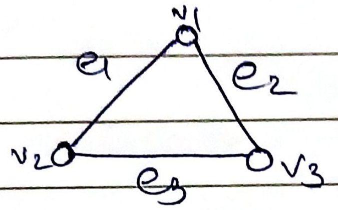
* Graph Theory

* **Graph** : let V be the non-empty set of points and E be the set of lines between points, then ordered pair (V, E) is known as graph.

- The element V is also known as vertex, node, point and elements E are known as Edge, line, arc.



$V = \{v_1, v_2, v_3\}$ is set of vertices.
$E = \{e_1, e_2, e_3\}$ is set of edges.

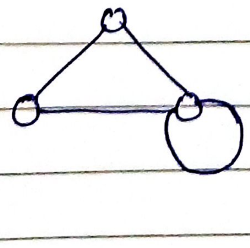
* **Trivial Graph** : A Graph have only one vertex and no edge is called Trivial graph

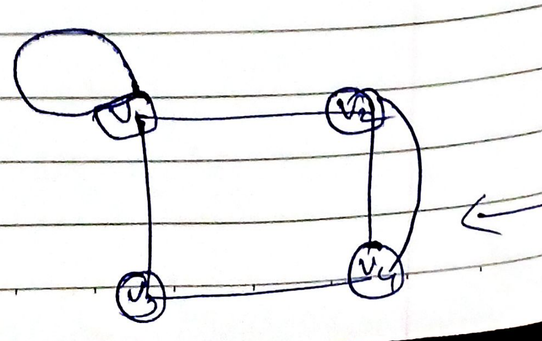
Ex o

* **Null Graph** : A Graph having number of n vertices and no edges is called Null Graph

Ex o o o o

* **Self Loop** : An Edges having same vertex as both its end vertices.

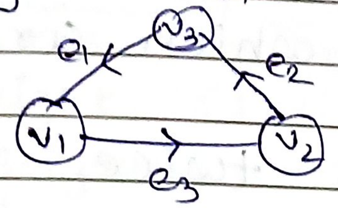




(Parallel edges)

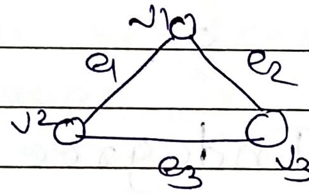
* **Multiedge** : In a Graph two or more edges having same end points. ~~then~~

* **Directed Graph** : Direction of edges in the Graph is known as Directed graph.

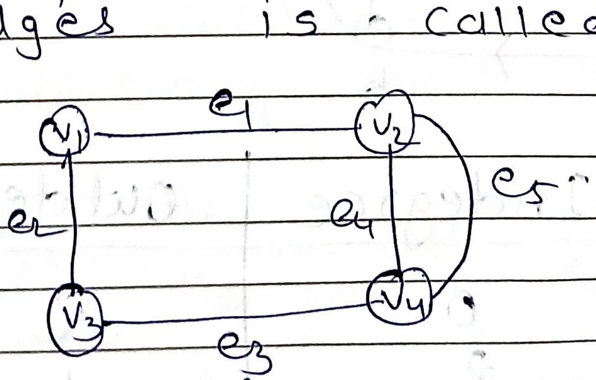


(In a graph, each edges has a direction, then --)

* **Simple Graph** :
A Graph does not contain any self loop and multiedges then the Graph is known as simple Graph.



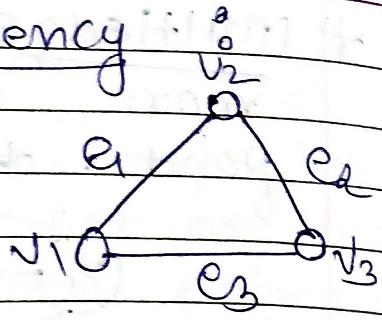
* **Multi Graph** : A Graph having multiedges is called Multi-Graph.



* **Pseudo Graphs** : If self loop as well as parallel edges are allowed, then the graph is known as Pseudo Graph.

- Two vertices are said to be adjacent if they are joined by an edge.

