empirical depreciation rate of the housing stock which equals 1.5% per year.[30] The linear transaction cost of housing adjustments κ_H equals 11% which falls within the 6-12% range estimated by Quigley (2002). Ghent (2012) finds a value of 13% and Ngai and Sheedy (2019) settle on 10% providing further support that a transaction cost of 11% aligns with empirical evidence.

Housing construction technology α is set at 0.6 so that the price elasticity of housing supply $\alpha/(1-\alpha)$ equals 1.5. Saiz (2010) estimates housing supply elasticities across MSAs and finds a median value equal to 1.5. The land permit parameter $\bar{L}$ pins down employment in the construction sector at 5% of total employment which is consistent with the 1998 employment share of construction measured by the Bureau of Labor Statistics.

[28]Kaplan and Violante (2014, footnote 28) explain that setting the discount factor to match aggregate net worth instead of median net worth has the benefit of better capturing aggregate price effects at the cost of overstating wealth holdings and understanding the marginal propensities to consume. Because this paper is more interested in prices than consumption movements, calibrating the discount factor to the aggregate net worth is more suitable than calibrating to the median.

[29]Kaplan and Violante (2014) provide several different deterministic life-cycle profiles and I use version number 1. To convert their quarterly values to a two-year frequency, I use the value from the quarter that corresponds to the start of each two-year period.

[30]Kaplan et al. (2019, footnote 18, page 18) use the Bureau of Economic Analysis' Table 7.4.5 which details the consumption of fixed capital of the housing sector divided by the stock of residential housing at market value.

Financial Instruments

The risk-free rate r is set at 3% per year and the lending wedge ι is set at 0.33 so that the interest rate on loans r_b is equal to 4% per year. This interest rate spread replicates the gap between the average rate on 30-year fixed-term mortgage and the 10-year Treasury bill rate in the late 1990s. Without default, all agents borrow at the same interest rate.

Government

The property tax τ_h is set to 1% per year which is the median tax rate across U.S. states according to the Tax Policy Center. The income tax function $\mathcal{T}(y) = y - \tau_0(y)^{1-\tau_1}$ in equation (6) follows the functional form of Heathcote et al. (2017). The parameter τ_y^0 indicates the average level of taxation and is set so that aggregate tax revenues are 20% of output. The parameter τ_y^1 measures the degree of progressivity of the tax and transfer system and is set at 0.15 based on the estimates of Heathcote et al. (2017). The social security payments detailed in equation (5) maintain income heterogeneity by scaling the last realization of earnings y^w_{Jret-1} by the replacement rate ρ_{ss}. Kaplan et al. (2019) compute the ratio of average benefits to average lifetime earnings and find a replacement rate equal to 0.4.

Aggregate Shocks

The economy faces aggregate shocks over productivity $\Theta(Z)$ and credit conditions $\theta^{LTV}(Z)$. These shocks are perfectly correlated and follow a two-state Markov process as detailed in section 3.1. Aggregate labor income $\Theta(Z)$ follows a discrete approximation of an AR(1) process estimated by Kaplan et al. (2019) from a linearly de-trended series of total U.S. labor productivity.

Shocks to credit conditions are represented by movements in the loan-to-value constraint $\theta^{LTV}(Z)$. Even though there were many shifts in mortgage finance occurring in late 1990s/early 2000s, higher loan-to-value constraints are the most common representation of looser credit conditions in macro models of the housing boom.[31] Loan-to-value constraints serve the technical purpose of ruling out Ponzi schemes in addition to the practical purpose of capturing shifts in market frictions.[32]

The loan-to-value constraint is equal to 0.8 in the low state which follows the estimates

[31] Dokko et al. (2019) find that 60% of all mortgages contained one non-traditional feature by 2005.

[32] Kiyotaki et al. (2011) explain that loan-to-value constraints are a reduced-form representation of contract enforcement frictions in credit markets. Because lenders fear that a household might not repay a mortgage loan, they require both a down payment and the house to be pledged as collateral. Even though this paper abstracts away from default, down payment requirements help discipline the dynamics of housing prices and quantities.

Interpretation	Parameter	Value	Annualized
Demographics			
Maximum age	J	30	N
Retirement age	J^{ret}	23	N
Preferences			
Discount factor	β	0.964	Y
Inverse elasticity of substitution	γ	0.8	N
Risk aversion	σ	2	N
Strength of bequest motive*	ψ	500	N
Extent of bequest as a luxury*	$\underline{b}$	3	N
Taste for housing	ϕ	0.12	N
Risk-free interest rate	r	0.03	Y
Interest rate wedge on borrowing	ι	0.33	N
Housing			
Depreciation rate of housing	δ_H	0.015	Y
Property tax on housing	τ_h	0.01	Y
Housing supply elasticity	$\alpha/(1-\alpha)$	1.5	N
New land permits	$\bar{L}$	0.311	N
Fixed housing adjustment cost*	κ	0.11	N

Table 1: Parameter values. All values are from Kaplan et al. (2019) unless followed by a *. The model period is two years and values that have been annualized are denoted by Y in the final column. 1=$52,000 which is the average value of income in the 1998 SCF.

of Duca et al. (2011) for cumulative loan-to-value ratios of first-time home buyers in the 1990s. The high state value of 1.1 follows Kaplan et al. (2019) who target a 15 percentage point rise in combined loan-to-value ratios from the late 1990s to 2006. Although Kaplan et al. (2019) use a value of 0.9 instead of 0.8 for the low state, my results are broadly similar with either value.

The transition probabilities for aggregate shocks $\pi_{Z,Z'}$ are from a Markov chain approximation of de-trended aggregate labor productivity. These values will overstate the probability of a relaxation of credit conditions by assuming that every economic expansion is accompanied by looser credit conditions.

Learning

Three parameters discipline adaptive learning: the gain parameter g_t, the initial values of coefficients $\boldsymbol{a}_{\mathcal{Z}_0}$, and the length of the training sample $t_{learn} - t_{burn}$.

The gain parameter g_t dictates the weight agents place on new information and thus the

Interpretation	Parameter	Value
Aggregate and income parameters		
Loan-to-value ratio	$\{\theta^{LTV}(high), \theta^{LTV}(low))\}$	$\{1.1, 0.8\}$
Aggregate productivity	$\{\Theta(high), \Theta(low)\}$	$\{1.035, 0.965\}$
Deterministic income	$\{\chi_j\}$	Kaplan and Violante (2014)
Annual persistence, ind. income	ρ_ϵ	0.97
Annual st. dev., ind. income	σ_ϵ	0.20
Initial st. dev. of ind. income	σ_{ϵ_0}	0.42
Income tax function	τ_y^0, τ_y^1	0.75,0.151
Social Security replacement rate	ρ_{SS}	0.4
Aggregate shock transition matrix	$\begin{bmatrix} \pi_{h,h} & \pi_{h,l} \\ \pi_{l,h} & \pi_{l,l} \end{bmatrix}$	$\begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$
Initial distributions		
Age distribution*	$j_1 \in [1, 30]$	Uniform
Loan distribution*	$b_1 = 0$	
Housing distribution*	$h_1 = [\min H'_h, \max H'_h]$	Normal
Learning		
Gain*	g_t	0.02
Initial coefficients*	$\boldsymbol{a}_{\mathcal{Z}_{t-1}}$	$\boldsymbol{a}_{Z_{low}, Z'_{low}}$
Training sample length	$t_{learn} - t_{burn}$	30

Table 2: Parameters governing income, credit conditions, learning and Markov processes. All values from Kaplan et al. (2019) unless followed by a *. The model period is two years and annualized values are noted in the parameter description in the first column.

speed of learning. Following the existing literature, I use a constant gain $g_t = g$ which is essentially a rolling window regression that puts more weight on recent observations. Because constant gain learning allows agents to forget past values, it better accommodates structural changes in the economy. Moreover, Branch and Evans (2006) compare the performance of recursive forecasting models and find that constant gain learning delivers the best out of sample forecast fit as well as the best match to the Survey of Professional Forecasters. Recursive least squares learning where $g_t = 1/(t + t_{learn})$ is a potential alternative to constant gain learning. By assigning a declining weight instead of a constant weight to more recent observations, least squares learning estimates a time varying regression instead of a rolling window regression as is the case with a constant gain learning. Least squares learning has the advantage of potentially guaranteeing convergence to rational expectations equilibria while constant gain learning may only converge to an erogodic distribution.[33] I find that the constant gain ergodic coefficients are close to their near-rational benchmark values for a range of values of g_t as discussed in the extension in section (6). The exact rational expectations evolution of house prices may not follow the form of an AR(1) forecasting rule shown in equation (10) suggesting that least squares learning could only guarantee convergence to the Krusell and Smith (1998) approximate equilibrium at best.

With $g_t = 0.02$, the gain parameter falls within the range of 0.002 to 0.062 established by the existing literature.[34] By setting $g_t = 0.02$, agents assign a weight of 0.6 on 50 year old observations ($[1-0.02]^{25} \approx 0.6$) which is close to the weight used by Eusepi and Preston (2011). Agents almost entirely forget observations that are 500 years old.

Lubik and Matthes (2016) note that there is no generally accepted way to choose initial beliefs $\boldsymbol{a}_{\mathcal{Z}_0}$. I will assume that agents know the true coefficients of the non-boom state $\boldsymbol{a}_{Z_{low},Z'_{low}}$ because they often experience this state. For the boom state agents rarely visit, I assume that agents naively set their beliefs to $\boldsymbol{a}_{Z_{low},Z'_{low}}$. Because this forecast does not internalize the positive response of future house prices to present aggregate shocks, agents will under predict house prices at first. Although this assumption imposes backwards looking house price forecasts, evidence suggests that expectations are sluggish and contain adaptive components. A survey conducted by Coibion et al. (2018) shows that firms are slow to update expectations in response to changes in economic conditions. Focusing on house price expectations specifically, Gelain et al. (2016), Glaeser and Nathanson (2017), Case and Shiller (1989), Pancrazi and Pietrunti (2019), and Granziera and Kozicki (2015) find that models with some form of backwards looking expectations better match aggregate house price data.

Coefficients are fixed at their initial values $\boldsymbol{a}_{\mathcal{Z}_0}$ for $t = 1, \ldots, t_{learn}$ periods by setting the gain parameter to zero $g_t = 0$. The length of the training sample $t_{learn} - t_{burn}$ is chosen so that agents experience 60 years of the economy prior to learning about house prices in the aggregate state characterized by higher productivity and looser credit conditions. These training sample realizations then pin down the initial covariance matrix,
$R_{\mathcal{Z}_{t-1}} = (t_{learn} - t_{burn})^{-1} \left(\sum_{t=t_{burn}}^{t_{learn}} \boldsymbol{x}_{\mathcal{Z}_t} \boldsymbol{x}'_{\mathcal{Z}_t}\right)$. If initial beliefs $\boldsymbol{a}_{\mathcal{Z}_0}$ are not equal to the values obtained from the near-rational benchmark, the training sample will be computed off-equilibrium.

