A joint normal-ordinal model for ordinal and continuous longitudinal data

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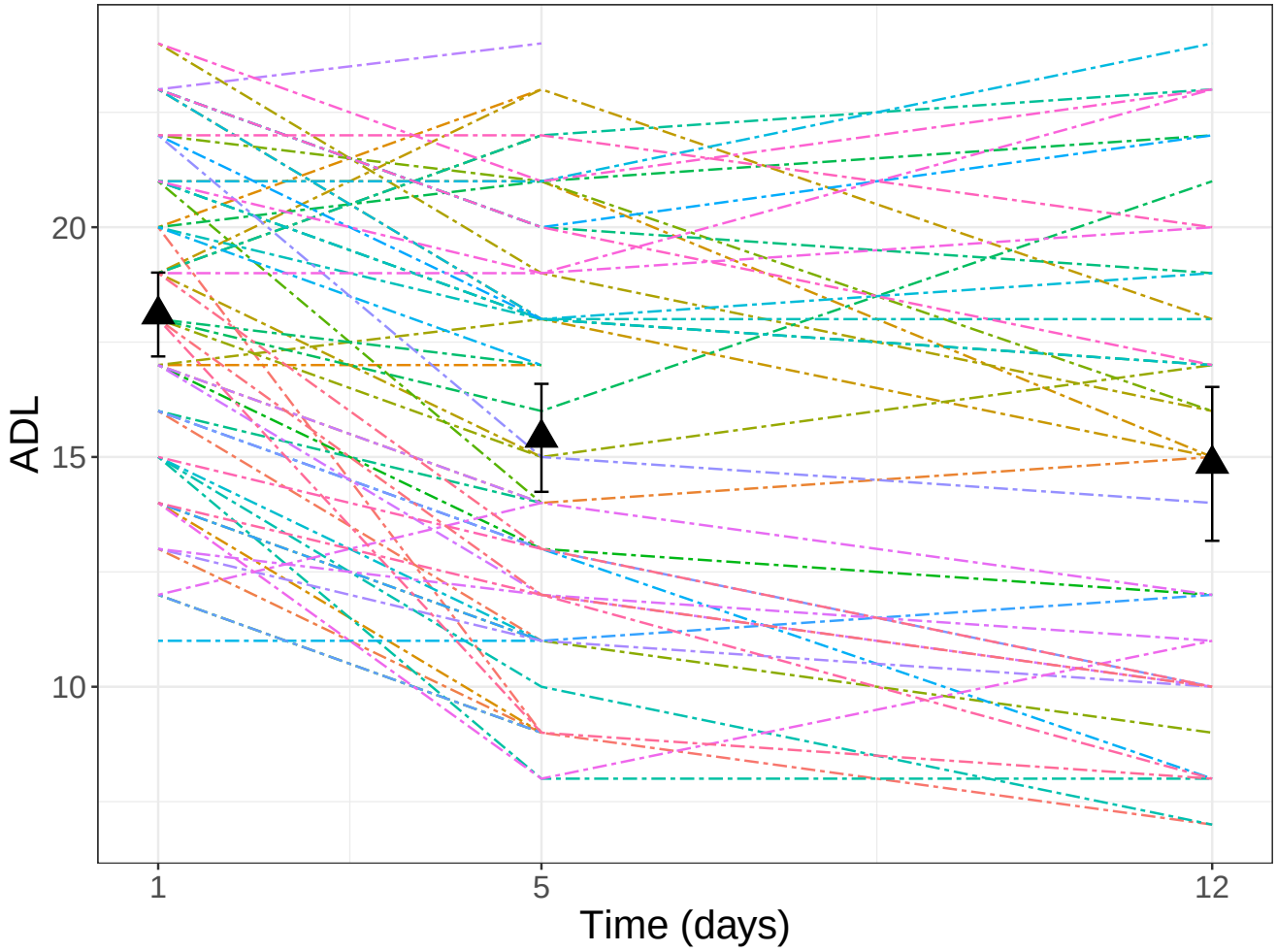
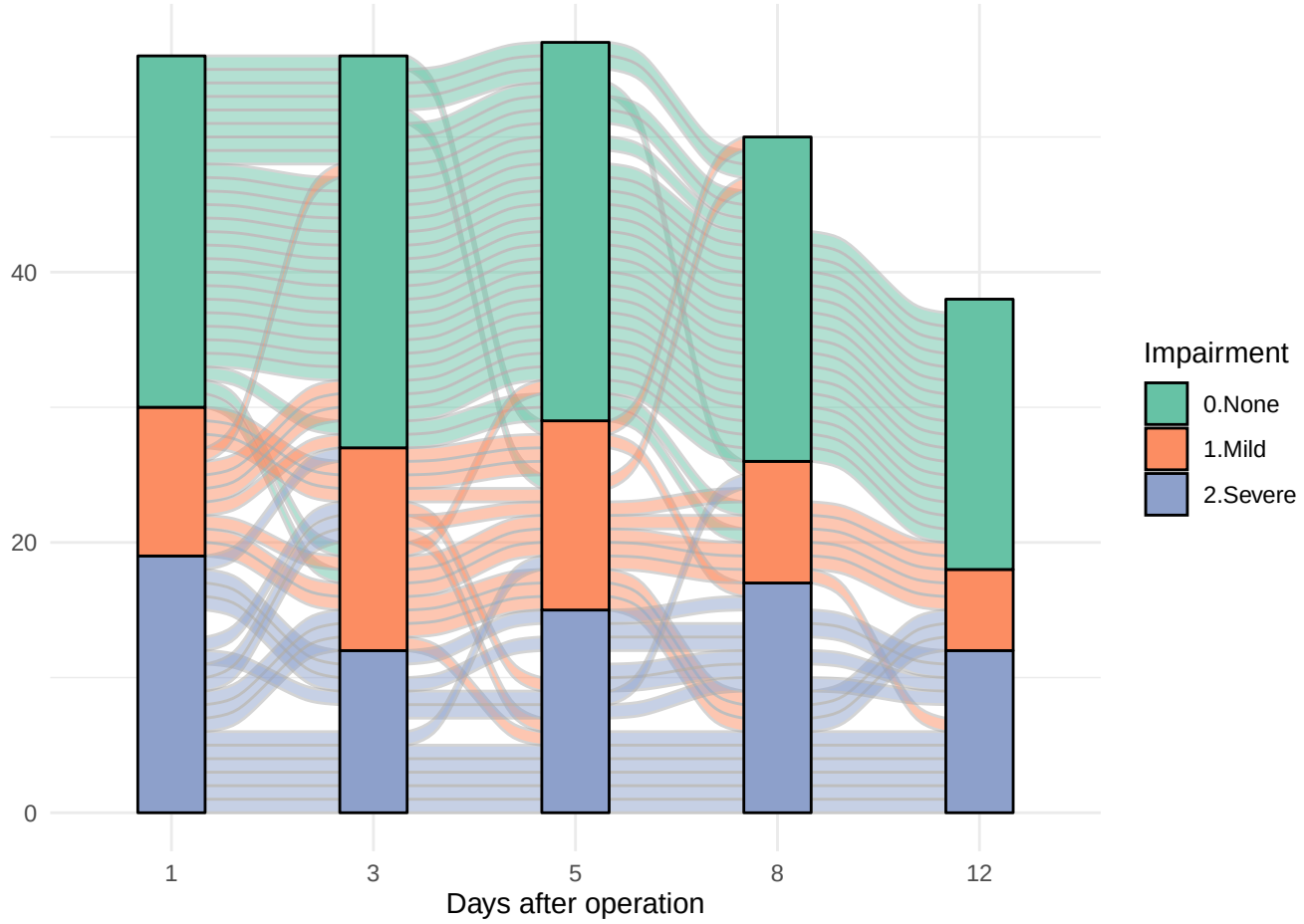
0 Case study

- 60 older patients with hip fractures
- Post-operative follow-up
- Activities of Daily Living (ADL)
- Cognitive impairment (MMSE)

0 Activities of daily living score



0 Mini Mental State Exam



0 Research question

What is the association between cognitive impairment and activities of daily living?

