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1. Let S be the set of all integers from 100 to 999 which are neither divisible by 3 nor divisible by 5 then find number of elements in S.

Solⁿ:-

Here given set S contain the integer from 100 to 999 then $n(S) = 900$.

$A = \{x / 100 \leq x \leq 999$ & divisible by $3\}$

$B = \{x / 100 \leq x \leq 999$ & divisible by $5\}$

$A \cap B = \{x / 100 \leq x \leq 999$ & divisible by 3 and $5\}$

$n(A) = \frac{900}{3} = 300$; $n(B) = \frac{900}{5} = 180$

$n(A \cap B) = \frac{900}{15} = 60$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 300 + 180 - 60$
$= 420.$

Now the number of element in S neither divisible by 3 nor 5 is $= 900 - 420$
$= 480$

2. Find the numbers of positive integer's ≤ 300 and divisible by 2 or 5.

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Sol^n:- Here we want to find number of positive integer ≤ 200.

$$A = \{x / x \in N \; \& \; x \leq 300 \; \& \text{ divisible by } 2\}$$

$$B = \{x / x \in N \; \& \; x \leq 300 \; \& \text{ divisible by } 5\}$$

$$A \cap B = \{x / x \in N \; \& \; x \leq 300 \; \& \text{ divisible by } 2 \text{ and } 5\}$$

$\therefore n(A) = \frac{300}{2}$; $n(B) = \frac{300}{5}$

$n(A) = 150$ $\qquad = 60$

$$n(A \cap B) = \frac{300}{10} = 30$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 150 + 60 - 30$$
$$= 180$$

3. A relation is define on set Z is $R = \{(x,y) / x-y$ divided by $7\}$ then check that R is equivalence relation also ~~be~~ check that R is poset or not.

Sol^n:-

Here given set A is integer set Z and relation

$$R = \{(x,y) / 7 \mid x-y\}$$

(i) Reflexive

$\forall \; x \in Z$, $x \in x$;

$$\Rightarrow \frac{x-y}{7} \Rightarrow \frac{x-x}{7} \Rightarrow 0$$

This equation contains reflexive. ~~property~~.

(ii) Symmetric

- $\forall\ x, y \in Z$

$\Rightarrow$ $x-y$ divided by 7 then $y-x$ is ~~devi~~ divided by 7

This relation contains symmetric ~~property~~.

(iii) Anti-symmetric :-

$\forall\ x, y \in Z$; $x, y \in Z$ and $y, x \in Z \Rightarrow x = y$

This relation fulfil this condition that's why it is anti-symmetric.

(iv) Transitive Relation:-

- Here a, b, c $\in Z$ then $A \to B \Rightarrow B \to C \Rightarrow A \to C$

$$\frac{x-y}{7} \Rightarrow \frac{y-z}{7}$$

$$\Rightarrow \frac{x-y+y-z}{7}$$

$$\Rightarrow \frac{x-z}{7}$$

This relation fulfill the last relation

→ This relation $7 | x-y$ is poset or equivalence relation

4. Let $f: Z \to Z$ be define by $f(x) = 9x+5$. Then check that f is one-one and onto

Solⁿ:- Here given function f is $f: Z \to Z$ define by

$f(x) = 9x+5$

$\therefore x_1, x_2 \in Z$, Now

$$f(x_1) = f(x_2)$$

$$9x_1 + 5 = 9x_2 + 5$$

$$9x_1 = 9x_2$$

$$x_1 = x_2$$

⇒ Hence f is one-one function. However, f is not onto function with relation Z.

Let, $4 \in Z$, For

if $f(x) = 4$ for some $x \in Z$

$$9x + 5 = 4$$

$$9x = -1$$

$$x = \frac{-1}{9} \notin Z$$

Hence f is not onto function with relation Z.

5. Prove that $f: R \to R$ define by $f(x) = ax + b, a \neq 0$ is bijective function.

Sol^n: Here we want to prove that $R \to R$ define by

$$f(x) = ax + b$$

Let, $x_1, x_2 \in R$. Now

$$f(x_1) = f(x_2)$$

$$ax_1 + b = ax_2 + b$$

$$ax_1 = ax_2$$

$$x_1 = x_2$$

Hence f is one-one function.