[33]See Evans and Honkapohja (2001) and Marcet and Sargent (1989) for details on the learnability of rational expectations equilibria.

[34]Eusepi and Preston (2011) choose a value of 0.002 so that forecast errors from their model are consistent with empirical forecast errors from the Survey of Professional Forecasters. Branch and Evans (2006) estimate the optimal constant gain parameters that provide the best out of sample forecasts for inflation and GDP. They find values equal to 0.007 for GDP and 0.062 for inflation. Orphanides and Williams (2005) estimate a structural model with a gain of 0.05. Milani (2007) estimates a gain value of 0.0187 from a monetary DSGE model.

4.2 Calibration

Table (3) details the targeted moments from the 1998 SCF used by Kaplan et al. (2019) as well as the values obtained from the model in this paper. Model simulated moments are generally in line with their empirical counterparts except for the counterfactually high ratio of the 90th percentile of housing expenditures to net worth (final row) and the counterfactually low net worth at age 75 relative to age 50 (third row). Lacking a savings instrument, wealthy households rely entirely on housing to build wealth resulting in an overaccumulation of housing relative to what is observed in the data.[35] Consequently, housing has a more pronounced hump shape over the life-cycle which then distorts the net worth at age 75 relative to net worth at age 50. The model is still able to match the lower percentiles of housing to total net worth because households in these middle and low percentiles would only hold small amounts of a liquid savings instrument if it were available.

Moment	Parameter	Empirical value	Model value
Agg. net worth/annual aggregate labor income	β	5.5	5.2
Median net worth: age 75/age 50	ψ	1.51	0.4
Fraction of bequests in bottom half of wealth dist.	$\underline{b}$	0	0
Employment in construction sector	$\bar{L}$	0.05	0.05
P10 housing/total net worth	housing	0.11	0.10
P50 housing/total net worth	housing	0.50	0.52
P90 housing/total net worth	housing	0.95	2.20

Table 3: Targeted moments in calibration corresponding to model parameters

4.3 Life-cycle Results

Figure (1) presents cross-sectional life-cycle profiles of the means and log variances of consumption, housing expenditures, and borrowing in the stochastic steady state. These profiles are largely consistent with a typical incomplete markets model and the discrepancies are subsequently discussed.

Average income and volatility (bottom right panel) rise over the life-cycle consistent with the data from the 1998 Survey of Consumer Finances. The non-stationary age-dependent variance for idiosyncratic income helps match the more volatile earnings in the later stages of economic life as is observed in the data. With minimal housing market frictions, average

[35]Following the findings of Kaplan and Violante (2014), wealthy households can be characterized as wealthy hand-to-mouth since they hold no liquid assets but a large quantity of durables. Using the 2001 Survey of Consumer Finances, they estimate that 7% to 26% of all households can be considered wealthy hand-to-mouth.

Figure 1: Life-cycle statistics in 1998 dollars. Top-left panel: goods consumption by age. Top-right panel: housing expenditures by age. Bottom-left panel: borrowing by age. Bottom-right panel: gross income by age

housing expenditures and average consumption (top panels) have life-cycle profiles that differ from Kaplan et al. (2019) but are broadly similar to those of Kaplan and Violante (2014).

All households accumulate housing as a buffer against income movements that arise from both the deterministic earnings profile as well as stochastic fluctuations. This accumulation motive contributes to the more pronounced hump shape of housing over the life-cycle (top right panel) compared to Kaplan et al. (2019) who develop a much more detailed housing market and allow for liquid savings. The volatility of housing expenditures is almost three times as large as either consumption volatility or income volatility because of the counterfactually high levels of housing expenditures by wealthy agents who rely on housing to store wealth. The spike in housing volatility in the later stages of economic life is due to bequest heterogeneity. Households who leave bequests maintain a near constant level of housing throughout the retirement phase of the life-cycle while non-bequesters drawn down their housing expenditures.

The patterns of average consumption and its log variance (top left panel) suggest that households succeed at partially insuring against income movements. Consumption volatility is lower than income volatility and average consumption is relatively smoother over the life-cycle compared to housing expenditures. The slight hump shape in average consumption, however, suggests that this insurance is only partial and households are unable to fully smooth consumption over the course of economic life.

Average household borrowing (bottom left panel) panel is highest in the early stages of life as households pull forward their future earnings to somewhat smooth consumption and housing expenditures. The slight pickup in average borrowing towards the end of economic life arises from the bequest motive where some households borrow to maintain their housing stock as a bequest rather than draw it down. Borrowing volatility is low throughout the life-cycle because there are many households who take on no debt. Among households with positive debt, borrowing is most volatile in the early stages of economic life.

5 Results

5.1 Simulating the Housing Boom

The model is solved on Indiana University's Karst and Carbonate high-throughput computing clusters using 50 gigabytes of virtual memory and 40 gigabytes of memory. Initial guesses for forecasting coefficients in equation (10) were obtained by solving the model without aggregate shocks. I first solve the benchmark model to obtain the forecasting coefficients corresponding to a fixed point before solving the model with adaptive learning. The benchmark forecasting coefficients converge in 7 iterations with parallelization on four cores.[36]

As explained in Appendix B, adaptive learning requires as many additional grids as there are aggregate states. If two belief grids corresponding to two aggregate states each have 3 grid points, the households' problem must be solved $3^2 = 9$ times. Fortunately, the increase in dimensionality is not a computational burden because the 9 versions of the household problem can be parallelized.[37] Each version is forked to its own core which then creates an additional pool of processors that also parallelizes the value function iteration step. With 9 cores at the outer level of parallelization and four cores at the inner level, this configuration requires a total of 36 cores. Nested parallelization reduces the computation time of value function iteration with the two additional belief grids from 34 minutes to 2 minutes.

[36]Parallelization reduces computation time of the value function iteration step from 4.7 minutes to 1.2 minutes. If I increase the number of cores, the speed improvements are only marginal because value function iteration is not parallelizable across the age grid and the house price grid.

[37]Allowing independent aggregate shocks to credit conditions and income would require defining four belief grids instead of two. With 3 points for each grid, the households' problem would then need to be solved $3^4 = 81$ times instead of $3^2 = 9$ times. Increasing the dimensions of the arrays requires large amounts of virtual memory ranging from 150 to 200 gigabytes and interpolation in two additional dimensions. Computation times increase drastically with larger approximation errors as a result of these additional grids.

Even though a savings instrument would help the cross-sectional results better reflect their empirical counterparts, determining an additional price in equilibrium would add more complication to the computation of equilibrium. Households would need to form beliefs and keep track of loan prices $q_j(\mu, Z)$ in addition to house prices $p(\mu, Z)$ if savings were introduced via bonds in zero net supply or capital rented to final goods firms. Solving for the fixed point of forecasting coefficients becomes more complicated when coefficients are 2x2 matrices instead of 2x1 vectors.[38] Allowing for bonds in zero net supply would also require another explicit market clearing step which would further add to the computational complexity. The most tractable strategy to introduce savings would be to allow agents to borrow one-period risk-free bonds from the non-modeled foreign owned financial sector.

Using the converged forecasting coefficients, I simulate the economy in the low aggregate state for 100 periods and then switch to the high state for 5 periods before contracting back to the low state. This sequence of shocks represents the housing boom-bust where labor productivity and borrowing constraints increase in 1997 and then contract in 2007 as shown in figure (2).

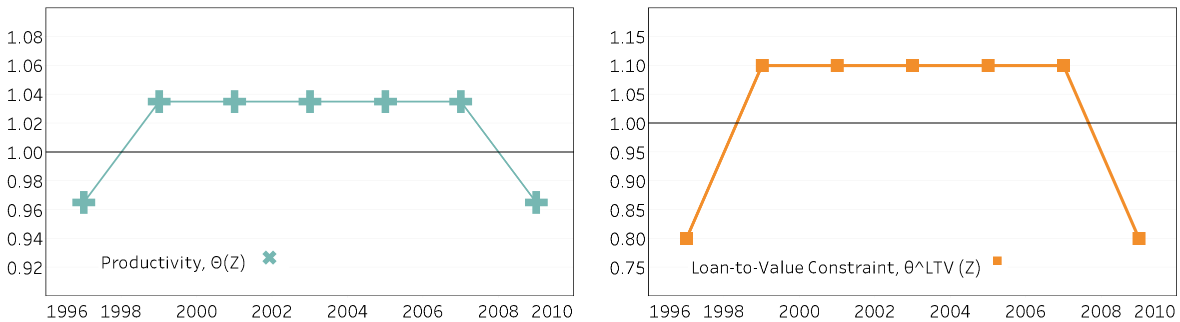

Figure 2: Aggregate shocks corresponding to values in table (2) for 1997-2009.

5.2 Results: Near-Rational Benchmark

The converged house price forecasting coefficients are shown in table (4) along with various accuracy measures. Kaplan et al. (2019) and Favilukis et al. (2017) obtain R^2 statistics close to 1 for all combinations of current and future aggregate states suggesting that my results fall short of the accuracy standards of similar models. The relatively larger forecast errors arise from variation in the marginal propensities to consume of individual agents. While Krusell and Smith (1998) find only small differences in marginal propensities to consume in

[38]See Favilukis et al. (2017) for an example of an incomplete markets model with aggregate risk that solves for the fixed point of a matrix of forecasting coefficients.

an infinite horizon economy, life-cycle frameworks typically find more variation. Den Haan (2010) and Chipeniuk et al. (2019) propose alternative accuracy tests and caution that the R^2 can be a misleading statistic. With a Den Haan (2010) statistic of 0.02 which is then improved to 0.01 via the auctioneer iteration method of Chipeniuk et al. (2019), my results fall within reasonable ranges of accuracy.

$\log P'_{Z,Z'}$	$=$	$a^0_{Z,Z'}$	$+$	$a^1_{Z,Z'} \log p(Z)$	R^2	Den Haan (2010) Statistic	Chipeniuk et al. (2019) Statistic
$\log P'_{high,high'}$	$=$	-0.13	$+$	$0.90 \log p(Z_{low})$	0.83	0.02	0.01
$\log P'_{high,low'}$	$=$	-0.08	$+$	$0.98 \log p(Z_{low})$	0.83		
$\log P'_{low,high'}$	$=$	-0.12	$+$	$0.87 \log p(Z_{high})$	0.84		
$\log P'_{low,low'}$	$=$	-0.10	$+$	$0.93 \log p(Z_{high})$	0.85		

Table 4: Converged forecasting coefficients and accuracy tests

Figure (3) compares the paths of house prices and consumption to their empirical counterparts. Model simulated house prices (left panel) only rise by 5.5% which is far below the nearly 40% empirical increase. The 7% increase in productivity is the main driver of higher house prices along with a slight boost from constrained households who demand higher housing in response to looser collateral constraints. Because house prices closely track productivity, they are relatively constant unless there is a shift in the aggregate state. Aggregate shocks, however, are relatively persistent and transitions are thus infrequent. As a result, annualized house price volatility is a negligible 0.014 which is far below its 0.28 annualized empirical value during the boom period.

