# Kadi Sarva Vishwavidyalaya

L.D.R.P. Campus, Sector-15, GANDHINAGAR

Faculty of Engineering

L.D.R.P. Institute of Technology and Research

# Assignment/ Tutorials (3rd Sem)

## Discrete Mathematics

## (CE, IT)

Department of Science and Humanities

L.D.R.P. Institute of Technology and Research

**2024-2025**

| Sr. No | Title of the Unit | Minimu Hours |
|---|---|---|
| 1 | Set, Relation & Function | 4 |
| 2 | Lattices | 6 |
| 3 | Propositional Logic | 7 |
| 4 | Algebraic Structures and Morphism | 16 |
| 5 | Graphs and Trees | 15 |

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## Syllabus

| Sr. No | Topic | Lecture | (%) |
|---|---|---|---|
| 1 | **Set, Relation & Function**: Operations and Laws of Sets, Cartesian Products, Binary Relation, Partial Ordering Relation, Equivalence Relation, Image of a Set, Sum and Product of Functions, Bijective functions, Inverse and Composite Function, Size of a Set, Finite and infinite Sets, Power set theorem. | 4 | 10% |
| 2 | **Lattices:** Hasse Diagrams, Lattices as poset, properties of lattices, complemented lattices, bounds of lattices, distributive lattice, complemented lattices. | 6 | 15% |
| 3 | **Propositional Logic:** Basic Connectives and Truth Tables, Logical Equivalence: The Laws of Logic, Logical Implication, Rules of Inference, The use of Quantifiers. Proof Techniques: Some Terminology, Proof Methods and Strategies, Forward Proof, Proof by Contradiction, Proof by Contrapositive, Proof of Necessity and Sufficiency. | 7 | 15% |
| 4 | **Algebraic Structures and Morphism:** Algebraic Structures with one Binary Operation, Semi Groups, Monoids, Groups, Congruence Relation and Quotient Structures, Permutation Groups, Normal Subgroups, cyclic groups, homomorphisms Algebraic Structures with two Binary Operation: Rings, Integral Domain and Fields (definition and properties). Boolean Algebra, Identities of Boolean Algebra, join-irreducible, meet-irreducible, atoms, anti atoms, Representation of Boolean Function, Disjunctive and Conjunctive Normal Form. | 16 | 30% |
| 5 | **Graphs and Trees:** Graphs and their properties, Degree, Connectivity, Path, Cycle, Sub Graph, Isomorphism, Eulerian and Hamiltonian Walks, Planar Graphs, Graph Colouring: Colouring maps, Colouring Vertices, Colouring Edges (definition, properties and Examples), trees, rooted trees and spanning trees, weighted trees and prefix codes, Shortest distances. | 15 | 30% |
| | **Total** | **48** | 100% |

**Text / Reference Books:**


1. " Discrete Mathematics and its Applications", Kenneth H. Rosen, Tata McGrawHILL .
2. "Discrete Mathematics with Applications,4th edition", Susanna S. Epp, Wadsworth Publishing Co. Inc.
3. "Elements of Discrete Mathematics A Computer Oriented Approach", C L Liu, D P Mohapatra, Tata McGraw-Hill.
4. "Discrete Mathematical Structure and It's Application to Computer Science", J.P. Trembla and R. Manohar, TMG Edition, Tata McGraw-Hill.

5"Discrete Mathematical Structure", Bernard Kolmann & others, Sixth Edition ,Pearson Education.

6.“Discrete Mathematics with Graph Theory”, Edgar G. Goodaire, Michael M. Parmenter. PHI

7“Logic and Discrete Mathematics”, , J. P. Tremblay and W. K. Grassman, Pearson Education.

8“Discrete Mathematics”, Norman L. Biggs, 2nd Edition, Oxford University Press.

9“Schaum’s Outlines Series”, Seymour Lipschutz, Marc Lipson.

# Kadi Sarva Vishwavidyalaya
# LDRP Institute of Technology and Research, Gandhinagar
## B.E. Sem III (CE, IT)
## Sub.: Discrete Mathematics

## Unit -1: Set, Relation & Function

### Task-1

1) Define 1) set 2) empty set 3) singleton set 4) power set 5) finite set 6) infinite set 7) cardinal number of set
2) If A = {a, b, c, d}, B = {c, d, e} and C = {e, f, g, h} then state the elements of the following sets.

   (1) A ∪ C (2) B ∩ A (3) B ∩ (A ∪ C) (4) (B ∩ A) ∪ (B ∩ C)

3) For universal set U={1,2,3,4,5,6,7,8,9,10} & sets $A = \{1,2,4,5\}$ & $B = \{4,5,6,7\}$ determine the following set.
   1. $A - B$
   2. $B - A$
   3. $A \Delta B$
   4. $A'$
   5. $B'$
4) For universal set U={1,2,3,4,5,6,7,8} & two sets $A = \{1,2,3,4\}$ & $B = \{4,5,6,7,8\}$ ,draw Venn diagram for the following condition.
   6. $A \cup B$
   2. $A'$
   3. $A \cap B$
   4. $B - A$
   5. $A - B$
   6. $A \Delta B$

5) Define following sets in tabular form. Also show that which of the following are null sets or singleton?
   7. $A = \{X: X \in \mathbb{R} \text{ and } X \text{ is a solutionof } X^2 + 2 = 0\}$.
   8. $B = \{X: X \in \mathbb{Z} \text{ and } X \textit{ is a solution of } X - 3 = 0\}$
   9. $C = \{X: X \in \mathbb{Z} \text{ and } X \textit{ is a solution of } X^2 - 2 = 0\}$

6) Let $A = \{X: X$ is a even natural number less then or equal to $10\}$ and $B = \{X: X$ is an odd natural number less then or equal to $10\}$. Find $(i) A - B$ $(ii) B - A$ $(iii)$ is $A - B = B - A$?

7) Let $\mathbb{N}$ be the universal set and $A,\ B,\ C,\ D$ be its subsets given by

$A = \{X : X$ is a even natural number $\}$
$B = \{X : X \in \mathbb{N}$ and $X$ is a multiple of $3\}$
$C = \{X : X \in \mathbb{N}$ and $X \geq 5\}$
$D = \{X : X \in \mathbb{N}$ and $X \leq 10\}$

Find the complement of A, B, C and D respectively.

