

Q. Let S be the set of all integers from 100 to 999 which are neither divisible by 3 nor divisible by 5 then find number of elements in S .

Sol:

Given set S contains the integers from 100 to 999 then $n(S) = 900$.

$$A = \{x \mid 100 \leq x \leq 999 \text{ & divisible by 3}\}$$

$$B = \{x \mid 100 \leq x \leq 999 \text{ & divisible by 5}\}$$

$$A \cap B = \{x \mid 100 \leq x \leq 999 \text{ & divisible by 3 and 5}\}$$

$$\therefore n(A) = \frac{900}{3} = 300$$

$$n(B) = \frac{900}{5} = 180$$

$$n(A \cap B) = \frac{900}{15} = 60$$

Now the number of element in S neither divisible by 3 nor 5 is $= 900 - 420 = 480$

Q. Find the number of positive integer's ≤ 300 and divisible by 2 or 5.

Sol: Here we want to find number of positive odd integers ≤ 300 . Odd is divisible by 2 and 2 is not divisible by 2 so it is not even.

$$A = \{x | x \in \mathbb{N} \text{ & } x \leq 300 \text{ & divisible by 2}\}$$

Sol: $B = \{x | x \in \mathbb{N} \text{ & } x \leq 300 \text{ & divisible by 5}\}$

$A \cap B = \{x | x \in \mathbb{N} \text{ & } x \leq 300 \text{ & divisible by 2 and 5}\}$
(i.e. x is divisible by $5 \times 2 = 10$)

$$\therefore n(A) = 300 ; n(B) = 300$$

$$\begin{aligned} \text{As x is divisible by 5} &\Rightarrow x \geq 10, 20, 30, \dots \\ n(A) &= 150 \end{aligned}$$

{As x is divisible by 10} $\Rightarrow x \geq 10, 20, 30, \dots$ $\therefore n(A) = 30$

$$n(A \cap B) = \frac{300}{10} = 30$$

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 150 + 60 - 30 \\ &= 180 \end{aligned}$$

Q3: A relation is defined on set \mathbb{Z} is $R = \{(x, y) | x-y$ divided by 7} then check that R is equivalence relation also check that R_A is poset or not.

Sol:

Here given set A is integer set \mathbb{Z} and relation R is defined as relation of $x-y$ is divisible by 7.

$$R = \{(x, y) | x-y \text{ is divisible by 7}\}$$

(i) Reflexive

Given $x \in \mathbb{Z}$ then $x-x = 0$ is divisible by 7.

$\therefore \forall x \in \mathbb{Z}, x-x \text{ is divisible by 7}$

$$\Rightarrow \frac{x-y}{7} \rightarrow \frac{y-x}{7} \text{ which is } 0 \text{ if } x=y \text{ and } 7 \text{ if } x \neq y$$

which implies this relation is reflexive and symmetric.

This equation contains reflexive, symmetric property.

(ii) Symmetric Relation :- If for A

from above diagram we can say that both study

$$\forall x, y \in Z$$

$$\Rightarrow x-y \text{ divided by 7 then } y-x \text{ is also divided by 7}$$

which is $x-y \equiv y-x \pmod{7}$

This relation contains symmetric property

(iii) Anti-symmetric :-

$$2+xp = 2+yp$$

$$\forall x, y \in Z ; x, y \in Z^P \text{ and } y, x \in Z \Rightarrow$$

$$x = y$$

Now this relation is fulfilling this condition that's why it is antisymmetric relation without proof.

(iv) Transitive Relation :- If for A, S ⊂ A, f : A → S

- Here $x, a, b, c \in Z$ then $f: A \rightarrow B \Rightarrow B \rightarrow C$

$$\Rightarrow A \rightarrow C$$

$$\frac{x-y}{7} \Rightarrow \frac{y-z}{7} \pmod{7}$$

$$\Rightarrow \frac{x-y+y-z}{7} \pmod{7}$$

$$\Rightarrow \frac{x-z}{7} \pmod{7}$$

This relation fulfill the last relation

→ This relation $\{x-y\}$ is poset or equivalence relation with min max property. not

Q. Let $f: \mathbb{Z} \rightarrow \mathbb{Z}$ be define by $f(x) = 9x + 5$. Then check that f is one-one and onto

Sol:- Here given function f is $f: \mathbb{Z} \rightarrow \mathbb{Z}$ define by

$$f(x) = 9x + 5$$

$\therefore x_1, x_2 \in \mathbb{Z}$, Now

$$f(x_1) = f(x_2)$$

$$9x_1 + 5 = 9x_2 + 5$$

$$\Rightarrow 9x_1 + 5 = 9x_2 + 5 \Rightarrow 9x_1 = 9x_2 \Rightarrow x_1 = x_2$$

⇒ Hence f is one-one function. However, f is not onto function with relation \mathbb{Z} .

