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Simple Monte Carlo in Proton Clouds

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1 INTRODUCTION

Although proton clouds make up only a small fraction of the Interstellar Medium (ISM), they can also be the densest. In this project, our aim is to develop a basic method of how to approach the radiative transfer problem through the cloud. We will do this by explaining our approximations in section 2. and presenting our results in section 3.

2 METHODS

A formal approach to investigating the path traveled by a photon emitted from the center of a proton cloud would encompass a solution to the radiative transfer equation :

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu. \tag{1}$$

The difficulty of this endeavor compels us to a series of approximations. Throughout this project, we will assume a cold, $T = 0K$, 2D cloud of circular shape with a constant number density of n_{e}. The assumption of only cold electrons in our system allows us to examine it by considering only scattering events. Thus, in the absence of emission and absorption, the radiative transfer equation reads:

$$\frac{dI_\nu}{ds} = -\sigma_\nu(I_\nu + J_\nu), \tag{2}$$

where

$$J_\nu = S_\nu = \frac{1}{4\pi}\int I_\nu d\Omega.$$

This, however, is still a difficult problem to solve. Fortunately enough, if we chose to treat the problem in probabilistic terms of individual photons rather than the statistical behavior of a photon beam, we can approximate it by means of random walks.

Thus, our goal is to determine the stochastic behavior of our random walk problem via a Monte Carlo simulation and correlate it to the optical depth from the center of the cloud. We do this by independently sampling the scattering angle θ from a uniform distribution in the range $[0, 2\pi)$ and our selection criterion from the probability distribution $P(x) = e^{-\tau}$. Depending on whether our random number ξ is smaller or greater than P(x) we assume that the photon either exited the cloud or interacted within it at a distance l. This criterion can be intuitively understood by examining the behavior of the exponential by varying τ. An increase in τ yields a smaller value of $e^{-\tau}$, decreasing the number of escaped photons. However, by doing so, we expect a greater number of interactions within the cloud, thus being self-consistent.

We consider various assumptions for this interaction length l, as well as the optical depth τ, and the absorption coefficient ϵ. In our simplest approximation, we assume our interaction length equal to the mean free path $l = \bar{l} = (n_{\mathrm{e}}\sigma_{\mathrm{T}})^{-1}$ and our optical depth as a variable $\tau = \tau_{\mathrm{center}}$, constant throughout the cloud.

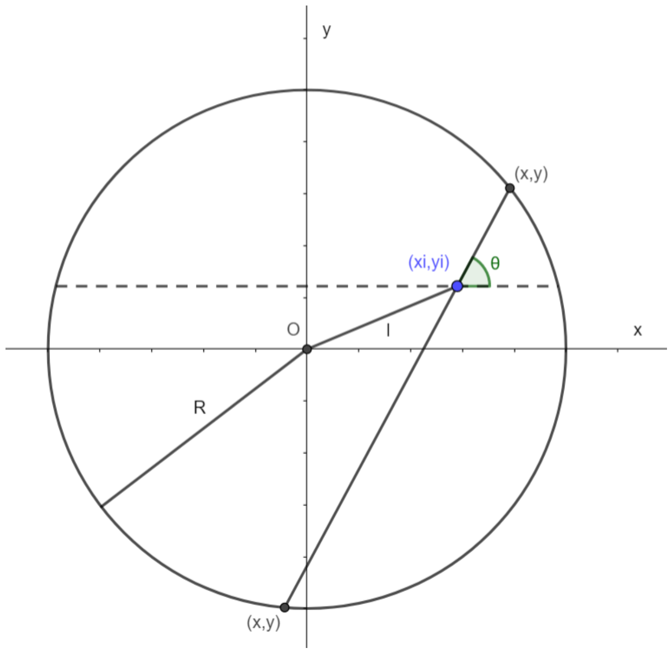

Figure 1. Geometry of Cloud

Our next step entails varying our interaction length as $l = -\bar{l} * ln(1-\zeta)$ via the random variable ζ in the range $[0, 1)$ by noting that

$$\int_0^1 (1 - e^{-\frac{l'}{\bar{l}}})dl' = \zeta \rightarrow l = -\bar{l} * ln(1-\zeta) \tag{3}$$

Here, it is noteworthy to comment that we can equivalently write $ln(1-\zeta) = ln\zeta$ for a random $\zeta \in (0, 1)$. However, since we will be using python to evaluate these lengths, where the random variables are given in [0,1), we will not adopt this simplification to avoid problems at zero.

