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2. Boolean Algebra and logic gates

1) Simplify the following Boolean functions to a maximum number of literals.

a) $xy + xy'$
$= x(y + y')$
$= x(1)$
$= x$

b) $(x+y)(x+y')$
$= x + yy'$
$= x + 0$
$= x$

c) $xyz + x'y + xyz'$
$= xy(z + z') + x'y$
$= xy(1) + x'y$
$= y(x + x')$
$= y(1)$
$= y$

d) $zx + zx'y$
$= z(x + x'y)$
$= z(x + x')(x + y)$
$= z(1)(x + y)$
$= z(x + y)$

e) $(a+b)'(a'+b')$
$(a'b')(a'+b')$

f) $y(w'z + wz) + xy$
$= y[w(z + z')] + xy$
$= y(w + x$

2) Reduce the following Boolean expression to the required number of Intervals.

a) $abc + a'b'c + abc' + a'bc + a'b'c'$ to 5 literals

$= ab(c+c') + a'b'(c+c') + a'bc$

$= ab + a'(b' + bc)$

$= ab + a'(b'+c).(b'+b)$

$= ab + a'(b'+c)$

b) $bc + ac' + ab + bcd$ to 4 literals

$= bc(1+d) + ac' + ab$

$= bc + ac' + ab$

$= bc + ac' + ab(c+c')$

$= bc + ac' + abc + abc'$

$= bc(1+a) + ac'(1+b)$

$= bc + ac'$

c) $[(CD') + A]' + A + CD + AB$ to three literals

$= (C' + D).a' + a + cd + ab$

$= a'c' + a'd + a + cd + ab$

$= a'c' + a'd + a(1+b) + cd$

$= a'c' + a'd + a + cd$

$= (a + a')(a + c') + a'd + cd$

$= 1(a + c') + a'd + cd$

$= (a+a').(a+d) + (c'+c).(c'+d)$

$= a + d + c' + d$

$= a + c' + d$

3) Find the complement of the following Boolean functions and reduce them into minimum numbers of literals

