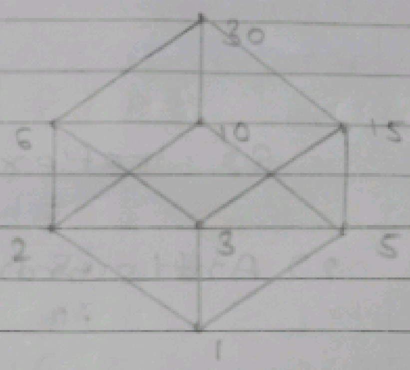
31) Define Join-irreducible elements, Meet-irreducible elements, Atom and Anti-atoms for the lattice.

Ans 1) $\langle S_{30}, D \rangle$

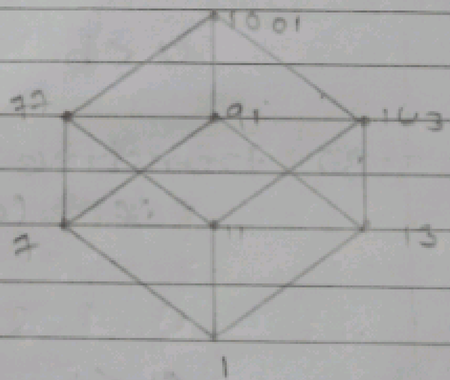
$S_{30} = \{1,2,3,5,6,10,15,30\}$

- join = 2,3,5
meet = 6,10,15
Atom = 2,3,5
Anti-atom = 6,10,15



2) $\langle S_{1001}, D \rangle$

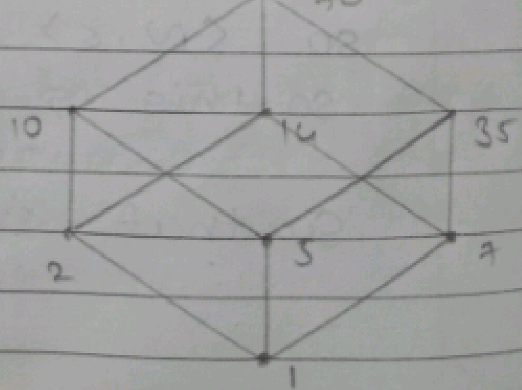
$S_{1001} = \{1,7,11,13,77,91,143,1001\}$



- join : 7,11,13
meet : 77,91,143
Atom : 7,11,13
Anti Atom : 77,91,143

3) $\langle S_{70}, D \rangle$

$S_{70} = \{1,2,5,7,10,14,35,70\}$

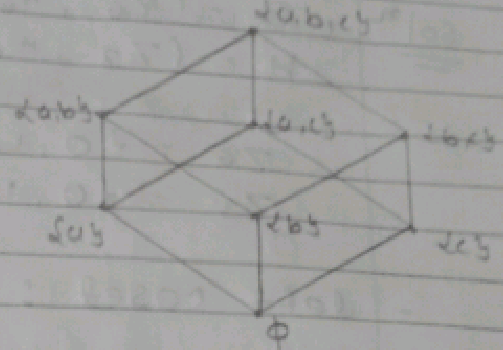


- join = 2,5,7
meet = 10,14,35
Atom = 2,5,7
Anti-atom = 10,14,35

4) $\langle P(A), \subseteq \rangle$ where $A = \{a, b, c\}$

$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$

- join : $\{a\}, \{b\}, \{c\}$
  meet : $\{a,b\}, \{b,c\}, \{a,c\}$
  Atom : $\{a\}, \{b\}, \{c\}$
  Anticatom : $\{a,b\}, \{b,c\}, \{a,c\}$



32) Define Right and left coset. Let $G = (Z, +)$ and $H = (4Z, +)$ Then find all the possible left and right coset.

Sol^n

$G = (Z, +)$

$H = (4Z, +)$

$= \{\dots, -4, 0, 4, \dots\}$

$a_H = \{a_h \mid h \in H\} \quad a \in G$

- left coset :-

$[0] + H = \{[0], [4]\}$

$[1] + H = \{[1], [5]\}$

$[2] + H = \{[2], [6]\}$

$[3] + H = \{[3], [7]\}$

- Right coset : $\{ha \mid h \in H\} \quad h \in G$

$H + 0 = \{\dots, -4, 0, 4, \dots\}$

$H + 1 = \{\dots, -7, -3, 1, 5, \dots\}$

$H + 2 = \{\dots, -6, -2, 2, 6, \dots\}$

$H + 3 = \{\dots, -1, 3, 7, 11, \dots\}$

33) Let $G = (Z_6, +_6)$ and $H = (Z_3, +_6)$ then find all the possible left and right coset.