Aggregate consumption (right panel) rises by about 4.5% which is just under its empirical counterpart for the early part of the boom but falls short of its 10% peak before the bust. Higher house prices may account for higher consumption in the later part of the boom due to a housing wealth effect. Because the model simulations do not fully capture the empirical increases in house prices, they may also fail to capture the subsequent pass through to consumption.

Figure (4) shows that higher productivity and looser credit conditions generate only a small uptick in housing expenditures and counterfactually low leverage which is defined as the ratio of borrowing to housing value. The rise in productivity makes all households wealthier resulting in a shift away from costly borrowing. Constrained households, however, *increase* their borrowing as credit constraints loosen offsetting some of the contraction in aggregate borrowing.

Model simulations show a counterfactually large drop in leverage which may suggest that the calibration understates the fraction of constrained households. Although looser credit

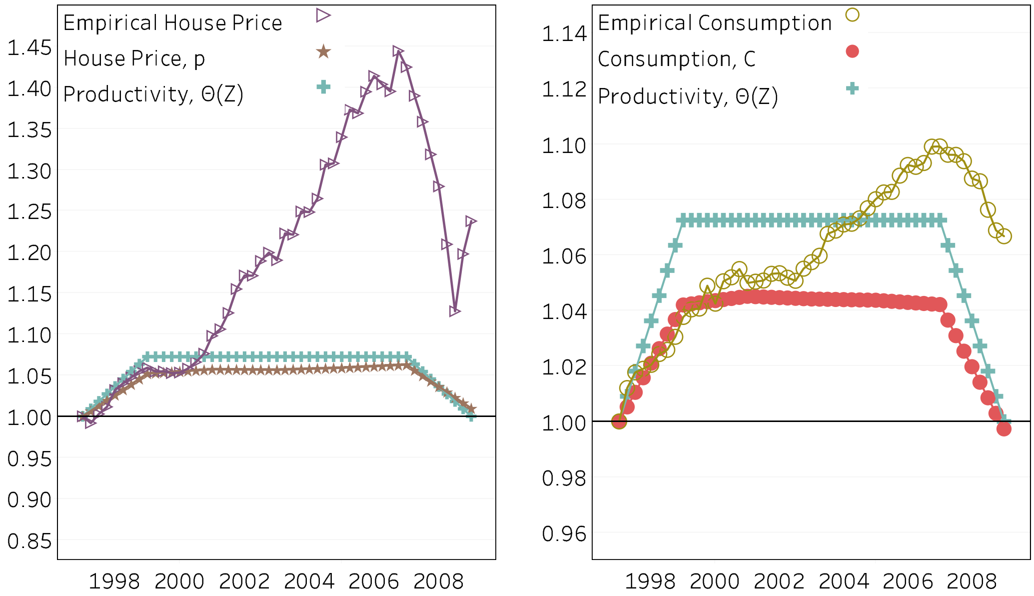

Figure 3: Housing boom simulations from the benchmark model along with their empirical counterparts. Left panel: house prices and productivity. Right panel: goods consumption and productivity. All values are expressed relative to their 1997 value. See Appendix (C) for sources and definitions of the data series.

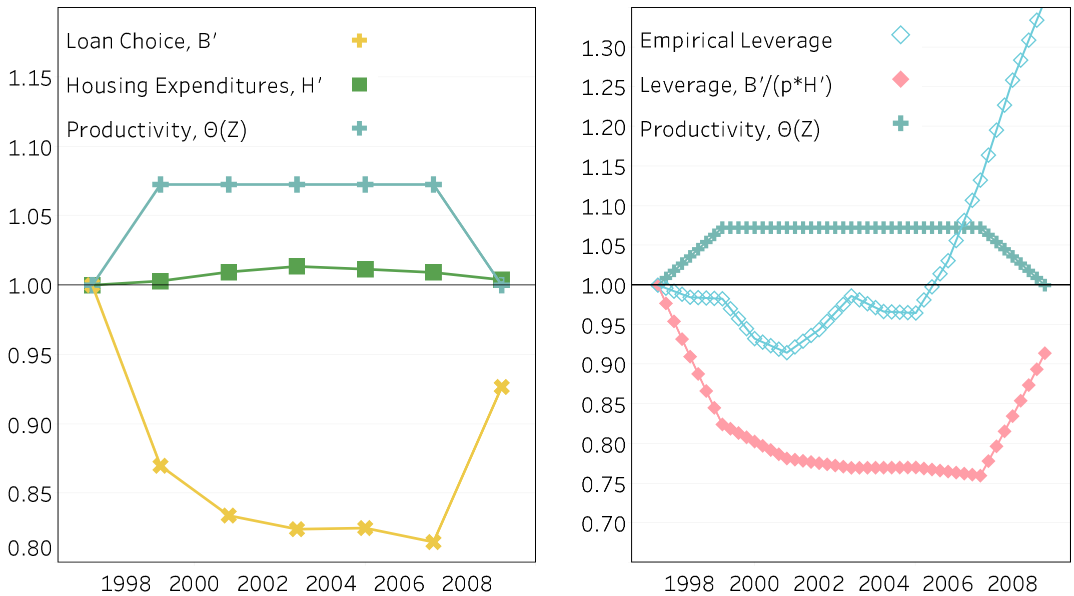

Figure 4: Housing boom simulations from the benchmark model along with their empirical counterparts. Left panel: model simulated productivity, borrowing, and housing expenditures. Right panel: model simulated and empirical leverage defined as the ratio of borrowing to housing value. All values are expressed relative to their 1997 value. See Appendix (C) for data definitions.

conditions would have more of a direct upward impact on house prices if the fraction of constrained agents were larger, house price volatility would still be relatively small without any additional internal propagation mechanisms.

Moreover, because aggregate fluctuations only account for a fraction of the empirical increase in house prices and struggle to generate any volatility, the next section introduces adaptive learning to allow for an explicit role for beliefs about future house prices. Credit conditions then take on the important indirect role of triggering learning dynamics.

5.3 Results: Adaptive Learning

The results in this section repeat the same simulations of the previous section but now require agents to learn the house price forecasting coefficients in both the transition to the boom state and the boom state itself. Figure (5) compares the paths of house prices, consumption, and leverage to their counterparts from the benchmark model where the values of the forecasting coefficients are time invariant and fully known.

With adaptive learning, house prices increase to a level that is almost 10% above their pre-boom values. Although this increase is still far below the 40% rise observed in the data, it is nearly double that of the benchmark model. The self-referential nature of these temporary adaptive learning equilibria help account for the sustained increase in house prices above their benchmark values. Because the house price forecast for the first realization of the boom state does not internalize the positive response of future house prices to present aggregate shocks, agents initially under predict house prices. This positive forecast error is incorporated into subsequent forecasts which pushes beliefs upward generating optimism. Overly optimistic expectations alter current period consumption, housing, and borrowing which then brings to fruition these relatively higher house prices as the boom state persists. This sustained optimism continues until agents learn the actual evolution of house prices. In this boom simulation, however, the economy contracts to the non-boom state before agents fully learn the forecasting coefficients.[39]

As aggregate shocks contract resulting in lower productivity and tighter credit conditions, agents underpredict the drop in house prices because they also lack a precedent for the transition out of the boom. Once arrived in the non-boom state, the role of beliefs diminishes because agents have a sufficient number of data points to understand the evolution of house prices. Forecast errors are quickly resolved as transitory noise making the learning mechanism proposed in this paper less suited for addressing housing dynamics in the bust.[40]

[39]Experiments with a longer boom period show that coefficients do converge to an ergodic distribution close to their benchmark values.

[40]For accounts and explanations of the housing bust see the handbook chapter of Guerrieri and Uhlig

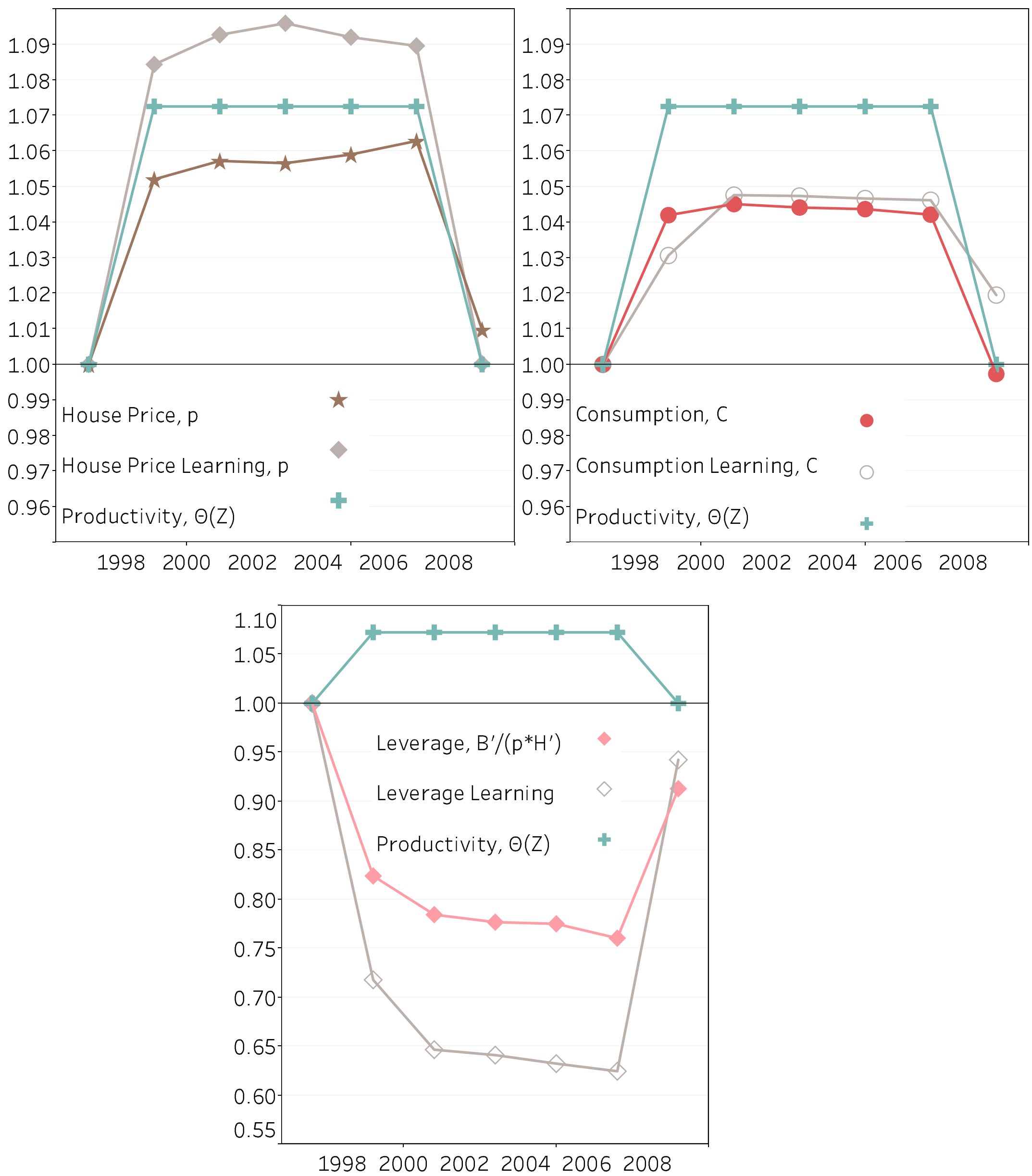

Figure 5: Housing boom simulations from the benchmark and adaptive learning versions of the model. Top-left panel: house prices and productivity. Top-right panel: goods consumption and productivity. Bottom panel: leverage defined as the ratio of borrowing to housing value and productivity. All values are expressed relative to their 1997 value.