a) $(BC' + A'D)(AB' + CD')$

$= [(BC' + A'D).AB'] + [(BC' + A'D).CD']$

$= AB'.BC' + AB'.A'D + BC'.CD' + A'D.CD'$

$= 0 + 0 + 0 + 0$

$F = 0$

$F' = 1$

b) $F = B'D + A'BC' + ACD + A'BC$

$= D(B' + AC) + A'B(C + C')$

$= D(B' + AC) + A'B$

$= DB' + ACD + A'B$

$= B'DA' + B'DB + ACD$

$= B'DA' + ACD$

$= D(A'B' + AC)'$

$= D.(A'B'A + A'B'C)'$

$= D.A'B'C$

$= A + B + C' + D'$

4) Obtain the truth table of the function

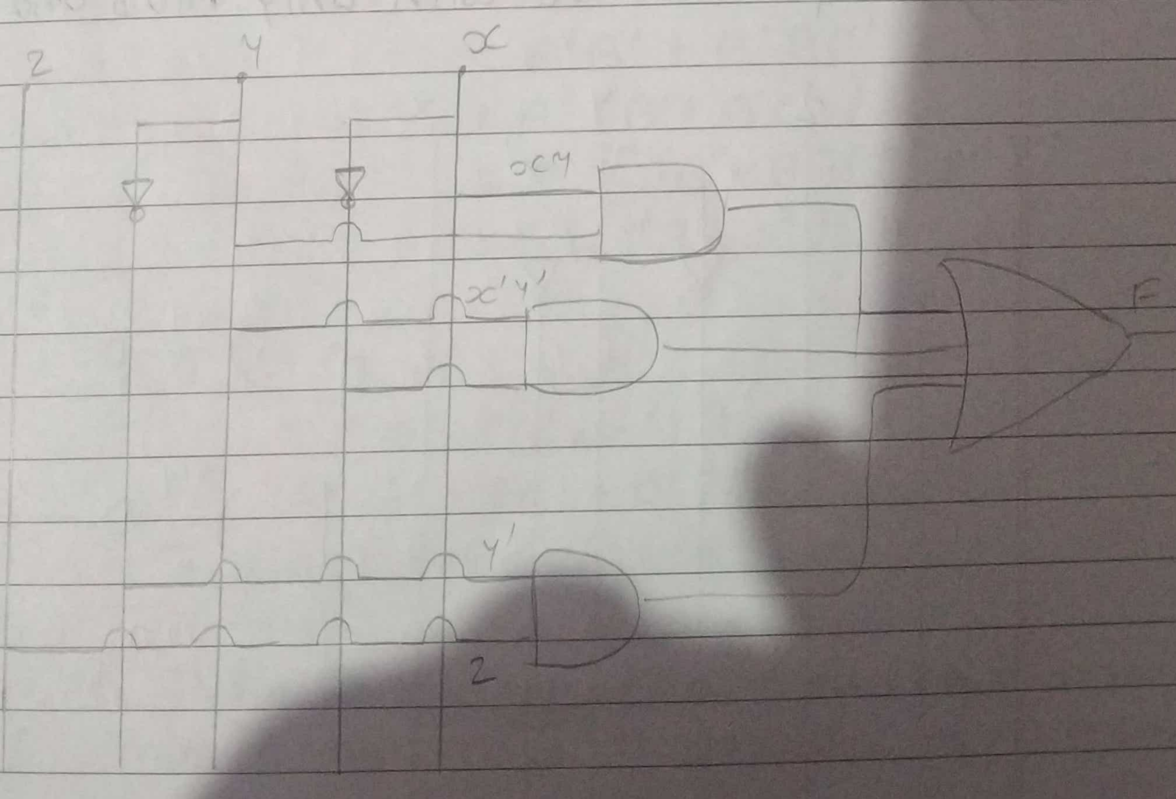
$F = xy + x'y' + y'z$

x	y	z	x'	y'	z'	xy	xy'	$y'z$	F
0	0	0	1	1	1	0	0	0	0
0	0	1	1	1	0	0	0	1	1
0	1	0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0	0	0
1	0	0	0	1	1	0	1	0	1
1	0	1	0	1	0	0	1	1	1
1	1	0	0	0	1	1	0	0	1
1	1	1	0	0	0	1	0	0	1

