Assignment : 2.

BOOLEAN ALGEBRA AND LOGIC GATES.

(1) Simplify the following Boolean functions to a minimum no. of literals.

(a) $\cancel{x}\ xy + xy'$

$= x(y + y')$

$= x(1) = \boxed{x}$

(b) $(x+y)(x+y')$

$= x + (y . y')$

$= x + 0 = \boxed{x}$

(c) $(xyz + x'y + xyz')$

$= xyz + xyz' + x'y$

$= xy(z+z') + x'y$

$= xy.1 + x'y$

$= xy + x'y$

$= y(x+x')$

$= y.1 = \boxed{y}$

(d) $(zx + zx'y)$

$= z(x + x'y)$

$= \cancel{z(x+y)}$

$\cancel{z(x.x+}$

$= z[(x+x').(x+y)]$

$= z[1.(x+y)]$

$= z(x+y)$

$= \boxed{zx + zy}$

(e) $((A+B)' \cdot (A'+B'))$

$(A+B)' = A'B'$

$= A'B'(A'+B')$

$= \boxed{A'B'}$

(f) $(y(wz' + wz) + xy)$

$y(wz' + wz) + xy$

$= y[w(z'+z)] + xy$

$= y[w.1] + xy$

$= yw + xy$

$= \boxed{y[w+x]}$

(2) Reduce the following Boolean expressions to the required no. of literals.

(a) $ABC + A'B'C + A'BC + ABC' + A'B'C'$ to five literals.

→ ~~This Boolean expression is already minimized.~~

$= AB(C+C') + A'B'(C+C') + A'BC$

$= AB + A'B' + A'BC$

$= AB + A'(B' + BC)$

$= AB + A'[(B'+B)(B'+C)]$

$= \boxed{AB + A'(B'+C)}$

(b) $BC + AC' + AB + BCD$ to four literals.

$= BC\,[1+D] + AC' + AB$

$= BC + AC' + AB$

$= \cancel{B[C+A]+}$ ↳ By Reduction theorem

$= \boxed{BC + AC'}$

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(c) $[(CD')+A]' + A + CD + AB$ to three literals

Doubt

$$[(CD')+A]' = (CD')' . A'$$
$$= C'+D . A'$$

$= [(C'+D).A'] + A + CD + AB$

$= C'A' + DA' + A + CD + AB$

$= \cancel{A} \, C'A' + DA' + A + [1+B] + CD$

$= C'A' + DA' + A + CD$

$= \cancel{A'}$

(3) Find the complement of the following Boolean functions & reduce them to a min. no. of literals.

(a) $(BC' + A'D)(AB' + CD')$

= By applying DE-MORGAN'S LAW,

$\overline{(BC' + A'D)(AB' + CD')}$

$= \overline{BC' + A'D} + \overline{(AB' + CD')}$

$= \overline{BC'}.\overline{A'D} + \overline{AB'}.\overline{CD'}$

$= [B'+C . A+D'] \cancel{+} [A'+B] . C'+D]$

$= [(B'+C).(A+D').(A'+B).(C'+D)]$

$= (B'+C)(A+D') = B'A + B'D' + CA + CD'$

$= (A'+B)(C'+D) = A'C' + A'D + BC' + BD$

$= (B'A + B'D' + CA + CD')(A'C' + A'D + BC' + BD)$.

$= B'AA'C' + B'AA'D + B'ABC' + B'ABD + B'D'A'C' + B'D'\cancel{A}A'D + B'D'BC' + B'D'BD + CAA'C' + CAA'D + CABC' + CABD + CD'A'C' + CD'A'D + CD'BC' + CD'BD$.