Although a reversal of beliefs in the bust is discussed by Kaplan et al. (2019) and Burnside et al. (2016), house price pessimism as an explanation for the busts receives relatively less attention than optimism as an explanation for the boom.

With adaptive learning, house price volatility throughout the housing boom is 0.06 which

(2016) as well as the research of Gerardi et al. (2009) and Garriga and Hedlund (2017).

is still below the 0.28 annualized empirical value but nearly four times as large as the 0.014 value obtained from the benchmark model. The model simulations generate higher volatility from both the bigger increase in house prices at the onset of the boom as well as the adjustments to forecasting coefficients within the boom state. In contrast, movements in house prices within the boom state are relatively minimal in the benchmark version of the model with fixed forecasting coefficients.

Optimistic beliefs and hence higher house prices result in consumption and housing expenditure responses that are higher than those of the benchmark models (top-right and bottom panels). These results, along with the more pronounced drop in leverage, are consistent with a house wealth effect. Because households are wealthier with higher house prices, they pay down debt and increase consumption and housing expenditures.

The responsiveness of consumption and the lack of responsiveness of housing expenditures to aggregate fluctuations in both the benchmark and adaptive learning versions of the model contradict consumption smoothing motives. Kaplan and Violante (2014) reconcile a similar result by noting that adjustments to goods consumption are less costly than adjustments to housing expenditures unless the underlying income shock is substantial. Agents thus prefer to maintain housing as a buffer against income movements instead of reducing consumption volatility. Introducing a savings instrument and developing a more detailed housing market may help discipline these mismatched responses to aggregate shocks.

6 Extension: Learning Sensitivity and Regional Variation

To assess the sensitivity of house prices and other model results to the adaptive learning parameters, I adjust the learning gain g_t which regulates the speed at which agents update their beliefs with new information. These sensitivity tests also help assure that the forecasting coefficients converge to an ergodic distribution near their benchmark counterparts with a variety of learning parameters. To provide a context, I ask if different speeds of learning can account for some of the regional variation in the U.S. housing boom. Figure (6) along with Ferreira and Gyourko (2017) and Charles et al. (2015) show that the housing boom varied in timing and magnitude across MSAs. Phoenix, AZ and Cleveland, OH have similar growth in house prices until late 2004 when house prices suddenly pick up in Phoenix but not Cleveland.

Empirical estimates of adaptive learning coefficients and transaction level expected capital gains motivate the hypothesis that a low gain parameter corresponding to slow information updating characterizes a no boom MSA like Cleveland and a high gain parameter with quick information updating explains a boom MSA like Phoenix. The relatively weaker long term local economic outlook in Cleveland makes agents more likely to see positive forecast errors as transitory and thus assign lower weight to them in subsequent forecasts. In Phoenix, however, the stronger long term local economic outlook results in forecast errors being perceived as signals of a structural shift in house prices.[41]

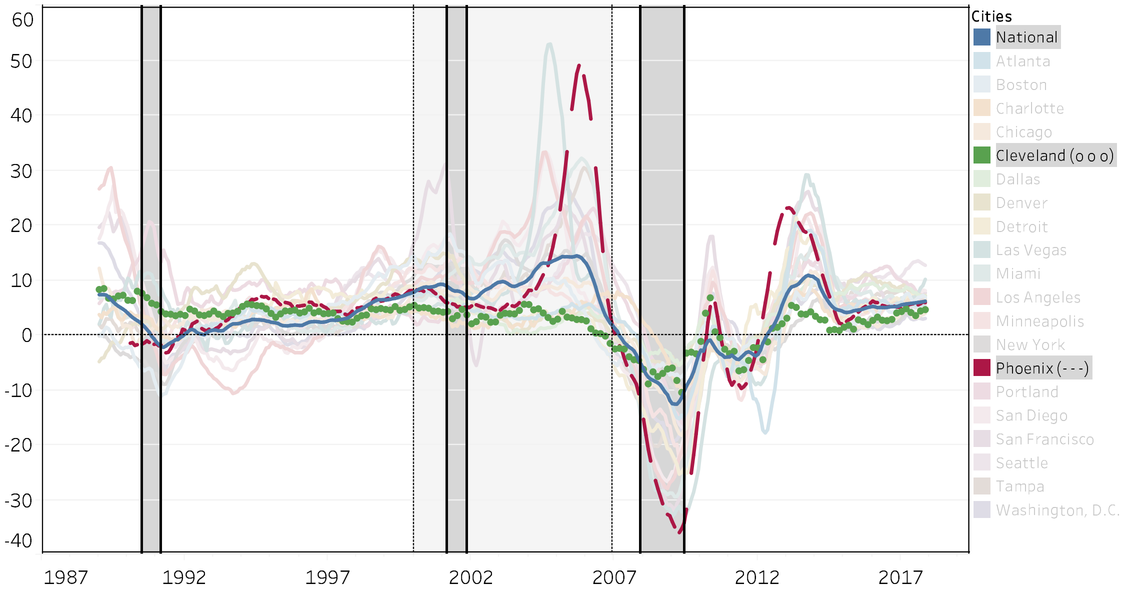

Figure 6: Year-over-year house price growth by select MSA, monthly frequency. Source: Corelogic/Case-Shiller accessed via Bloomberg.

Figure (7) shows estimates of an AR(1) process using a recursive least squares adaptive learning filter shown in equations (12)-(13) on year-over-year house price data for Phoenix, AZ and Cleveland, OH. Rather than estimate the constant gain learning parameter for each series which would be problematic with the short sample of MSA-level house price data, I simply run a time-varying regression with a training sample from April, 1991 to December, 1997 with learning starting in January, 1998 so that $g_t = 1/(t+t_{\{Apr.\,1991\}})$.[42] The persistence of house price growth is relatively constant for both Phoenix and Cleveland in the early stages of the housing boom. Starting in 2004, the estimated persistence for Phoenix (left panel) is rapidly revised upward in conjunction with a large and positive forecast error (right panel). Even though forecast errors are sometimes positive, the estimated persistence for Cleveland (left panel) remains constant until the end of the boom when house prices begin to decline. While the forecast errors for Phoenix coincide with large movements in house prices, the forecast errors for Cleveland do not which supports the hypothesis that beliefs in

[41]Factors contributing to these regional differences in economic outlooks include population growth which is high in Phoenix and low in Cleveland, the unemployment rate, and GDP.

[42]I have experimented by taking out and including the various recessionary periods as well as the start date of the adaptive learning filter and found little change to these estimates.

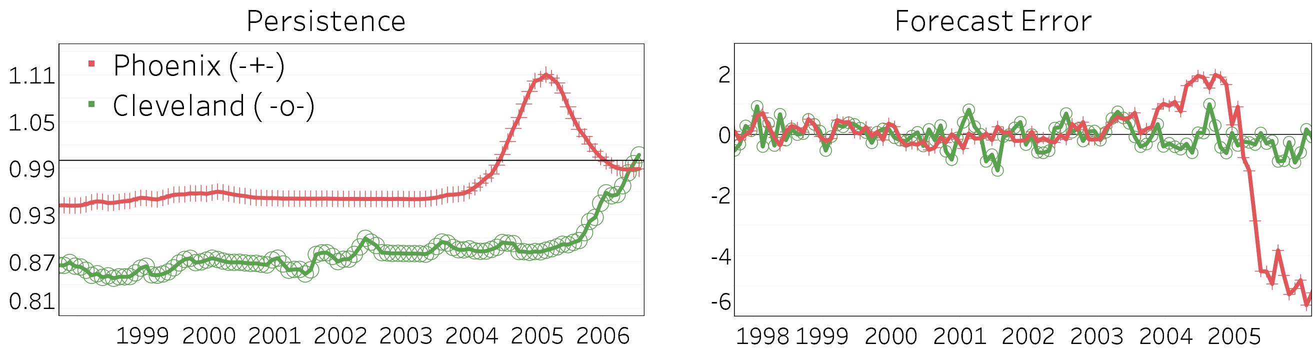

Figure 7: AR(1) estimates of year-over-year house price growth using recursive least squares adaptive learning. Left panel: persistence of house price growth for Phoenix, AZ and Cleveland, OH. Right panel: forecast error for Phoenix, AZ and Cleveland, OH. Source: Corelogic/Case-Shiller accessed via Bloomberg.

Phoenix are more aggressively updated with new information and beliefs in Cleveland are less aggressively updated.

To provide additional evidence of higher expected house price growth in Phoenix relative to Cleveland, I follow the methodology of Landvoigt et al. (2015) and estimate expected capital gains using transaction level data on repeat housing sales.[43] Figure (8) shows that the expected levels of house price growth in Cleveland and Phoenix are consistent with the MSA-level adaptive learning estimates in figure (7). Both MSAs experience positive expected capital gains throughout most of the housing boom with a notable increase in Phoenix in 2005 at the height of the local boom in house prices. In contrast, expected capital gains in Cleveland never experience a large uptick but adjust downward starting in 2006.