For onto $y \in R$

$$y = f(x)$$

$$y = ax + b$$

$$x = \frac{y-b}{a}$$

$$f(x) = f\left(\frac{y-b}{a}\right)$$

$$= a\left(\frac{y-b}{a}\right) + b$$

$$= y - b + b$$

$$= y$$

Hence given function is one-one and onto, so f is bijective.

6. Let $f: Z \rightarrow Z$ be define by $f(x) = x^2+1$. Then check that f is one-one and onto.

sol^n: Here given function is $Z \rightarrow Z$ be define by

$f(x) = x^2+1$.

let $x_1, x_2 \in Z$,

$$f(x_1) = f(x_2)$$

$$x_1^2+1 = x_2^2+1$$

$$x_1^2 = x_2^2$$

$$x_1 = x_2$$

Hence f is one-one function.

let, $4 \in Z$

if, $f(x) = 4$

$$x^2+1 = 4$$

$$x^2 = 3$$

$$x = \sqrt{3}$$

Hence f is not onto function.

7. Suppose A is any non-empty set and P(A) is set of all subset of A then show that $\langle P(A), \leq \rangle$ is partially ordered ~~set~~ relation where

" $\leq$ " means " $\subseteq$ ".

Sol^n: Here given set is all the subset of A is any non-empty set.

(i) Reflexive :-

$\forall A_1 \in P(A)$ we write $A_1 \subseteq A_1$

That is why P(A) fulfill this condition.
$\langle P(A), \subseteq \rangle$ is Reflexive.

(ii) Anti symmetric :-

$\forall A_1, A_2 \in P(A)$ suppose $A_1 \subseteq A_2$ and $A_2 \subseteq A_1$
then always possible $A_1 = A_2 \Rightarrow$

$\subseteq$ is Antisymmetric.

(iii) Transitive :-
$\forall A_1, A_2, A_3 \in P(A)$,

If $A_1 \subseteq A_2$; $A_2 \subseteq A_3$ $A_1 \subseteq A_3$ is always true.

" $\subseteq$ " is transitive.

from ①, ②, ③ relation " $\leq$ " is partially ordered relation.

Hence proved that $\langle P(A), \leq \rangle$ is Poset.

8. Under the divisiblity relation CaRb means, a divides b that is $a|b$ } show that set of natural number N is Poset. If we replace N by Z then this

then this set is poset ?

sol^n:- Here given that sets N and relation "a|b".
Now for Poset we check following properties.

(i) Reflexive:-
The relation is reflexive as every natural numbers is divisor of it self $\forall a \in N$ we write $a|a$
$\Rightarrow$ "$\leq$" is reflexive.

(ii) Anti-symmetric :-

- The relation is antisymmetric as $\forall a, b \in N$, suppose $a|b$ and $b|a$ then always possible $a = b$

$\Rightarrow$ "$\leq$" is antisymmetric

(iii) Transitive :-

- The relation is transitive as $\forall a, b, c \in N$,

Suppose $a|b$ and $b|c \Rightarrow b = k_1 a$ & $c = k_2 b$ where $k_1, k_2 \in Z$

$\Rightarrow c = k_1 k_2 a$
$\Rightarrow c = ka$ take ; $k_1 k_2 = k$

$\Rightarrow a|c$

then we get $a \leq c$

$\Rightarrow$ "$\leq$" is transitive.

- from ①, ② and ③ relation " ≤ " is partially ordered set.

q. Prove that $\langle \{1, 2, 2^2, 2^3, \ldots\}, D \rangle$ are Poset and chain.

Sol^n Here given set $A = \{1, 2, 2^2, 2^3, \ldots\}$ and relation "D" ("|") is divisibility.

(i) Reflexive:-

$\forall\ a \in A$ we write $a|a$

$\Rightarrow$ "D" is reflexive

(ii) Antisymmetric:-

- Here we have $2^r, 2^s \in A$, with $r < s$ then it is clear that $2^r D 2^s$ but $2^s \not D 2^r$ if possible then $2^r = 2^s$

$\Rightarrow$ "D" is antisymmetric.