## Task-2

1) Let A = {a, b, d, e}, B = {b, c, e, f} and C = {d, e, f, g}

   (i) Verify $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

   (ii) Verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

2) Let A and B be two finite sets such that n(A) = 20, n(B) = 28 and $n(A \cup B)$ = 36, find $n(A \cap B)$.
3) In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both coffee and tea?

4) There are 35 students in art class and 57 students in dance class. Find the number of students who are either in art class or in dance class.

   • When two classes meet at different hours and 12 students are enrolled in both activities.

   • When two classes meet at the same hour.

5) In a group of students, 65 play foot ball, 45 play hockey, 42 play cricket, 20 play foot ball and hockey, 25 play foot ball and cricket, 15 play hockey and cricket and 8 play all the three games. Find the total number of students in the group (Assume that each student in the group plays at least one game.)

6) Write down the power set of A= {a, b, c}. What is the cardinal number of P(A)?

7) Write the power set of each of the following sets:
   10. $A = \{X : X \in \mathbb{Z}$ and $X^2 - 9 = 0\}$
   11. $B = \{y : y \in \mathbb{N}$ and $1 \leq y \leq 3\}$

8) Prove that $A \cap (B - C) \subset A - (B \cap C)$

9) Use the properties of sets to prove that for all the sets A and B ,

$$A - (A \cap B) = A - B$$

Ans: 2) 12 3) 9 4) 80 , 92 5) 100

## Task-3

1) Let A={1,2,3} & B={a, b} then find $A \times B$, $A \times A$ & $B \times B$, $B \times A$
2) If A = { 1, 3, 5} and B = {2, 3}, then

   Find: (i) A × B (ii) B × A (iii) A × A (iv) (B × B)
3) If A × B = {(a, 1); (a, 2); (b, 1); (b, 2); (c, 1); (c, 2)}, find A and B.
4) If P × Q = {(x, 2); (x, 6); (x, 3); (y, 3); (y, 6); (y, 2)}, find Q × P.
5) If A = {1, 2, 3} and B = {4, 5, 6}, state which of the following is a relation from A to B.

   (a) $R_1$= { (1,4);(2,5);(6,3)} (b) $R_1$= { (2,5);(6,3)}

   (c) $R_1$= { (4,1);(5,2);(6,3)}

   (d) $R_1$= { (1,5);(1,6);(2,4);(2,6);(3,4);(3,5)}
6) Write the domain and range of the following relations.

   (a) $R_1$ = {(4, 3); (6, 8); (4, 8); (0, 9); (7, 5); (0, 10)}

   (b) $R_2$ = {(a, 2); (b, 3); (c, 2); (a, 3); (d, 4); (b, 4)}

## Task-4

1) For each of the following relation on A = {1, 2, 3, 4}, determine whether it is reflexive, symmetric or transitive
   (1) R = {(1, 4), (4, 1)}
   (2) R = {(1, 1)}
   (3) R = {(1, 1), (2, 2), (3, 3), (4, 4), (2, 3), (3, 2)}
   (4) R = {(1, 3), (1, 4)}

2) Let X = {1, 2, 3, 4, 5, 6, 7} and R = {< x, y > /x−y is divisible by 3} ,then show that R is an equivalence relation.
3) On the set ℤ of all integer , define the relation R by R = {(a, b) ∈ ℤ × ℤ/a −bis divisible by 5 } then show that R is an equivalence relation.

4) Let A = {1, 2, 3, 4, 5} and let R be a relation on A define

by R = {(1, 3), (2, 1), (2, 2), (2, 5), (3, 4), (4, 3), (4, 4), (5, 1), (5, 3)} then compute $R^2$, $R^3$, $R^{-1}$, $RoR^{-1}$, $R^{-1}oR$.

5) Draw the arrow diagrams to represent the following relations.

(a) $R_1$ = {(3, 3); (3, 6); (3, 9); (5, 8); (6, 3)}

(b)$R_2$ = {(4, 10); (4, 13); (4, 16); (5, 13); (6, 16)}

## Task-5

1)Let N be the set of natural number if f: N → N by f (x) = 3x +2, then

find f (1), f (2), f (-3), f (-4).

2)Let A = {a, b, c, d} and B= {c, d, e, f, g}

Let $R_1$ = {(a, c) (b, d) (c, e)}

$R_2$ = {(a, c) (a, g) (b, d) (c, e) (d, f)}

$R_3$ = {(a, c) (b, d) (c, e) (d, f)}

Justify which of the given relation is a function from A to B.

3) Determine which of the following function are one-one, on-to or both.
(1) $f : \mathbb{N} \to \mathbb{Z} - \{0\}$ define by $f(n) = -n$ for all $n \in \mathbb{N}$.
(2) $f : \mathbb{Z} \to \mathbb{Z}$ define by $f(x) = x - 4$ for all $x \in \mathbb{Z}$.
(3) $f : \mathbb{R} \to \mathbb{R}$ define by $f(x) = |x| + x$ for all $x \in \mathbb{R}$.
(4) $f : \mathbb{R} \to \mathbb{R}$ define by $f(x) = x^3$ for all $x \in \mathbb{R}$.

4) Let $f : \mathbb{Q} \to \mathbb{Q}$ be the function defined by $f(x) = 3x + 4$ for all $x \in \mathbb{Q}$ then find the inverse of f if it exist.

5) Let $f : \mathbb{N} \to \mathbb{N}$ be defined by $f(n) = n + 3$ for all $n \in \mathbb{N}$ then show that f is one-one but not on-to.

## Task-6

1) A = {1, 2, 3, 4} and B = {p, q, r, s} and R = {(1, p), (1, q), (1, r), (2, q), (2, r), (2, s)} then find matrix relation $M_R$ .

2) Let A = {1, 4, 5} and R = {(1, 4), (1, 5), (4, 1), (4, 4), (5, 5)}. Draw a diagraph for R.

3) Let A= { a,b,c,d } and B = {1,2,3}.Let R be relation define from set A to set B and is given as R = { (a,1),(a,2) ,(b,1) ,(c,2) ,(d,1) }. Find matrix relation $M_R$. Also draw arrow diagram.

4) Let A={ 1,2,3,4 } and B = { 1,4,6,8,9 }. Let R be a relation from set A to set B and defined as $_aR_b$ if and only if $b = a^2$. Define relation R &find matrix relation $M_R$. Also draw arrow diagram.

5) Let A={ 1,2,3,4,6 } be a set and R be a relation on set A defined as $_aR_b$ if and only if a is multiple of b . Represent this relation in diagraph form.