The estimates of adaptive learning forecast coefficients and transaction level capital gains both suggest that forecasts of house price growth in Phoenix were more likely to be revised with incoming data than those in Cleveland. These results then inform the sensitivity tests of the model where I adjust the adaptive learning gain parameter but keep all other model parameters the same. Although adjustments to other model parameters would help capture some of differences between the local economies of Phoenix and Cleveland, keeping all parameters the same allows for a cleaner assessment of the effect of learning speeds on model results.[44]

[43]See Appendix (D) for details on the data set, specification, and GMM estimation.

[44]Solving a model with migration across multiple regions is beyond the scope of this paper and for this reason I compare the two regions by solving different versions of an aggregate model. I could also vary house price elasticity $\alpha/(1-\alpha)$ to match the estimates of Saiz (2010) for Phoenix and Cleveland. Although the share of employment in the construction sector is relatively higher in Phoenix, construction is still a small amount of total employment. Empirically plausible construction employment would likely do little to alter the results.

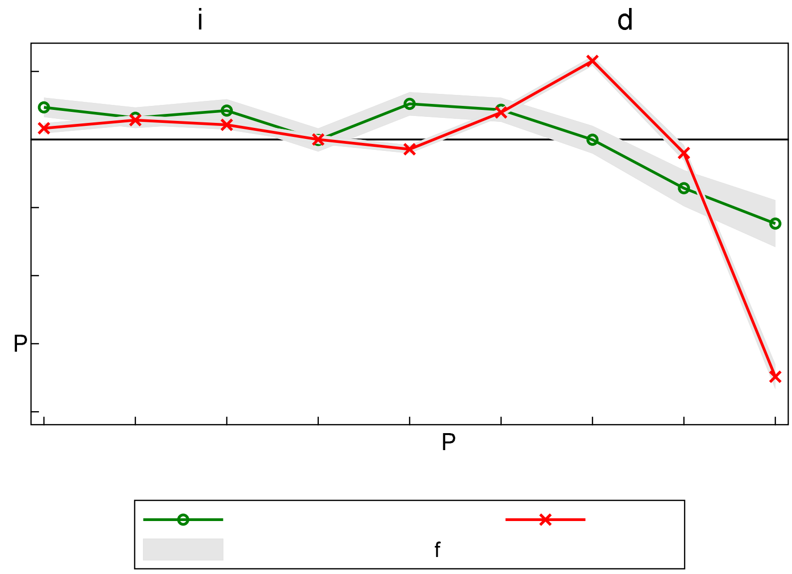

Figure 8: Estimated level of expected capital gains for repeat sales of single family homes in Cleveland, OH and Phoenix, AZ following the methodology of Landvoigt et al. (2015). The data consist of 48,968 repeat sales in Cleveland, OH and 148,842 repeat sales in Phoenix, AZ.

The results in figure (9) compare the path of house prices with the gain parameters set at the lowest and highest values within the established range. The left panel shows model simulations with a low gain parameter ($g_t = 0.002$) and year-over-year house price growth for Cleveland. Although the units of the model and data are not quite comparable, the exercise suggests that model simulations from both the benchmark and learning versions *over* predict empirical house price growth in Cleveland. With the small learning gain, house prices rise by about 7% which is lower than the 10% increase generated by the learning gain in the main text ($g_t = 0.02$) but higher than the 5.5% increase of the benchmark model. These results suggest that even with a slight gain parameter, house prices are still relatively higher with adaptive learning than without. Consequently, some updating of beliefs occurs even if forecast errors receive a relatively low weight when forming beliefs.

The right panel shows model simulations with the gain parameter set at the highest value in the accepted range ($g_t = 0.062$) and empirical year-over-year house price growth for Phoenix, AZ. The larger gain results in an increase in house prices around 11% which is only slightly higher than the 10% generated with the baseline gain of $g_t = 0.02$. Model simulated house prices initially overshoot house price growth in Phoenix and fail to capture the massive run up in the final years of the boom. Some other mechanism not modeled in

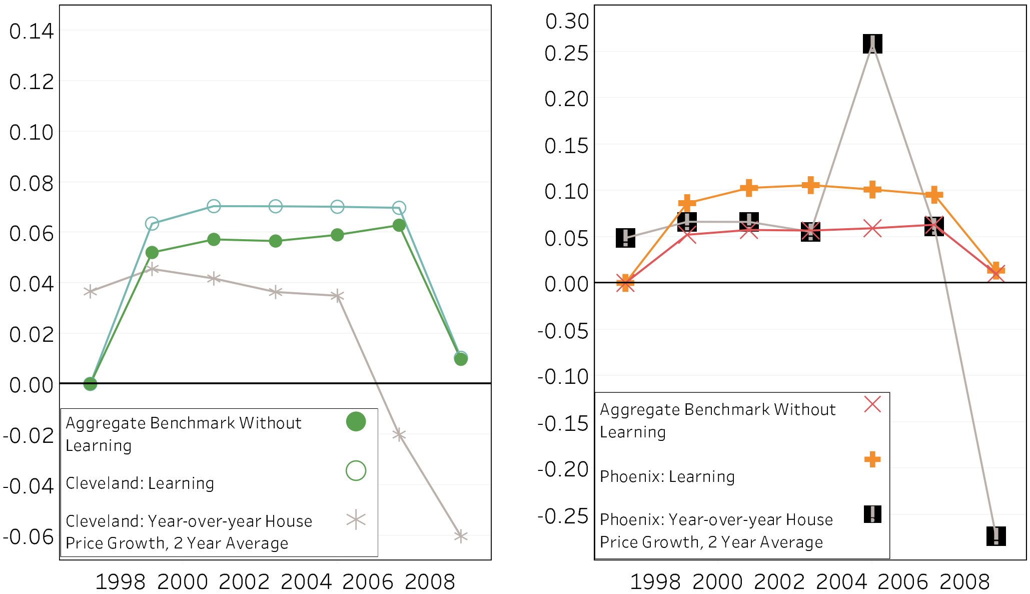

Figure 9: House prices during the housing boom, adaptive learning with various gain parameters. All model values are expressed relative to their 1997 value minus 1, data are expressed in growth rates.

this paper may better account for the massive run up in house growth in Phoenix.[45]

Overall, the results with smaller and larger learning gains suggest that learning speeds can help account for some variation in house prices. Relative to the baseline gain of $g_t = 0.02$, a larger gain results in marginally higher house prices and a smaller gain results in house prices that are closer to but still higher than the benchmark model. Allowing for any adjustments to beliefs thus generates house prices that are higher than the benchmark model without learning.

7 Conclusion

This paper address *why* beliefs about future house prices shifted in the late 1990s to push up house prices throughout the 1998-2006 U.S. housing boom. It postulates that there was a lack of historical precedent for forecasting house prices in a state of the economy with high productivity and loose credit conditions. Agents are thus more likely to perceive forecast errors as permanent shifts in house prices and under predict the extent to which house prices mean revert. The resulting optimistic beliefs help generate 20% of the empirical volatility of

[45]Nathanson and Zwick (2018) argue that land speculation accounts for the similar sized boom in house prices in Las Vegas. Chinco and Mayer (2016) suggest that a large influx of out of town speculators during the height of the boom pushed up house prices in Phoenix. Ben-David et al. (2019) find that shocks to expectations are the primary driver of house prices in boom states.

house prices observed throughout the housing boom which improves upon the 5% generated from a baseline model without learning. These results suggest that incomplete information may be key to understanding movements in beliefs and hence house prices.

This paper embeds adaptive learning about future house prices into a general equilibrium model with incomplete markets and aggregate risk. Aggregate fluctuations in productivity and credit conditions capture the nation-wide shift in economic conditions in the late 1990s and trigger adaptive learning about house price forecasts. By linking optimistic beliefs and loose credit conditions, this framework complements growing efforts to unify the most common explanations of the U.S. housing boom.

Assessing if expectations about future house prices actually correspond to a constant gain adaptive learning mechanism is an important issue to be addressed in future work. Using an information survey, Coibion et al. (2018) also find that firms update inflation expectations in a Bayesian manner when presented new information. Bayesian learning allows agents to internalize that they are learning which improves upon the recursive adaptive learning assumption that current period beliefs will be in place forever [Cogley and Sargent (2008)]. Adam and Marcet (2011a) and Adam et al. (2017) show how Bayesian learning can generate boom-bust episodes in asset markets suggesting that this approach may be useful for exploring the role of beliefs in the U.S. housing boom. Similar approaches relying on Kalman filtering techniques assume that forecasting parameters are truly time varying in contrast to the recursive adaptive learning approach where they are time-invariant but unknown. Because these more sophisticated updating procedures introduce additional layers of complexity, first understanding model dynamics with a simpler learning mechanism is an important base to build upon.

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A Appendix: Recursive Problems

A.1 Recursive Households' Problem

Households solve the value function V_j:

$$V_j(b, h, \epsilon; \mu, Z) = \max_{\{b', h', c\}} \left\{ U_j(c, h') + \beta \mathbb{E}_{Z', \epsilon' | Z, \epsilon}[V_{j+1}(b', h', \epsilon'; \mu', Z')] \right\} \tag{14}$$

$$\begin{aligned}
s.to. \quad c =& y - \mathcal{T}(y) + p(\mu, Z)[h(1 - \delta_h - \tau_h - \mathbb{1}_{h' \neq h}\kappa)h - h'] + q_j(\mu, Z)b' - b \\
b' \leq& \theta^{LTV} p(\mu, Z)h' \\
0 \leq& b', h', c \\
\mu' =& \Gamma_\mu(\mu; Z, Z') \\
Z' \sim& \Gamma_Z(Z) \\
\epsilon_{j+1} \sim& \Upsilon_{j+1|j}(\epsilon_j)
\end{aligned}$$

When $j = J$ and households exit economic life, the continuation value has the bequest motive: $V_{J+1} = v(\flat)$ where $\flat = p'(\mu'; Z, Z')[h'(1 - \delta_h - \tau_h - \mathbb{1}_{h' \neq h}\kappa)]$

The households' problem yields the following objective function with Lagrange multiplier λ on the budget constraint and $U_c \lambda^{LTV}$ on the loan to value constraint, and $U_c \lambda^S$ on the short sale constraint for loans. The short sale constraints for housing h' and consumption c will not bind due to the Inada conditions on the functional form of the utility function. The Lagrangian is:

$$\begin{aligned}
V_j(b, h, \epsilon; \mu, Z) = \max_{c, b', h'} \Bigg\{ & U_j(c, h') + \beta \mathbb{E}_{Z', \epsilon' | Z, \epsilon}[V_{j+1}(b', h', \epsilon'; \mu', Z')] \\
\ldots - \mathbb{E}_{Z', \epsilon' | Z, \epsilon} \Bigg[& \lambda(c - y + \mathcal{T}(y) - p(\mu, Z)[(1 - \delta_h - \tau_h - \mathbb{1}_{h' \neq h}\kappa)h - h'] + b - q_j(\mu, Z)b') \ldots \\
\ldots - U_c \lambda^{LTV}(b' - \theta^{LTV} p(\mu, Z)h') - U_c \lambda^S(-b')] \Bigg] \Bigg\}
\end{aligned}$$