* Incidence and Adjacency :

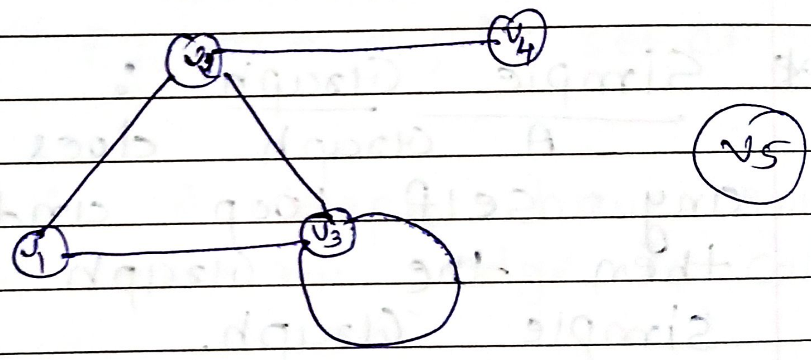


v_1 & v_2 = Adjacent

e_1 = Incident

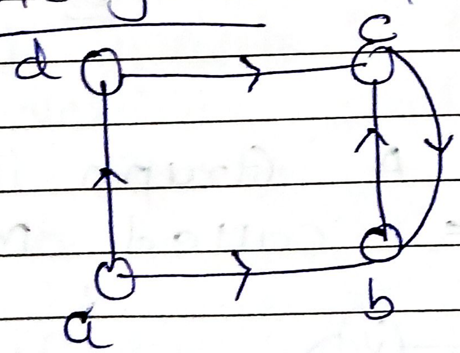
* Degree of Vertex :

- Degree of vertex v is equal to Number of edges which are incident on v.
- self loop counted twice



⇒ $d(v_1) = 2$
$d(v_2) = 3$
$d(v_3) = 4$
$d(v_4) = 1$
$d(v_5) = 0$

* Indegree & Outdegree

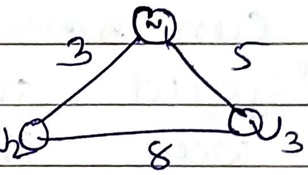


vertices	Indegree	outdegree
a	0	2
b	2	1
c	2	1
d	1	1

* **Isolated vertex** : A vertex with degree 0 is called isolated vertex.

* **Pendant vertex** : A vertex with degree 1 is called pendant vertex

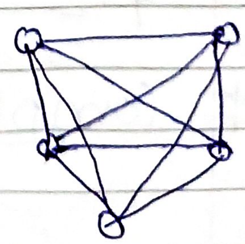
* **Weighted Graph** : A Graph is known as weighted graph if some weight (Positive real number) is assigned to each vertices or each edges or both.

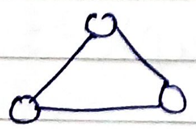


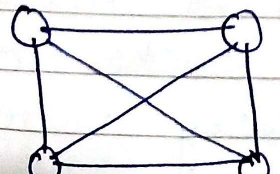
* **Complete Graph** : A simple graph with n vertices is known as complete graph if degree of each vertex is (n-1).
  i.e Every pair is adjacent.
- A complete graph with n-vertices is denoted by K_n.

[A graph in which each vertex connected to every other vertex].

For $n=2 \Rightarrow$  K_2

$n=5 \Rightarrow$ 

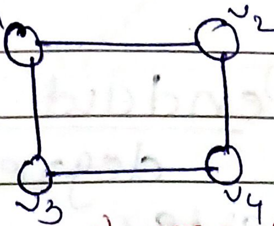
$n=3 \Rightarrow$  K_3

$n=4 \Rightarrow$ 

K_4 $\Rightarrow$ Number of edges in complete graph K_n are $n(n-1)/2$

* **Regular Graph :** In a graph if degree of each vertex is same then the graph is known as Regular Graph.