Let, $4 \in \mathbb{Z}$, For, $\exists x \in \mathbb{Z}$ such that

x if $f(x) = 4$ for some $x \in \mathbb{Z}$ -

$$9x + 5 = 4$$

$$9x = -1$$

$$x = \frac{-1}{9} \notin \mathbb{Z}$$

Hence f is not onto function with relation \mathcal{Z} .

Q.5. Prove that $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = ax + b, a \neq 0$ is a bijective function.

Solⁿ: Here we want to prove that $\mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = ax + b$$

Let $x_1, x_2 \in \mathbb{R}$. Now

$$f(x_1) = f(x_2)$$

$$ax_1 + b = ax_2 + b$$

$$ax_1 = ax_2$$

$$x_1 = x_2$$

Hence f is one-one function.

For onto $y \in \mathbb{R}$

$$y = f(x)$$

$$y = ax + b \Rightarrow x = \frac{y-b}{a}$$

$$x = \frac{y-b}{a}$$

$$f(x) = f\left(\frac{y-b}{a}\right)$$

$$= a\left(\frac{y-b}{a}\right) + b$$

$$= y - b + b$$

$$= y$$

Hence given function is one-one and onto, so f is bijective.

6. Let $f: \mathbb{Z} \rightarrow \mathbb{Z}$ be defined by $f(x) = x^2 + 1$. Then check that f is one-one and onto.

Sol: Here given function is $\mathbb{Z} \rightarrow \mathbb{Z}$ defined by

$$f(x) = x^2 + 1$$

Let $x_1, x_2 \in \mathbb{Z}$,

$$d + x_1^2 = x_2^2$$

Given function $f(x_1) = f(x_2)$ i.e., $x_1^2 + 1 = x_2^2 + 1$

$$x_1^2 + 1 = x_2^2 + 1$$

$$x_1^2 = x_2^2$$

$$x_1 = x_2$$

Hence f is one-one function.

Let, $4 \in \mathbb{Z}$ such that $f(x) = 4$

$$f(x) = 4$$

$$x^2 + 1 = 4$$

$$x^2 = 3$$

$$\frac{d - b}{2} = x$$

$$x = \sqrt{\frac{d - b}{2}} = \sqrt{3}$$

Hence f is not onto function.

7. Suppose A is any non-empty set and $P(A)$ is

the set of all subsets of A then show that

$\langle P(A), \subseteq \rangle$ is partially ordered set relation where

" \subseteq " means " \subseteq ".

Sol: Here given set is all the subset of A is any non-empty set the last one will be empty which is also in the set so it is reflexive.

(i) Reflexive:

$\forall A_i \in P(A)$ we write $A_i \subseteq A_i$.

That is why $P(A)$ fulfill this condition.

$\langle P(A), \subseteq \rangle$ is Reflexive.

(ii) Antisymmetric:

$\forall A_1, A_2 \in P(A)$, suppose $A_1 \subseteq A_2$ and $A_2 \subseteq A_1$, then always possible $A_1 = A_2 \Rightarrow$

\subseteq is Antisymmetric relation.

(iii) Transitive:

$\forall A_1, A_2, A_3 \in P(A)$,

If $A_1 \subseteq A_2$; $A_2 \subseteq A_3$; $A_1 \subseteq A_3$ is always true.

\subseteq is transitive.

from (i), (ii), (iii) relation " \subseteq " is partially ordered relation.

Hence proved that $\langle P(A), \subseteq \rangle$ is Poset.

8. Under the divisibility relation (a) b means, a divides b that is a/b show that set of natural numbers N is Poset. If we replace N by Z then this

then this set is poset?

Sol: Here given that sets \mathbb{N} and relation "a|b".
Now for Poset we check following properties.

