Chapter : 3

Task : 2

* Propositional Function :-

→ A Propositional Fun is a Sentence that can be true or false, but is not determined it contains a Variable that is not defined

Que-1

① $(2n+1)^2$ is an odd integer.

Ans→ Let P(n) is a Propositional Function. $n \in R$ because it is true or False based on the input.

It's truth set $T_p = \{n \in z\}$

② $1+3=4$

Ans→ It is not a Propositional Function because it doesnot have any free variable.

→ It is constant statement which is always true.

③ $x \geq x^2$

Ans→ It is a Propositional Function and $x \in R$,
$T_x \in x : 0 \leq x \leq 1$

④ Let x be a real no.

Ans → The given statement is not a Proporsitional because it is always true that any number is a real no.

∴ so ~~P(x)~~ ~~let~~ the given statement is not a proporsitional Function.

Que-2

$P(n) = n$ divides 77 ; $n \in Z$

ⓐ $P(11)$

Ans → 11 is divides with 77 means true.

ⓑ $P(8)$

Ans → False because 8 doesnot divides with 77 means False

ⓒ $\forall n P(n)$

Ans → Here given Function $P(n) = n$ divides 77.

→ Here $\forall n P(n)$ states For every $n \in Z^+$ $P(n)$

Because, There are various values which return P(n) true and various values which return P(n) False

$\therefore \forall n P(n)$ is False For $\forall n \in Z^+$

(d) $\exists n P(n)$

Ans → Here $\exists n P(n)$ state that there exists $n \in Z^+$, For which P(n) is true.

Now, P(n) = n divides 77

So we can see that $\exists n \in Z^+$ For which P(n) is true. There are various values of $n \in Z^+$ which given P(n) as true.

$\therefore \exists n P(n)$ is true For $\exists n \in Z^+$.

Que-3

$P(x)$ = x is doing walk in morning ; $x \in$ All people

1. $\forall x P(x)$

Ans → Here the given statement is " x is doing walk in morning." $\forall x P(x)$ state that for every people belonging to domain of discourse is true.

"All people are doing walk in morning".

2. $\exists x P(x)$

Ans → some people are doing walk in the morning.

3. $\sim(\exists x\, P(x))$

Ans → It is not true that some people are doing walk in the morning.

4. $\exists x(\sim P(x))$

Ans → Some people are not doing walk in the morning.

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(5) $\sim(\forall_x P(x))$

Ans- ~~$\exists x$~~ $\sim(\forall_x P(x)) = \exists_x(\sim P(x))$ (demorgan's law)

$\therefore$ It is not true that all people are doing walk in the morning.

Que-4

$P(x) =$ x spends more than six hours every weekday in class ; ~~$x \in$~~

x = set of all students.

(1) $\exists x P(x)$

Ans Some students spends more than six hours every weekday in class.

(2) $\forall_x P(x)$

Ans All the student spends more than six hours every weekday in class.

③ $\exists_x \sim P(x)$

Ans some student are not spends more than six hours every weekday in class.

④ ~~$\forall_x$~~ $\forall_x \sim P(x)$

Ans All the student are not spends more than six hours every weekday in class.

Que- 5

$P(x) = x$ is an accountant

$Q(x) = x$ owns a porsche.

① All accountants own Porsches.

Ans $\forall_x (P(x) \rightarrow Q(x))$

② Some accountants owns a porsche.

Ans $\exists_x (P(x) \land Q(x))$

③ All owners of Porsches are accountants.

Ans $\forall x\,(Q(x) \rightarrow P(x))$

④ Someone who owns a Porsche is an accountant.

Ans $\exists x\,(Q(x) \wedge P(x))$

⑤ Write the negation of each proposition in (1), (2), (3) and (4) symbolically and in words.

Ans ① $\exists x\,(P(x) \wedge \sim Q(x))$. Some accountants do not own a Porsche.

② $\forall x\,(P(x) \rightarrow \sim Q(x))$. All accountants do not own a porsche.

③ $\exists x\,(Q(x) \wedge \sim P(x))$. Someone who owns a Porsche is not an accountant.

④ $\forall x\,(Q(x) \rightarrow \sim P(x))$. No accountant owns a Porsche

Que-6

$P(x)$ = x can speak Punjabi
$Q(x)$ = x knows the computer language C++
x = set of all student

① There is a student at your school who can speak punjabi and who knows C++.

Ans $\exists_x (P(x) \wedge Q(x))$

② There is a student at your school who can speak Punjabi but whose does not know C++.

Ans $\exists_x (P(x) \wedge \sim Q(x))$

③ Every student at your school either can speak Punjabi or knows C++.

Ans $\forall_x (P(x) \vee Q(x))$

④ No student at your school can speak Punjabi or knows C++.

Ans $\forall x \sim (P(x) \vee Q(x))$

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Que-7

$P(x) = x$ is a Proffessor
$Q(x) = x$ is ignorent.
$R(x) = x$ is vain
$x =$ set of all people

1. NO Professors are ignorent.

Ans $\forall x (P(x) \rightarrow \sim Q(x))$

2. All ignorent pepole are vain.

Ans $\forall x (Q(x) \rightarrow R(x))$

3. NO professors are vain.

Ans $\forall x (P(x) \rightarrow \sim R(x))$

4. Is 3rd statement valid from 1st and 2nd statement

Ans NO Professore are vain

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Que-8

$P(x)$ = x is a baby
$Q(x)$ = x is logical
$R(x)$ = x is able to manage a crocodile.
$S(x)$ = x is despised.

x = set of all people.