The first order and envelope conditions are:

$$\text{FOC}_c: \quad U_c = \lambda \tag{15}$$

$$\text{FOC}_{b'}: \quad \beta\mathbb{E}_{Z',\epsilon'|Z,\epsilon}\left[\frac{\partial V_{j+1}(b',h',\epsilon';\mu',Z')}{\partial b'}\right] + q_j(\mu,Z)\lambda = U_c[\lambda^S - \lambda^{LTV}] \tag{16}$$

$$\text{FOC}_{h'}: \quad \lambda p(\mu,Z) - U_c\lambda^{LTV}\theta^{LTV}p(\mu,Z) = U_{h'} + \mathbb{E}_{Z',\epsilon'|Z,\epsilon}\left[\frac{\partial V_{j+1}(b',h',\epsilon';\mu',Z')}{\partial h'}\right] \tag{17}$$

$$\text{Envelope}_b: \quad \frac{\partial V_j(b,h,\epsilon;\mu,Z)}{\partial b} = \lambda \tag{18}$$

$$\text{Envelope}_h: \quad \frac{\partial V_j(b,h,\epsilon;\mu,Z)}{\partial h} = \lambda p(\mu,Z)(1-\delta_h-\tau_h-\mathbb{1}_{h'\neq h}\kappa) \tag{19}$$

Iterating forward the envelope condition for h given by equation (19) and substituting it into the first order condition for housing h' in equation (17) yields an expression for housing demand:

$$p(\mu,Z)(1-\theta^{LTV}\lambda^{LTV}) = \frac{U_{h'}}{\lambda} + \frac{\beta}{\lambda}\mathbb{E}_{Z',\epsilon'|Z,\epsilon}\Big[\lambda' p'(\mu',Z')(1-\delta_h-\tau_h-\mathbb{1}_{h'\neq h}\kappa)\Big]$$

Letting $\mathcal{M}' \equiv \lambda'/\lambda = (U_{c'}/U_c)$ and $\frac{U_{h'}}{\lambda} = \frac{U_{h'}}{\lambda'}\frac{\lambda'}{\lambda} = \frac{U_{h'}}{\lambda'}\mathcal{M}' = \mathcal{H}'\mathcal{M}'$, housing demand can be written as:

$$p(\mu,Z)(1-\theta\lambda^{LTV}) = \underbrace{\beta\mathbb{E}_{Z',\epsilon'|Z,\epsilon}[\mathcal{M}'\mathcal{H}']}_{\text{Intrinsic value}} + \underbrace{\mathbb{E}_{Z',\epsilon'|Z,\epsilon}[\mathcal{M}'p'(\mu',Z')(1-\delta_h-\tau_h-\mathbb{1}_{h'\neq h}\kappa)]}_{\text{Expected future price}} \tag{20}$$

The loan demand of each individual household can be obtained by iterating forward the envelope condition b in equation (18) and substituting into the first order condition for b' in equation (16):

$$q_j(\mu,Z) = \beta\mathbb{E}_{Z',\epsilon'|Z,\epsilon}[\mathcal{M}'] + \lambda^{LTV} - \lambda^S \tag{21}$$

A.2 Recursive Final Goods and Construction Firms' Problems

V_c denotes the value function of firms in the final goods sector.

$$\begin{aligned} V_c(N_c;\mu,Z) =& \max_{N_c}\{Y - w(\mu,Z)N_c\} \\ s.to. \qquad Y =& \Theta(Z)N_c \\ 0 \leq& N_c \\ \mu' =& \Gamma_\mu(\mu;Z,Z') \end{aligned}$$

Because final goods consumption C_b is the numeraire, the price for final goods has been normalized to one. Taking the first order condition with respect to labor N_c pins down the equilibrium wage:

$$w(\mu,Z) = \Theta(Z)$$

V_h denotes the value function of housing construction firms.

$$\begin{aligned} V_h(N_h;\mu,Z) =& \max_{N_h}\{p(\mu,Z)H_h - w(\mu,Z)N_h\} \\ s.to. \qquad H_h =& [\Theta(Z)N_h]^\alpha \bar{L}^{1-\alpha} \\ 0 \leq& N_h \\ \mu' =& \Gamma_\mu(\mu;Z,Z') \end{aligned}$$

Taking the first order condition with respect to labor N_h and using the above equilibrium expression for wage $w(\mu,Z)$ yields:

$$\alpha\Theta(Z)p(\mu,Z)[\Theta(Z)N_h]^{\alpha-1}\bar{L}^{1-\alpha} = \Theta(Z)$$

Canceling $\Theta(Z)$ from both sides and re-arranging:

$$[\Theta(Z)N_h]^{\alpha-1}\bar{L}^{1-\alpha} = [\alpha p(\mu,Z)]^{-1}$$

Taking both sides to the $\frac{\alpha}{\alpha-1}$ power:

$$[\Theta(Z)N_h]^{\alpha}\bar{L}^{-\alpha} = [\alpha p(\mu,Z)]^{\frac{\alpha}{1-\alpha}}$$

Multiplying both sides by $\bar{L}$ delivers the expression for housing supply:

$$\underbrace{[\Theta(Z)N_h]^{\alpha}\bar{L}^{1-\alpha}}_{\equiv H_h} = [\alpha p(\mu,Z)]^{\frac{\alpha}{1-\alpha}}\bar{L}$$

B Appendix: Computational Algorithm

The algorithm is adapted from Kaplan et al. (2019) and Favilukis et al. (2017) who use a variation of the Krusell and Smith (1998) algorithm. Agents keep track of house prices via a log-linear forecasting rule for each combination of current and future aggregate states $\mathcal{Z} = \{Z, Z'\}$ instead of the entire distribution over individual states μ and its law of motion $\Gamma_\mu(\mu; \mathcal{Z})$.

$$p'(p(Z); \mathcal{Z}) = a^0_{\mathcal{Z}} + a^1_{\mathcal{Z}} \log p(Z) \qquad \Longleftrightarrow \qquad \mu' = \Gamma_\mu(\mu; \mathcal{Z})$$

1. Define grids over ages $j = 1, \ldots, J$, loans b, loan choices, b', housing h, housing expenditures h', aggregate productivity $\Theta(Z) \in \{\Theta(Z_{high}), \Theta(Z_{low})\}$, aggregate loan-to-value constraints $\theta^{LTV}(Z) \in \{\theta^{LTV}(Z_{high}), \theta^{LTV}(Z_{low})\}$, individual income ϵ, and house prices p.

2. Define coefficients:

   - Benchmark: guess fixed coefficients $a^0_{\mathcal{Z}}$ and $a^1_{\mathcal{Z}}$ for each $\mathcal{Z}$ for a total of $\#Z^2 = 4$ vectors of coefficients to forecast next period house prices

   $$\log p'(p(Z); \mathcal{Z}) = a^0_{\mathcal{Z}} + a^1_{\mathcal{Z}} \log p(Z) \tag{22}$$

   - Adaptive learning: define additional grids over future house prices in each future state $\{p'(Z'_{low}), p'(Z'_{high})\}$ for a total of $\#Z' = 2$ additional grids.

3. With a risk-neutral lending sector, the loan price $q_j(p(Z), Z)$ is pinned down by the lenders' pricing function: $q_j(p(Z), Z) = 1/(1 + r_b)$

4. Solve the individual households' problem at each point on the house price grid p. I use value function with iteration grid search and trilinear interpolation over $p'(p(Z); \mathcal{Z})$, h', and b'.[46]

5. Simulate a long time series of aggregate states Z_t for $t = 1, \ldots, T$ where $T = 2,000$ with a burn in period of 500.

6. Fix an initial distribution of loans, housing, idiosyncratic incomes, and ages $\mu_t \in \mathcal{B} \times \mathcal{H} \times \mathcal{E} \times \mathcal{J}$ for $t = 1$ and $N = 15,000$ households. $\mathcal{B}$ is the set of all possible beginning of period loan realizations, $\mathcal{H}$ is the set of all possible beginning of period housing realizations, $\mathcal{E}$ is the set of individual income realizations, and $\mathcal{J}$ is the set of all possible beginning of period age realizations.

[46] Berger and Vavra (2015, footnote 54) note that linear interpolation has speed advantages relative to cubic splines in their model that also has expenditures on durables.

Some notes on the initial distribution of individual states:

(a) Initial household loan endowments are set to zero for all households $b_{i,1} = 0$

(b) Initial housing endowments are draw from a normal distribution $h_{i,1} \sim \mathcal{N}(0,1)$. Each $h_{i,1}$ is re-scaled to lie in the bounds of housing supply $h_{i,1} \in [\min H'_h, \max H'_h]$
$h_{b,1} = \max H'_h + \frac{|\lfloor \min h_1 \rfloor| + h_{b,1}}{|\lfloor \min h_1 \rfloor| + \lceil \max h_1 \rceil}(\max H'_h - \min H'_h)$

(c) Initial household ages drawn from a uniform distribution[47] $j \sim [1, \ldots, J]$

(d) After household i reaches the final period of economic life such that $j_{i,t} = J$, a new household replaces them with $j_{i,t+1} = 1$. New households begin economic life with no debt $b_{i,t} = 0$ and inherit the bequested housing stock of the household they are replacing.

7. Compute period $t = 1$ coefficients to pin down $p'(p(Z); \mathcal{Z})$

- Benchmark: coefficients are already fixed at the beginning of the problem
- Adaptive learning: period t coefficients will be calculated at each point on the house price grid p because equilibrium house prices $p_t(Z_t) = y_{\mathcal{Z}_{t-1}}$ are not yet determined. Agents need two vectors of coefficients $\boldsymbol{a}_{Z_t, Z'_{low}}$ and $\boldsymbol{a}_{Z_t, Z'_{high}}$ because the value of Z' is also not yet known. For this reason, adaptive learning requires as many additional grids as there are aggregate states.

$$\begin{aligned} \boldsymbol{a}_{\mathcal{Z}_t}(p) =& \boldsymbol{a}_{\mathcal{Z}_{t-1}} + g_t R^{-1}_{\mathcal{Z}_t} \boldsymbol{x}_{\mathcal{Z}_{t-1}} (y_{\mathcal{Z}_{t-1}}(p) - \boldsymbol{x}'_{\mathcal{Z}_{t-1}} \boldsymbol{a}_{\mathcal{Z}_{t-1}}) \\ R_{\mathcal{Z}_t} =& R_{\mathcal{Z}_{t-1}} + g_t (\boldsymbol{x}_{\mathcal{Z}_{t-1}} \boldsymbol{x}'_{\mathcal{Z}_{t-1}} - R_{\mathcal{Z}_{t-1}}) \end{aligned}$$

Setting $g_t = 0$ is equivalent to solving the benchmark model with coefficients set at the initial coefficients $\boldsymbol{a}_{\mathcal{Z}_0}$. For $t < t_{burn}$, I set $g_t = 0$ to obtain the adaptive learning training sample. When $t = t_{learn}$ and learning begins, such that $g_t > 0$ and $\boldsymbol{a}_{\mathcal{Z}_{t-1}} = \boldsymbol{a}_{\mathcal{Z}_0}$. The covariance matrix then becomes $(t_{learn} - t_{burn})^{-1} R_{\mathcal{Z}_t} = \left(\sum_{t=burn}^{learn} \boldsymbol{x}_{\mathcal{Z}_t} \boldsymbol{x}'_{\mathcal{Z}_t} \right)$.