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## B.E. SEM III (CE, IT)
## Sub.: Discrete Mathematics

**Unit -2: Lattices**

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**Task-1**

1) Define poset with example.
2) Consider the set Z of integers. Define a R b if there is a positive integer r such that $b=a^r$.Is R a partial ordering of Z?
3) Let N={1,2,3….} be ordered by divisibility. State Whether each of the following subset of N are linearly (totally) ordered.

   (a){24,2,6} (b) {3,15,5} (c) {1,2,3,….} (d) {2,8,32,4} (e) {7} (f) {15,5,30}
4) Show that $\langle p(x), \subseteq\rangle$ is a poset where x is a nonempty set and p(x) is a poset set of x.
5) Which of the following are posets? Give reasons.
   (a)$\langle Z, =\rangle$ (b) $\langle Z, \neq\rangle$ (c) $\langle Z, >\rangle$ (d) $\langle Z, <\rangle$
6) Show that $\langle\{1,5,5^2,5^3,5^4,\ldots..\}, D\rangle$ is a chain.
   Answer: 2) yes; 3) (a) yes (b) no (c) no (d) yes (e) yes (f) yes 5) a) yes b) no c) no d) no

**Task-2**

1) Draw the Hass diagram of following
   a) $\langle S_{30}, D\rangle$
   b) $\langle S_{90}, D\rangle$
   c) $\langle (A), \subseteq\rangle$where A={a,b,c,d}
   d) $\langle S_{210}, D\rangle$
   e) $\langle S_{72}, D\rangle$
   f) $\langle S_{36}, D\rangle$
   g) $\langle S_{24}, D\rangle$
   h) $\langle S_{63}, D\rangle$
   i) $\langle S_{81}, D\rangle$

2) Draw the Hass diagram of $\langle L^2, \leq_2 \rangle$ ; where $L^2 = L \times L$ ,L={0,1} and (a ,b) $\leq_2$ (c ,d) if a$\leq$ c and b $\leq d$.

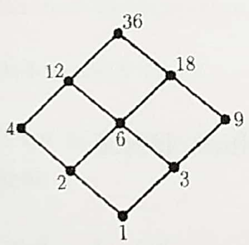
3) Draw the Hass diagram of $\langle L^4, \leq_4 \rangle$ ; where $L^4 = L \times L \times L \times L$ , L={0,1} and (a, b,c,d) $\leq_4$ (e,f,g,h) if a$\leq$ e, b $\leq f$ ,c $\leq$ g and d $\leq$ h.

**Task-3**

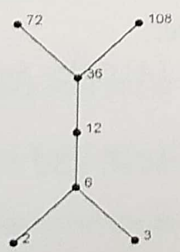
1) Define properties of Lattice.

2) Give an example of poset which is not Lattice.

3) Let P = {1, 2, 3, 4, 6,9,12,18,36}with divisibility relation. Find GLB and LUB of {3,9} and {2,3}.

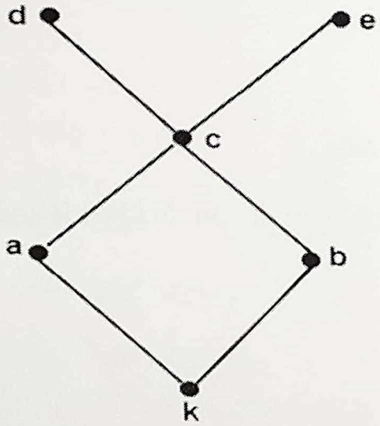



4) Let A= {2,3,6,12,36,72,108} with divisibility relation. Find upper bound, lower bound, GLB, LUB, maximal elements and minimal elements.



5) Let A= {k, a, b, c, d, e} be ordered and represented by hasse diagram.

If A= {k, a, b} then find upper bound and LUB of A.




3) Give an example of complete lattice which is an infinite lattice.

4) Show that $< s_{1001}, D >$ is a lattice.

5) Show that $< P(x), \subseteq >$ is a bounded lattice where x={a,b,c}.

6) Show that $< s_{30}, D >$ is a bounded lattice.

**Task-4**

1) find complement of each element of lattice $< s_{10}, GCD, LCM, 1,10 >$
2) Find complement of each element of lattice $< s_{30}, GCD, LCM, 1,30 >$
3) Is $< s_{75}, D >$ is a Bounded lattice? Justify your answer.
4) Show that S = $\{\varphi, \{a\}, \{b\}, \{a, b\}\}$ is a sub lattice of $< (A), \cap, \cup >$ for A= {a,b,c}.
5) Show that S = {1,2,3,6} is a sub lattice of $< s_{30}, GCD, LCM, 1,30 >$.

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# Sub.: Discrete Mathematics

**Unit -3: Propositional Logic**

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**Task – 1 Logic**

**Q.1** : Translate the following propositions into their symbolic form:

1. Aditya is a musician and Rita is a photographer.
2. Krishna will attend the class or will go to picnic.
3. It is not true that Vivek Arya is a Doctor and Sushil Gupta is an Engineer.
4. Discrete Mathematics is interesting or Mathematics is not difficult in general.
5. 3+4=7 and 5+8=13.

**Q.2** : Let p: Today is Monday, q: it is raining, r: it is hot. Formulate the following expressions in words:

1. $p \to q$   2. $\sim q \to (r \wedge p)$   3. $\sim p \to (q \vee r)$   4. $p \wedge (q \vee r) \to r \vee (q \vee p)$

5. $\sim (p \vee q) \leftrightarrow r$

**Q.3:** State inverse, converse and contrapositive of the following implications:

1. if 5x+1=11 then x=2.   2. if you work hard then you can earn money.

**Q.4:** Determine the truth value of each of the following propositions if 3+5>2 is p and 1+3=4 is q:

1. If 3+5<2 then 1+3=4   2. If 3+5<2 then 1+3≠4   3. If 3+5>2 then 1+3≠4

4. 3+5<2 if and only if 1+3=4   5. 3+5>2 if and only if 1+3=4   6. 3+5<2 if and only if 1+3≠4

**Q.5** : Represent the given proposition symbollically by letting p: you run 10 laps daily    q: you are healthy    r: you take multivitamins

1. If you run 10 laps daily, then you will be healthy.
2. If you do not run 10 laps daily or do not take multivitamins then you will not be healthy.
3. Taking multivitamins is sufficient for being healthy.
4. You will be healthy if and only if you run 10 laps daily and take multivitamins.
5. If you are healthy then you run 10 laps daily or you take multivitamins.
6. If you are healthy and run 10 laps daily then you do not take multivitamins.