(iii) Transitive :-

- Here we take $\forall\ 2^r, 2^s, 2^t \in A$,

Suppose $2^r / 2^s$ and $2^s | 2^t$

$\Rightarrow 2^s = k_1 2^r$ & $2^t = k_2 2^s$ where $k_1, k_2 \in \mathbb{Z}$

$\Rightarrow 2^t = k_1 k_2 2^r$

$\Rightarrow 2^t = k 2^r$ take $k_1 k_2 = k$

$\Rightarrow 2^r | 2^t$

$\Rightarrow$ then we get $2^2 D 2^t$

$\Rightarrow$ "D" is transitive.

- from ①, ② and ③ the set $\langle \{1, 2, 2^2, 2^3, \ldots\}, D \rangle$ is partially ordered set For chain we proved only comparable property.

(iv) Comparable:-

- Let $x, y \in A$ with $x = 1$ & $y = 1$.

Now let $x = 2^r$ & $y = 2^s$ for some $r, s \in N$, we have either $r \leq s$ or $r > s$ that is $(2^r D 2^s$ or $2^s D 2^r$ that is x y or y x, so for any element $x, y \in A$ that are comparable. Hence given set is chain.

10 Prove that $\langle \{1, 3, 3^2, 3^3, \ldots\}, D \rangle$ are Poset and chain.

Solⁿ:- Here given set $A = \{1, 3, 3^2, 3^3, \ldots\}$ and relation "D" ("|") is divisibility.

(i) Reflexive:-

- $\forall a \in A$ we write $a | a$.

$\Rightarrow$ "D" is reflexive.

(ii) Anti symmetric:-

Here we have $3^r, 3^s \in A$, with $r < s$ and if then it is clear that $3^r D 3^s$ but $3^s \not{D} 3^r$ and if Possible then

$$2^r = 2^s$$

$\Rightarrow$ "D" is antisymmetric.

(iii) Transitive :-

- Here we take $3^r, 3^s, 3^t \in A$

Suppose $3^r | 3^s$ and $3^s | 3^t$

$3^s = k_1 3^r$ & $3^t = k_2 3^s$ where $k_1, k_2 \in \mathbb{Z}$

$\Rightarrow 3^t = k_1 k_2$

$\Rightarrow 3^t = k 3^r$ take $k_1 k_2 = k$

$\Rightarrow 3^r | 3^t$

then we get $3^r D 3^t$

$\Rightarrow$ "D" is transitive.

- from ①, ② and ③ the set $\langle \{1, 3, 3^2, 3^3, \ldots\}, D \rangle$ is partially ordered set for chain we proved only comparable property.

(iv) Comparable :-

- Let $x, y \in A$ with $x = 1$ & $y = 1$.

Now, let $x = 3^r$ & $y = 3^s$ for some, $r, s \in \mathbb{N}$, we we have either $r \leq s$ or $r > s$ that is 3^r 3^s or 3^s 3^r that is x y or y x, so for any element $x, y \in A$ that are comparable. Hence given set is chain.

11. $\langle \{\emptyset\}, \{a\}, \{b\}, \{a,b\}\}, \subseteq \rangle$ is not a chain"