(i) Reflexive-

The relation is reflexive as every natural number is divisor of itself $\forall a \in \mathbb{N}$ we write also

$\Rightarrow "a|a"$ is reflexive. (A) is path if $a|a$

$\text{min}(a) \in \langle a, (A) \rangle$

(ii) Anti-symmetric :-

if $a|b$ and $b|a$ then $a = b$

- The relation is antisymmetric as $\forall a, b \in \mathbb{N}$,

suppose $a|b$ and $b|a$ then it's always possible

$a=b$ $\Leftrightarrow a|a$ (dividing symbol result)

$\Rightarrow "$ is antisymmetric" is true \Rightarrow

(iii) Transitive -

- The relation is transitive as $\forall a, b, c \in \mathbb{N}$,

such that $a|b$ and $b|c$ $\Rightarrow a|c$ (A)

Suppose $a|b$ and $b|c \Rightarrow b = k_1 a$ & $c = k_2 b$ where

$k_1, k_2 \in \mathbb{Z}$

$\Rightarrow c = k_1 k_2 a$ (1, 2, 3, 4 result)

$\Rightarrow c = k a$ take $k = k_1 k_2$ then

$\Rightarrow a|c$ (A) is path through result

then we get $a \leq c$ (already right result)

hence $a|c$ and $c|c$ both of which

$\Rightarrow "$ is transitive" is true in \mathbb{N} relation

- from (1), (2) and (3) relation "≤" is partially ordered set.

d. Prove that $\langle \{1, 2, 2^2, 2^3, \dots\}, D \rangle$ are Poset

(i) Partially ordered chain: for any $a, b \in A$ has D , i.e. either $a \leq b$ or $b \leq a$

Sol: Here given set $A = \{1, 2, 2^2, 2^3, \dots\}$ and relation "D" ("≤") is divisibility.

(ii) Reflexive:-

$\forall a \in A$ we write $a|a$

$\Rightarrow a|a$ for $a = k \cdot a$ with $k = 1$

$\Rightarrow D$ is reflexive for $a \geq a$ condition and

since $a \geq a$ & $a \leq a$ both $a \geq a$

(iii) Antisymmetric:- for $A \neq \emptyset$ from above

- Here we have $2^2, 2^3 \in A$, with $2^2 < 2^3$ then
but find it is clear that $2^2 \nmid 2^3$ but $2^3 \mid 2^2$ if
possible then $2^2 = 2^3$

$\Rightarrow D$ is antisymmetric for divisibility in $(\{1\}) \cup A$

(iv) Transitive:-

- Here we take $\forall 2^s, 2^t, 2^r \in A$,

Suppose $2^s \mid 2^t$ and $2^t \mid 2^r$

$\Rightarrow 2^s = k_1 2^t$ & $2^t = k_2 2^r$ where $k_1, k_2 \in \mathbb{Z}$

$\Rightarrow 2^s = k_1 k_2 2^r$ \therefore $2^s \mid 2^r$ (i.e.)

$\Rightarrow 2^s \mid 2^r$ take $k_1 k_2 = k$

$\Rightarrow 2^s \mid 2^r$

\Rightarrow then we get $2^2 D 2^1$ and $\textcircled{1}, \textcircled{2}$ must be $2^2 D 0$

$\Rightarrow "0"$ is transitive.

- from $\textcircled{1}, \textcircled{2}$ and $\textcircled{3}$ the set $\{1, 2, 2^2, 2^3, 0\}$, D is partially ordered set for chain we proved only comparable property.

iv) Comparable

- Let $x, y \in A$ with $x = 1 \neq y = 1$.

Now let $x = 2^i \neq y = 2^s$ for some $i, s \in \mathbb{N}$, we have either $i < s$ or $i > s$ that is $2^i D_2 2^s$ or $2^s D_2 2^i$ that is $x \mid y$ or $y \mid x$, so for any element $x, y \in A$ that are comparable. Hence given set is chain.

- Q Prove that $\{1, 3, 3^2, 3^3, 3^5, 3^7\}, D$ are Poset and chain.

Sol:- Here given set $A = \{1, 3, 3^2, 3^3, 3^5, 3^7\}$ and "relation D " ("1") is divisibility.

(i) Reflexive:-

- $\forall a \in A$ we write $a \mid a$ as $a = a \cdot 1$

$\Rightarrow "D"$ is reflexive.

(ii) Anti symmetric :-

Here we have $3^z, 3^s \in A$, with $z < s$
 then it is clear that $3^z D 3^s$ but $3^s \not D 3^z$ and if
 possible then $3^z = 3^s$ which contradicts given

$\Rightarrow "D"$ is antisymmetric.