→ $d(v_1) = 2$, $d(v_3) = 2$
$d(v_2) = 2$, $d(v_4) = 2$

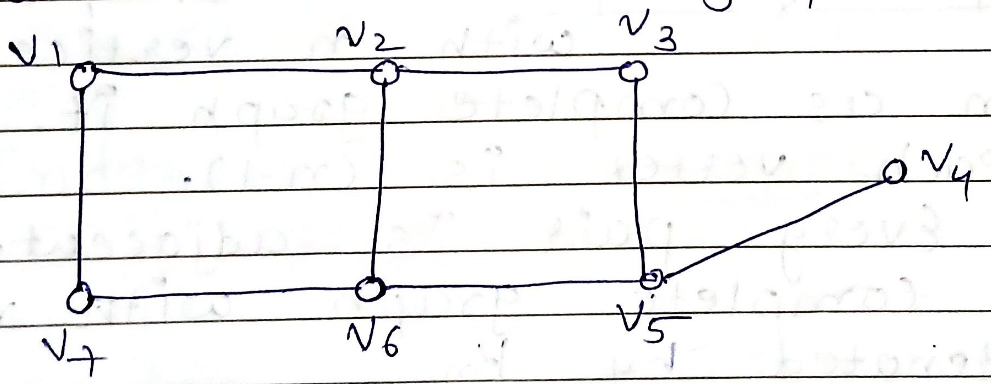


2-regular graph


1-Regular graph

* **Bigraph (Bipartite)**

If the vertex set V of a graph G can be Partioned into two non-empty disjoint subset X and Y in such a way that edge of G has one end in X and one end in Y, then G is called Bigraph.



⇒ $V = \{v_1, v_2, \text{—}, v_7\}$.
$X = \{v_1, v_6, v_3, v_4\}$.
$Y = \{v_2, v_7, v_5\}$.

* **Complete Bipartite Graph :**

A Bipartite Graph is known as complete bipartite if each vertex of V_1 is joined with every vertex of V_2 Y.

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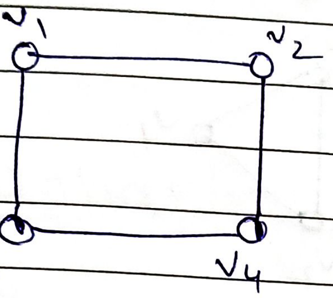
- A complete bipartite graph is denoted by $K_{m,n}$ where number of vertices of X are m and number of vertices of Y are n.

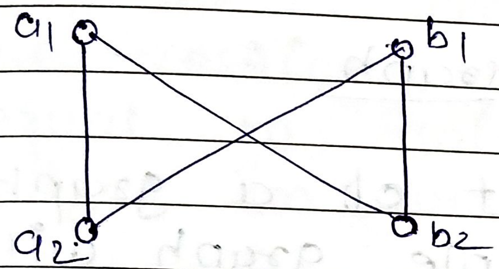
∴ The total no. of edges in a complete bipartite graph $K_{m,n}$ is $m.n$.

Ex ① 

$X = \{v_1, v_4\}$

$Y = \{v_3, v_2\}$.

- It is denoted by $K_{2,2}$

② 

$X = \{a_1, b_1\}$.

$Y = \{a_2, b_2\}$

- $K_{2,2}$.

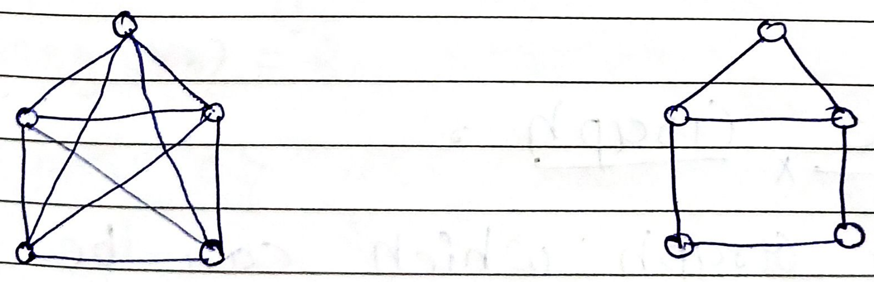
- no. of edges are 4.

* Subgraph :

∵ Let $G = (V, E)$ be a graph

∴ A graph $H = (V', E')$ is said to be a subgraph of G if E' is a subset of E and V' is a subset of V.