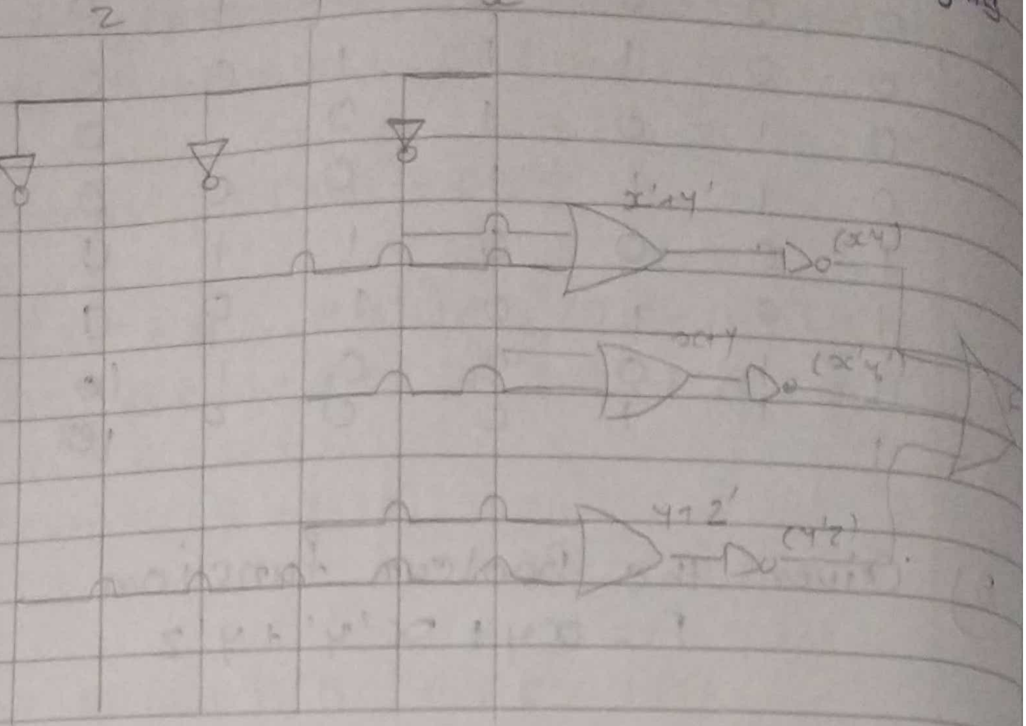
5) Given the Boolean function

$$F = xy + x'y' + y'z$$

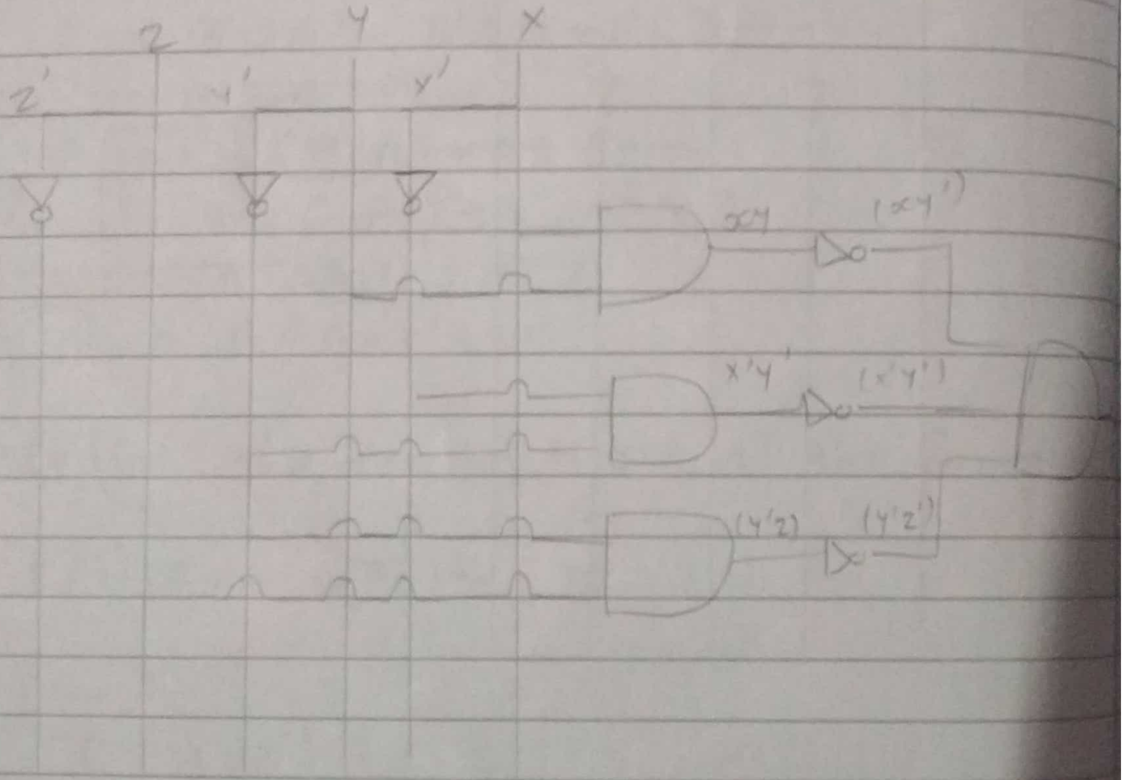
a) Implement



b) Implement it with only OR and NOT gates



c) Implement it with only AND and NOT gate



6) Simply the functions T_1 & T_2 to a maximum number of literal

A	B	C	T_1	T_2
0	0	0	$1 = m_0$	$0 = M_0$
0	0	1	$1 = m_1$	$0 = M_1$
0	1	0	$1 = m_2$	$0 = M_2$
0	1	1	$0 = m_3$	$1 = M_3$
1	0	0	$0 = m_4$	$1 = M_4$
1	0	1	$0 = m_5$	$1 = M_5$
1	1	0	$0 = m_6$	$1 = M_6$
1	1	1	$0 = m_7$	$1 = M_7$

$$T_1 = \Sigma(0,1,2) = A'B'C' + A'B'C + A'BC'$$
$$= A'B'(C'+C) + A'BC'$$
$$= A'B' + A'BC'$$
$$= A'(B' + BC')$$
$$= A'[(B'+B)(B'+C')]$$
$$= A'.(B'+C')$$

$$T_2 = (T_1)'$$
$$= [A'.(B'+C')]'$$
$$= A + (B'+C')'$$
$$T_2 = A + BC$$

7) Express the following functions in a sum of minterms and a product of maxterms

a) $F(A,B,C,D) = D(A'+B) + B'D$
$$= A'D + BD + B'D$$

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→ $A'D = A'D(C+C')$
$= A'DC + A'DC'$
$= A'D(B+B') + A'DC'(B+B')$
$= A'BCD + A'B'CD + A'BC'D + A'B'C'D$