(iii) Transitive :-

Here we take $3^z, 3^s, 3^t \in A$

Suppose $3^z | 3^s$ and $3^s | 3^t$

$3^s = k_1 3^z \& 3^t = k_2 3^s$ where $k_1, k_2 \in \mathbb{Z}$

$$\Rightarrow 3^t = k_1 k_2$$

$$\Rightarrow 3^t = k_1 3^z \text{ take } k_1 k_2 = k$$

$$\Rightarrow 3^z | 3^t$$

then we get $3^z D 3^t$

$\Rightarrow "D"$ is transitive.

From ①, ② and ③ the set $\{1, 3, 3^2, 3^3, \dots\}, 0\}$ is
 partially ordered set for chain we proved only
 comparable property.

(iv) Comparable :-

Let $x, y \in A$ with $x=1$ & $y=1$.

Now, let $x = 3^z$ & $y = 3^s$ for some $z, s \in \mathbb{N}$, we

∴ we have either $x \leq s \leq y$ or $y \leq s \leq x$ that is s is between x and y , that is $x, y \in A$ that are comparable. Hence given set is chain.

ii. $\langle \{a, (a, b), (a, b)\}, \subseteq \rangle$ is not a chain.

Sol:-

$A \ni e \in E$ such that $e \in e$

$e \in e$ but $e \in e$ is wrong

$S \ni x, y$ such that $x \neq y$ & $x \in y$

$x \in y \neq y$
 $x \in y$ not $y \in x$

$e \in e$

$e \in e$ top sw art

question in "Q"

i. $\langle A, \{ \subseteq, \in, \neq \} \rangle$ is art $\textcircled{1}$ true $\textcircled{2}$ not

where \subseteq is many to one function called as

subset relation

of subset relation

so \subseteq is art $\textcircled{1}$ true $\textcircled{2}$ not

so \subseteq is art $\textcircled{1}$ true $\textcircled{2}$ not

12. Draw the Hasse diagram of the following poset also find cover of element.

1. $\{S_{30}, D\}$

Solⁿ: Here we have $S_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$

Cover of 1 = 2, 3, 5

Cover of 2 = 6, 10

Cover of 3 = 6, 15

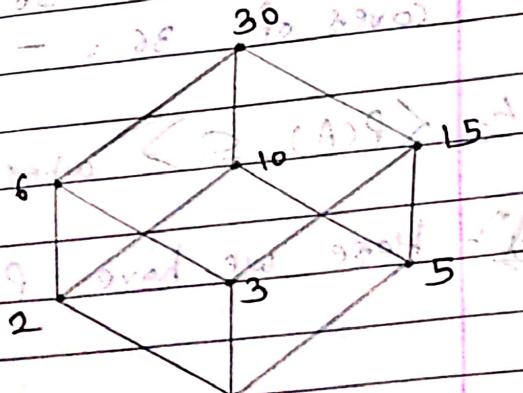
Cover of 5 = 10, 15

Cover of 6 = 30

Cover of 10 = 30

Cover of 15 = 30

Cover of 30 = -



2. $\{S_{1001}, D\}$

Solⁿ: Here we have $S_{1001} = \{1, 7, 11, 13, 77, 91, 143, 1001\}$

Cover of 1 = 7, 11, 13

Cover of 7 = 77, 91

Cover of 11 = 77, 143

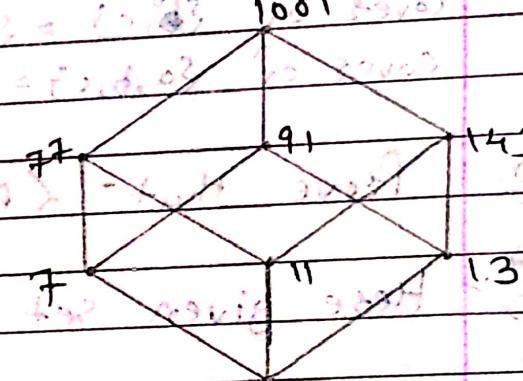
Cover of 13 = 91, 143

Cover of 77 = 1001

Cover of 91 = 1001

Cover of 143 = 1001

Cover of 1001 = -



3. $\{S_{36}, D\}$

Solⁿ: Here we have $S_{36} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$



for ex. $\text{Cover of } 1 = \{2, 3\}$ with $2 \leq 1, 3 \leq 1$ and $2 \leq 3$ $\Rightarrow 36 \text{ is } G$

