14. Show that $\langle S_{1001}, D \rangle$ is lattice.

Sol^n:- Here we know that $\langle S_n, D \rangle$ is a poset $\forall n \in N$, so

$S_{1001} = \{1, 7, 11, 13, 77, 91, 143, 1001\}$ $= x * y$

and glb $\{x, y\}$ = gcd $\{x, y\}$ $= x \oplus y$

$x * y$	1	7	11	13	77	91	143	1001
1	1	1	1	1	1	1	1	1
7	1	7	1	1	7	1	1	7
11	1	1	11	1	11	1	13	11
13	1	1	1	13	1	7	11	13
77	1	7	11	1	77	7	11	77
91	1	1	1	7	7	91	13	91
143	1	1	13	11	11	13	143	143
1001	1	7	11	13	77	91	143	1001

$x \oplus y$	1	7	11	13	77	91	143	1001
1	1	7	11	13	77	91	143	1001
7	7	7	77	91	77	91	1001	1001
11	11	77	11	143	77	1001	143	1001
13	13	91	143	13	1001	91	143	1001
77	77	77	77	1001	77	1001	1001	1001
91	91	91	1001	1001	1001	91	1001	1001
143	143	1001	143	143	1001	1001	143	1001
1001	1001	1001	1001	1001	1001	1001	1001	1001

$\langle S_{1001}, D \rangle$ is a lattice.

15. Show that $\langle \{\emptyset, \{a\}, \{b\}, \{a, b\}\}, \leq \rangle$ is a sub lattice of $\langle P(A), \leq \rangle$ where $A = \{a, b, c\}$

Solⁿ:- $S = \{\phi, \{a\}, \{b\}, \{a,b\}\}$ is sub lattice of $\langle P(A), \leq \rangle$
$A = \{a, b, c\}$

$x \wedge y$	ϕ	$\{a\}$	$\{b\}$	$\{a,b\}$
ϕ	ϕ	ϕ	ϕ	ϕ
$\{a\}$	ϕ	$\{a\}$	ϕ	$\{a\}$
$\{b\}$	ϕ	ϕ	$\{b\}$	$\{b\}$
$\{a,b\}$	ϕ	$\{a\}$	$\{b\}$	$\{a,b\}$

$x \vee y$	ϕ	$\{a\}$	$\{b\}$	$\{a,b\}$
ϕ	ϕ	$\{a\}$	$\{b\}$	$\{a,b\}$
$\{a\}$	$\{a\}$	$\{a\}$	$\{a,b\}$	$\{a,b\}$
$\{b\}$	$\{b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$
$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$

- lub $\{x,y\} = x \vee y \in S$; $\forall\ x, y \in S$ and
glb $\{x,y\} = x \wedge y \in S$; $\forall\ x, y \in S$
S is a sub lattice of $\langle P(A), \leq \rangle$

16. Show that $\langle \{1, 3, 6, 15\}, \leq \rangle$ is not a sub lattice of

17 Show that $\langle S_{30}, D \rangle$ is bounded and Complemented lattice.

Sol^n: $S_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$

Now

$1 \; D \; 1 \; D \; 30$ $\qquad$ $1 \; D \; 6 \; D \; 30$

$1 \; D \; 2 \; D \; 30$ $\qquad$ $1 \; D \; 10 \; D \; 30$

$1 \; D \; 3 \; D \; 30$ $\qquad$ $1 \; D \; 15 \; D \; 30$

$1 \; D \; 5 \; D \; 30$ $\qquad$ $1 \; D \; 30 \; D \; 30$

Thus we get $1 \; D \; x \; D \; 30, \; \forall \; x \in S_{30}$

glb $S_{30} = 1 = 0$ - element of S_{30} and lub $S_{30} = 30$
$= 1$ - element of S_{30}.

$\therefore$ 0 & $1 \in S_{30}$

$\therefore$ $\langle S_{30}, D \rangle$ is bounded lattice.

- Now we want to prove that $\langle S_{30}, D\rangle$ is complemented lattice for that we find the complement element of every element.

$1 * 30 = 1$ and $1 \oplus 30 = 30$,

So complement of 1 is 30 and complement of 30 is 1.
$2 * 15 = 1$ and $2 \oplus 15 = 30$,
So complement of 2 is 15, and complement of 15 is 2
$3 * 10 = 1$ and $3 \oplus 10 = 30$,
So complement of 3 is 10, and complement of 10 is 3
$5 * 6 = 1$ and $5 \oplus 6 = 30$,
So complement of 5 is 6 and complement of 6 is 5

$\therefore \langle S_{30}, D\rangle$ is complemented lattice.