$F = 0$. $\boxed{F' = 1}$

Doubt

(b) $B'D + A'BC' + ACD + A'BC$

Doubt

$= D(B' + AC) + A'B(C + C')$

$= D(B' + AC) + A'B$

$= DB' + ACD + A'B$

$= \cancel{B'DA' + B'DB + ACD}$

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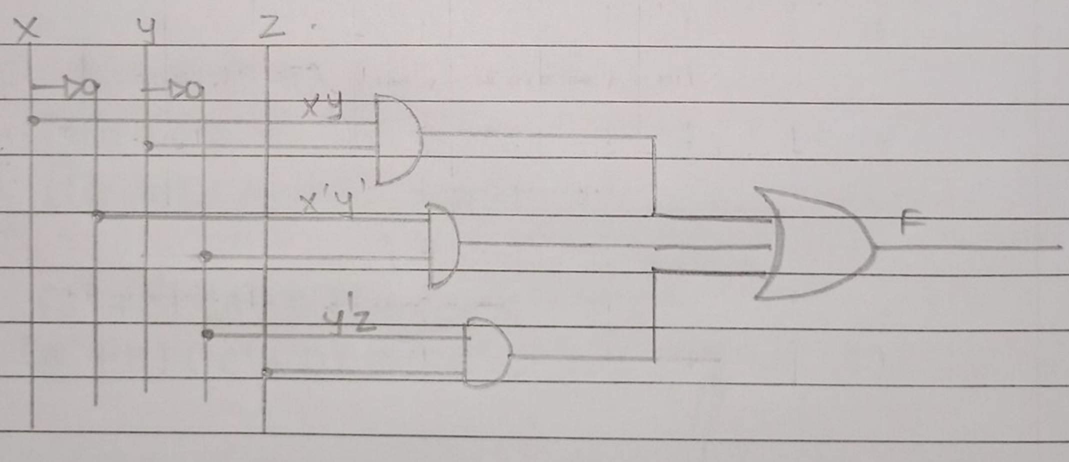
(4) Obtain the truth table of the function.
$F = xy + x'y' + y'z$

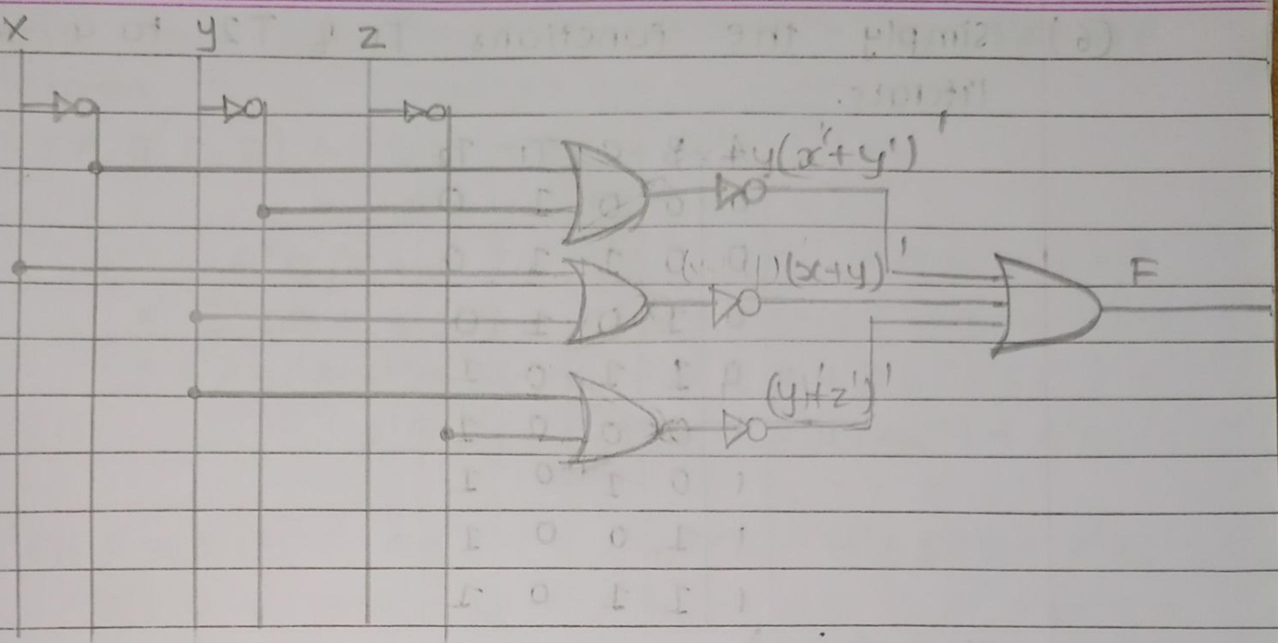
x	y	z	xy	xy'	y'z	(F)
0	0	0	0	0	0	0
0	0	1	0	0	1	1
0	1	0	0	0	0	0
0	1	1	0	0	0	0
1	0	0	0	1	0	1
1	0	1	0	1	1	1
1	1	0	1	0	0	1
1	1	1	1	0	0	1

(5) Given the Boolean function:
$F = xy + x'y' + y'z$
(a) Implement it with AND, OR & NOT Gates.