→ $BD = BD(A+A') = ABD + A'BD$
$= ABD(C+C') + A'BD(C+C')$
$= ABCD + ABC'D + A'BCD + A'BC'D$

→ $B'D = B'D(A+A') = A'D + A'B'D$
$= AB'D(C+C') + A'B'D(C+C')$
$= AB'CD + AB'C'D + A'B'CD + A'B'C'D$

→ $F(A,B,C,D) = A'BCD + A'BCD + A'BC'D + A'B'C'D$
$+ ABCD + ABC'D + AB'CD + AB'C'D$
$+ A'BCD + A'BCD + A'B'C'D + A'B'C'D$
(Here $x + x = x$)

$$F(A,B,C,D) = \underset{9}{\underline{A'BCD}} + \underset{3}{\underline{A'B'CD}} + \underset{5}{\underline{A'BC'D}} + \underset{1}{\underline{A'B'C'D}}$$
$$+ \underset{15}{\underline{ABCD}} + \underset{13}{\underline{ABC'D}} + \underset{11}{\underline{AB'CD}} + \underset{9}{\underline{AB'C'D}}$$

$F(A,B,C,D) = \Sigma(1,3,5,7,9,11,13,15)$ (SOP)

$F'(A,B,C,D) = m(0,2,4,6,8,10,12,14)$ (POS)

$F'(A,B,C,D) = (A+B+C+D).(A+B+C'+D).$
$(A+B'+C+D).(A+B'+C'+D).(A'+B+C+D)$
$(A'+B+C'+D)(A'+B'+C+D).(A'+B'+C'+D)$

b) $F(w,x,y,z) = y'z + wxy' + wxz' + w'x'z$

$y'z = y'z(x+x') = xy'z + x'y'z$
$= xy'z(w+w') + x'y'z(w+w')$
$= wxy'z + w'xy'z + wx'y'z + w'x'y'z$

$wxy' = wxy'(z+z')$
$= wxy'z + wxy'z'$

$\rightarrow wxz' = wxz'(y+y')$
$= wxyz' + wxy'z'$

$\rightarrow w'x'z = w'x'z(y+y')$
$= w'x'yz + w'x'y'z$

$F(w,x,y,z) = wxy'z + w'xy'z + wx'y'z + wx'yz +$
$wxy'z + wx'y'z' + wxyz' + wxy'z'$
$+ w'x'yz + w'x'y'z$
$(\because x + x = x)$

$F(w,x,y,z) = \Sigma(1,3,5,9,12,13,15)$ (SOP)

$F'(w,x,y,z) = \pi(0,2,4,6,\cancel{8},7,8,10,11,15)$ (POS)

c) $F(x,y,z) = 1$

$\rightarrow 1 = x + x' = x(y+y') + x'(y+y')$
$= xy + xy' + x'y + x'y'$
$= xy(z+z') + xy'(z+z') + x'y(z+z)$
$+ x'y'(z+z')$
$= xyz + xyz' + xy'z + xy'z' + x'yz$
$+ x'yz' + x'y'z + x'y'z'$

$F(x,y,z) = \Sigma(0,1,2,3,4,5,6,7) \rightarrow SOP$

No. Maxterms $\Rightarrow$ so No POS.