8. At $t = 1$, compute the aggregate demand schedule for housing at each point on the house price grid p given Z_t, μ_t, and the policy functions from the individual households' problem.

[47] This assumption follows Kaplan and Violante (2014). Matching the distribution of ages in the 1998 SCF would introduce generational demographic changes into equilibrium dynamics which is beyond the scope of this paper.

- Benchmark: interpolate over $b_{i,t}$ and $h_{i,t}$

$$H_{j,t+1}(p, Z_t) = \frac{1}{N} \sum_{i=1}^{N} h'(b_{i,t}, h_{i,t}, \epsilon_{i,t}, j_{i,t}; p, Z_t)$$

- Adaptive learning: interpolate over $b_{i,t}$, $h_{i,t}$, $p'(Z'_{low})$, and $p'(Z'_{high})$

$$H_{j,t+1}(p, Z_t) = \frac{1}{N} \sum_{i=1}^{N} h'(b_{i,t}, h_{i,t}, \epsilon_{i,t}, j_{i,t}; p, Z_t, p'(Z'_{low}), p'(Z'_{high}))$$

Compute the excess demand for aggregate housing at each point on the housing grid p and interpolate the resulting excess demand function to find $p_t^*(Z_t)$ such that the housing market clears:

$$H_{j,t+1}(p_t^*(Z_t), Z_t) - (1 - \delta_h) H_{j,t} = H_{h,t}(p_t^*(Z_t), Z_t)$$

9. Find the equilibrium quantities of aggregate loans $B^*_{j,t+1}$ and aggregate housing $H^*_{j,t+1}$ by interpolating the individual policy functions b' and h' at $p_t^*(Z_t)$.

   - Benchmark: re-aggregate across all households

$$B^*_{j,t+1}(p_t^*(Z_t), Z_t) = \frac{1}{N} \sum_{i=1}^{N} b'(b_{i,t}, h_{i,t}, \epsilon_{i,t}, j_{i,t}; p_t^*(Z_t), Z_t)$$
$$H^*_{j,t+1}(p_t^*(Z_t), Z_t) = \frac{1}{N} \sum_{i=1}^{N} h'(b_{i,t}, h_{i,t}, \epsilon_{i,t}, j_{i,t}; p_t^*(Z_t), Z_t)$$

   - Adaptive learning: re-compute coefficients at the equilibrium price

$$\boldsymbol{a}_{\mathcal{Z}_t}(p_t^*(Z_t)) = \boldsymbol{a}_{\mathcal{Z}_{t-1}} + g_t R_{\mathcal{Z}_t}^{-1} \boldsymbol{x}_{\mathcal{Z}_{t-1}} (y_{\mathcal{Z}_{t-1}}(p_t^*(Z_t)) - \boldsymbol{x}'_{\mathcal{Z}_{t-1}} \boldsymbol{a}_{\mathcal{Z}_{t-1}})$$

   Pin down equilibrium house price forecasts

$$\boldsymbol{p'}(p_t^*(Z_t)) = \begin{pmatrix} \exp\{a^0_{Z_t, Z'_{low}} + a^1_{Z_t, Z'_{low}} \log(p_t^*(Z_t))\} \\ \exp\{a^0_{Z_t, Z'_{high}} + a^1_{Z_t, Z'_{high}} \log(p_t^*(Z_t))\} \end{pmatrix}$$

   And re-aggregate across all households

$$B^*_{j,t+1}(p_t^*(Z_t), Z_t, \boldsymbol{p'}(p_t^*(Z_t))) = \frac{1}{N} \sum_{i=1}^{N} b'(b_{i,t}, h_{i,t}, \epsilon_{i,t}, j_{i,t}; p_t^*(Z_t), Z_t, \boldsymbol{p'}(p_t^*(Z_t)))$$
$$H^*_{j,t+1}(p_t^*(Z_t), Z_t, \boldsymbol{p'}(p_t^*(Z_t))) = \frac{1}{N} \sum_{i=1}^{N} h'(b_{i,t}, h_{i,t}, \epsilon_{i,t}, j_{i,t}; p_t^*(Z_t), Z_t, \boldsymbol{p'}(p_t^*(Z_t)))$$

10. Simulate for $t = 1, \ldots, T$ periods by repeating steps (7)-(9).

11. Compare coefficients:

 - Benchmark: partition the time series of market clearing house prices $\{p_t^*(Z_t)\}_{t=1}^T$ by $\mathcal{Z} = \{Z, Z'\}$ to generate $\#Z^2 = 4$ sub-samples. Estimate new forecasting coefficients for each $\mathcal{Z}$ via ordinary least squares regression:

$$\boldsymbol{a}_{\mathcal{Z}}^{new} = \left(\sum_{t=1}^{T} \boldsymbol{x}_{\mathcal{Z}_t} \boldsymbol{x}'_{\mathcal{Z}_t}\right)^{-1} \sum_{t=1}^{T} \boldsymbol{x}_{\mathcal{Z}_t} y_{\mathcal{Z}_t}$$

   Repeat steps (2) - (11) until coefficients converge:

$$\left(a_{\mathcal{Z}_t}^0, a_{\mathcal{Z}_t}^1\right)' \approx \left(a_{\mathcal{Z}_t}^{0,new}, a_{\mathcal{Z}_t}^{1,new}\right)'$$

 - Adaptive learning: adjust the initial coefficients $\boldsymbol{a}_{Z_0}$, the starting time for learning t_{learn}, or the gain parameter g_t and verify that the time series of coefficients $\boldsymbol{a}_{\mathcal{Z}_t}$ computed in step (9) converge to their near-rational benchmarks $\boldsymbol{a}_{\mathcal{Z}}$.

Chipeniuk et al. (2019) propose a solution check called auctioneer iteration to avoid relying on potentially misleading R^2 statistics to gauge the accuracy of the solution method. Their method can be implemented by adding the following steps to the benchmark computational algorithm:

12. Use the converged coefficients $\boldsymbol{a}_{\mathcal{Z}}$ and set the price grid to a subset of the simulated price data, $p =\subset \{p_t^*(Z_t)\}_{t=1}^T$.

13. Repeat steps (1)-(11) to obtain a new set of converged coefficients and a new series of prices $\{p_t^{new}(Z_t)\}_{t=1}^T$

14. Stop if $\{p_t^{new}(Z_t)\}_{t=1}^T \approx \{p_t^*(Z_t)\}_{t=1}^T$, otherwise go to step (1).

C Appendix: Data Definitions

The aggregate data definitions follow those in Appendix E.1 of Kaplan et al. (2019).

- **Consumption**: Quarterly nominal nondurable expenditures (line 8 of NIPA Table 2.3.5 Personal Consumption Expenditures by Major Type of Product) divided by the price index for nondurable consumption (line 8 of NIPA Table 2.3.4. Price Indexes for Personal Consumption Expenditures by Major Type of Product).
- **House Prices**: House price index for the entire United States (Federal Housing Finance Agency downloaded via FRED USSTHPI) divided by the price index for nondurable consumption (line 8 of NIPA Table 2.3.4. Price Indexes for Personal Consumption Expenditures by Major Type of Product).
- **Leverage**: Flow of Funds Table B.101 Balance Sheet of Households and Nonprofit Organizations. Home mortgage liabilities (FL163165505) divided by the sum of household owner occupied housing at market value (LM155035015) and nonprofit organization real estate at Market Value (LM165035005)

House prices for Phoenix and Cleveland are year-over-year percentage change of the Corelogic/Case Shiller monthly house price indices.

D Appendix: Transaction Level Housing Data

This appendix details the data cleaning procedures used by Landvoigt et al. (2015) to obtain market transactions of single family homes and replicates their expected capital gains estimates in San Diego county from 1999 to 2008. Table (5) and figure (10) show that I replicate their estimates with only some slight discrepancies likely attributable to differing underlying data sources.

While Landvoigt et al. (2015) use transaction data from Trulia, my data set consists of Zillow Ztrax assessment and transaction data accessed through the Bureau of Economic Analysis.[48] I modify their methodology used for San Diego slightly to create datasets of market transactions of single family homes for zip codes in the Phoenix, AZ and Cleveland, OH metropolitan statistical areas.

The housing transactions are cleaned according to three criteria: deed type, buyer or house characteristics, and outliers. Individual properties are denoted by the `rowid` variable and years are denoted by the year in the `recordingdate` variable. A repeat sale is a pair of sales of the same property.

First, deeds (`documenttypes`) that are not typically used in arms length transfers of homes –intrafamily deeds (`INTR`), foreclosures (`SHDE`)– are dropped. Landvoigt et al. (2015) keep only grant deeds (`GRDE`), condo deeds (`CDDE`), corporate deeds (`CPDE`), and individual deeds (`IDDE`). For Cleveland and Phoenix, the list of deeds denoting arms length transactions is more exhaustive.[49] Other non "arms length" deeds are also dropped as well as those indicating that a sale is for only a share of a house.[50]

Second, deeds are dropped based on buyer or house characteristics. Deeds without a latitude (`propertyaddresslatitude`) or longitude (`propertyaddresslongitude`) are dropped as are deeds that transfer multiple parcels as identified by the APN number (`alternateparcelnumber`!=0). Second homes (`secondhomeriderflag`==Y) and trailers are dropped (`propertylandusecode`==MB). Foreclosed properties (`foreclosure`!=.) are also dropped. Buyers (`buyercode_1` or `buyercode_2`) that are not a couple or single per-

[48]The Ztrax transaction and assessment data sets are merged by the variable `rowid`.