18 : Show that $\langle P(x), \subseteq\rangle$ is a bounded and complemented lattice. where $x = \{a, b, c\}$

Soln: Here given set $X = \{a, b, c\}$
$\therefore P(X) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$

from the set $P(X)$ we can write $\phi \subseteq A \subseteq X$,

$\forall A \in P(x)$

$\therefore$ glb $P(x) = \phi$ = 0- element of $P(x)$ and lub $P(x)$
$= X$
$= 1$- element of $P(x)$.

$\therefore$ 0 & $1 \in P(x)$.
$\langle P(x), \subseteq\rangle$ is a bounded lattice.

Now we want to prove that is complemented lattice for that we find the complement element of every element.

$\phi * X = \phi$ and $\phi \oplus x = X$
$= \{a,b,c\}$

So complement of ϕ is $\{a,b,c\}$ and complement of $\{a,b,c\}$ is ϕ

$\{a\} * \{b,c\} = \phi$ and $\{a\} \oplus \{b,c\} = \{a,b,c\}$

So complement of $\{a\}$ is $\{b,c\}$ and complement of $\{b,c\}$ is $\{a\}$

$\{b\} * \{a,c\} = \phi$ and $\{b\} \oplus \{a,c\} = \cancel{\phi}\{a,b,c\}$

So complement of $\{b\}$ is $\{a,c\}$ and complement of $\{a,c\}$ is $\{b\}$

$\{c\} * \{a,b\} = \phi$ and $\{c\} \oplus \{a,b\} = \{a,b,c\}$

So complement of $\{c\}$ is $\{a,b\}$ and complement of $\{a,b\}$ is $\{c\}$.

$\therefore \langle P(x), \subseteq \rangle$ is a complemented lattice.

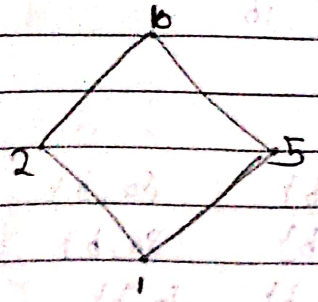
19. Show that lattice $\langle S_{10}, *_1, \oplus_1 \rangle$ and $\langle P(x), *_2, \oplus_2 \rangle$ are isomorphic lattice for $X = \{a,b\}$.

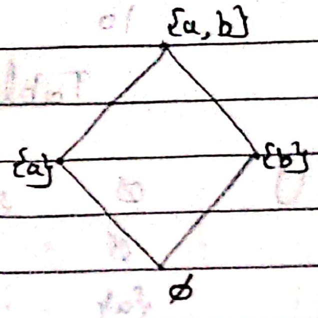
Solⁿ Here

$$\langle S_{10}, *_1, \oplus_1 \rangle$$
$$\langle P(X), *_2, \oplus_2 \rangle$$
$$X = \{a,b\}$$

$S_{10} = \{1, 2, 5, 10\}$

$P(X) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$





- $f : S_{10} \rightarrow P(X)$ by

$f(1) = \emptyset$
$f(2) = \{a\}$
$f(5) = \{b\}$
$f(10) = \{a,b\}$

$*_1 = \gcd$	1	2	5	10
1	1	1	1	1
2	1	2	1	2
5	1	1	5	5
10	1	2	5	10

Table :- 1

$*_2 = \cap$	$\emptyset$	$\{a\}$	$\{b\}$	$\{a,b\}$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\{a\}$	$\emptyset$	$\{a\}$	$\emptyset$	$\{a\}$
$\{b\}$	$\emptyset$	$\emptyset$	$\{b\}$	$\{b\}$
$\{a,b\}$	$\emptyset$	$\{a\}$	$\{b\}$	$\{a,b\}$

Table :- 2

- $f(x *_1 y) = f(x) *_2 f(y)$, $\forall\ x, y \in S_{10}$

$\oplus_1$ = lcm	1	2	5	10
1	1	2	5	10
2	2	2	10	10
5	5	10	5	10
10	10	10	10	10

Table :- 3

$\oplus_2 = \cup$	ϕ	{a}	{b}	{a, b}
ϕ	ϕ	{a}	{b}	{a, b}
{a}	{a}	{a}	{a, b}	{a, b}
{b}	{b}	{a, b}	{b}	{a, b}
{a, b}	{a, b}	{a, b}	{a, b}	{a, b}

Table :- 4

- From the table :- 3 and table 4 we have

$$f(x \oplus_1 y) = f(x) \oplus_2 f(y)$$

$$\forall\ x, y \in S_{10}$$

$\therefore$ f preserves binary operation meet ($*$) and join ($\oplus$).