Furthermore, now we reevaluate τ anew at each scattering iteration as $\tau_{\mathrm{i}} = n_{\mathrm{e}}\sigma_{\mathrm{T}}D_{\mathrm{i}}$, where D_{i} the distance from the photon's current position $(x_{\mathrm{i}}, y_{\mathrm{i}})$ to the boundary. The geometry can be seen in Fig 1. This can easily be calculated by noticing that

$$\frac{y - y_{\mathrm{i}}}{x - x_{\mathrm{i}}} = tan\theta \tag{4}$$

and

$$x^2 + y^2 = R^2 \tag{5}$$

where R is the radius of the cloud given by $R = \bar{l} * \tau_{\mathrm{center}}$, and θ is again uniformly sampled from $[0, 2\pi)$. Substituting and solving for y we get:

$$y_{1,2} = -\frac{x_i tan\theta - y_i}{tan^2\theta + 1} \pm \frac{\sqrt{(x_i tan\theta - y_i)^2 - (x_i tan\theta - y_i)(tan^2\theta + 1)}}{tan^2\theta + 1} \tag{6}$$

To make a choice between the solutions we have to look at θ for each instance. Therefore, if $0 < \theta < \pi \rightarrow y = max(y_1, y_2)$, otherwise $y = min(y_1, y_2)$. Of particular note are the instances of $\theta = n\frac{\pi}{2}, n = [0, 1, 2, 3]$, where we have horizontal or vertical lines, making the distance calculation especially easy as our system of unknowns reduces from 2 to 1.

In our last scenario, we include absorption through the absorption coefficient $\varepsilon = \frac{\alpha}{\alpha+\sigma}$. However, this analysis goes only so far as to consider the photons lost after absorption without examining emission phenomena.

3 RESULTS

Following the above steps, and using a number density of $n_e = 10^3 cm^{-3}$ we can arrive at a series of histograms illustrating the stochastic nature of the scattering phenomenon inside the cloud. Furthermore, by repeating the process for a range of optical depths we can derive conclusions on the dependency between $< N >$ and $\tau^2_{\rm center}$. Therefore, by examining Fig. 2 - 5 we can draw a number of conclusions. First and foremost, as was expected, one can easily see that for a greater optical depth, the mean number of scatterings increases, while for a fixed optical depth the distribution rapidly falls off for large scattering numbers. We can also see a clear relation of $< N >\sim \tau^2_{\rm center}$. What is especially of note here, is how we can get similar results with both approximations with only a fraction of the computational cost.

In the case where we include absorption, we can easily see how small changes can lead to drastic behaviors. In the case of $\varepsilon = 0.4$, the ratio of escaped to total photons is approximately 7%. While $\varepsilon = 0.1$ gives a more even split of $\sim$ 37%, and $\varepsilon = 0.01$ gives $\sim$ 87%. In Fig. 7 we again compare the mean value of steps to the optical depth. However, now that absorption is included there is no longer a linear correlation between the 2. The form of the curve can be understood by examining how an increasing number of scatterings increases evermore the probability of an absorption event.

The next logical step of this analysis would be to consider the re-emission of the absorbed photons and correlate them with a temperature gradient with the ultimate goal of a spectra analysis. However, this is outside the scope of this project.

4 CONCLUSIONS

To conclude, through a series of simplifications we approximated a simple case of a random walk problem in a proton cloud. We constructed histograms and derived a linear correlation between $< N >$ and $\tau_{\rm center}$. We showed how even the simplest approximation can yield great results, and lastly considered the case of lost photons through absorption showing how the linear correlation of $< N >\approx \tau^2$ no longer holds true.

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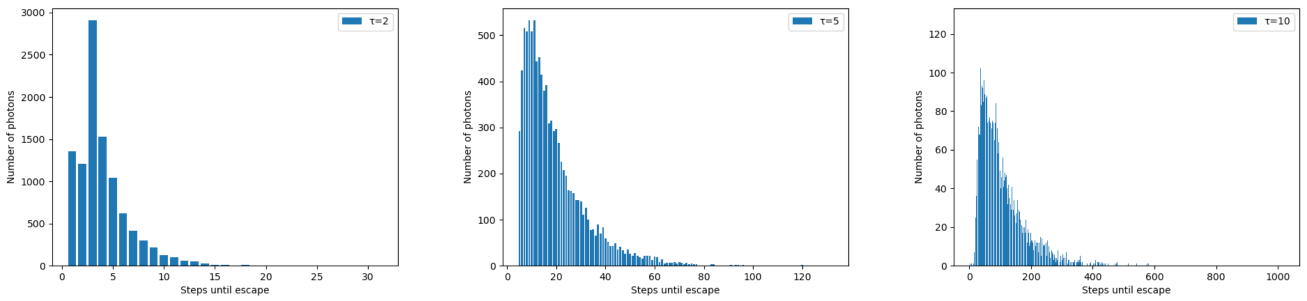

Figure 2. Histograms for $l = \bar{l} = (n_{\mathrm{e}}\sigma_{\mathrm{T}})^{-1}$ and $\tau = \tau_{\mathrm{center}}$. for 3 different optical depths. Drawn for 10.000 photons.