**Q.6** : Write for the following statements

1. If 4<6 then 9>12    2. |4|<3 if -3<4<3

(a) Each conditional proposition symbollically

(b) Write the converse and contrapositive of each proposition symbollically and in words. Also find the truth values of each conditional proposition, its converse and its contrapositive

**Q.7:** State whether P≡Q or not for each proposition P and Q : –

1. P=p, Q= $p \vee q$    2. P= $p$ A $q$, Q= $\sim p \vee \sim q$    3. P= $p \rightarrow q$, Q= $\sim p \vee q$

4. P= $p$ A $(\sim q \vee r)$ , Q= $p \vee (q$ A $\sim r)$    5. P= $p \rightarrow q$, Q=$\sim q \rightarrow \sim p$

**Q.8:** Show that the binary operation disjunction (V) over the set of statements is commutative. Also show by truth table that the statement $(p \vee q) \leftrightarrow (q \vee p)$ is Tautology.

**Q.9:** Prove that the statement $(p \vee q)$ A $(\sim p$ A$\sim q)$ is a contradiction.

**Q.10:** Examine the validity of the arguments:

1.
$$\begin{array}{l} p \vee q \\ \underline{\sim q} \\ \therefore p \end{array}$$

2.
$$\begin{array}{l} p \vee q \\ p \rightarrow \sim q \\ \underline{p \rightarrow r} \\ \therefore p \end{array}$$

3.
$$\begin{array}{l} p \rightarrow q \\ \underline{\sim p} \\ \therefore \sim q \end{array}$$

**Q.11:** Test the validity of the argument:-

1. If the morning is fine, I go for a walk
I do not go for a walk
∴ The morning is not fine

2. . If income tax rates are lowered, income tax collection increases
Income tax collection increases

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$\therefore$ income tax rates are lowered

3. **Hypothesis:** If there is gas in the car, then I will go to the store. If I go to the store, then I will get a soda. There is gas in the car. **Conclusion:** I will get a soda.

## Answers

## Task 1: LOGIC

**A.1:** 1. $p \text{ A } q$ 2. $p \vee q$ 3. $\sim (p \text{ A } q)$ 4. $p \vee \sim q$ 5. $p \text{ A } q$

**A.2:** 1. If today is Monday then it is raining.

2. If it is not raining then it is hot and today is Monday.

3. If today is not Monday then it is raining or it is hot.

4. If today is Monday and it is raining or hot then it is hot or it is raining or not Monday today.

5. It is not true that today is Monday or it is raining if and only if it is hot.

**A.3:** 1. **Inverse:** if 5x+1≠11 then x≠2. **Converse:** if x=2 then 5x+1=11.

**Contrapositive:** if x≠2 then 5x+1≠11.

2. **Inverse:** if you do not work hard then you can not earn money.

**Converse:** if you can earn money then you work hard.

**Contrapositive:** if you can not earn money then you do not work hard.

**A.4:** **1.** T **2.** T **3.** F **4.** F **5.** T **6.** T

**A.5:** **1.** $p \rightarrow q$ **2.** $(\sim p \vee \sim r) \rightarrow \sim q$ **3.** $r \rightarrow q$ **4.** $q \leftrightarrow (p \text{ A } r)$ **5.** $q \rightarrow (p \vee r)$ **6.** $(q \text{ A } p) \rightarrow \sim r$

**A.6:** **1.** (a) $\rightarrow$ , for p: 4<6 and q: 9>12, truth value: false (b) **Converse:** $q \rightarrow p$ ; if 9>12 then4<6 , truth value:True; **Contrpositive:** $\sim q \rightarrow \sim p$ ; if 9≤12 then 4≥6 , truth value: false

**2.** (a) $\rightarrow p$ , for p: 4<|3| and q: -3<4<3; truth value: true

(b) **Converse:** $p \rightarrow$ ; if 4<|3| then -3<4<3 , truth value: true

**Contrpositive:** $\sim p \rightarrow \sim$ ; if 4≥|3| then -3≥4 or 4≥3, truth value: true

**A.7:** **1.** not equivallent **2.** equivallent **3.** Equivallent **4.** not equivallent **5.** equivallent

**A:10.** **1.** Valid **2.** Invalid **3.** Invalid

**A:11.** **1.** Valid **2.** Invalid **3.** Valid

## Task-2 Quantifiers

**Q.1:** Check whether the following statements are propositional function and for each propositional function, give a domain of discourse and truth set;

1. $(2n+1)^2$ is an odd integer 2. 1+3=4 3. $x \geq x^2$ 4. Let x be a real no.

**Q.2:** Let P(n) be the propositional function "n divides 77". Write following propositions in words and tell whether it is true or false. The domain of discourse is $Z^+$.

1. P(11) 2. P(8) 3. $\forall n\ P(n)$ 4. $\exists n\ P(n)$

**Q.3:** Let P(x) denote the statement "x is doing walk in morning". The domain of discoourse is the set of all people. Write each proposition in words.

1. $\forall x\ P(x)$ 2. $\exists x\ P(x)$ 3. $\sim (\exists x\ P(x))$ 4. $\exists x \sim P(x)$ 5. $\sim (\forall x\ P(x))$

**Q.4:** Let P(x) denote the statement "x spends more than six hours every weekday in class". The domain of discoourse is the set of students. Express each of the following quantifications in words.

1. $\exists x\ P(x)$ 2. $\forall x\ P(x)$ 3. $\exists x \sim P(x)$ 5. $\forall x \sim P(x)$

**Q.5:** Let P(x) denote the statement "x is an accountant" and let Q(x) denote the statement "x owns a Porsche". Write each statement symbolically:-

1. All accountants own Porsches.

2. Some accountants owns a Porsche.

3. All owners of Porsches are accountants.

4. Someone who owns a Porsche is an accountant.

5. Write the negation of each proposition in (1), (2), (3) and (4) symbolically and in words.

**Q.6:** Let P(x) denote the statement "x can speak Punjabi" and let Q(x) denote the statement "x knows the computer language C++". Express each of the following statement in terms of P(x) and Q(x), quantifiers and logical connectives. For the universe of discourse for quantifiers use the set of all students at your school.