Ex



* Decomposition of Graph :

A graph G is said to be decomposition into two subgraph

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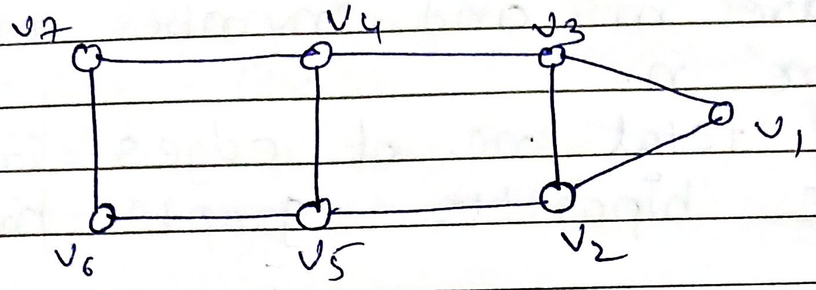
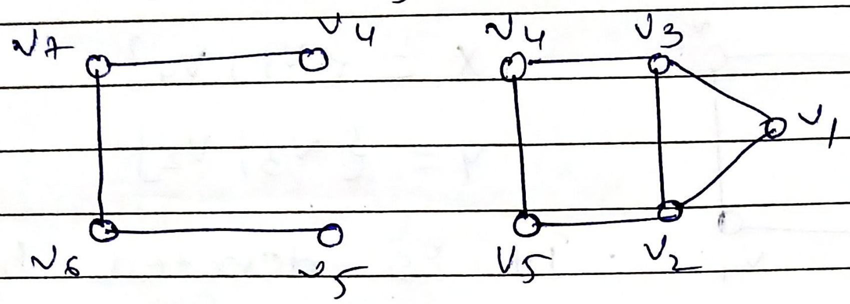
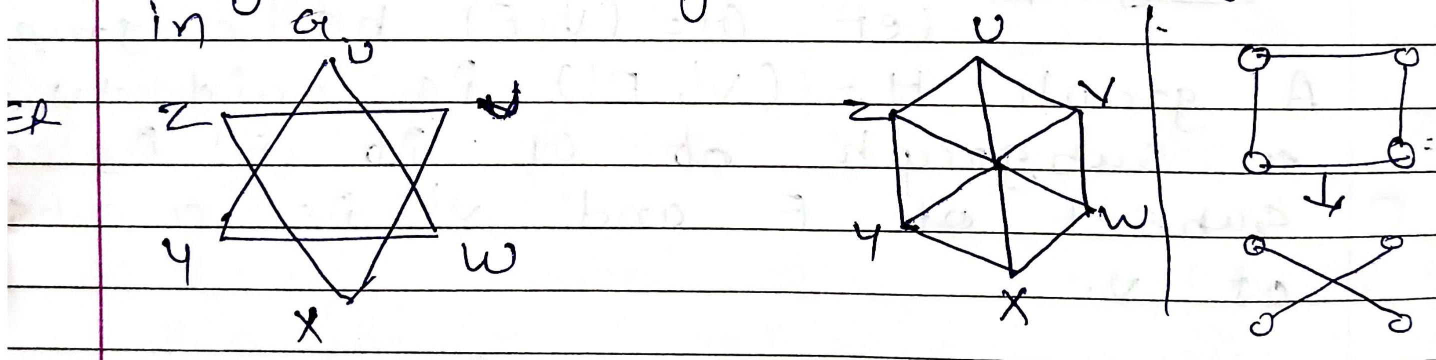
G1 and G2 if $G_1 \cup G_2 = G$ &
$G_1 \cap G_2$ = null Graph.





