Exam Seat no. ....

KADI SARVA VISHWAVIDHYALAYA,

BE Semester–III (December 2023)

Discrete Mathematics (CC302B-N)

Date: 14-12-23 **Max Marks: 70**

Duration: 3 hr

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work**.

Section- I

Q.1 (a) Show that $< s_{1001}, D >$ is a lattice. [5]

(b) Define group and show that set of fourth root of unity form abelian group under the binary operation multiplication. [5]

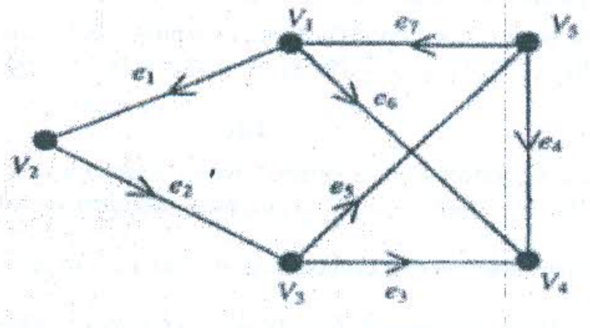
(c) Define with appropriate examples: i) Loop ii) Multigraph iii) pendent vertex iv) complete Graph v) Regular graph [5]

OR

(c) A relation is define on set $\mathbb{Z}$ is R = {(x , y) / x-y divided by 3} then check that R is equivalence relation [5]

Q.2 (a) Define Join-Irreducible Element and Meet-Irreducible Element. Find Join-Irreducible & Meet-Irreducible elements for lattice $\langle S_{30}, D\rangle$. [5]

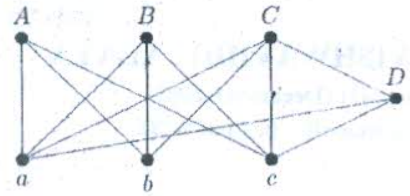
(b) Define adjacent matrix and incident matrix for directed simple graph. Find the adjacency Matrix and incident matrix of following graph. [5]



OR

Q.2 (a) Define Homomorphism of the Group. Let Z be the additive group of integers and G = {1,-1, i,-i} be the multiplicative group. Prove that the function $f: Z \to G$ defined by $f(n) = i^n, \quad \forall n \in Z$ is a homomorphism from Z to G. [5]

(b) State Handshaking Lemma & verify it for the following graph. [5]



Q.3 (a) Prove that G = (Z_6, $+_6$) is a cyclic group and find all its generators. [5]

(b) Let A= { a,b,c,d } and B = {1,2,3}.Let R be a relation define from set A to set B and is given as R = { (a,1),(a,2) ,(b,1) ,(c,2) ,(d,1) }. Find matrix relation M_R. Also draw arrow diagram. [5]

OR

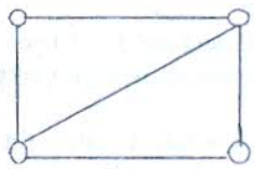
Q.3 (a) Define the order of the group and order of an element. Also find order of the group and order of each element of the group ($Z_7 - \{0\}$, $\times_7$). [5]

(b) Prove that $\langle\{1,5,5^2,5^3,....\},D\rangle$ is a chain. [5]

Section- II

Q.4 (a) Define the following terms (1) Semi group (2) Monoid (3) Abelian Group (4) Subgroup (5) index of a subgroup [5]

(b) Define the following terms (1) Vertex coloring (2) Edge coloring (3) Chromatic number. Also find chromatic number of the following graph. [5]



(c) Let P(x) denote the statement "x is doing walk in morning". The domain of discourse is the set of all people. Write each proposition in words. [5]
1. $\forall x\ P(x)$ 2. $\exists x\ P(x)$ 3. $\sim(\exists x\ P(x))$ 4. $\exists x \sim P(x)$ 5. $\sim(\forall x\ P(x))$

OR

(c) State inverse, converse and contrapositive of the following implications: [5]
1. If 5x+1=11 then x = 2. 2. If you work hard then you can earn money.

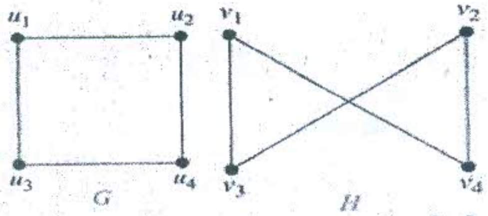
Q.5 (a) Find all the subgroup of a cyclic group of order 12 with generator 'a'. [5]

(b) Draw the Hasse diagram of $\langle s_{45}, D\rangle$ where D means 'a divides b' and find the cover of each element of s_{45}. [5]

OR

Q.5 (a) Prove that following graphs are Isomorphic. [5]

2/3



(b) Find fog & gof for given $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$ & $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$ and show that $fog \neq gof$. [5]

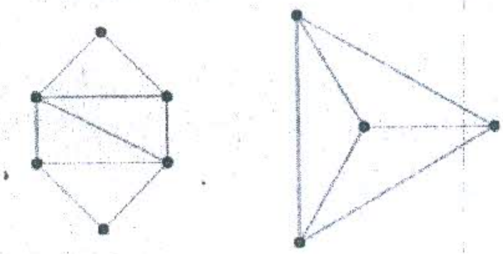
Q.6 (a) Define right coset and left coset. [5]
Consider group $(Z, +)$ and its subgroup $H = (3Z, +)$. Find all left cosets of H in G.

(b) Define the following terms (1) Walk (2) Path (3) Trail (4) circuit(5) Tree [5]

OR

(a) Prove that $(S_3, \circ)$ is non-abelian permutation group. [5]

(b) State Euler's formula & verify it for the each of the following graphs: [5]



ALL THE BEST

3|3

Exam Seat no. ....