$\therefore$ f is an isomorphism $\langle S_{10}, *_1, \oplus_1 \rangle$ to $\langle P(x), *_2, \oplus_2 \rangle$

$$\langle S_{10}, *_1, \oplus_1 \rangle = \langle P(x), *_2, \oplus_2 \rangle$$

Q.20 Construct the truth table for the statement formula A, A : $(\sim(P \vee q)) \rightarrow (q \wedge P)$

P	q	$(P \vee q)$	$(\sim(P \vee q))$	$(q \wedge P)$	$A = (\sim(P \vee q)) \rightarrow (q \wedge P)$
T	T	T	F	T	T
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	T	F	F

PAGE NO.
DATE

Q.21 Let P, q & ~~or~~ r be the statement then construct the truth table for the statement formula A, A: $(\sim P \wedge q) \to r$.

$\Rightarrow$ A: $(\sim P \wedge q) \to r$

$2^3 = 8$

A: $(\sim P \wedge q) \to r$

P	q	r	~P	~P ∧ q	A: (~P∧q) → r
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	T
F	F	F	T	F	T

Que22:- If P and q are any two statement then verify $\sim (P \leftrightarrow q) = \sim P \leftrightarrow q = P \leftrightarrow \sim q$

Soln:- $\sim (P \leftrightarrow q) = \sim P \leftrightarrow q = P \leftrightarrow \sim q$

P	q	P ↔ q	~P ↔ q	~P	~P ↔ q	~q	P ↔ ~q
T	T	T	F	F	F	F	F
T	F	F	T	F	T	T	T
F	T	F	T	T	T	F	T
F	F	T	F	T	F	T	F

Que23:- Show that statement formula A = $\sim(P \wedge q)$ and B = $(\sim P) \vee (\sim q)$ is logically equivalent.

Soln:-

p	q	$p\wedge q$	$\sim(p\wedge q)$	$\sim p$	$\sim q$	$(\sim p)\vee(\sim q)$	$A\leftrightarrow B$
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

Que 24. Show that statement formula $A = \sim(p\wedge q)$ and $B = (\sim p)\vee(\sim q)$ is logically equivalent.

Que 23. Let A denote the statement formula $p\wedge(p\to q)$ and B be q then show that A logically implies to B.

Solⁿ :-

p	q	$p\to q$	$p\wedge(p\to q)$	$A\to B$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

From the truth table it follows that $A\to B$ is a tautology and hence A logically implies B.

Que 25. Show that cube roots of unity from a group under multiplication.

Solⁿ :- Here we want to show that cube roots of unity is group under multiplication, for that

$$x^3 = 1$$
$$\therefore x^3 - 1 = 0$$
$$\therefore (x-1)(x^2+x+1) = 0$$

$\therefore x = 1, \frac{-1 \pm \sqrt{3}i}{2}$

let $w = \frac{-1+\sqrt{3}i}{2}$ and $w^2 = \frac{-1-\sqrt{3}i}{2}$. Hence

$G = \{1, w, w^2\}$, to check the properties of group we prepare composition table as follow.

*	1	w	w^2
1	1	w	w^2
w	w	w^2	1
w^2	w^2	1	w

Now we check the properties of group.

(i) Closer

- From the composition table closer operations are hold: that is, $a*b \in G$, for all $a, b \in G$.

(ii) Associativity

- From the composition table associative operations are hold : that is, $(a*b)*c = a*(b*c)$ for all $a,b,c \in G$.

(iii) Identity

- From ~~of~~ the compotion table the identity element of group G is 1, this identity element hold the operation:

$$a*1 = 1*a = a, \text{ for all } a \in G.$$

(iv) Inverses :-

- From the composition table the inverse of each elements are as follow:

Inverse of 1 is 1.
Inverse of ω is ω^2.
Inverse of ω^2 is ω.

Hence given set $G = \{1, \omega, \omega^2\}$ is group under the multiplication.

Q.ue. 26 Prove that the set $G = \{0, 1, 2, 3, 4\}$ is an abelian group under addition modulo 5 and multiplication modulo 5.

