BLM2031/COM2031 Discrete Structures
Sample Questions for Final

1. How many ways are there to distribute 40 balls to 4 people Hasan, Mehmet, Ayla, and Büşra if Ayla and Büşra together get no more than 30 balls and Mehmet gets at least 9?

2. Given a box containing five balls numbered 1, 2, 3, 4, 5. Let X be the bigger number when two balls are randomly drawn from the box. Determine $E(X)$.

3. Use mathematical induction to prove that 43 divides $6^{n+1} + 7^{2n-1}$ for every positive integer n.

4. Use mathematical induction to prove $\sum_{i=1}^{n} i^2.2^i = n^2.2^{n+1} - n.2^{n+2} + 3.2^{n+1} - 6$ for every positive integer n.

5. Suppose that we roll a fair die until a 6 comes up.
a) What is the probability that we roll the die n times?
b) What is the expected number of times we roll the die?

6. Let R be the relation defined on $A = Z \times Z$ in the following way :

$$\big((x_1, y_1), (x_2, y_2)\big) \in R \Leftrightarrow x_1.y_2 = x_2.y_1$$

Determine whether the relation R is an equivalence relation on A or not.

7. Let A be a set and let R and S be symmetric relations defined on A. Determine whether $R \circ S$ (the composition of R and S) is symmetric or not.

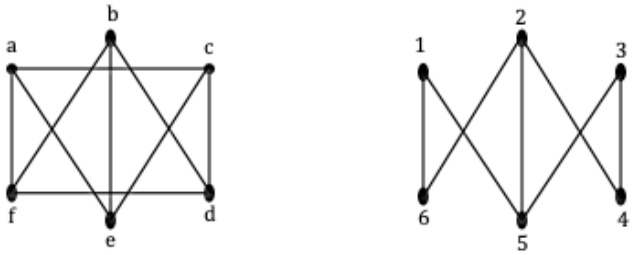
8. What is the maximum possible number of vertices for a connected undirected graph with 19 edges such that each vertex has degree at least 4 ? Draw a graph to demonstrate one possible case.

9. Consider the poset $\big(\{\{1\},\{2\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,3,4\},\{2,3,4\}\},\subseteq\big)$.

a) Find the maximal elements of the poset.
b) Find the minimal elements of the poset.
c) Find the all upper bounds of $\{\{2\},\{4\}\}$.
d) Find the all lower bounds of $\{\{1,3,4\},\{2,3,4\}\}$.

10. Let $n \in \mathbb{Z}^+$ and $n \leq 500$. How many such n are there which are not divisible by 3, 5, or 8?

11.



Determine whether two given graphs are isomorphic or not.

12. Let S be a subset of Z^+ and $|S| \geq 3$. Show that there exist distinct $x, y \in S$ such that $x + y$ is even.

13. Let A be a nonempty set and B be a fixed subset of A. Define a relation R on $P(A)$ such that for any $X, Y \in P(A)$, $(X, Y) \in R$ if $B \cap X = B \cap Y$. Show that R is an equivalence relation.

14. Find the number of permutations of the letters a b c d e … x y z (26 letters) in which none of the patterns 'spin' or 'net' occurs.

15. Let R be the relation defined on Z in the following way :

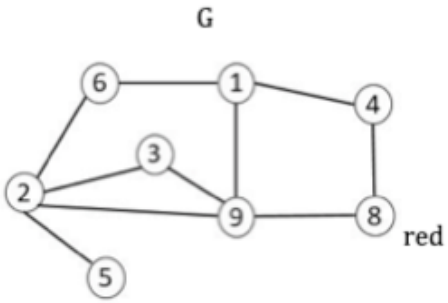
$$(x, y) \in R \Leftrightarrow x - y \; is \; a \; multiple \; of \; 3$$

Determine which properties (reflexive, symmetric, antisymmetric, transitive) the relation satisfies. Justify your answer.

16. Consider an ordinary deck of 52 playing cards such that there are 4 suits: diamond, heart, spade, and club, and there are 13 kinds for each suit: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K. The cards are to be drawn successively at random and without replacement. What is the probability that the second diamond (not second of diamonds) appears on the sixth draw?

17. Suppose that a Bayesian spam filter is trained on a set of 10,000 spam messages and 5000 messages that are not spam. The word "enhancement" appears in 1500 spam messages and 20 messages that are not spam, while the word "herbal" appears in 800 spam messages and 200 messages that are not spam. Estimate the probability that a received message containing both the words "enhancement" and "herbal" is spam.Will the message be rejected as spam if the threshold for rejecting spam is 0.9?

18.



a) Give the adjacency matrix of the graph G

b) Does the graph have an Euler path? If so, give such a path. If not, determine the minimum number edges that must be deleted to form a graph which has an Euler path.

c) What is the Chromatic number χ(G) of G? To get full credit, you need to write the color you assign to each node (as shown in the graph for the node 8).

d) What is κ(G), the minimum number of vertices in a vertex cut of G? To get full credit, you need to write the corresponding vertex cut set.

e) What is λ(G), the minimum number of edges in an edge cut of G? To get full credit, you need to write the corresponding edge cut set.