A joint normal-binary (probit) model for high-dimensional data.

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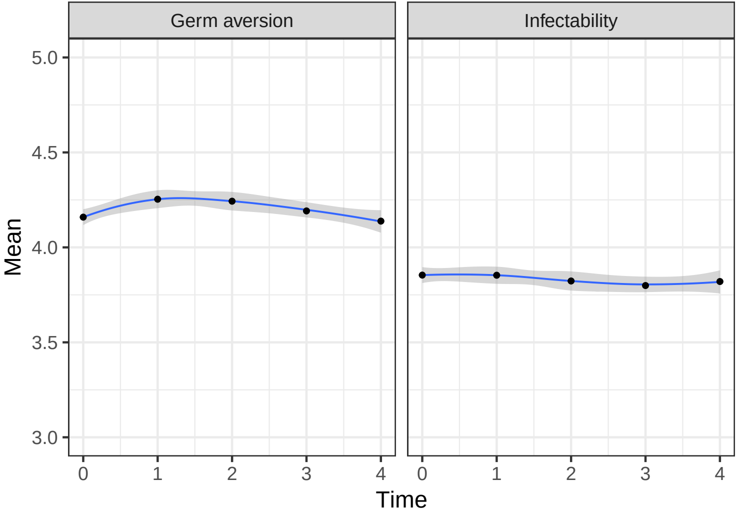
0 Dataset



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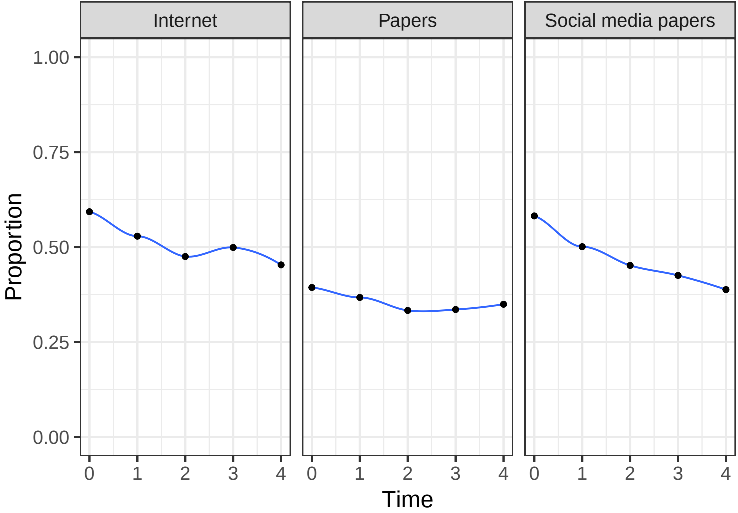


Internet
Papers
Social media papers
1.00
0.75
0.50
0.25
0.00
Proportion
0 1 2 3 4
Time

0 Time dependent covariates

$$y_{ij} = (\beta_0 + b_{i0}) + (\beta_1 + b_{i1})\mathsf{t}_{ij} + \beta_2 \mathsf{X}_{1i(j-1)} + \beta_3 \mathsf{X}_{2i} + \epsilon_{ij}$$

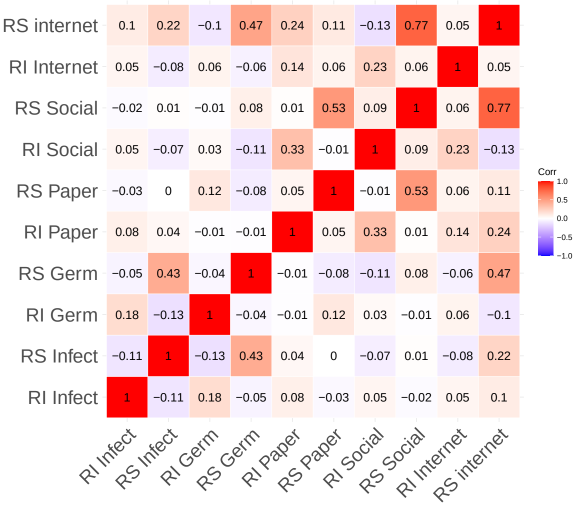
- Endogenous/exogenous
- Lag relationship

0 Joint mixed model

$$
\begin{array}{rcl}
Y_{1ij} & = & \boldsymbol{X}_{1ij}\boldsymbol{\beta}_1 + b_{10i} + b_{11i}t_{ij} + \epsilon_{1ij}, \\
Y_{2ij} & = & \boldsymbol{X}_{2ij}\boldsymbol{\beta}_2 + b_{20i} + b_{21i}t_{ij} + \epsilon_{2ij}, \\
\Phi^{-1}(P(Y_{3ij} = 1)) & = & \boldsymbol{X}_{3ij}\boldsymbol{\beta}_3 + b_{30i} + b_{31i}t_{ij}, \\
\Phi^{-1}(P(Y_{4ij} = 1)) & = & \boldsymbol{X}_{4ij}\boldsymbol{\beta}_4 + b_{40i} + b_{41i}t_{ij}, \\
\Phi^{-1}(P(Y_{5ij} = 1)) & = & \boldsymbol{X}_{5ij}\boldsymbol{\beta}_5 + b_{50i} + b_{51i}t_{ij}, \\
\epsilon_i & \sim & N(\mathbf{0}, \Sigma_i).
\end{array}
$$

The random intercepts ($b_{10i}, b_{20i}, b_{30i}$,$b_{40i}$ and b_{50i}) and the random slopes ($b_{11i}, b_{21i}, b_{31i}$,$b_{41i}$ and b_{51i}) follow a multivariate normal distribution with mean $\mathbf{0}$ and variance-covariance $\boldsymbol{D}$.