Here given set ~~in~~ is $G = \{0, 1, 2, 3, 4\}$ & operation is addition modulo 5

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

(i) closure :-

- From the composition table closure operations are hold : that is, $a +_5 b \in G$,

$$\forall\ a, b \in G$$

(ii) Associativity

- From the composition table associative operations are hold: that is $(a +_5 b) +_5 c = a +_5 (b +_5 c)$

$$\forall\ a, b, c \in G$$

(iii) Identity

From the composition table the identity ~~of~~ element of group G is 0, this identity element hold the operation :

$$a +_5 0 = 0 +_5 a = a, \quad \forall a \in G.$$

(iv) Inverse

From the composition table the inverse of each elements are as follow.

- Inverse of 0 is 0.
- Inverse of 1 is 4.
- Inverse of 2 is 3.
- Inverse of 3 is 2.
- Inverse of 4 is 1.

Also for any two elements $a, b \in G$ we have $a +_5 b = b +_5 a$, So given G is abelian group under addition modulo m.

Here given set is $G = \{1, 2, 3, 4\}$ & operation is multiplication modulo 5 and we want to prove that G is abelian group.

$\times_5$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

(i) Closer

- From the composition table closer operations are hold

$$a \times_5 b \in G ; \forall a, b \in G$$

(ii) Associativity

- From the composition table associative operations are hold:

$$(a \times_5 b) \times_5 c = a \times_5 (b \times_5 c) ; \forall a, b, c \in G.$$

(iii) Identity

- From the composition table the identity element of group G is 1, this identity element hold the operation.

$$a \times_5 1 = 1 \times_5 a = a ; \forall a \in G$$

(iv) Inverse

- From the composition table the inverse of each elements are as follow.

- Inverse of 1 is 1.
- Inverse of 2 is 3.
- Inverse of 3 is 2.
- Inverse of 4 is 1.

- Also for any two elements $a, b \in G$ we have $a \times_5 b = b \times_5 a$, so given set G is ~~abelian~~ abelian group under multiplication modulo m.

27 Show that (S

Sol^n: Here given se

$$P_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

$$P_4 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

Now we chec

(i) Closer

- From the c

$P_i \circ$

(ii) Associativi

- From the hold:

(P

PAGE NO.
DATE

27 Show that $(S_3, \circ)$ is not an abelian group.

Sol^n:- Here given set $S_3 = \{P_1, P_2, P_3, P_4, P_5, P_6\}$, where

$$P_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix};\ P_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix};\ P_3 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

$$P_4 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix};\ P_5 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix};\ P_6 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

$\circ$	P_1	P_2	P_3	P_4	P_5	P_6
P_1	P_1	P_2	P_3	P_4	P_5	P_6
P_2	P_2	P_1	P_6	P_5	P_4	P_3
P_3	P_3	P_5	P_1	P_6	P_2	P_4
P_4	P_4	P_6	P_5	P_1	P_3	P_2
P_5	P_5	P_3	P_4	P_2	P_6	P_1
P_6	P_6	P_4	P_2	P_3	P_1	P_5

Now we check the properties of group:

(i) Closer

- From the composition table closer operation are hold

$$P_i \circ P_j \in S_3 \ ; \forall\ P_i, P_j \in S_3 \ ; \ 1 \leq i, j \leq 3$$

(ii) Associativity

- From the composition table associative operations are hold:

$$(P_i \circ P_j) \circ P_k = P_i \circ (P_j \circ P_k) \ ; \forall\ P_i, P_j, P_k \in S_3$$

$$1 \leq i, j, k \leq 3.$$

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(iii) Identity

- From the composition table the identity element of group S_3 is P_1, this identity element hold

$$P_i . P_1 = P_1 . P_j = P_j ,$$

$$\forall \; P_j \in S_3 \; ; \; 1 \leq j \leq 3.$$

(iv) Inverses:-

- From the composition table the inverse of each elements are as follow.

- Inverse of P_1 is P_1
- Inverse of P_2 is P_2
- Inverse of P_3 is P_3
- Inverse of P_4 is P_4
- Inverse of P_5 is P_6
- Inverse of P_6 is P_5

- Also from composition table, for any two elements $P_i, P_j \in S_3$ we have $P_3 . P_4 = P_6$ and $P_4 . P_3 = P_5$,

- Hence $P_3 . P_4 \neq P_4 . P_3$

- So given set is non abelian group under the composition table.

Que 25: Show that the set of all positive rational number Q^+ form a group under the composition operation define by $a * b = \frac{ab}{5}$.

Sol: Here given set $G = Q^+$ the set of positive rational number and $*$ be binary operation of G defined as

(i) closer

- Let $a, b \in G = Q^+$

- Because of a if a & ... then $\frac{ab}{5}$ is also ...