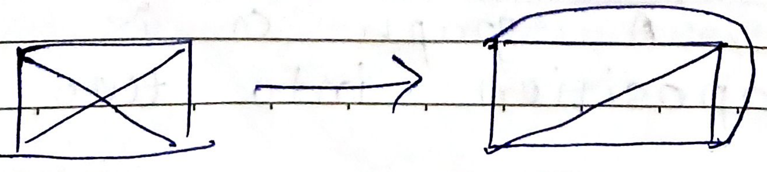
* Complement of Graph :

The complement of a graph G is defined as a simple graph with the same vertex set as G and where two vertices u and v are adjacent only when they are not adjacent in G

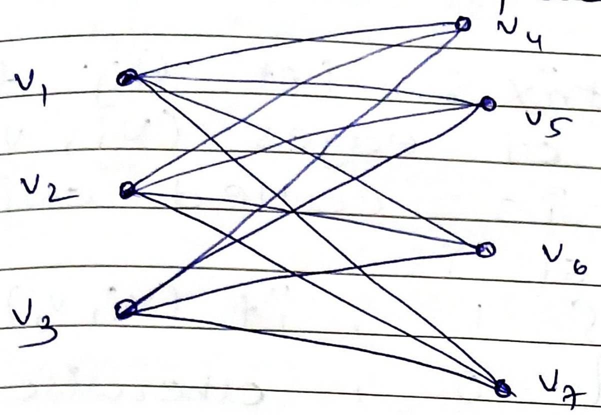


* Planare Graph :

A Graph which can be drawn in the plane so that its edges do not cross is called planner graph.



* Draw $K_{3,4}$ bipartite graph.

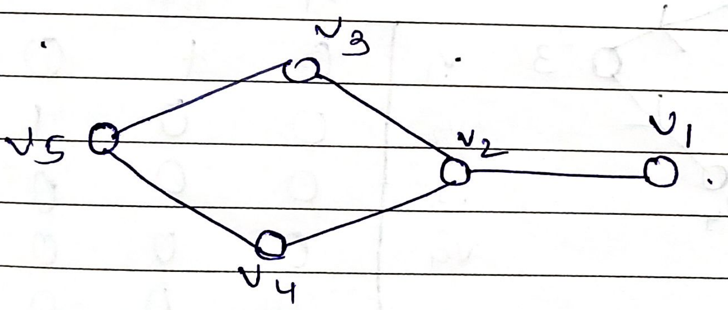


* Handshaking Theorem :

The sum of the degree of the vertices of a graph G is equal to twice the number of edges in G.

i.e $\sum_{i=1}^{n} deg(v_i) = 2 \times$ Number of edges.

Ex



→ Here, $deg(v_1) = 1$, $deg(v_2) = 3$, $deg(v_3) = 2$, $deg(v_4) = 2$, $deg(v_5) = 2$

∴ Sum of degree of all vertices.

$1 + 3 + 2 + 2 + 2 = 10$.

Also, number of edges are 5.

∴ $\sum_{i=1}^{5} deg(v_i) = 2 \times 5 = 10$.

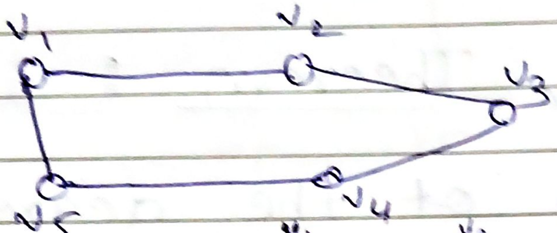
Hence, thm is verified

* Matrix Representation of a Graph :-

① **Adjacency Matrix :** Let a_{ij} denote the number of edges (v_i, v_j) then $A = [a_{ij}]_{m \times m}$ is called adjacency matrix of G if

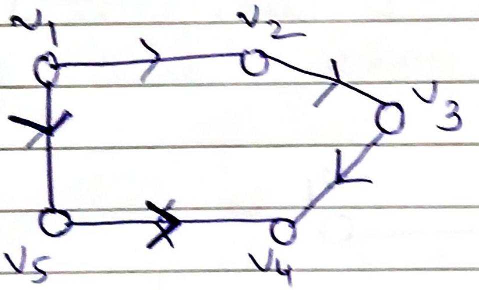
$$a_{ij} = \begin{cases} 1 , & \text{if } (v_i, v_j) \text{ is an edges.} \\ 0 , & \text{otherwise} \end{cases}$$

