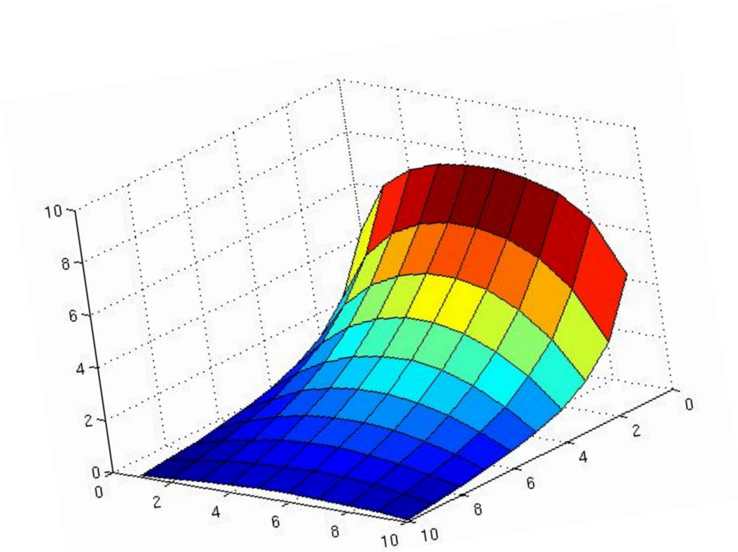
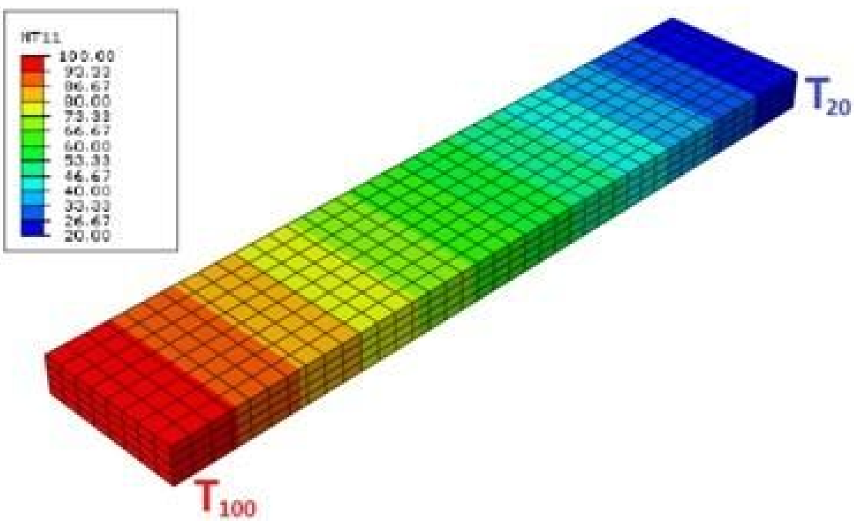
Masson Margaux

Groupe A
3 ETI

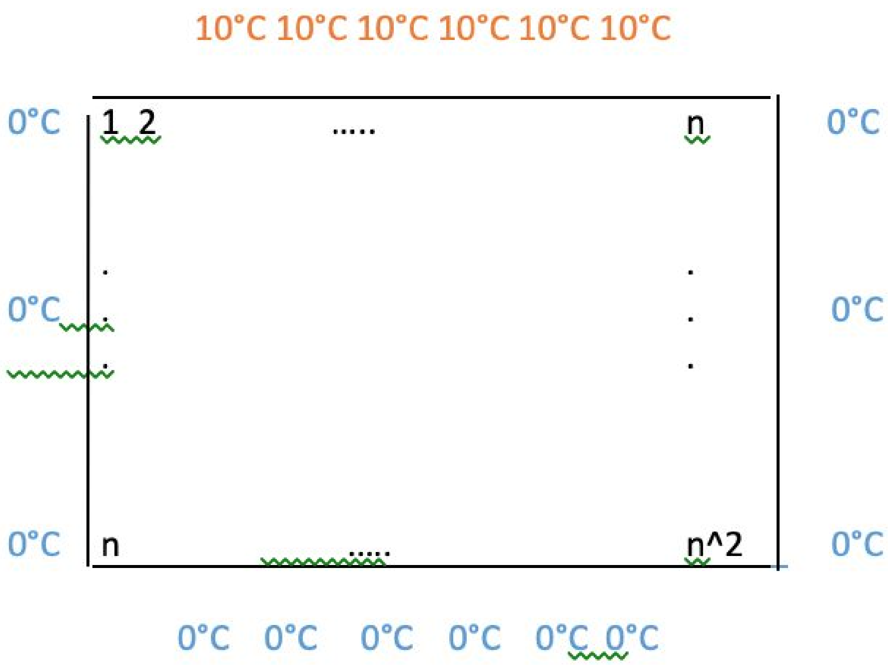
Report n°5
Heat equation





Heat equation

The main goal of this project is to simulate (2D) the thermal equilibrium in a conductive plate.
The first step here is to construct the plate in a grid n * n with the following boundary conditions: the temperature is equal to 10°C at the North border, 0°C at the other borders.