1. There is a student at your school who can speak Punjabi and who knows C++.

2. There is a student at your school who can speak Punjabi but who does not know C++.

3. Every student at your school either can speak Punjabi or knows C++.

4. No student at your school can speak Punjabi or knows C++.

**Q.7:** Let P(x), Q(x) and R(x) be the statements "x is a Proffessor", "x is ignorent" and "x is vain" respectively. Express each of the following statement in terms of P(x), Q(x) and R(x), quantifiers and logical connectives. For the universe of discourse for quantifiers use the set of all people.

1. No professors are ignorent.

2. All ignorent people are vain.

3. No professors are vain.

4. Is 3rd statement valid from 1st and 2nd statement.

**Q.8:** Let P(x), Q(x), R(x) and S(x) be the statements "x is a baby", "x is logical", "x is able to manage a crocodile and "x is despised" respectively. Express each of the following statement in terms of P(x), Q(x ), R(x) and S(x), quantifiers and logical connectives. For the universe of discourse for quantifiers use the set of all people.

1. Babies are illogical.

2. Nobody is despised who can manage a crocodile.

3. Illogiacal persons are despised.

4. Babies cannot manage a crocodile.

5. Does 4th statement follows from 1st, 2nd and 3rd statement.

REDMI NOTE 8
PATEL VANDAN

**Q.9:** Give an argument using rules of inference to show that the conclusion follows from the hypothesis:-

1. **Hypothesis:** Everyone in the class has a graphic calculator. Everyone who has a graphic calculator understands the trigonometric functions. **Conclusion:** Raphie who is in the class, understand the trigonometric functions.

2. For every real number x, if x is an integer then x is a rational number. The number $\sqrt{2}$ is not rational. Therefore $\sqrt{2}$ is not an integer.

3. Everyone loves Microsoft or Apple. Lynn does not love Microsoft. Show that the conclusion, Lynn loves Apple.

## Answers

## Task 2: Quantifiers

**A:1.** **1.** Yes, DOC- $n \in$ , Truth set- $n \in Z$ **2.** No

**3.** Yes, DOC: $x \in$ , Truth set=$\{x: 0 \leq x \leq 1\}$ **4.** NO

**A:2.** **1.** T **2.** F **3.** F **4.** T

**A:3.** **1.** All people are doing walk in the morning.

**2.** Some people are doing walk in the morning.

**3.** It is not true that Some people are doing walk in the morning.

**4.** Some people are not doing walk in the morning.

**5.** It is not true that all people are doing walk in the morning.

**A:4.** 1. There is a student who spends more than six hours every weekday in class.

2. Every student spends more than six hours every weekday in class.

3. There is a student who does not spends more than six hours every weekday in class.

4. No student spends more than six hours every weekday in class.

**A:5.** **1.** $\forall x(P(x) \rightarrow Q(x))$ **2.** $\exists x(P(x)\ A\ Q(x))$ **3.** $\forall x(Q(x) \rightarrow P(x))$ **4.** $\exists x(Q(x)\ A\ P(x))$

5. **1.** $\exists x(P(x)\ A \sim Q(x))$. Some accountants do not own a Porsche.

2. $\forall x(P(x) \rightarrow \sim Q(x))$. All accountants do not own a Porsche.

3. $\exists x(Q(x) \wedge \sim P(x))$. Someone who owns a Porsche is not an accountant.

4. $\forall x(Q(x) \rightarrow \sim P(x))$. No accountant owns a Porsche.

**A:6.** 1. $\exists x(P(x) \wedge Q(x))$ **2.** $\exists x(P(x) \wedge \sim Q(x))$ **3.** $\forall x(P(x) \vee Q(x))$ **4.** $\forall x \sim (P(x) \vee Q(x))$

**A:7.** 1. $\forall x(P(x) \rightarrow \sim Q(x))$ **2.** $\forall x(Q(x) \rightarrow R(x))$ **3.** $\forall x(P(x) \rightarrow \sim R(x))$ **4.** No

**A:8.** 1. $\forall x(P(x) \rightarrow \sim Q(x))$ **2.** $\forall x(R(x) \rightarrow \sim S(x))$ **3.** $\forall x(\sim Q(x) \rightarrow S(x))$

4. $\forall x(P(x) \rightarrow \sim R(x))$ **5.** Yes

**A:9.** 1. Valid by Modus Ponens **2.** Valid by Modus Tollends **3.** Valid by Disjunctive syllogism

## Task- 3 Proof Techniques

**Q.1:** Prove that the product of two odd integers is an odd integer. Give the name of method of proof.

**Q.2:** Prove that between two rational numbers, there is always a rational number.

**Q.3:** Let n be an integer. Prove that if $n^2$ is odd, then n is odd.

**Q.4:** Prove that there is no rational number p/q whose square is 2, that is, show that $\sqrt{2}$ is an irrational number.

**Q.5:** Prove or disprove that $P(n)=n^2-n+41$ is prime for all $n \in N$.

## Answers

### Task 3: Proof Techniques

**A:1.** 1. By direct proof 2. By direct proof 3. By contrapositive proof
4. By contradiction 5. For n=41, not satisfied

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**Unit -4: <u>Algebraic Structures and Morphism</u>**

**<u>Task -1: Binary Composition and its Properties</u>**

**1.** Define binary operation with example.

**2.** Write all the properties of binary operations with examples.

**3.** Test the following properties for the given structure.

Properties: Closure, Associative, Existence of identity, Existence of inverse

Structure: (1) $(Z,*)$; $\forall\, a,b \in Z$ , $a * b = a + b + 5$

(2) $(Z^{+}, *)$; $\forall\, a,b \in Z^{+}$ , $a * b = a^{b}$

(3) $(R, *)$; $\forall\, a,b \in$ , $a * b = \frac{a}{a+b}$

**<u>Task -2: Various Algebraic Structures & Order of element</u>**

**1.** Define the following terms (1) Semi group (2) Monoid (3) Group (4) Abelian Group(5) Order of the group (6) Order of element.

**2.** Prove that set of positive rational number forms an Abelian group under the composition defined by $a * b = \frac{ab}{2}$.

**3.** Check whether $(R, \times)$ is group or not.

**4.** Classify the algebraic structure for

(1) $(Z_7 - \{0\}, \times_7)$

(2) $(M_{22}, \times)$ ; Where $M_{22}$ is set of all 2×2 Matrices.

**5.** Prove that the set of cubic root of unity under multiplication is Abelian group. Find the order of the group, and order of each element.