(b) Implement it with only OR and NOT Gates
$F = xy + x'y' + y'z$
= By De Morgan's theorem

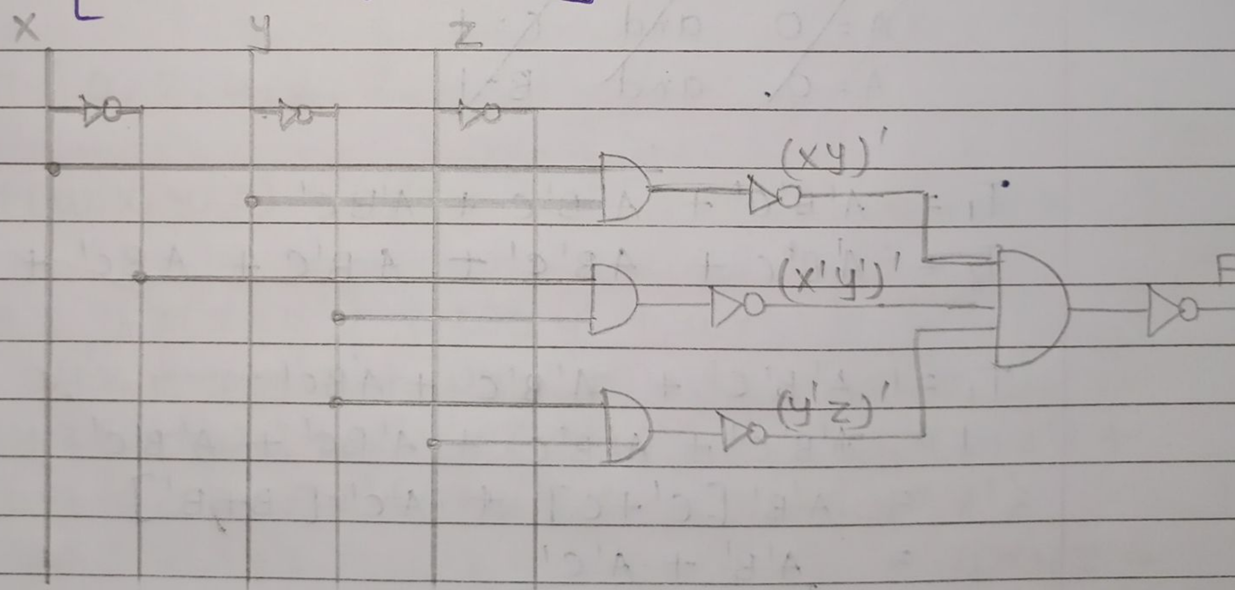
$(x'+y')' + (x+y)' + (y+z')'$



(c) Implement it with only AND and NOT Gates.

$F = xy + x'y' + y'z$

$= \left[(xy)'(x'y')'(y'z)'\right]'$



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(6) simply the functions T1 & T2 to a ~~max~~ min. no. of literals.

A	B	C	T_1	T_2
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	0	1

FOR (T1).

~~From table (T1) = 1 when~~
~~A = 0 and C = 0~~
~~A = 0 and C = 1~~
~~A = 0 and C = 1~~
~~A = 0 and B = 1~~

$$T_1 = A'B'C' + A'B'C + A'BC'$$
$$T_2 = A'BC + AB'C' + AB'C + ABC' + ABC$$

$$\begin{aligned}
T_1 &= A'B'C' + A'B'C + A'BC' \\
&= A'B'C' + A'B'C + A'BC' + A'B'C' \\
&= A'B'[C'+C] + A'C'[B+B'] \\
&= A'B' + A'C' \\
&= \boxed{A'[B'+C']}
\end{aligned}$$

$$\begin{aligned}
T_2 &= A'BC + AB'C' + AB'C + ABC' + ABC + ABC \\
&= BC[A'+A] + AB'[C'+C] + AB[C'+C] \\
&= BC + AB' + AB \\
&= BC + A[B'+B] \\
&= \boxed{A+BC}
\end{aligned}$$

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(7) Express the following functions in a sum of minterms & a product of maxterms.