For this, we create the auxiliary matrix Aij such as: $Aij = n(i-1)+j$ that is to say, for n = 2 :

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

The method to get the simulation is:

We need to create an incidence matrix M such as for every group
A(i,j) A(i,j+1) or A(i,j) A(i+1,j)   we have :
M(A(i,j),A(i,j+1))= M(A(i,j+1),A(i,j))= 1 and
M(A(i+1,j),A(i,j))= M(A(i,j) ,A(i+1,j))= 1.

Which leads to the Matlab code following:

```
clear all;close all;
n= input('n = ');

for i=1:n
    for j=1:n
        A(i,j)=n*(i-1)+j;    %we create the Aij matrix auxiliary which
represents the plate
    end
end
A
M=zeros(n^2,n^2);  %initialization of the matrix M
for i=1:n
    for j=1:n-1
        I1 = A(i,j);  %coeff of A(i,j) which become the indexes of M when M=1
        J1 = A(i,j+1);  %coeff of A(i,j+1) which become the indexes of M when
M=1
        M(I1,J1) = 1;
        M(J1,I1) = 1;
    end
end

for i=1:n-1
    for j=1:n
        I2 = A(i,j);   %coeff of A(i,j) which become the indexes of M when
M=1
        J2 = A(i+1,j);  %coeff of A(i,j+1) which become the indexes of M when
M=1
        M(I2,J2) = 1;
        M(J2,I2) = 1;
    end
end
M
```

Which gives us for n=3 :

M =

0	1	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	1	0	1	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	1	0	1
0	0	0	0	0	0	0	1	0

Then in order to obtain the temperatures at the different areas of the plate in a matrix T, we know that T= 0.25 * V * M * T + B

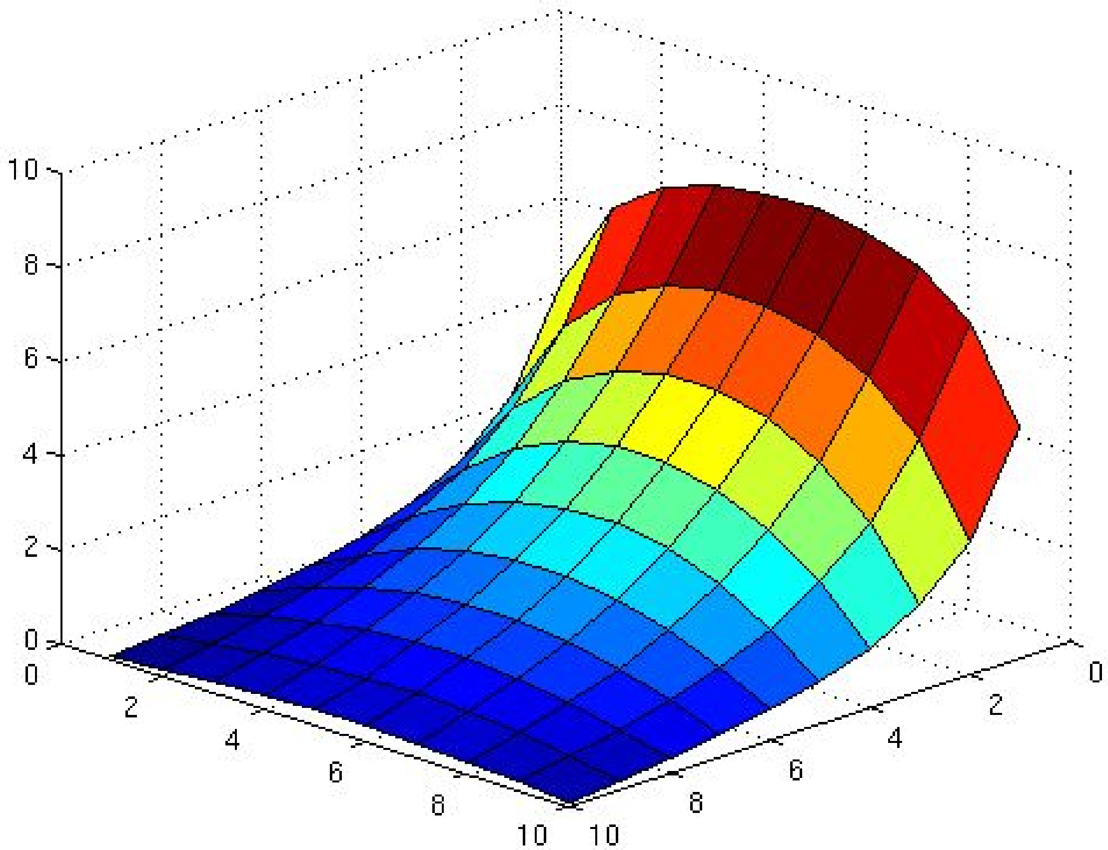
With: T the temperature column matrix, the previously defined matrix M and the column matrix B of the boundary conditions,
for instance fo n=3 we have : $B = (10/4\ 10/4\ \ 10/4\ 0\ 0\ 0\ 0\ 0\ 0 \quad)$
We then complete the previous Matlab program and also implement the display of thermal equilibrium in the plate:

```
clear all;close all;
n= input('n = ');

for i=1:n
    for j=1:n
        A(i,j)=n*(i-1)+j;    %we create the Aij matrix auxiliary which
represents the plate
    end
end
A
M=zeros(n^2,n^2);  %initialization of the matrix M
for i=1:n
    for j=1:n-1
        I1 = A(i,j);  %coeff of A(i,j) which become the indexes of M when M=1
        J1 = A(i,j+1);  %coeff of A(i,j+1) which become the indexes of M when
M=1
        M(I1,J1) = 1;
        M(J1,I1) = 1;
    end
end

for i=1:n-1
    for j=1:n
        I2 = A(i,j);   %coeff of A(i,j) which become the indexes of M when
M=1
        J2 = A(i+1,j);  %coeff of A(i,j+1) which become the indexes of M when
M=1
        M(I2,J2) = 1;
        M(J2,I2) = 1;
    end
end

T= zeros(n^2,1);   %initialization of the temperatures matrix T
B=zeros(n^2,1);    %initialization of the boundaries conditions matrix
i=[1:n];
B(i)=10/4 ;
```