6. Find the order of the group and order of each element for

(1) $(Z_7 - \{0\}, \overset{6}{\times_7})$ (2) $(Z_5, \overset{5}{+_5})$.

## Task -3: Sub Group & Coset of a Subgroup

1. Define the following term (1) Sub Group (2) Improper Sub group (3) Coset (left as well as right) of a sub group (4) Index of Sub group.

2. State Lagrange's theorem. State the necessary and sufficient condition for a subset to be sub group.

3. Find all Subgroups of the following groups:

(1) $(Z_6, +_6)$ (2) $(G = \{\pm 1, \pm i\}, \times)$.

4. Prove that H is sub group of the given group G in each of the following. Find all cosets for H, Also find index of H (G : H).

(i) $(G = Z_6, +_6)$ & (H={0,3},$+_6$) (ii) $(G = \{\pm 1, \pm i\}, \times)$, $(H = \{1, -1\}, \times)$.

(iii) $(G = \ , +)$ & $(H = 2Z, +)$

## Task -4: Various Groups - I

1. Define the following term (1) Permutation Group (2) Cyclic Group (3) Generator.

2. State Cayley's Theorem. Prove that $(S_3, \circ)$ is non-abelian permutation group.

3. Prove that following structure is cyclic. Find all its generators:

(1) $(G = \{1, -1, i, -i\}, \times)$ (2) $(G = Z_8, +_8)$

4. Find all generators of a cyclic group of order 16.

5. If $, \mu$ & $r \in S_6$ defined by $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 4 & 6 & 5 \end{pmatrix}$ $\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 2 & 6 & 5 & 4 \end{pmatrix}$ & $r = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 3 & 2 & 5 & 1 \end{pmatrix}$. Find the following

(1) $\sigma^2 r$ (2) $\mu^{-1} r$ (3) O $(\sigma)$ (4) O ( ) .

Verify (1) $(\sigma\ )^{-1} = \mu^{-1}\,\sigma^{-1}$ (2) $\sigma \circ (\mu \circ r) = (\sigma \circ \mu) \circ r$

**Task -5: Various Groups - II**

**1.** Define the following term (1) Normal Sub Group (2) Quotient Group.

**2.** Prove that Sub group H = $\{p_1, p_2, p_3\}$ is normal for the group $(S_3,\circ)$ .

Check Whether Sub group $H_1 = \{p_1, p_4\}$ is normal for the group $(S_3,\circ)$ ?

**3.** Prove that Sub group H $=\{1,-1\}$ is normal for the group$(G = \{1, -1,\ i, -i\},\times)$.

**4.** Prove that G/ H is Quotient (Factor) Group for the following group G and normal subgroup H, Using composition table.

(i) $(G = \{\pm 1, \pm i\},\times)$& $(H = \{1,-1\}, \times)$

(ii) $(G = S_3,\circ)$& $(H = \{p_1, p_2, p_3\},\circ)$

**Task -6: Homomorphism of Groups**

**1.** Define the following term (1) Homomorphism of groups (2) Kernel of homomorphism.

**2.** Prove each of the following function is homomorphism. Find kernel of the homomorphism.

(i) $f : ( ,+) \to (R,+)$ Defined by $f\ (x + iy) = x$.

(ii) $f : (Z_5 - \{0\},\times_5) \to (Z_4, +_4)$ Defined by $f = \{(1,0), (2,3), (3,1), (4,2)\}$.

(iii) $f : (G = \{\pm 1, \pm i\},\times) \to (Z_4, +_4)$ Defined by $f = \{(1,0), (-1,2), (i, 1), (-i, 3)\}$.

**Task -7: Rings, Integral Domain and Fields**

**1.** Define the following term with Properties and example

1) Ring

2) Intergal Domain

3) Field

## Task -8 Introduction to Boolean algebra

Define the following term with example

1) Boolean Algebra
2) Sub Boolean algebra
3) Boolean Ring

## Task -8: Meet and Join

1. Define the following with example

   1) Join-irreducible 2) Meet-irreducible 3)Atoms 4)Anti-atoms

2. Prove the following Boolean Identities

   1) $\oplus (a' * b) = a \oplus b$ 2) $a * (a' \oplus b) = a * b$

3. Find Join-irreducible, Meet – irreducible, atoms and anti atoms of following lattices.

   1) $\langle S_{30}, D\rangle$ 2) $\langle S_{150}, D\rangle$ 3) $\langle (A), \subseteq\rangle$where A={a,b,c} 4) $\langle S_4 \times S_9, D\rangle$ 5) $\langle S_{210}, D\rangle$

## Task -9: Boolean Expression, Disjunctive Normal Form(DNF) or Sum of Product(SOP) and Conjunctive Normal Form(CNF) or Product of Sum(POS) Canonical form

1. Find the sum of products canonical and product of sum canonical expansions of the following Boolean functions.

   1) $(x, y, z) = x + y + z$
   2) $(x, y, z) = (x + z)y$
   3) $(x, y, z) = x + (yz')$
   4) $(x, y, z) = (x + y)' + (x'z)$

5) $(x, y, z, w) = (xy') + w$

Ans.**SOP**

1) $\sum(1,2,3,4,5,6,7)$
2) $\sum(3,6,7)$
3) $\sum(2,4,5,6,7)$
4) $\sum(0,1,3)$
5) $\sum(0,1,3,5,7,8,9,10,11,13,15)$

**POS**

1) $\prod(0)$
2) $\prod(0,1,2,4,5)$
3) $\prod(0,1,3)$
4) $\prod(2,4,5,6,7)$
5) $\prod(2,4,6,12,14)$

## Task -10: Boolean Expression and Equivalence

**1.** In any Boolean algebra, show that

1) $a = b \Leftrightarrow ab' + a'b = 0$
2) $a = 0 \Leftrightarrow ab' + a'b = b$
3) $(a + b')(b + c')(c + a') = (a' + b)(b' + c)(c' + a)$

**2.** Simplify the following Boolean expressions

1) $(a * b)' \oplus (a \oplus b)'$
2) $(a' * b' * c) \oplus (a * b' * c) \oplus (a * b' * c')$