KADI SARVA VISHWAVIDHYALAYA,

BE Semester–III (December 2022)

Discrete Mathematics (CC302B-N)

Date: 12/12/2022 **Max Marks: 70**

Duration: 3 hr

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work**.

Section- I

Q.1 (a) Draw the Hasse diagram of $\langle S_{36}, D\rangle$ where D means a divides b. and find the cover of each element of S_{36}. **[5]**

(b) Define order of an element & Find order of each element of multiplicative group $G = \{1, -1, i, -i\}$. **[5]**

(c) A = {1, 2, 3, 4} and B = {p, q, r, s} and R = {(1, p), (1, q), (1, r), (2, q), (2, r), (2, s)} then find matrix relation M_R. Also, draw an arrow diagram. **[5]**

OR

(c) Define i) Group ii) Monoid iii) Cyclic Group iv) Right Coset v) Semi Group. **[5]**

Q.2 (a) Find the sum of products canonical and product of sum canonical expansions of the Boolean functions $f(x, y, z) = x + y + z$. **[5]**

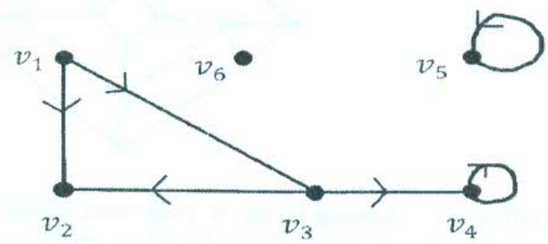
(b) Let R be the additive group of real numbers and R^+ be the multiplicative group of positive real numbers. Prove that the function $f: R \to R^+$ defined by $f(x) = e^x$, $\forall x \in R$ is an isomorphism from R onto R^+. **[5]**

OR

Q.2 (a) Check whether $\langle S_{15}, *, \oplus, 0, 1, '\rangle$ is a Boolean algebra or not where S_{15} is given as $S_{15} = \{1,3,5,15\}$. **[5]**

(b) Show that $\langle S_{30}, D\rangle$ is a complemented lattice. **[5]**

Q.3 (a) Find out degree & in degree of each vertex for following directed graph **[5]**



(b) Show that the set $S = \langle\{\emptyset, \{a\}, \{b\}, \{a, b\}\}, \subseteq\}\rangle$ is a sub lattice of the lattice $\langle P(A), \subseteq\rangle$, where $A = \{a, b, c\}$. **[5]**

OR

Q.3 (a) Define the following terms for Undirected graphs with example. **[5]**
(i) Walk (ii) Cycle (iii) Path (iv) Length of path (v) Closed walk.

(b) Find Join-Irreducible elements, Meet-Irreducible elements, Atoms and Anti-atoms for lattices $\langle S_{60}, D\rangle$ and $\langle S_{90}, D\rangle$. **[5]**

Section –II

Q.4 (a) Prove that set of positive rational number forms an abelian group under the composition defined by $a * b = \frac{ab}{2}$. [5]

(b) Check whether $\langle S_{24}, D \rangle$ is a lattice or not. [5]

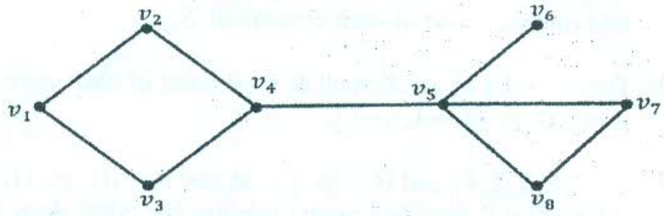
(c) On the set $\mathbb{Z}$ of all integers, define the relation R by $R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} / a - b$ is divisible by $5\}$ then show that R is an equivalence relation. [5]

OR

(c) Prove that the conditional statement $p \to q$ is equivalent to $(\sim p \vee q)$. [5]

Q.5 (a) Consider group $(Z, +)$ and its subgroup $H = (4Z, +)$. Find all right cosets of H in G. [5]

(b) State Handshaking Lemma & verify it for the following graph. [5]

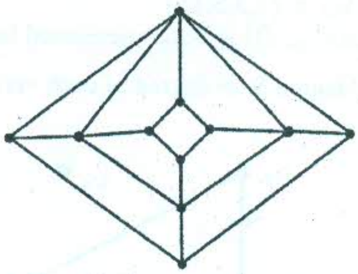


OR

Q.5 (a) If the permutations f_1, f_2 and f_3 are defined by $f_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 4 & 6 & 5 \end{pmatrix}$, $f_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 2 & 6 & 5 & 1 \end{pmatrix}$ and $f_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 3 & 2 & 5 & 1 \end{pmatrix}$ then find (i) $f_1 \circ f_2$ and (ii) $f_3 \circ f_2$ [5]

(b) Find all the minterms of a Boolean algebra with three variables x_1, x_2 and x_3. [5]

Q.6 (a) State Euler's formula & verify it for the following graph: [5]

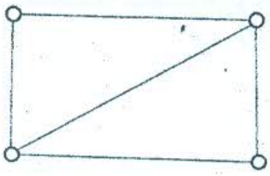


(b) Define the following terms of undirected graph with example. (i) Tree (ii) Spanning tree (iii) Forest (iv) height of tree (v) Leaf. [5]

OR

(a) Define vertices coloring and chromatic number and find the chromatic number of the following graphs [5]

(i)



(ii)

(b) Construct the truth table for the following statement, $(\sim(p \vee q)) \to (q \wedge p)$. [5]

ALL THE BEST