Sol^n

$G = (Z_6, +_6)$
$H = (Z_3, +_6)$

$Z_6 = \{0,1,2,3,4,5\}$
$Z_3 = \{0,1,2\}$

- left coset : $a_H = \{a_h \mid h \in H\} \quad a \in G$

$[0] + H = \{[0],[3]\}$
$[1] + H = \{[1],[4]\}$
$[2] + H = \{[2],[5]\}$
$[3] + H = \{[3],[0]\}$
$[4] + H = \{[4],[1]\}$
$[5] + H = \{[5],[2]\}$

- Right coset : $Ha = \{ha \mid h \in H\} \quad h \in G$

$H + 0 = \{\ldots, -3, 0, 3, \ldots\}$
$H + 1 = \{\ldots, -5, -2, 1, 4, \ldots\}$
$H + 2 = \{\ldots, -1, 2, 5, \ldots\}$
$H + 3 = \{\ldots, 0, 3, 6, \ldots\}$
$H + 4 = \{\ldots, 1, 4, 7, \ldots\}$
$H + 5 = \{\ldots, 2, 5, 8, \ldots\}$

34) Define cyclic group. Show that $(Z_7, +_7)$ and $(Z_6, +_6)$ is a cyclic group also find the generator of this group.

Sol^n

Cyclic group : Group G is called cyclic group if there exists an element $a \in G$ such that every element of G can be express power of a.

$$G = \langle a \rangle \text{ or } \{a^n \mid n \in z\}$$

- $(Z_6, +_6)$

$Z_6 = \{0, 1, 2, 3, 4, 5\}$

$\langle 1 \rangle = \{1, 2, 3, 4, 5, 0, \ldots\}$

$\langle 2 \rangle = \{2, 4, 0, \ldots\}$

$\langle 3 \rangle = \{3, 0, 3, 0, \ldots\}$

$\langle 5 \rangle = \{5, 4, 3, 2, 1, 0\}$

$(Z_6, +_6) = \{1, 5\}$

- $(Z_7, +_7)$

$Z_7 = \{0, 1, 2, 3, 4, 5, 6\}$

$\langle 1 \rangle = \{0, 1, 2, 3, 4, 5, 6\}$

$\langle 2 \rangle = \{2, 4, 6, 1, 3, 5\}$

$\langle 3 \rangle = \{3, 6, 2, 1, 4\}$

$\langle 4 \rangle = \{4, 1\}$

$(Z_7, +_7) = \{1, 6\}$

35) expressed the boolean ~~funct~~ expression $x_1 * x_2$ in an equivalent sum of products canonical form of three Variable.

Sol^n

x_1	x_2	x_3	$x_1 * x_2$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

36) Expressed the boolean expression $x_1 \oplus x_2$ in an equivalent sum of product canonical form.

Sol^n

x_1	x_2	x_1'	x_2'	$x_1'x_2$ +	x_1x_2'	$x_1 \oplus x_2$
0	0	1	1	0	0	0
0	1	1	0	1	0	1
1	0	0	1	0	0	0
1	1	0	0	0	0	0

37) expressed the boolean expression $x_1 * x_2$ is an equivalent product of sum canonical form.

Sol^n

x_1	x_2	$x_1 * x_2$
0	0	0
0	1	0
1	0	0
1	1	1

39) expressed boolean expression $(x_1 \oplus x_2)' * x_3$ is an equivalent product of sum canonical form.

Ans

x_1	x_2	x_3	$x_1 \oplus x_2$	$(x_1 \oplus x_2)'$	$(x_1 \oplus x_2)' * x_3$
0	0	0	0	1	0
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	1	0	0
1	0	0	0	1	0
1	0	1	0	1	1
1	1	0	0	1	0
1	1	1	0	1	1

40) Define boolean algebra. show that following 1 to 4 are boolean algebra.

1) $\langle S_{30}, D \rangle$ $S_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$

table ①

*	1	2	3	5	6	10	15	30
1	1	1	1	1	1	1	1	1
2	1	2	1	1	2	2	1	2
3	1	1	3	1	3	1	3	3
5	1	1	1	5	1	5	5	5
6	1	2	3	1	6	2	3	6
10	1	2	1	5	2	10	5	10
15	1	1	3	5	3	5	15	15
30	1	2	3	5	6	10	15	30

⊕	1	2	3	5	6	10	15	30
1	1	2	3	5	6	10	15	30
2	2	2	6	10	6	10	30	30
3	3	6	3	15	6	30	15	30
5	5	10	15	5	30	10	15	30
6	6	6	6	30	6	10	15	30
10	10	10	30	10	10	10	15	30
15	15	30	15	15	15	15	15	30
30	30	30	30	30	30	30	30	30

Table ②

table ③

element	1	2	3	5	6	10	15	30
complement	30	15	10	6	5	3	2	1

properties : 1) lattice : from table 1 and 2 we can say that $\langle S_{30}, *, \oplus \rangle$ is a lattice.