$\Rightarrow a * b \in G \; ; \; \forall \; a$ ...

(ii) Associativity

- let $a, b, c \in G$

L.H.S

R.H.S

From the ... hold. (a ...

of G defined as $a * b = \frac{ab}{5}$.

(i) closer

- Let $a, b \in G = Q^+$ then $a * b = \frac{ab}{5} \in G = Q^+$

- Because ~~of~~ a if a & b are positive rational number then $\frac{ab}{5}$ is also positive rational numbers.

$\Rightarrow$ $a * b \in G$; $\forall\ a, b \in G$

(ii) Associativity

- let $a, b, c \in G = Q^+$; $(a*b) * c = a*(b*c)$

$$L.H.S = (a*b) * c$$

$$= \frac{ab}{5} * c$$

$$= \frac{\left(\frac{ab}{5}\right) c}{5}$$

$$= \frac{abc}{25} \quad \text{——— ①}$$

$$R.H.S. = a * (b*c)$$

$$= a * \frac{bc}{5}$$

$$= \frac{a\left(\frac{bc}{5}\right)}{5}$$

$$= \frac{abc}{25} \quad \text{——— ②}$$

From the equation ① and ② the associative properties hold. $(a*b) * c = a*(b*c)$; $\forall\ a, b, c \in G$

(iii) Identity

- Let e be the identity of Q^+ under the given operation.

$\therefore$ for any $a \in G \Rightarrow a * e = a$

$$\Rightarrow \frac{ae}{5} = a$$

$$\Rightarrow e = 5 \in Q^+$$

here we get the identity element $e = 2$ in G such that $a * e = e * a = a$, $\forall a \in G$

(iv) Inverses

Let b be the inverse of $a \in Q^+$ under the given operation

$\therefore$ for any $a \in G \Rightarrow a * b = e$

$$\Rightarrow \frac{ab}{5} = 5$$

$$\Rightarrow b = \frac{25}{a} \in Q^+$$

- here we get the inverse element of a is $b = \frac{25}{a}$ Such that $a * b = b * a = e$.

$\Rightarrow$ Hence, given set $G = Q^+$ is group under the composition operation $a * b = \frac{ab}{5}$.

Que 21 Define the following term with example and truth table

Sol^n:- (i) Negation:-

- Let p be a statement. The negation of p, written $\neg p$ (or $\neg p$), is the statement obtained by

negating statement

Consider the follo

p : 2

q : I

$\Rightarrow$ We see that state

false. Statement

p, and the tru

$\Rightarrow$ statement q is

(2) Conjunction:-

- Let p and q b

p and q, with

joining statemen

p : 2

q :

$\Rightarrow$ Now consider

integers and

The truth ta

p
T
T
F
F

negating statement p.

Consider the following statements:

p : 2 is positive.
q : It is not the case that 2 is positive.

⇒ We see that statement p is true and statement q is false. Statement q is obtained by negating statement p, and the truth values of p and q are opposite.
⇒ statement q is the negation of statement p.

(2) Conjunction :-

- Let p and q be statements. The conjunction of p and q, written $p \wedge q$, is the statement formed by joining statements p and q using the word "and".

p : 2 is an even integer.
q : 7 divides 14.

⇒ Now consider the sentence q : 2 is an even integer and 7 divides 14.

The truth table of $p \wedge q$ is given by:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(3) Disjunction :-

- Let p and q be statements. The disjunction of p and q, written $p \vee q$ is the statement

⇒ formed by putting statements p and q together using the word "or".

p: 2^2+3^3 is an even integer.
q: 2^2+3^3 is an odd integer.

⇒ Then

$p \vee q$: 2^2+3^3 is an even integer or 2^2+3^3 is an odd integer.

Sometimes, for better readability we write $p \vee q$ as :

$p \vee q$: Either 2^2+3^3 is an even integer or 2^2+3^3 is an odd integer.

$p \vee q$: 2^2+3^3 is an even integer or odd integer.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

(4) Contradiction :-

- A statement formula A is said to be a contradiction if the truth value of A is F for any

- assignment of the truth ~~table~~ values T and F to the statement variables occurring in A

Ex Let A be the statement formula $\sim P \wedge P$. we construct the truth table for A.

P	$\sim P$	$\sim P \wedge P$
T	F	F
F	T	F

⇒ From the table, it follows that A is a contradiction

Que. 30 Show that $\langle N, \leq \rangle$ is chain.

Solⁿ:-

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