Exam Seat no. .....

KADI SARVA VISHWAVIDHYALAYA,

BE Semester–III (December 2023)

Discrete Mathematics (CC302B-N)

Date: 14-12-23 **Max Marks: 70**

Duration: 3 hr

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) **Indicate clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work**.

Section- I

Q.1 (a) Show that $< S_{1001}, D >$ is a lattice. [5]

(b) Define group and show that set of fourth root of unity form abelian group under the binary operation multiplication. [5]

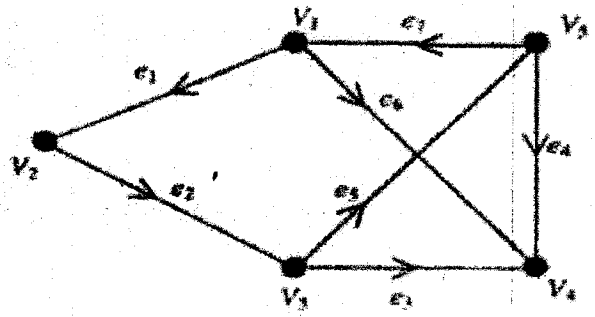
(c) Define with appropriate examples: i) Loop ii) Multigraph iii) pendent vertex iv) complete Graph v) Regular graph [5]

OR

(c) A relation is define on set $\mathbb{Z}$ is R = {(x , y) / x-y divided by 3} then check that R is equivalence relation [5]

Q.2 (a) Define Join-Irreducible Element and Meet-Irreducible Element. Find Join-Irreducible & Meet-Irreducible elements for lattice $\langle S_{30}, D \rangle$. [5]

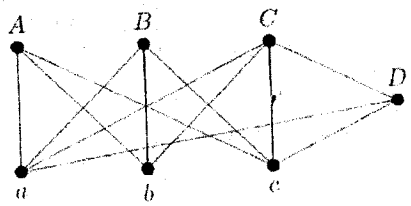
(b) Define adjacent matrix and incident matrix for directed simple graph. Find the adjacency Matrix and incident matrix of following graph. [5]



OR

Q.2 (a) Define Homomorphism of the Group. Let Z be the additive group of integers and G = {1,-1, i,-i} be the multiplicative group. Prove that the function $f: Z \to G$ defined by $f(n) = i^n$, $\forall n \in Z$ is a homomorphism from Z to G. [5]

(b) State Handshaking Lemma & verify it for the following graph. [5]



Q.3 (a) Prove that G = (Z_6, $+_6$) is a cyclic group and find all its generators. [5]

(b) Let A= { a,b,c,d } and B = {1,2,3}.Let R be a relation define from set A to set B and is given as R = { (a,1),(a,2) ,(b,1) ,(c,2) ,(d,1) }. Find matrix relation M_R. Also draw arrow diagram. [5]

OR

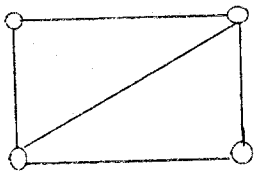
Q.3 (a) Define the order of the group and order of an element. Also find order of the group and order of each element of the group ($Z_7 - \{0\}$, $\times_7$). [5]

(b) Prove that $\langle\{1,5,5^2,5^3,\ldots.\},D\rangle$ is a chain. [5]

Section- II

Q.4 (a) Define the following terms (1) Semi group (2) Monoid (3) Abelian Group (4) Subgroup (5) index of a subgroup [5]

(b) Define the following terms (1) Vertex coloring (2) Edge coloring (3) Chromatic number. Also find chromatic number of the following graph. [5]



(c) Let P(x) denote the statement "x is doing walk in morning". The domain of discourse is the set of all people. Write each proposition in words. [5]
1. $\forall x$ P(x) 2. $\exists x$ P(x) 3. $\sim (\exists x$ P(x)) 4. $\exists x \sim$ P(x) 5. $\sim (\forall x$ P(x))

OR

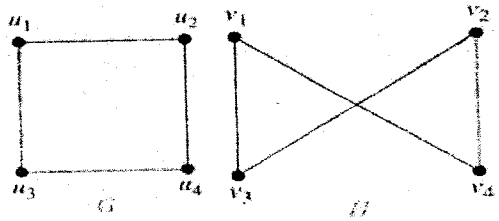
(c) State inverse, converse and contrapositive of the following implications: [5]
1. If $5x+1=11$ then $x = 2$. 2. If you work hard then you can earn money.

Q.5 (a) Find all the subgroup of a cyclic group of order 12 with generator 'a'. [5]

(b) Draw the Hasse diagram of $\langle S_{45}, D\rangle$ where D means 'a divides b' and find the cover of each element of S_{45}. [5]

OR

Q.5 (a) Prove that following graphs are Isomorphic. [5]



(b) Find fog & gof for given $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$ & $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$ and show that $fog \neq gof$. [5]

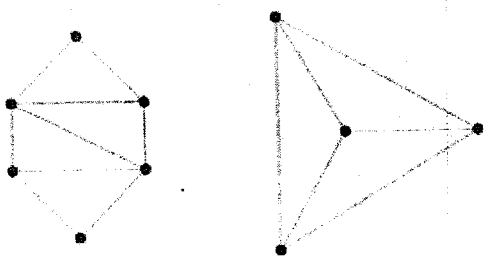
Q.6 (a) Define right coset and left coset. [5]
Consider group $(Z, +)$ and its subgroup $H = (3Z, +)$. Find all left cosets of H in G.

(b) Define the following terms (1) Walk (2) Path (3) Trail (4) circuit(5) Tree [5]

OR

(a) Prove that $(S_3, \circ)$ is non-abelian permutation group. [5]

(b) State Euler's formula & verify it for the each of the following graphs: [5]



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