Table: *Number of measurements at each time point for MMSE and ADL.*

Response	Day 1	Day 3	Day 5	Day 8	Day 12
MMSE	59	58	60	52	38
ADL	60	0	60	0	40

0 Time dependent covariates

$$Y_{ij} = (\beta_0 + b_{i0}) + (\beta_1 + b_{i1})\mathsf{t}_{ij} + \beta_2 \mathsf{X}_{1i(j-1)} + \beta_3 \mathsf{X}_{2i} + \epsilon_{ij}$$

- Lag relationship
- Endogenous/exogenous

0 Joint mixed model

$$
\begin{aligned}
Y_{1ij} &= \boldsymbol{X}_{1ij}\boldsymbol{\beta}_1 + b_{10i} + b_{11i}t_{ij} + \epsilon_{1ij}, \\
\Phi^{-1}(P(Y_{2ij} \leq c)) &= \gamma_c - (\boldsymbol{X}_{2ij}\boldsymbol{\beta}_2 + b_{20i} + b_{21i}t_{ij}) .
\end{aligned}
$$

The random intercepts (b_{10i}, b_{20i}) and the random slopes (b_{11i}, b_{21i}) follow a multivariate normal distribution with mean $\mathbf{0}$ and variance-covariance $\boldsymbol{D}$.

0 Latent correlations

Table: *Latent correlations [CI] between the random effects of impairment and ADL.*

	b_{10i}	b_{11i}	b_{20i}	b_{21i}
b_{10i}	1			
b_{11i}	.12 [-.44;.61]	1		
b_{20i}	-.70[-.89;-.31]	-.38 [-.77;.21]	1	
b_{21i}	.38 [-.80;.95]	-.07 [-.98;.98]	-.72 [-1;.95]	1

0 Manifest correlations

Table: *Correlations between ADL (higher: lower functioning) and severe cognitive impairment for 78-year-old men.*

	t(Impaired)				
t(ADL)	1	3	5	8	12
1	.44[.28;.58]	.44[.29;.57]	.44[.29;.57]	.43[.28;.57]	.42[.26;.57]
5	.48[.34;.60]	.48[.34;.60]	.48[.34;.60]	.48[.34;.61]	.48[.31;.61]
12	.47[.26;.65]	.48[.28;.64]	.48[.29;.64]	.49[.29;.64]	.49[.28;.65]

0 Manifest correlations

Table: *Correlations between ADL (higher: lower functioning) and cognitive impairment for 78-year-old men.*

	t(Impaired)				
t(ADL)	1	3	5	8	12
1	.47[.31;.60]	.47[.32;.60]	.47[.33;.60]	.48[.34;.60]	.48[.32;.61]
5	.51[.38;.62]	.52[.39;.62]	.52[.41;.62]	.53[.41;.63]	.54[.41;.65]
12	.50[.30;.66]	.51[.32;.66]	.52[.34;.66]	.54[.37;.67]	.55[.37;.70]

0 Predictions

Table: *Prediction of cognitive impairment based on the history of ADL at time* 1 *and* 5 *for* 78*-year-old women.*

History ADL (day 1 – day 5)	P(Impairment) (day 12)		P(Severe Impairment) (day 12)	
14.57-10.87	0.14	[0.05;0.33]	0.01	[0.00;1.00]
18.10-15.42	0.57	[0.41;0.72]	0.17	[0.08;0.34]
21.63-19.97	0.93	[0.74;0.98]	0.63	[0.39;0.82]

0 Summary

- Multiple longitudinal responses can be analyzed with joint models
- Formulas for computing the correlations between the responses on the observed scale
- Formulas for predictions of one response given the other response(s)

0 More detailed information

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