BLM2031/COM2031 Discrete Structures

Sample Questions for Midterm

1. Let $f, g, h : \mathbb{N} \longrightarrow \mathbb{N}$ be the functions defined as $f(n) = 3n$, $g(n) = \lfloor (n+1)/3 \rfloor$, $h(n) = \lfloor (n-1)/3 \rfloor$. Determine $(gof)(n)$ and $(hof)(n)$.

2. Solve the recurrence relation $a_n = 7a_{n-1} - 12a_{n-2}$ where $n \geq 2$, $a_0 = 5$, $a_1 = 18$

3. Give a big-O estimate for each of the following functions:

a) $(x^3 + x^2 \log x)(\log x + 5) + (15 \log x + 7)(x^4 + 3)$

b) $x \log(x^2 + 1) + x^2 \log x$

(For the function g of your estimate $f(x)$ is $O\big(g(x)\big)$, use a simple function g of smallest order)

4. Prove that there is no integer a such that $a \equiv 2\ (mod\ 6)$ and $a \equiv 7\ (mod\ 9)$. (Use proof by contradiction)

5. How many bit strings (that consist of the symbols '0' and '1') of length 6 have more zeroes than ones?

6. Prove that if $2n^2 + 3n$ is even integer, then n is even integer.

7. You are given two algorithms, Power and NewPower, that compute the n-th power of a given positive integer. Calculate and compare the worst-case complexity of both algorithms in terms of big-O notation.

POWER (x,n)
input : $x, n \in Z^+$
output : x^n

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temp = x
for i=2 to n
    temp = temp * x
return temp
```

NEWPOWER(x,n)
input : $x, n \in Z^+$
output : x^n

```
pow = x
temp = 1
while ( n > 0)
      if ( n is odd)
         { temp = pow * temp }
      pow = pow * pow
      n = ⌊n/2⌋
return temp
```

8. In how many ways can 12 different books be distributed among 4 people so that each gets exactly 3 books?

9. For the functions $f(x) = x^2 + 3x^2 + 1$ and $g(x) = x^3$

a) Determine whether $f(x) = O(g(x))$ or not.

b) Determine whether $g(x) = O(f(x))$ or not.

10. Sixteen people are to be seated around two circular tables, one with 10 chairs and the other with 6 chairs. How many different seating arrangements are possible?