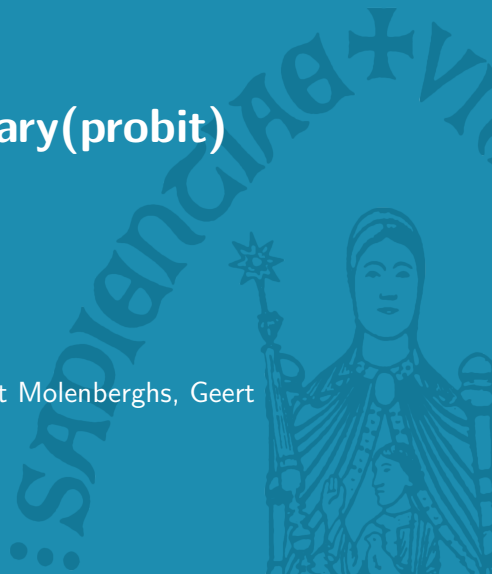


# A joint normal-binary(probit) model

IBC 2022 - 14th of July

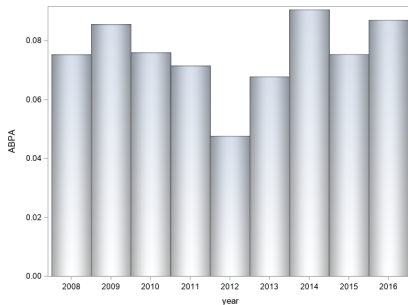
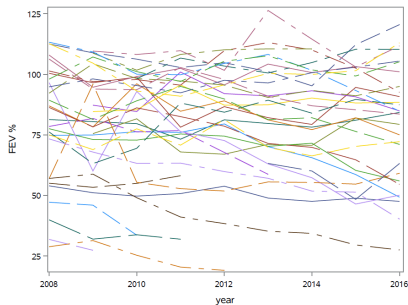
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Co-authors: Steffen Fieuws, Geert Molenberghs, Geert Verbeke



# 1 Introduction

- ▶ Repeated measurement of multiple responses
- ▶ Joint analysis of the outcomes

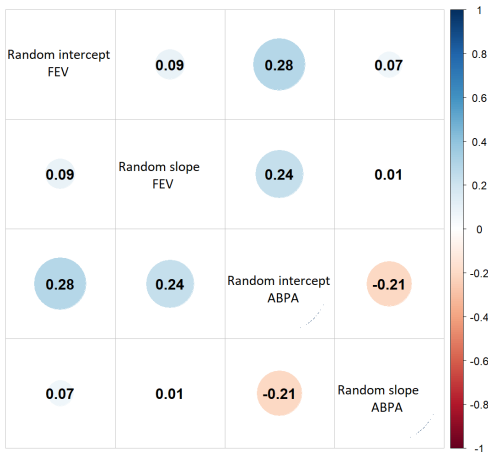


## 2 Existing methodology

$$\begin{aligned}Y_{1ij} &= b_{10i} + b_{11i}t_{ij} + X_{1i}\beta + \epsilon_{ij} \\ \Phi^{-1}(P(Y_{2ik} = 1)) &= b_{20i} + b_{21i}t_{ik} + X_{2i}\beta \\ (b_{10i}, b_{11i}, b_{20i}, b_{21i})' &\sim N \left[ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} d_{11} & d_{21} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{13} & d_{23} & d_{33} & d_{34} \\ d_{14} & d_{24} & d_{34} & d_{44} \end{pmatrix} \right] \\ \epsilon_i &\sim N(\mathbf{0}, \Sigma_i)\end{aligned}$$

### 3 Joint model

Allow correlation between the random effects of the mixed models



## 4 Results

year(FEV)	year(ABPA)								
	0	1	2	3	4	5	6	7	8
0	0.146	0.148	0.15	0.151	0.151	0.151	0.15	0.149	0.147
1	0.155	0.157	0.159	0.159	0.159	0.159	0.158	0.157	0.155
2	0.163	0.165	0.166	0.167	0.167	0.166	0.165	0.163	0.161
3	0.169	0.171	0.172	0.173	0.172	0.171	0.17	0.168	0.166
4	0.174	0.177	0.177	0.177	0.177	0.176	0.174	0.172	0.17
5	0.179	0.18	0.181	0.181	0.18	0.179	0.177	0.175	0.173
6	0.182	0.183	0.184	0.184	0.183	0.181	0.18	0.177	0.175
7	0.184	0.185	0.186	0.185	0.184	0.183	0.181	0.178	0.176
8	0.185	0.186	0.187	0.186	0.185	0.184	0.181	0.179	0.176

## 4 Results

$j$	$Y_{1i(j-3)}$	$Y_{1i(j-2)}$	$Y_{1i(j-1)}$	Acute ABPA		Chronic ABPA	
				$E[Y_{1ij}]$	PI $Y_{1ij}$	$E[Y_{1ij}]$	PI $Y_{1ij}$
3	...	...	...	62.67	[49.9; 78.7]	62.43	[49.2; 79.3]
4	...	...	...	62.38	[49.7; 78.4]	62.18	[48.9; 79.1]
5	...	...	...	62.14	[49.4; 78.1]	61.98	[48.7; 78.9]
6	...	64.9	...	61.96	[49.3; 77.9]	61.82	[48.6; 78.7]
7	...	...	...	61.82	[49.1; 77.8]	61.72	[48.5; 78.6]
8	...	...	...	61.73	[49.1; 77.6]	61.66	[48.4; 78.5]

## 4 Results

$j$	$Y_{1i(j-3)}$	$Y_{1i(j-2)}$	$Y_{1i(j-1)}$	Acute	Chronic
				$E[Y_{1ij}]$	$E[Y_{1ij}]$
3	...	...	...	62.67	62.43
4	...	...	...	62.38	62.18
5	...	64.9	...	62.14	61.98
6	...	...	...	61.96	61.82
7	...	...	...	61.82	61.72
8	...	...	...	61.73	61.66
3	...	...	...	81.65	81.48
4	...	...	...	81.6	81.47
5	...	84	...	81.59	81.48
6	...	...	...	81.6	81.51
7	...	...	...	81.64	81.57
8	...	...	...	81.71	81.65
3	...	...	...	95.33	95.21
4	...	...	...	95.45	95.36
5	...	97.7	...	95.61	95.52
6	...	...	...	95.75	95.7
7	...	...	...	95.92	95.88
8	...	...	...	96.08	96.05

## 5 Conclusion and discussion

- ▶ Latent versus manifest correlations
- ▶ Time dependent covariates
  - Missing data
  - Characterization of the lag relationship
  - Endogenous or exogenous
  - Intermediate variable