$\text{Cover of } 2 = \{4, 6\}$ $\Rightarrow 16 \text{ is } G$ and $6 \leq 2$

$\text{Cover of } 3 = \{6, 9\}$

$\text{Cover of } 4 = \{12\}$

$\text{Cover of } 6 = \{12, 18\}$

$\text{Cover of } 9 = \{18\}$

$\text{Cover of } 12 = \{36\}$

$\text{Cover of } 18 = \{36\}$

$\text{Cover of } 36 = -$

Ex. $\langle P(A), \leq \rangle$ where $A = \{a, b, c\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100\}$

Sol. Here we have $P(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$, $- = 36 \text{ to } 39, 40$

$\text{Cover of } \emptyset = \{\{a\}, \{b\}, \{c\}\}$

$\text{Cover of } \{a\} = \{\{a, b\}, \{a, c\}\}$

$\text{Cover of } \{b\} = \{\{a, b\}, \{b, c\}\}$

$\text{Cover of } \{c\} = \{\{a, c\}, \{b, c\}\}$

$\text{Cover of } \{a, b\} = \{\{a, b, c\}\}$

$\text{Cover of } \{a, c\} = \{\{a, b, c\}\}$

$\text{Cover of } \{b, c\} = \{\{a, b, c\}\}$

$\text{Cover of } \{a, b, c\} = -$

Prove that $\langle P(A), \leq \rangle$ is a lattice for $A = \{a, b, c\}$

Sol. Given set $A = \{a, b, c\}$ (lattice \Rightarrow poset)

$P(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

Also we know that $\langle P(A), \leq \rangle$ is a poset. Now we prepare table as follow.

| $\text{glb}\{\text{x,y,z}\} = \text{x} \cap \text{y} \cap \text{z}$ | \emptyset | $\{\text{a}\}$ | $\{\text{b}\}$ | $\{\text{c}\}$ | $\{\text{a,b}\}$ | $\{\text{a,c}\}$ | $\{\text{b,c}\}$ | \emptyset |
|---|-------------|--|--|----------------|------------------|------------------|------------------|--------------------|
| \emptyset | \emptyset | \emptyset | \emptyset | \emptyset | \emptyset | \emptyset | \emptyset | \emptyset |
| $\{\text{a}\}$ | \emptyset | $\{\text{a}\}$ | \emptyset | \emptyset | $\{\text{a,b}\}$ | $\{\text{a,c}\}$ | $\{\text{b,c}\}$ | \emptyset |
| $\{\text{b}\}$ | \emptyset | $\{\text{a,b}\}$ | \emptyset | \emptyset | $\{\text{b}\}$ | $\{\text{b,c}\}$ | $\{\text{a,b}\}$ | \emptyset |
| $\{\text{c}\}$ | \emptyset | $\{\text{a,c}\}$ | \emptyset | \emptyset | $\{\text{c}\}$ | \emptyset | $\{\text{a,c}\}$ | \emptyset |
| $\{\text{a,b}\}$ | \emptyset | $\{\text{a,b}\}$ | $\{\text{a,b}\}$ | \emptyset | \emptyset | \emptyset | $\{\text{a,b}\}$ | \emptyset |
| $\{\text{a,c}\}$ | \emptyset | $\{\text{a,c}\}$ | \emptyset | \emptyset | $\{\text{a,c}\}$ | \emptyset | \emptyset | \emptyset |
| $\{\text{b,c}\}$ | \emptyset | $\{\text{b}\}$ | \emptyset | \emptyset | $\{\text{b,c}\}$ | $\{\text{b,c}\}$ | \emptyset | \emptyset |
| $\{\text{a,b,c}\}$ | \emptyset | $\{\text{a}\}$ | $\{\text{b}\}$ | $\{\text{c}\}$ | $\{\text{a}\}$ | $\{\text{b}\}$ | $\{\text{c}\}$ | $\{\text{a,b,c}\}$ |