8) Convert the following to the order canonical form

a) $F(x,y,z) = \Sigma(1,3,7)$

$F' = \Sigma(0,2,4,5,6)$

$F' = m_0 + m_2 + m_4 + m_5 + m_6$

$F = (F')' = (m_0)'(m_2)'(m_4)'(m_5)'(m_6)'$

$= M_0 . M_2 . M_4 . M_5 . M_6$

$F(x,y,z) = \Pi(0,2,4,5,6)$

b) $F(w,x,y,z) = \Sigma(0,2,6,11,13,14)$

$F'(w,x,y,z) = \Sigma(1,3,4,5,7,8,9,10,12,15)$

$F' = m_1 + m_3 + m_4 + m_5 + m_7 + m_8 + m_9 + m_{10} + m_{12} + m_{15}$

$F = (F')' = (m_1)'(m_3)'(m_4)'(m_5)'(m_7)'(m_8)'(m_9)'(m_{10})'(m_{12})'(m_{15})'$

$= M_1 . M_3 . M_4 . M_5 . M_7 . M_8 . M_9 . M_{10} . M_{12} . M_{15}$

$F(w,x,y,z) = \Pi(1,3,4,5,7,8,9,10,12,15)$

(c) $F(x, y, z) = \Pi(0, 3, 6, 7)$

$F' = \Pi(1, 2, 4, 5)$
$= M_1 . M_2 . M_4 . M_5$

$F' = (F')' = m_1 + m_2 + m_4 + m_5$
$F(x, y, z) = \Sigma(1, 2, 4, 5)$

Q) Obtain the simplified expressions in sum of products for the following functions

a) $F(x, y, z) = \Sigma(2, 3, 6, 7)$

x \ yz	00	01	11	10
0	0 0	0 1	1 3	1 2
1	0 4	0 5	1 7	1 6

$F = y$

b) $F(A, B, C, D) = \Sigma(7, 13, 14, 15)$

AB \ CD	00	01	11	10
00	0	0 1	0 3	0 2
01	0 4	0 5	1 7	0 6
11	0 12	1 13	1 15	1 14
10	0 8	0 9	0 11	0 10

BCD
ABC
ABD

$F = ABC + ABD + BCD$

10) Obtain the simplified expression in sum of products for the following boolean functions

a) $xy + x'y'z' + x'yz'$

$= xy(z+z') + x'y'z' + x'yz'$

$= \underset{7}{\underline{xyz}} + \underset{6}{\underline{xyz'}} + \underset{0}{\underline{x'y'z'}} + \underset{2}{\underline{x'yz'}}$

$F = (0, 2, 6, 7)$

$x \backslash yz$	00	01	11	10
0	1 0	1	3	1 2
1	4	5	1 7	1 6

$\rightarrow x'z'$

xy

$F = xy + x'z'$

b) $F = A'B + BC' + B'C'$

$= A'B(C+C') + BC'(A+A') + B'C'(A+A')$

$= \underset{3}{\underline{A'BC}} + \underset{2}{\underline{A'BC'}} + \underset{6}{\underline{ABC'}} + \underset{4}{\underline{AB'C'}} + \underset{0}{\underline{A'B'C'}}$

$f = \Sigma(0, 2, 3, 4, 6)$

$A \backslash BC$	00	01	11	10
0	1 0	1	1 3	1 2
1	1 4	5	7	1 6

$F = A'B + C'$

11) obtain the simplified expressions in Sum of product for the following boolean functions.

a) $F = D(A'+B) + B'(C + AD)$

$= A'D + BD + B'C + AB'D$

$= A'D(B+B') + BD(A+A') + B'C(A+A') + AB'D$

$= A'BD + A'B'D + ABD + A'BD + AB'C + A'B'C + AB'D$

$= A'\underset{7}{\underline{BCD}} + A'\underset{5}{\underline{B'C'D}} + A'\underset{3}{\underline{B'CD}} + A'\underset{1}{\underline{B'C'D}}$

$+ \underset{15}{\overline{ABCD}} + \underset{13}{\overline{ABC'D}} + A'B\cancel{CD} + A'B\cancel{C'D} + \underset{11}{\underline{AB'CD}}$

$+ \underset{10}{\underline{AB'CD'}} + A'B'\cancel{CD} + \underset{2}{\underline{A'B'CD'}} + \underset{9}{\underline{AB'C'D}}$

$F = \Sigma(1, 2, 3, 5, 7, 9, 10, 11, 13, 15)$

AB \ CD	00	01	11	10
00	0	1_1	1_3	1_2
01	4	1_5	1_7	6
11	12	1_{13}	1_{15}	14
10	8	1_9	1_{11}	1_{10}

B'C

D

$F = D + B'C$

$F = D + B'C$.