2) bounded : glb of $S_{30} = 1$
$= 0$ - element of S_{30}

lub of $S_{30} = 30 = 1$-

that's why $\langle S_{30}, *, \oplus, 0, 1 \rangle$ is bounded.

3) Complemented : from table ③ we can say that every element of S_{30} has unique complement in S_{30}.

$\therefore \langle S_{30}, *, \oplus, 0, 1, ' \rangle$ is a bounded complement lattice.

4) Distrubutive : from table 1 and 2 element of S_{30} is distrubutive.

$\therefore \langle S_{30}, *, \oplus \rangle$ is distributive lattice.

hence $\langle S_{30}, *, \oplus, 0, 1, ' \rangle$ is a boolean algebra.

2) $\langle S_{1001}, D \rangle$

$S_{1001} = \{1, 7, 11, 13, 77, 91, 143, 1001\}$

table ①

*	1	7	11	13	77	91	143	1001
1	1	1	1	1	1	1	1	1
7	1	7	1	1	7	7	1	7
11	1	1	11	1	11	1	11	11
13	1	1	1	13	1	13	13	13
77	1	7	11	1	77	1	1	77
91	1	7	1	13	1	91	1	1
143	1	7	11	13	1	1	143	143
1001	1	7	11	13	77	91	143	1001

table ②

⊕	1	7	11	13	77	91	143	1001
1	1	7	11	13	77	91	143	1001
7	7	7	77	91	77	91	1001	1001
11	11	77	11	143	77	1001	143	1001
13	13	91	143	13	77	91	143	1001
77	77	77	77	1001	77	1001	1001	1001
91	91	91	1001	1001	1001	91	1001	1001
143	143	1001	143	[illegible]	[illegible]	[illegible]	143	1001
1001	1001	1001	1001	1001	1001	1001	1001	1001

table ③

element	1	7	11	13	77	91	143	1001
complement	1001	143	91	77	13	11	7	1

properties : ① lattice : from table 1 and 2 we can say that $\langle S_{1001}, *, \oplus \rangle$ is a lattice.

2) bounded : glb of $S_{1001} = 1$
lub of $S_{1001} = 1001$

that's why $\langle S_{1001}, *, \oplus, 0, 1 \rangle$ is bounded.

3) complemented : from the table ③ we can say that every element of S_{30} has unique complement.

$\therefore \langle S_{1001}, *, \oplus, 0, 1, ' \rangle$ is a bounded complemented lattice.

4) Distrubutive : from table 1 and 2 element of S_{1001} is distrubutive.

$\therefore \langle S_{1001}, *, \oplus \rangle$ is a distrubutive lattice.

hence $\langle S_{1001}, *, \oplus, 0, 1, ' \rangle$ is a boolean algebra.

3) $\langle S_{70}, D \rangle$

$S_{70} = \{1, 2, 5, 7, 10, 14, 35, 70\}$

Table ①

*	1	2	5	7	10	14	35	70
1	1	1	1	1	1	1	1	1
2	1	2	1	1	2	2	1	2
5	1	1	5	1	5	1	5	5
7	1	1	1	7	1	7	7	7
10	1	2	5	1	10	2	5	10
14	1	2	1	7	2	14	7	14
35	1	1	5	7	5	7	35	35
70	1	2	5	7	10	14	35	70

Table ②

⊕	1	2	5	7	10	14	35	70
1	1	2	5	7	10	14	35	70
2	2	2	10	14	10	14	70	70
5	5	10	5	35	10	70	35	70
7	7	14	35	7	70	14	35	70
10	10	10	10	70	10	14	35	70
14	14	14	70	14	10	14	35	70
35	35	70	35	35	35	35	35	70
70	70	70	70	70	70	70	70	70

Table ③

element	1	2	5	7	10	14	35	70
Complement	70	35	14	10	7	5	2	1

properties : ① lattice : from table 1 and 2 we can say that $< S_{70}, *, \oplus >$ is lattice.

2) bounded : glb of $S_{70} = 1 = 0$
lub of $S_{70} = 70 = 1$

that's why $< S_{70}, *, \oplus, 0, 1 >$ is bounded.