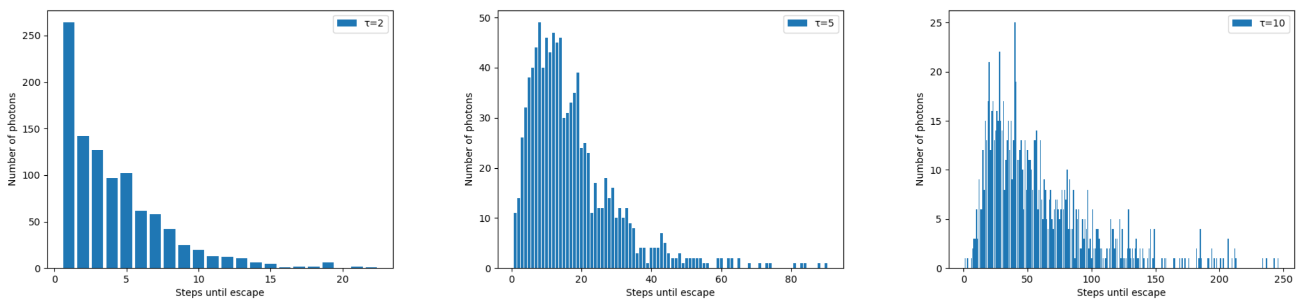

Figure 3. Histograms for $l = -\bar{l} * ln(1 - \zeta)$ and $\tau_{\mathrm{i}} = n_{\mathrm{e}}\sigma_{\mathrm{T}}D_{\mathrm{i}}$. for 3 different initial optical depths τ_{center}. Drawn for 1000 photons.

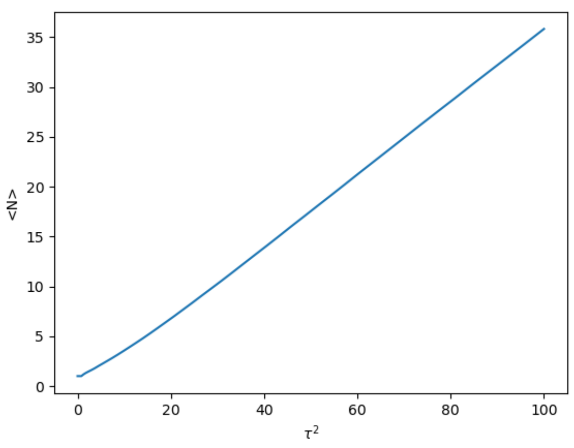

Figure 4. Correlation between the mean number of scattering steps to the optical depth for $l = \bar{l} = (n_{\mathrm{e}}\sigma_{\mathrm{T}})^{-1}$ and $\tau = \tau_{\mathrm{center}}$.

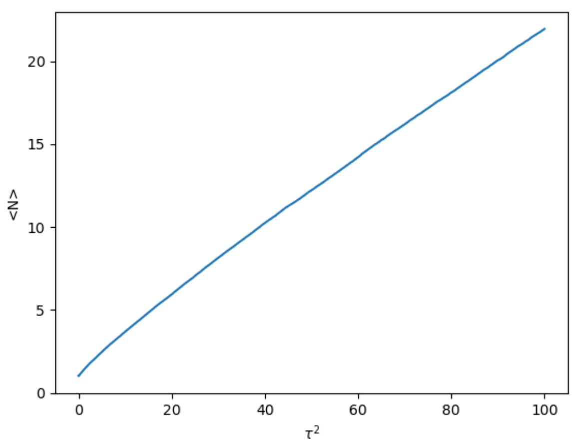

Figure 5. Correlation between the mean number of scattering steps to the optical depth for $l = -\bar{l} * ln(1 - \zeta)$ and $\tau_{\mathrm{i}} = n_{\mathrm{e}}\sigma_{\mathrm{T}}D_{\mathrm{i}}$. Here τ refers to τ_{center}.