| $\text{x} \cap \text{y} \cap \text{z}$ | $\{\text{a}\}$ | $\{\text{b}\}$ | $\{\text{c}\}$ | $\{\text{a,b}\}$ | $\{\text{a,c}\}$ | $\{\text{b,c}\}$ | $\{\text{a,b,c}\}$ |
|--|----------------|--------------------------------------|----------------|------------------|------------------|------------------|--------------------|
| \emptyset | \emptyset | \emptyset | \emptyset | \emptyset | \emptyset | \emptyset | \emptyset |
| $\{\text{a}\}$ | \emptyset | $\{\text{a}\}$ | \emptyset | \emptyset | $\{\text{a}\}$ | $\{\text{a}\}$ | \emptyset |
| $\{\text{b}\}$ | \emptyset | \emptyset | \emptyset | \emptyset | $\{\text{b}\}$ | $\{\text{b}\}$ | \emptyset |
| $\{\text{c}\}$ | \emptyset | \emptyset | \emptyset | $\{\text{c}\}$ | \emptyset | $\{\text{c}\}$ | \emptyset |
| $\{\text{a,b}\}$ | \emptyset | $\{\text{a}\}$ | $\{\text{b}\}$ | \emptyset | $\{\text{a,b}\}$ | $\{\text{a}\}$ | $\{\text{b}\}$ |
| $\{\text{a,c}\}$ | \emptyset | $\{\text{a}\}$ | \emptyset | $\{\text{c}\}$ | \emptyset | $\{\text{a,c}\}$ | \emptyset |
| $\{\text{b,c}\}$ | \emptyset | $\{\text{b}\}$ | \emptyset | $\{\text{c}\}$ | $\{\text{b}\}$ | \emptyset | $\{\text{b,c}\}$ |
| $\{\text{a,b,c}\}$ | \emptyset | $\{\text{a}\}$ | $\{\text{b}\}$ | $\{\text{c}\}$ | $\{\text{a,b}\}$ | $\{\text{a,c}\}$ | $\{\text{b,c}\}$ |

| $\text{x} \cap \text{y} \cap \text{z}$ | \emptyset | $\{\text{a}\}$ | $\{\text{b}\}$ | $\{\text{c}\}$ | $\{\text{a,b}\}$ | $\{\text{b,c}\}$ | $\{\text{a,c}\}$ | $\{\text{a,b,c}\}$ |
|--|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| \emptyset | \emptyset | $\{\text{a}\}$ | $\{\text{b}\}$ | $\{\text{c}\}$ | $\{\text{a,b}\}$ | $\{\text{b,c}\}$ | $\{\text{a,c}\}$ | $\{\text{a,b,c}\}$ |
| $\{\text{a}\}$ | $\{\text{a}\}$ | $\{\text{a}\}$ | $\{\text{b}\}$ | $\{\text{c}\}$ | $\{\text{a,b}\}$ | $\{\text{a,b,c}\}$ | $\{\text{a,c}\}$ | $\{\text{a,b,c}\}$ |
| $\{\text{b}\}$ | $\{\text{b}\}$ | $\{\text{a,b}\}$ | $\{\text{b}\}$ | $\{\text{c}\}$ | $\{\text{a,b}\}$ | $\{\text{b,c}\}$ | $\{\text{a,b,c}\}$ | $\{\text{a,b,c}\}$ |
| $\{\text{c}\}$ | $\{\text{c}\}$ | $\{\text{a,c}\}$ | $\{\text{b,c}\}$ | $\{\text{c}\}$ | $\{\text{a,b,c}\}$ | $\{\text{b,c}\}$ | $\{\text{a,c}\}$ | $\{\text{a,b,c}\}$ |
| $\{\text{a,b}\}$ | $\{\text{a,b}\}$ | $\{\text{a,b}\}$ | $\{\text{a,b}\}$ | $\{\text{a,b,c}\}$ | $\{\text{a,b}\}$ | $\{\text{a,b,c}\}$ | $\{\text{a,b,c}\}$ | $\{\text{a,b,c}\}$ |
| $\{\text{b,c}\}$ | $\{\text{b,c}\}$ | $\{\text{a,b,c}\}$ | $\{\text{b,c}\}$ | $\{\text{b,c}\}$ | $\{\text{a,b,c}\}$ | $\{\text{b,c}\}$ | $\{\text{a,b,c}\}$ | $\{\text{a,b,c}\}$ |
| $\{\text{a,c}\}$ | $\{\text{a,c}\}$ | $\{\text{a,c}\}$ | $\{\text{a,b,c}\}$ | $\{\text{a,c}\}$ | $\{\text{a,b,c}\}$ | $\{\text{a,b,c}\}$ | $\{\text{a,c}\}$ | $\{\text{a,b,c}\}$ |
| $\{\text{a,b,c}\}$ | $\{\text{a,b,c}\}$ | $\{\text{a,b,c}\}$ | $\{\text{a,b,c}\}$ | $\{\text{a,b,c}\}$ | $\{\text{a,b,c}\}$ | $\{\text{a,b,c}\}$ | $\{\text{a,b,c}\}$ | $\{\text{a,b,c}\}$ |