[49]Entries for the `documenttype` variable that are dropped include conservator's deed (`CVDE`), deed in lieu of foreclosure (`DELU`), gift deed (`GFDE`), intrafamily transfer (`INTR`), partnership deed (`PTDE`), personal representative's deed (`PRDE`), sheriff's deed (`SHDE`), trustee's deed (`TRFC`). Deeds that are kept have values for `documenttype` that include administrator's deed (`ADDE`), agreement of sale (`AGSL`), bargain and sale deed (`BSDE`), condominium deed (`CPDE`), court order/action (`COCA`), corporation deed (`CPDE`), correction deed (`CRDE`), deed (`DEED`), fiduciary deed (`FDDE`), guardian's deed (`GDDE`), grant deed (`GRDE`), individual deed (`IDDE`), joint tenancy deed (`JTDE`), land contract (`LDCT`), other (`OTHR`), quitclaim deed (`QCDE`), re-recorded deed (`RRDE`), tax deed (`TXDE`), warranty deed (`WRDE`).

[50]Partial sales are denoted as `partialinteresttransferstndcode`==Y or `partialinteresttransferpercent`> 0.

son are dropped eliminating buyers that are a corporation or partnership (CO,PT), a trust (FT,IT,LV,RL,RT,TE), or a beneficiary (BF).[51]

Single family homes are denoted by the propertylanduse variable from the assessment data[52] and observations kept are equal to single family residences (RR101), condominiums (RR106)[53], cooperatives (RR107), row houses (RR108), planned unit developments(RR109), bungalows (RR113), zero lot lines (RR114), manufactured, modular and prefabricated homes (RR115), patio homes (RR116), garden homes (RR119), landominiums (RR120), and inferred single family homes (RR999).[54]

Lastly, to control for outliers, transactions with prices below $15,000 (salespriceamount), combined loan-to-value ratios (first plus second mortgage) above 120 percent[55], and annualized capital gains above 50% are dropped.[56] To avoid the influence of house flipping, all pairs of sales that are less than 180 days apart[57] are dropped. Some properties that were sold twice in the same year, but more than 180 days apart remain in the sample. If the property was only sold twice in the same year, then both transactions are dropped. If the property was sold in another year then the earlier sale date in the year is dropped. Given that house prices are rising through this period, keeping the later sale should bias capital gains downward. Given the high house price growth observed at the MSA-level in Phoenix, robustness checks were run to increase the annualized capital gain threshold from 50% to 60% and 70% with little change to the estimates. Similarly, second family homes were included in a separate robustness check with littler alteration to the estimates.

Following Landvoigt et al. (2015) expected capital gains of house i at time t vary by their

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[51] This is achieved by keeping observations with buyercode equal to domestic partners (DP), formerly known as (FK), her husband (HH), husband and wife (HW), individual (ID), married man (MM), minor (MN), married person (MP), married woman (MW), single man (SM), single person (SP), single woman (SW), unmarried man (UM), unmarried woman (UW), widowed (WW) and dropping those with buyercode equal to affiant (AF), borrower or trustor in default (BR), estate (ES), executor (EX), government (borough, city, village, etc.) (GV), surviving joint tenant (SJ), personal representative (PR), agent (AG), not provided (NP).

[52] The propertylanduse variable form the assessment data and propertyusestndcode variable from the transaction data are mostly but not always consistent. I use the assessment data as the main variable to denote property use because it has less blank observations and is more stable across time.

[53] Although it may be debatable as to whether or not condominiums should be included as single family homes, If I drop condominiums the number of observations drops to 53,463 from 84,076 for San Diego county and the estimates of expected capital gains differ to a larger extent from those of Landvoigt et al. (2015).

[54] Dropped observations have a propertylanduse variable from the assessment data equal to rural residences including farms/productive land (RR102), mobile homes (RR103), residential common areas (RR110), time shares (RR111), seasonal, cabin, vacation residences (RR112), residential parking garages (RR117), and other improvements (RR118). Observations from the transaction data are also dropped as well including those where the propertyusestndcode variable from the transaction data equals agricultural (AG), apartment (AP), commercial (CM), mobile homes (MB), mixed use (MX), unimproved (UL), multifamily (MF).

[55] [loanamount+secondmtgl]/salespriceaamount

[56] ((salespriceamount[_n]/salespriceamount[_n-1])**(365/(recordingdate[_n]-recordingdate[_n-1]))-1)> 0.5

[57] recordingdate[_n]-recordingdate[_n-1]< 180

current price p_t^i where the idiosyncratic shocks e_{t+1}^i have mean zero and are such that the law of large number holds in the cross section of houses. The coefficient a_t captures the expected capital gains on all houses and the coefficient b_t determines the variation of expected capital gains in the cross section. If $b_t = 0$ then the expected capital gains on all houses is equal to a_t. A negative coefficient $b_t < 0$ suggests that houses of lower quality homes will have higher capital gains on average and vice versa for a positive coefficient.

The statistical model of price changes by quality for property i between dates t and $t+1$ can be written as:

$$\log p_{t+1}^i - \log_t^i = a_t + b_t \log p_t^i + e_{t+1}^i \tag{23}$$

Modifying (23), yields a formula to capture the expected capital gains between years $t+k$ and $t+\ell$.

$$\log p_{t+k}^i - \log_{t+\ell}^i = a_{t+\ell,t+k} + b_{t+\ell,t+k} \log p_{t+\ell}^i + \epsilon_{t+\ell,t+k}^i \tag{24}$$

Where the coefficients $a_{t+\ell,t+k}$ and $b_{t+\ell,t+k}$ are obtained by iterating (23). The parameters a_t and b_t are thus estimated by GMM with the sum of squared prediction errors as objective function, weighted by the inverse of their variance.

	2000	2001	2002	2003	2004	2005	2006	2007
a_t^{LPS}	1.29	1.41	1.30	0.87	0.60	-0.56	-1.09	-3.18
	(0.04)	(0.04)	(0.04)	(0.05)	(0.06)	(0.07)	(0.10)	(0.12)
b_t^{LPS}	-0.093	-0.10	-0.09	-0.05	-0.04	0.04	0.07	0.22
	(0.003)	(0.003)	(0.003)	(0.004)	(0.004)	(0.01)	(0.01)	(0.01)
a_t^{author}	1.04	1.37	1.23	0.71	0.55	-0.33	-1.84	-3.16
	(0.04)	(0.04)	(0.04)	(0.05)	(0.05)	(0.07)	(0.10)	(0.12)
b_t^{author}	-0.07	-0.10	-0.08	-0.04	-0.03	0.02	0.13	0.21
	(0.003)	(0.003)	(0.003)	(0.004)	(0.004)	(0.005)	(0.008)	(0.009)

Table 5: Upper panel contains estimates from Landvoigt et al. (2015) of 70,315 repeat sales in San Diego County during the years 1999-2008 using transaction data from Trulia. The lower panel contains a replication of their estimates using Zillow ZTRAX assessment and transaction data for 84,076 repeat seals in San Diego County. The numbers in parentheses are standard errors.

Table (5) estimates equation (23) for San Diego county and compares the estimates to those of Landvoigt et al. (2015). Although there are some discrepancies among the coefficient estimates of (23), figure (10) suggests that these are relatively small except for the year 2006. Given that the data used by Landvoigt et al. (2015) are from a different data set (Trulia instead of Zillow) it is not surprising that there are some slight discrepancies. My sample is larger than theirs with 84,076 repeat sales compared to their 70,315 which may account for some of the slight variation in coefficients. The estimates of standard errors are largely similar for both samples.

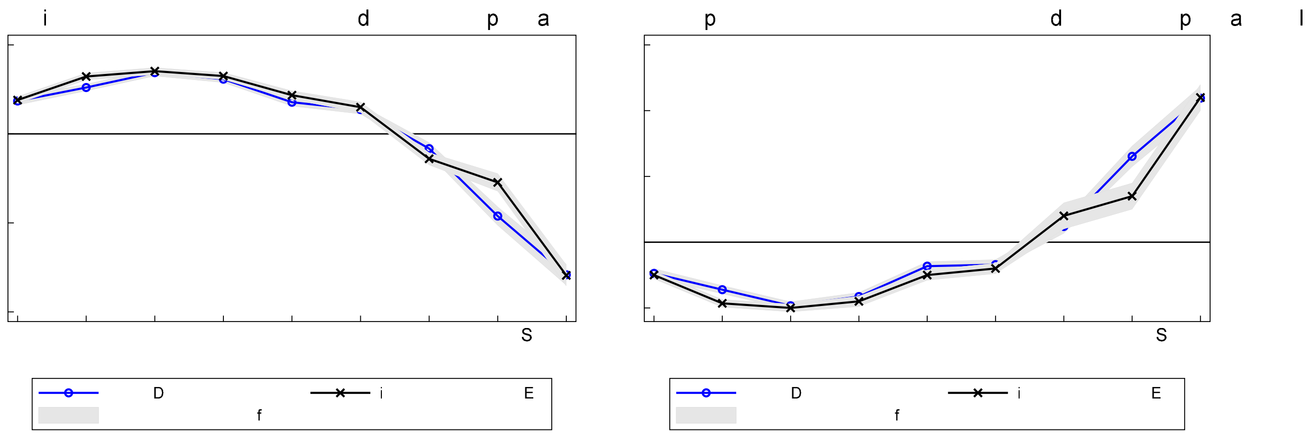

Figure 10: Table (5) in graphical form.

Table (6) details the GMM estimates of (23) for Phoenix and Cleveland. The overall level of capital gains a_t is lower in the boom period for both Phoenix and Cleveland relative to the estimates for San Diego, except for the year 2005 when there is a substantial pick up in Phoenix. With b_t^{PHX} close to zero, all houses are experiencing similar expected capital gains. In contrast, lower valued homes in Cleveland experience relatively higher capital gains $b_t^{CLE} < 0$.

	2000	2001	2002	2003	2004	2005	2006	2007
a_t^{PHX}	0.28	0.22	0.00	-0.14	0.40	1.15	-0.20	-3.48
	(0.04)	(0.04)	(0.04)	(0.03)	(0.03)	(0.04)	(0.06)	(0.09)
b_t^{PHX}	-0.02	-0.01	0.01	0.02	0.01	-0.09	0.01	0.24
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.03)	(0.005)	(0.007)
a_t^{CLE}	0.31	0.42	-0.01	0.52	0.44	-0.00	-0.72	-1.23
	(0.08)	(0.09)	(0.09)	(0.09)	(0.09)	(0.10)	(0.14)	(0.18)
b_t^{CLE}	-0.02	-0.03	0.00	-0.04	-0.03	-0.00	0.05	0.09
	(0.006)	(0.007)	(0.007)	(0.007)	(0.007)	(0.009)	(0.011)	(0.015)

Table 6: Upper panel contains estimates using the same statistical model and data cleaning as Landvoigt et al. (2015) for zip codes in the Phoenix, AZ MSA using 148,842 repeat sales during the years 1999-2008. The lower panel contains estimates for zip codes in the Cleveland, OH MSA using 48,968 repeat seals. The numbers in parentheses are standard errors.