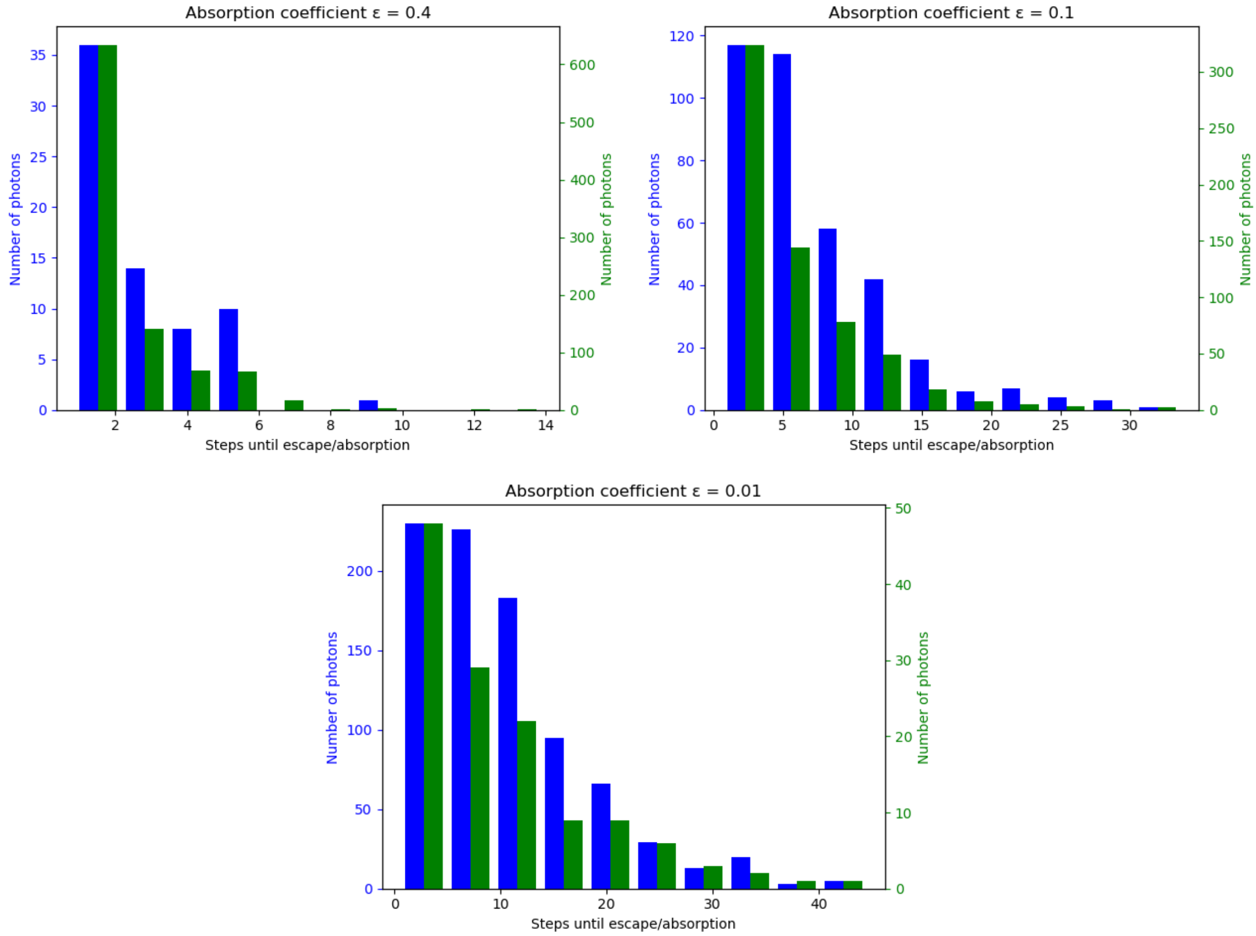

Figure 6. Histograms with the inclusion of absorption for 3 different values of ε. Blue: Number of escaped photons. Green: Number of absorbed photons.

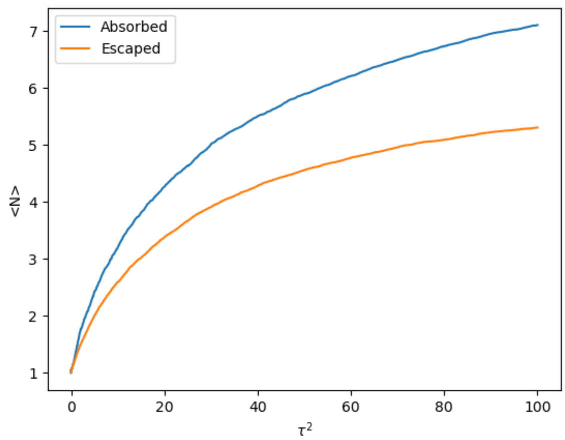

Figure 7. Correlation between the mean number of scattering steps to the optical depth if absorption is added to Fig. 5.