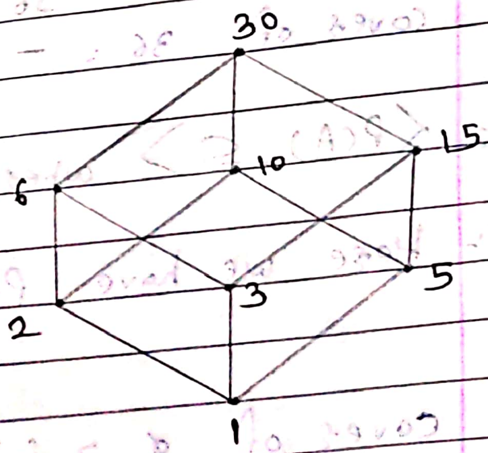
Sol^n:-

12. Draw the Hasse diagram of the following poset also find cover of element.

1. $\langle S_{30}, D \rangle$

Sol^n:- Here we have ~~P(A)~~ S_{30} = {1, 2, 3, 5, 6, 10, 15, 30}

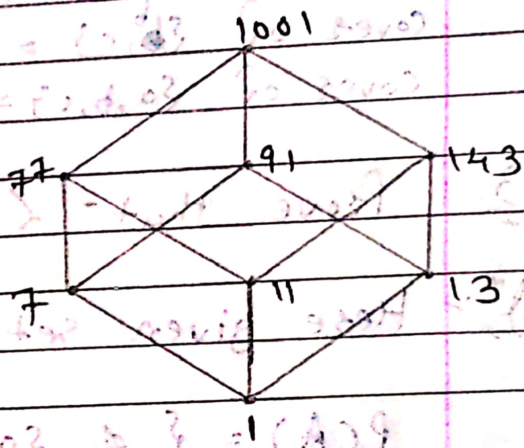
Cover of 1 = 2, 3, 5
Cover of 2 = 6, 10
Cover of 3 = 6, 15
Cover of 5 = 10, 15
Cover of 6 = 30
Cover of 10 = 30
Cover of 15 = 30
Cover of 30 = –



2. $\langle S_{1001}, D \rangle$

Sol^n:- Here we have S_{1001} = {1, 7, 11, 13, 77, 91, 143, 1001}

Cover of 1 = 7, 11, 13
Cover of 7 = 77, 91
Cover of 11 = 77, 143
Cover of 13 = 91, 143
Cover of 77 = 1001
Cover of 91 = 1001
Cover of 143 = 1001
Cover of 1001 = –



3. $\langle S_{36}, D \rangle$

Here we have S_{36} = {1, 2, 3, 4, 6, 9, 12, 18, 36} cover of 1 = 2, 3

cover of 2 = 4, 6

cover of 3 = 6, 9

cover of 4 = 12

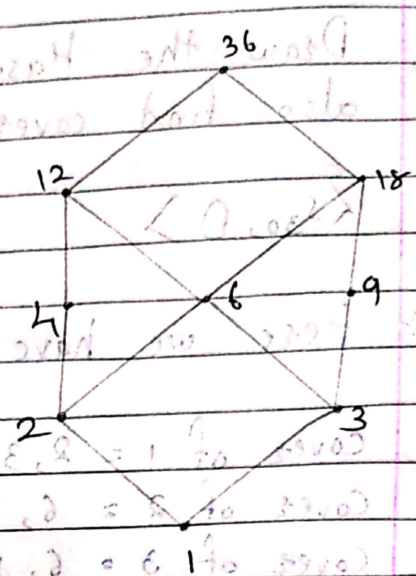
cover of 6 = 12, 18

cover of 9 = 18

cover of 12 = 36

cover of 18 = 36

cover of 36 = –



4. $\langle P(A), \subseteq \rangle$ where $A = \{a, b, c\}$

sol^n:- Here we have $P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

cover of ϕ = $\{a\}, \{b\}, \{c\}$

cover of $\{a\}$ = $\{a,b\}, \{a,c\}$

cover of $\{b\}$ = $\{a,b\}, \{b,c\}$

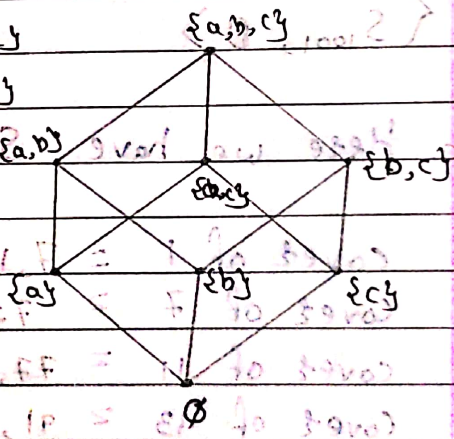
cover of $\{c\}$ = $\{a,c\}, \{b,c\}$ $\{a,b\}$

cover of $\{a,b\}$ = $\{a,b,c\}$

cover of $\{a,c\}$ = $\{a,b,c\}$

cover of $\{b,c\}$ = $\{a,b,c\}$

cover of $\{a,b,c\}$ = –



13. Prove that $\langle P(A), \leq \rangle$ is lattice for $A\{a,b,c\}$.

sol^n:- Here given set $A \{a, b, c\}$ then

$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

Also we know that $\langle P(A), \leq \rangle$ is Poset. Now we prepare table as follow.