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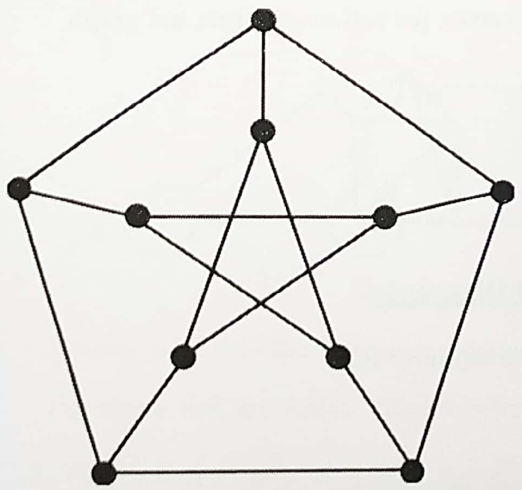
**Unit -5: Graph Theory**

**Task -1: Basic Concepts of Graphs and Diagraphs**

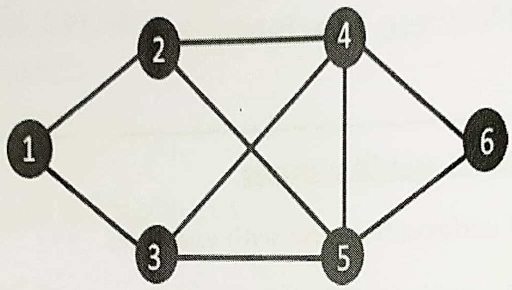
**1.** Define the following terms of undirected graph with example.

(i) Simple graph (ii) Multiple Graph (iii) Regular Graph (iv) Null Graph (v) Complete Graph (vi) Directed graph (vii) Undirected graph (viii) Bipartite Graphs (ix) Complete Bipartite Graph (x) Cycle Graph (xi) Wheel Graph (xii) Pendant Vertex (xiii) Labelled Graph.

**2.** State Handshaking lemma & verify it for the following graph.



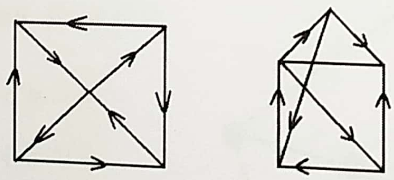
**3.** State Handshaking lemma & verify it for the following graph.




3. Draw the following graphs:

i. The complete graph $K_6$
ii. The complete bipartite graph $K_{5,7}$
iii. Cycle graph $C_4$.
iv. Wheel graph $W_5$.

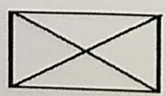
4. Find out degree & in degree of each vertex for following directed graph:



**Task -2: Isomorphism of Graphs and Diagraphs**

**1.** Prove that following pair of graphs are isomorphic.

(1)



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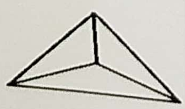

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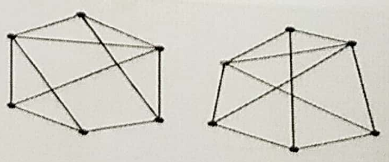
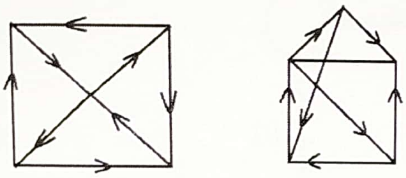
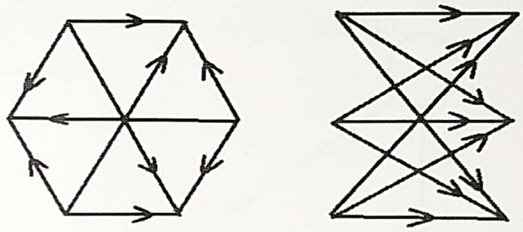
(2)



(3)



(4)



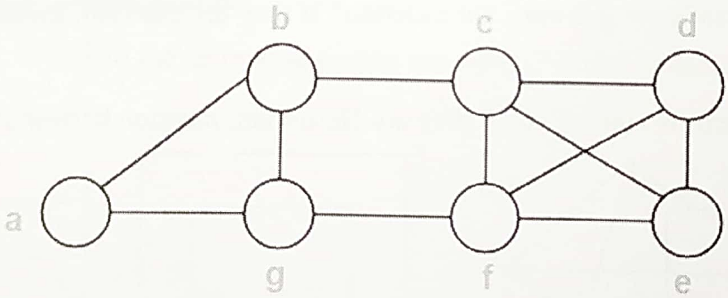
**Task -3: Walk, Path & Circuit of Graphs and Diagraphs**

**1.** Define the following terms for Undirected graphs with example.

(i) Walk (ii) Path (iii) Circuit (iv) Length of path (v) Trail (vi) Euler Graph

(vii) Hamiltonian Path

**2.** Consider the following graph.

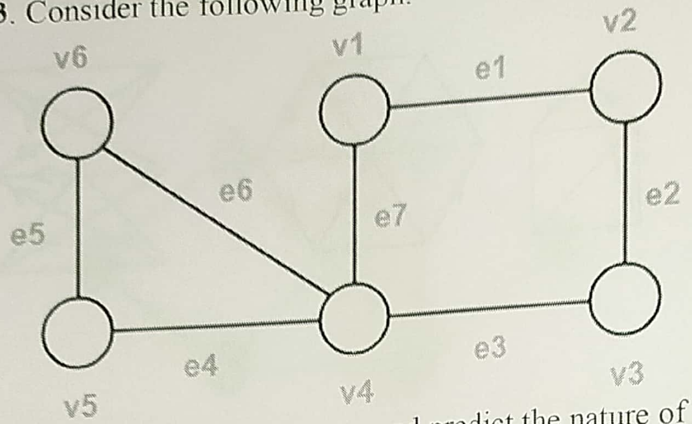



Decide which of the following sequences of vertices determine walks.

For those that are walks, decide whether it is a circuit, a path, a cycle or a trail.

1. a , b , g , f , c , b
2. b , g , f , c , b , g , a
3. c , e , f , c
4. c , e , f , c , e
5. a , b , f , a
6. f , d , e , c , b

3. Consider the following graph.



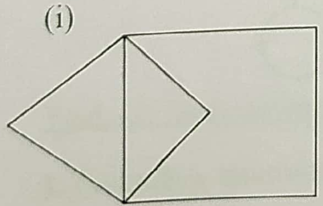

Observe the given sequences and predict the nature of walk in each case-

1. v1e1v2e2v3e2v2
2. v4e7v1e1v2e2v3e3v4e4v5
3. v1e1v2e2v3e3v4e4v5
4. v1e1v2e2v3e3v4e7v1
5. v6e5v5e4v4e3v3e2v2e1v1e7v4e6v6

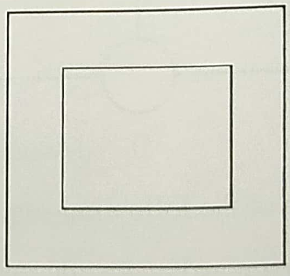
3. Which of the following graphs are Eulerian? If they are eulerian, mention Euler circuit.