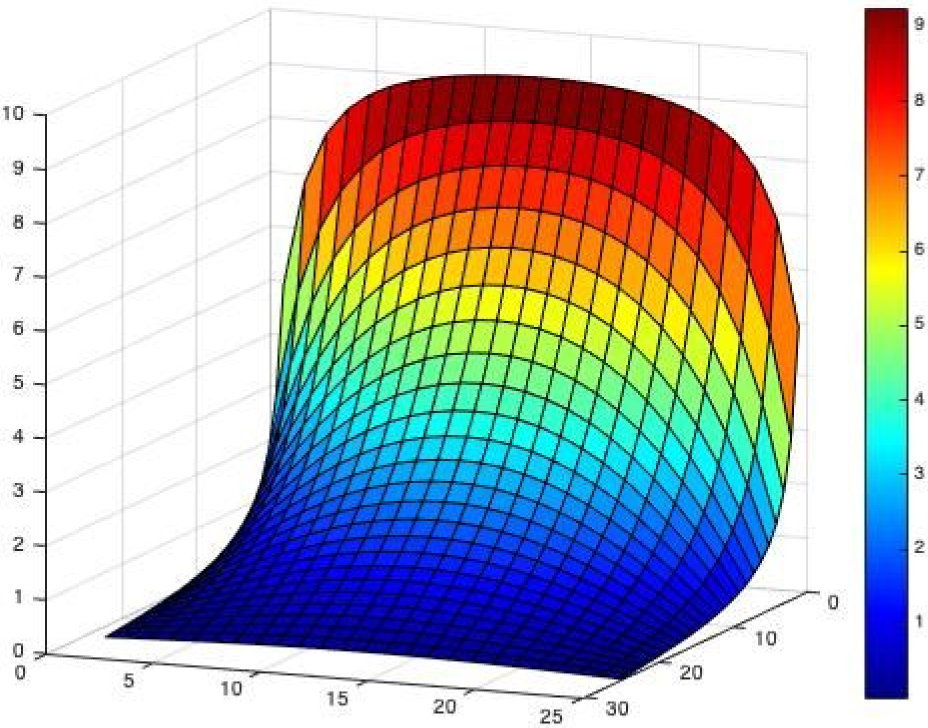
```
for m=1:100*n
    T= 0.25*M*T + B;   %matrix composed by the temperatures at the different
areas of the plate
```

```
end

TXY=reshape(T,n,n);
[X,Y]=meshgrid(1:n); surf(X,Y,TXY);  %plot the plate with the different
temperatures
```

And then we got the plot following:



Thermal equilibrium in a conductive plate (n = 10)



Thermal equilibrium in a conductive plate (n = 25)

We thus see on this graph the evolution of the temperature in the conductive plate, that is to say that the temperature is high (10 ° C) at the top of the plate, then the temperature decreases progressively downwards where it reaches the lowest temperature (0 ° C).