b) $F = ABD + A'C'D' + A'B + A'CD' + AB'D$

$= ABD + A'C'D' + A'BC + A'BC' + A'CD' + AB'D$

$= ABD(C+C') + A'C'D'(B+B') + A'BC(D+D') +$
$A'BC'(D+D') + A'CD'(B+B') + AB'D'(C+C')$

$= \underbrace{ABCD}_{15} + \underbrace{ABC'D}_{13} + \underbrace{A'BC'D'}_{4} + \underbrace{A'B'C'D'}_{0} + \underbrace{A'BCD}_{7}$
$+ \underbrace{A'BCD'}_{6} + \underbrace{A'BC'D}_{5} + A'BC'D' + A'BCD'$

$+ \underbrace{A'B'CD'}_{2} + \underbrace{AB'CD'}_{10} + \underbrace{AB'C'D'}_{8}$.

$F = \Sigma(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)$

$\rightarrow A'D'$

AB \ CD	00	01	11	10
00	1	0	0	1
01	1	1	1	1
11	0	1	1	0
10	1	0	0	1

$\rightarrow B'D'$
$\rightarrow BD$

$F = A'D' + BD + B'D'$

12) Obtain the simplified expressions in product of sums :-

a) $f(x, y, z) = \pi(0, 1, 4, 5)$

$F = \Sigma(2, 3, 6, 7)$

x \ yz	00	01	11	10
0	0_0	0_1	1_3	1_2
1	0_4	0_5	1_7	1_6

$\rightarrow y$

$F = y$
$f' = y'$
$f = (F')' = y$

b) $F = (A, B, C, D) = \pi(0, 1, 2, 3, 4, 10, 11)$

$f = \Sigma(5, 6, 7, 8, 9, 12, 13, 14, 15)$

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	1 5	1 7	1 6
11	1 12	1 13	1 15	1 14
10	1 8	1 9		

BC

$F = BD + BC + AC' = B'C + A'C'D' + A'B'$

$F = (F')' = (B + C')\cdot(A + C + D)\cdot(A + B)$

13) Simplify the booleun functian F in sum of products using don't care conditian

a) $F = y' + x'z'$
$d = yz + xy$

$f = y'(x + x') + x'z'$
$= xy'(z + z') + x'zy'(z + z') + x'z'(y + y')$
$= \underset{5}{xy'z} + \underset{4}{xy'z'} + \underset{1}{x'y'z} + \underset{0}{x'y'z'} + \underset{2}{x'z'y} + x'z'y'$

$f = \Sigma(0, 1, 2, 4, 5)$

$$d = yz + xy$$
$$= yz(x+x') + xy(z+z')$$
$$= \underset{7}{\underline{xyz}} + \underset{3}{x'\underline{yz}} + xyz + \underset{6}{\underline{xyz'}}$$

$$d = \Sigma(3, 6, 7)$$

x \ yz	00	01	11	10
0	0 1	1 1	x 3	1 2
1	1 4	1 5	x 7	x 6

$\rightarrow 1$

$$F = 1$$

b) $F = B'C'D' + BCD' + ABCD'$

$d = B'CD' + A'BC'D$

$$F = B'C'D'(A+A') + BCD'(A+A') + ABCD'$$

$$F = \underset{8}{AB'C'D'} + \underset{0}{\underline{A'B'C'D'}} + \underset{14}{\underline{ABCD'}} + \underset{6}{A'\underline{BC}D'} + ABCD'$$

$$f = \Sigma(0, 6, 10, 8, 14)$$

$$d = B'CD'(A+A') + A'BC'D$$
$$= \underset{10}{\underline{AB'CD'}} + \underset{2}{A'\underline{B'}CD'} + \underset{5}{A'B\underline{C'}D}$$

$$d = \Sigma(2, 5, 10)$$

AB \ CD	00	10	11	10
00	1 0	1	3	x 2
10	4	x 5	7	1 6
11	12	13	15	1 14
01	1 8	9	11	10

$\rightarrow A'BC'D'$

$\rightarrow B'D'$

$$F = B'D' + CD' + A'BC'D$$