0 Latent correlations



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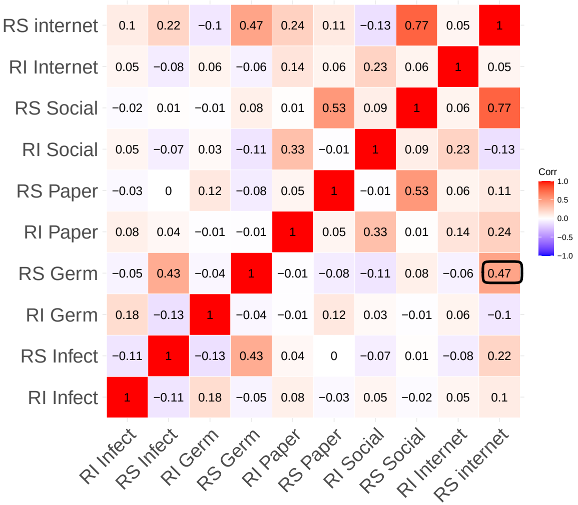

0 Manifest correlations

Table 4: Manifests correlations between perceived infectability and quality newspaper.

	Paper 0	Paper 1	Paper 2	Paper 3	Paper 4
Infect 0	**0.072 [0.009 ; 0.134]**	**0.068 [0.005 ; 0.130]**	0.059 [-0.004 ; 0.120]	0.053 [-0.009 ; 0.115]	0.048 [-0.014 ; 0.110]
Infect 1	**0.076 [0.013 ; 0.138]**	**0.071 [0.009 ; 0.133]**	0.062 [0.000 ; 0.124]	0.056 [-0.006 ; 0.118]	0.051 [-0.011 ; 0.113]
Infect 2	**0.079 [0.016 ; 0.141]**	**0.075 [0.012 ; 0.137]**	**0.065 [0.002 ; 0.127]**	0.059 [-0.004 ; 0.120]	0.054 [-0.008 ; 0.116]
Infect 3	**0.082 [0.019 ; 0.144]**	**0.077 [0.015 ; 0.139]**	**0.067 [0.005 ; 0.129]**	0.061 [-0.001 ; 0.123]	0.056 [-0.006 ; 0.118]
Infect 4	**0.084 [0.021 ; 0.146]**	**0.079 [0.016 ; 0.141]**	**0.069 [0.007 ; 0.131]**	0.063 [0.000 ; 0.124]	0.058 [-0.004 ; 0.120]

0 Predictions

Table 5: Predicted probability of reading the newspaper on time point 2 conditional on the history of perceived infectability, germ aversion and consumption of quality newspapers, social media of quality newspapers and internet.

Infectability	Germ aversion	Paper	Internet	Social media	Prediction	Prediction interval
Mean	Mean	0	0	0	0.035	[0.004 ; 0.234]
Mean	-1 SD	0	0	0	0.033	[0.004 ; 0.225]
Mean	+1SD	0	0	0	0.036	[0.004 ; 0.243]
-1 SD	Mean	0	0	0	0.031	[0.004 ; 0.211]
-1 SD	-1 SD	0	0	0	0.030	[0.004 ; 0.204]
-1 SD	+1SD	0	0	0	0.033	[0.004 ; 0.219]
+1SD	Mean	0	0	0	0.039	[0.004 ; 0.257]
+1SD	-1 SD	0	0	0	0.037	[0.004 ; 0.248]
+1SD	+1SD	0	0	0	0.040	[0.004 ; 0.266]
Mean	Mean	1	0	0	0.643	[0.171 ; 0.668]
Mean	-1 SD	1	0	0	0.635	[0.168 ; 0.666]
Mean	+1SD	1	0	0	0.652	[0.173 ; 0.670]
-1 SD	Mean	1	0	0	0.633	[0.167 ; 0.667]
-1 SD	-1 SD	1	0	0	0.624	[0.164 ; 0.665]
-1 SD	+1SD	1	0	0	0.641	[0.169 ; 0.67]
+1SD	Mean	1	0	0	0.655	[0.175 ; 0.669]
+1SD	-1 SD	1	0	0	0.647	[0.173 ; 0.667]
+1SD	+1SD	1	0	0	0.663	[0.177 ; 0.671]

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0 Summary

- Multiple longitudinal responses can be analyzed with joint models
- We have derived formulas for computing the manifest correlations
- Conditional models can be used to make predictions for one response conditional on the other response(s)

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