Ex



$$\Rightarrow A = \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} & \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} & \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \end{array}$$

Ex



$$\begin{array}{c|ccccc} & v_1 & v_2 & v_3 & v_4 & v_5 \\ \hline v_1 & 0 & 1 & 0 & 0 & 1 \\ v_2 & 0 & 0 & 1 & 0 & 0 \\ v_3 & 0 & 0 & 0 & 1 & 0 \\ v_4 & 0 & 0 & 0 & 0 & 0 \\ v_5 & 0 & 0 & 0 & 1 & 0 \end{array}$$

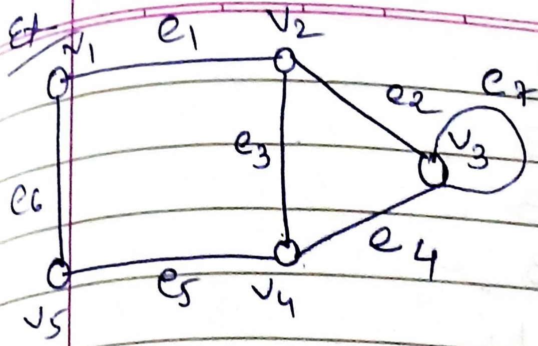
② **Incidence Matrix :**

Let G be a graph with m vertices $v_1, v_2, \ldots, v_m$ and n edges $e_1, e_2, \ldots, e_n$.

then matrix $M = [m_{ij}]_{m \times m}$ defined by

$$m_{ij} = \begin{cases} 1 ; & \text{vertex } v_i \text{ is incident on } e_j. \\ 0 ; & v_i \text{ is not incident on } e_j. \\ 2 ; & v_i \text{ is an end of the loop } e_j \end{cases}$$

Ex



	e_1	e_2	e_3	e_4	e_5	e_6	e_7
v_1	1	0	0	0	0	1	0
v_2	1	1	1	0	0	0	0
v_3	0	1	0	1	0	0	2
v_4	0	0	1	1	1	0	0
v_5	0	0	0	0	1	1	0

* Isomorphism of Graph :

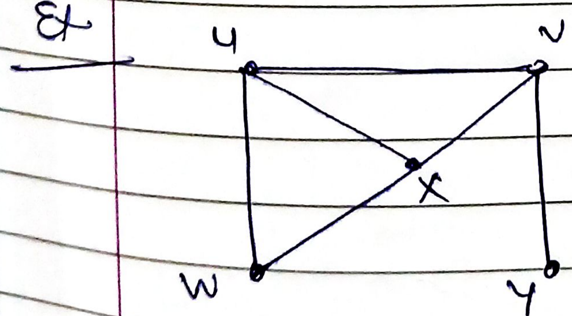
Two Graphs G_1 and G_2 are said to be Isomorphic if there is a one to one correspondence between their vertices and their edges.

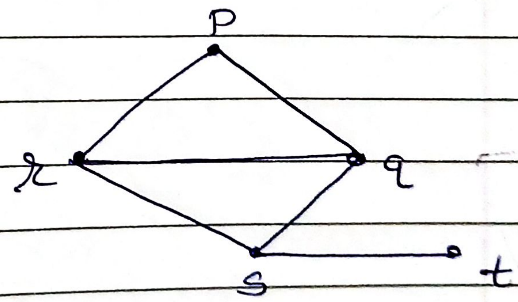
- In other words,

Two Graph G_1 and G_2 are Isomorphic if

1) Number of vertices are same
2) " " edges " "
3) An equal number of vertices with given degree
4) Vertex correspondence & edge corres- valid

Ex





1) Number of vertices are same
2) Number of edges are same
3) An equal number of vertices with given degree

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$d(u) = 3$	$d(p) = 2$
$v = 3$	$q = 3$
$w = 2$	$r = 3$
$x = 3$	$s = 3$
$y = 1$	$t = 1$.

(4). One-one correspondence on vertex & edge

vertex		edges
	$y - t$	
	$w - p$	
	$v - s$	
q, r	$u - q$	
	$x - r$	