$glb\{x,y,z\} = x \cap y \cap z$	$\emptyset$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a,b\}$	$\{a,c\}$	$\{b,c\}$	$\{a,b,c\}$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\{a\}$	$\emptyset$	$\{a\}$	$\{a,b\}$	$\{a,c\}$	$\{a,b\}$	$\{a,c\}$	$\{b,c\}$	
$\{b\}$	$\emptyset$	$\{a,b\}$	$\{b\}$	$\{b,c\}$	$\{a,b\}$	$\emptyset$		
$\{c\}$	$\emptyset$	$\{a,c\}$	$\{b,c\}$	$\{c\}$	$\emptyset$	$\{a,c\}$		
$\{a,b\}$	$\emptyset$	$\{a,b\}$	$\cancel{\{b\}}\{a,b\}$	$\emptyset$	$\{a,b\}$			
$\{a,c\}$	$\emptyset$	$\{a,c\}$	$\emptyset$	$\{a,c\}$	$\emptyset$			
$\{b,c\}$	$\emptyset$	$\emptyset$	$\{b,c\}$	$\{b,c\}$	$\emptyset$			
$\{a,b,c\}$	$\emptyset$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a,b\}$			

$x \cap y \cap z$		$\{a\}$	$\{b\}$	$\{c\}$	$\{a,b\}$	$\{a,c\}$	$\{b,c\}$	$\{a,b,c\}$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\{a\}$	$\emptyset$	$\{a\}$	$\emptyset$	$\emptyset$	$\{a\}$	$\{a\}$	$\emptyset$	$\{a\}$
$\{b\}$	$\emptyset$	$\emptyset$	$\{b\}$	$\emptyset$	$\{b\}$	$\emptyset$	$\{b\}$	$\{b\}$
$\{c\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\{c\}$	$\emptyset$	$\{c\}$	$\{c\}$	$\{c\}$
$\{a,b\}$	$\emptyset$	$\{a\}$	$\{b\}$	$\emptyset$	$\{a,b\}$	$\{a\}$	$\{b\}$	$\{a,b\}$
$\{a,c\}$	$\emptyset$	$\{a\}$	$\emptyset$	$\{c\}$	$\{a\}$	$\{a,c\}$	$\{c\}$	$\{a,c\}$
$\{b,c\}$	$\emptyset$	$\emptyset$	$\{b\}$	$\{c\}$	$\{b\}$	$\{c\}$	$\{b,c\}$	$\{b,c\}$
$\{a,b,c\}$	$\emptyset$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a,b\}$	$\{a,c\}$	$\{b,c\}$	$\{a,b,c\}$

$X \cup Y \cup Z$	$\emptyset$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a,b\}$	$\{b,c\}$	$\{a,c\}$	$\{a,b,c\}$
$\emptyset$	$\emptyset$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a,b\}$	$\{b,c\}$	$\{a,c\}$	$\{a,b,c\}$
$\{a\}$	$\{a\}$	$\{a\}$	$\{a,b\}$	$\{a,c\}$	$\{a,b\}$	$\{a,b,c\}$	$\{a,c\}$	$\{a,b,c\}$
$\{b\}$	$\{b\}$	$\{a,b\}$	$\{b\}$	$\{b,c\}$	$\{a,b\}$	$\{b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$
$\{c\}$	$\{c\}$	$\{a,c\}$	$\{b,c\}$	$\{c\}$	$\{a,b,c\}$	$\{b,c\}$	$\{a,c\}$	$\{a,b,c\}$
$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b,c\}$	$\{a,b\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$
$\{b,c\}$	$\{b,c\}$	$\{a,b,c\}$	$\{b,c\}$	$\{b,c\}$	$\{a,b,c\}$	$\{b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$
$\{a,c\}$	$\{a,c\}$	$\{a,c\}$	$\{a,b,c\}$	$\{a,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,c\}$	$\{a,b,c\}$
$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$