3) Complemented : from table ③ we can say that every element of S_{70} has unique complement.

$\therefore \langle S_{70}, *, \oplus, 0, 1, ' \rangle$ is a bounded complemented lattice.

4) Distributive : from table 1 and 2 element of S_{70} is distributive.

$\therefore \langle S_{70}, *, \oplus \rangle$ is a distributive lattice.

hence $\langle S_{70}, *, \oplus, 0, 1, ' \rangle$ is a boolean algebra.

4) $\langle P(A), \subseteq \rangle$ where $A = \{a, b, c\}$

$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$

table ①

$*$	ϕ	$\{a\}$	$\{b\}$	$\{c\}$	$\{a,b\}$	$\{b,c\}$	$\{a,c\}$	$\{a,b,c\}$
ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
$\{a\}$	ϕ	$\{a\}$	ϕ	ϕ	$\{a\}$	ϕ	$\{a\}$	$\{a\}$
$\{b\}$	ϕ	ϕ	$\{b\}$	ϕ	$\{b\}$	$\{b\}$	ϕ	$\{b\}$
$\{c\}$	ϕ	ϕ	ϕ	$\{c\}$	ϕ	$\{c\}$	$\{c\}$	$\{c\}$
$\{a,b\}$	ϕ	$\{a\}$	$\{b\}$	ϕ	$\{a,b\}$	$\{b\}$	$\{a\}$	$\{a,b\}$
$\{b,c\}$	ϕ	ϕ	$\{b\}$	$\{c\}$	$\{b\}$	$\{b,c\}$	$\{c\}$	$\{b,c\}$
$\{a,c\}$	ϕ	$\{a\}$	ϕ	$\{c\}$	$\{a\}$	$\{b\}$	$\{a,c\}$	$\{a,c\}$
$\{a,b,c\}$	ϕ	$\{a\}$	$\{b\}$	$\{c\}$	$\{a,b\}$	$\{b,c\}$	$\{a,c\}$	$\{a,b,c\}$

table ②

$\oplus$	ϕ	$\{a\}$	$\{b\}$	$\{c\}$	$\{a,b\}$	$\{b,c\}$	$\{a,c\}$	$\{a,b,c\}$
ϕ	ϕ	$\{a\}$	$\{b\}$	$\{c\}$	$\{a,b\}$	$\{b,c\}$	$\{a,c\}$	$\{a,b,c\}$
$\{a\}$	$\{a\}$	$\{a\}$	$\{a,b\}$	$\{a,c\}$	$\{a,b\}$	$\{a,b,c\}$	$\{a,c\}$	$\{a,b,c\}$
$\{b\}$	$\{b\}$	$\{a,b\}$	$\{b\}$	$\{b,c\}$	$\{a,b\}$	$\{b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$
$\{c\}$	$\{c\}$	$\{a,c\}$	$\{b,c\}$	$\{c\}$	$\{a,b,c\}$	$\{b,c\}$	$\{a,c\}$	$\{a,b,c\}$
$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b,c\}$	$\{a,b\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$
$\{b,c\}$	$\{b,c\}$	$\{a,b,c\}$	$\{b,c\}$	$\{b,c\}$	$\{a,b,c\}$	$\{b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$
$\{a,c\}$	$\{a,c\}$	$\{a,c\}$	$\{a,b,c\}$	$\{a,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,c\}$	$\{a,b,c\}$
$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$

table ③

element	ϕ	$\{a\}$	$\{b\}$	$\{c\}$	$\{a,b\}$	$\{b,c\}$	$\{a,c\}$	$\{a,b,c\}$
Complement	$\{a,b,c\}$	$\{b,c\}$	$\{a,c\}$	$\{a,b\}$	$\{c\}$	$\{a\}$	$\{b\}$	ϕ

1) lattice : from table 1 and 2 we can say that $\langle P(A), *, \oplus \rangle$ is lattice.

2) bounded : glb of $P(A) = \phi = 0$
lub of $P(A) = \{a,b,c\} = 1$

that's why $\langle P(A), *, \oplus, 0, 1 \rangle$ is bounded.

3) Complemented : from table ③ we can say that every element of $P(A)$ has unique complement.

$\therefore \langle P(A), *, \oplus, 0, 1, ' \rangle$ is bounded Complemented lattice.

4) Distributive : from table 1 and 2 element of $P(A)$ is distributive.
$\therefore \langle P(A), *, \oplus \rangle$ is distrubutive lattice.

hence $\langle P(A), *, \oplus, 0, 1, ' \rangle$ is boolean algebra.