Also which of them is Hermitian? If they are Hermitian, mention hermitian circuit.

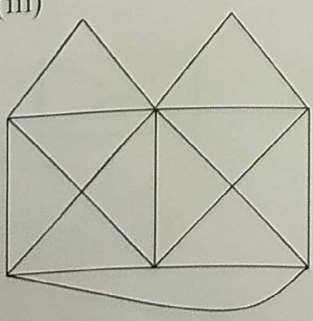
(i)



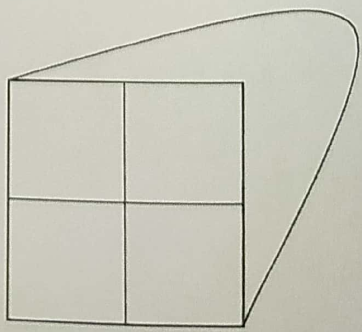
(ii)



(iii)



(iv)



**4.** Give one example of a graph which is Eulerian but not Hemiltonian.

**5.** Give one example of a graph which is Hemiltonian but not Eulerian.

Solution: 2)

1. Trail
2. Walk
3. Cycle
4. Walk
5. Not a walk
6. Path

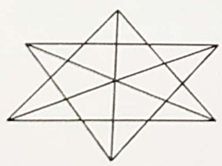
3)

1. Open walk
2. Trail (Not a path because vertex v4 is repeated)
3. Path
4. Cycle
5. Circuit (Not a cycle because vertex v4 is repeated)

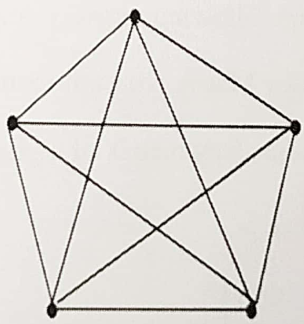
**Task -4: Planar graphs**

1. Which of the following graphs are planar? If planar draw

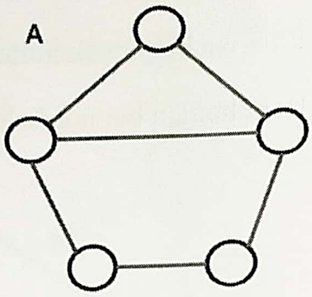
i)



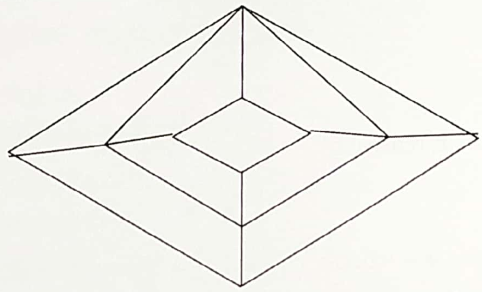
ii)



iii)




2. State Euler's formula & verify it for the following graph:



3. Is $\mathbf{K}_{3,3}$ a planar graph,Justify your answer?
4. Prove that a complete graph $\mathbf{K}_n$ is planar if $n \leq 4$.
5. Is $\mathbf{K}_5$ a planar graph?

## Task -5: Matrix Representation of Graphs and Diagraphs

**1.** Define the following terms of undirected & directed graph with example.

(i) Adjacency Matrix (ii) Path matrix (iii) Incident Matrix

**2.** Let the adjacency matrix of a graph G=$\langle V,E\rangle$ be

$$A=\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}\begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Find 1) Out degree of $v_2$ 2) The number of paths of length 2 from $v_1$ to $v_2$.

3) Total number of paths of length 2.

3. Construct Adjacency Matrix of all the graphs shown in Task No.2

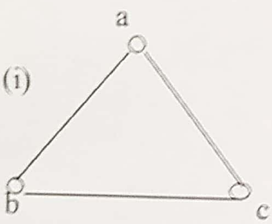
## Task -6: Coloring of Graphs:-

1. Define Following terms:-

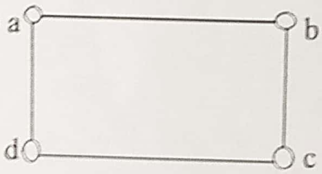
    i. Vertex Coloring
    ii. Edge Coloring
    iii. Chromatic Number

2. Coloring Following Graphs :-

(i)



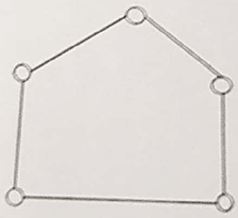

(ii)



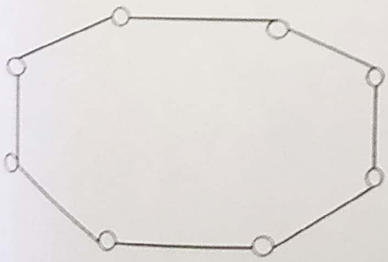

3. Find the chromatic number of the following Graphs

(i)

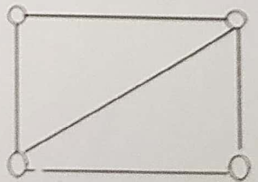
(ii)



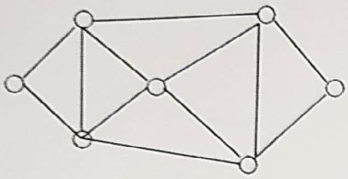
(iii)



(iv)



(v)



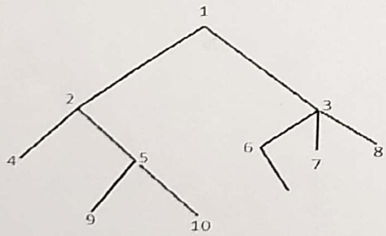
4. Prove That Every Bipartite Graph is 2 colorable.

**Task -7: Tree**

**1.** Define the following terms of undirected & directed graph with example.

(i) Tree (ii) Binary tree (iii) terminal & internal vertex (iv) height of tree

**2.** Represent the following tree in Circle, loop & Map forms.




**3.** Obtain the binary tree corresponding to the tree given in Question No. 2.