① Babies are illogical

Ans $\forall x\,(P(x) \rightarrow \sim Q(x))$

② Nobody is despised who can manage a crocodile.

Ans $\forall x\,(R(x) \rightarrow \sim S(x))$

③ Illogical persons are despised.

Ans $\forall x\,(\sim Q(x) \rightarrow S(x))$

④ Babies cannot manage a crocodile.

Ans $\forall x\,(P(x) \rightarrow \sim R(x))$

⑤ Does 4th statement follows from 1st, 2nd and 3rd statement.

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Ans Yes.

Que-9

(1) $P(x)$ = x has a graphic calculator.
$Q(x)$ = x Understands than trigonometrix function.

→ Let R stands for Raphine.

$$\frac{\begin{array}{l} P(x) \rightarrow q(x) \\ \therefore P(x) \end{array}}{q(R)} \longrightarrow \frac{\begin{array}{c} p \rightarrow q \\ p \end{array}}{\therefore q}$$

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

Here, Critical Row has all the true value.

So, this argument is valid by Modus Ponens.

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② $P(x)$ = x is an integer
$Q(x)$ = x is a rational number

→ Let x stands for $\sqrt{2}$

$$\begin{array}{l} \therefore\ P(x) \to q(x) \\ \quad \sim q(x) \\ \hline \therefore \sim P(x) \end{array} \longrightarrow \begin{array}{c} p \to q \\ \sim q \\ \hline \sim p \end{array}$$

p	q	p→q	~p	~q
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Here, critical Row has all the true value.

So this argument is valid by Modus Tollends.

③ $P(x) = x$ loves Microsoft
$Q(x) = x$ loves Apple.

→ x stands for Lynn.

$$\begin{array}{r} P(x) \vee q(x) \\ \sim p(x) \\ \hline \therefore q(x) \end{array} \longrightarrow \begin{array}{c} p \vee q \\ \sim p \\ \hline q \end{array}$$

p	q	$p \vee q$	$\sim p$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	T

Here critical row has all the true value.

So this argument is valid by Disjunctive syllogism.

Task 3: Proof Techniques

Que-1 Prove that the product of two odd integers is an odd integer. Give the name of method of proof

→ Let a and b are two odd integers.

Suppose : $a = 2m+1$
$b = 2n+1$
where $m, n \in Z$

We have to do two odd number product.

So, $ab = (2m+1)(2n+1)$

$= 4mn + 2m + 2n + 1$

$= 2(mn + m + n) + 1$

$= 2P + 1$
where $P = mn + m + n$

Here, we get odd number.
So, Hence Prove that the product of two odd integers is an odd integer

→ This is prove by direct Proof method.

Que-2 Prove that between two rational numbers, there

→ Let a and b be rational numbers since the set of rationals is closed under the addition and division.

So, between the a and b rational number is $\frac{a+b}{2}$

Assume that, $a<b$ then,

→ First we add the a both side

$\therefore a+a < b+a$

$\therefore 2a < a+b$

$\therefore a < \frac{a+b}{2}$ — ①

→ similarlly we add the b both side,

$\therefore a+b < b+b$

$\therefore \frac{a+b}{2} < b$ —(2)

→ By eqⁿ –(1) & (2)

$$a < \frac{a+b}{2} < b$$

Thus between two rational numbers, there is always a rational number and prove method is direct proof.

Que-3

→ Here, n is a odd number.

Suppose $n = 2k+1$ for some integer k.

then $n^2 = (2k+1)^2$

$$= 4k^2 + 4k + 1$$

$$n^2 = 2(2k^2+2k)+1$$

Here n^2 number is also prove odd.

So, If n^2 is odd then n is odd.

Hence, Prove if n^2 is odd then n is odd.

Que-4

→ Assume that a rational number $x = \frac{p}{q}$ is equal to 2.

Let take x square.

$$\therefore \frac{p^2}{q^2} = 2$$

$$\therefore p^2 = 2q^2 \quad -(1)$$

→ p^2 is an even integer and it can expressed in the form of $2k$.

$$\therefore 2k = 2q^2$$

$$\therefore \boxed{q^2 = k}$$

→ The square of an odd integer is always odd, so p cannot be an odd.

So, p can be expressed in the form 2k.

$\therefore p = 2k$

By eq^n - 1

$\therefore (2k)^2 = 2q^2$

$\therefore 4k^2 = 2q^2$

$\therefore q^2 = 2k^2$

$\therefore q^2$ is an even integer.

So, p and q are both the even integers.

→ This contradiction means that $x = p/q$ whose square is not equal to 2.

Que-5

→ For Prime number, $n \in (m+1)$ is a prime number.

→ Let $n \in k+1$ and It is Prime.

$$\therefore P(k+1) = (k+1)^2 - (k+1) + 41$$

$$\therefore P(k+1) = k^2 + 2k + 1 - k - 1 + 41$$

$$\therefore P(k+1) = k^2 - k + 41$$

which is true for $n \in k+1$

$\therefore P(n) = n^2 - n + 41$ is Prime number.