(a) $F(A, B, C, D) = D(A' + B) + B'D$.

$= B'D + A'D + BD$.

$= B'D(C+C') + A'D(B+B') + BD(A+A')$

$= B'CD + B'C'D + A'BD + A'B'D + ABD + A'BD$.

$= B'CD(A+A') + B'C'D(A+A') + A'BD(C+C') +$
$A'B'D(C+C') + ABD(C+C') + A'BD(C+C')$

$= AB'CD + A'B'CD + AB'C'D + A'B'C'D + A'BCD + A'BC'D +$
$A'B'CD + A'B'C'D + ABCD + ABC'D + A'BCD +$
$A'BC'D$

$= \Sigma(11, 3, 9, 1, 7, 5, 15, 13)$

$= \Sigma(1, 3, 5, 7, 9, 11, 13, 15)$

$= \Pi(0, 2, 4, 6, 8, 10, 12, 14)$.

(b) $F(w, x, y, z) = y'z + wxy' + wxz' + w'x'z$

$= y'z(x+x') + wxy' + wxz' + w'x'z$

$= xy'z + x'y'z + wxy' + wxz' + w'x'z$

$= xy'z(w+w') + x'y'z(w+w') + wxy'(z+z') +$
$wxz'(y+y') + w'x'z(y+y')$

$= wxy'z + w'xy'z + wx'y'z + w'x'y'z + wxy'z +$
$wxy'z' + wxyz' + wxy'z' + w'x'yz +$
$w'x'y'z$.

$= \Sigma(1, 3, 5, 9, 12, 13, 14)$

$= \Pi(0, 2, 4, 6, 7, 8, 10, 11, 15)$.

(c) $F(X,Y,Z) = 1$

$= x(y+\bar{y}) + \bar{x}(y+\bar{y})$

$= xy + x\bar{y} + \bar{x}y + \bar{x}\bar{y}$

$= xy(z+\bar{z}) + x\bar{y}(z+\bar{z}) + \bar{x}y(z+\bar{z}) + \bar{x}\bar{y}(z+\bar{z})$

$= xyz + xy\bar{z} + x\bar{y}z + xy\bar{z} + \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$

$=$

$\Sigma(0,1,2,3,4,5,6,7)$

$\boxed{\Pi = 0}$.

(8) Convert the following to the other canonical form.

(a) $F(X,Y,Z) = \Sigma(1,3,7)$

$F' = \Sigma(0,2,4,5,6)$

$F' = m_0 + m_2 + m_4 + m_5 + m_6$.

$F = (F')' = (m_0)'(m_2)'(m_4)'(m_5)'(m_6)'$

$= M_0 \cdot M_2 \cdot M_4 \cdot M_5 \cdot M_6$

$F(x,y,z) = \Pi(0,2,4,5,6)$

(b) $F(w,x,y,z) = \Sigma(0,2,6,11,13,14)$

$F'(w,x,y,z) = \Sigma(1,3,4,5,7,8,9,10,12,15)$

$F' = m_1 + m_3 + m_4 + m_5 + m_7 + m_8 + m_9 + m_{10} + m_{12} + m_{15}$

$F = (F')' = (m_1)'(m_3)'(m_4)'(m_5)'(m_7)'(m_8)'(m_9)'(m_{10})'(m_{12})'(m_{15})'$

$= M_1 \cdot M_3 \cdot M_4 \cdot M_5 \cdot M_7 \cdot M_8 \cdot M_9 \cdot M_{10} \cdot M_{12} \cdot M_{15}$

$F(w,x,y,z) = \Pi(1,3,4,5,7,8,9,10,12,15)$

(c) $F(x,y,z) = \prod(0,3,6,7)$

$$F'(x,y,z) = \overset{\Pi}{\cancel{\Sigma}}(1,2,4,5)$$

$$F' = M_1 \cdot M_2 \cdot M_4 \cdot M_5$$

$$F = (F')' = M_1 + M_2 + M_4 + M_5$$

$$F(x,y,z) = \Sigma(1,2,4,5)$$

(9) Obtain the simplified expⁿ in SOP for the following Boolean expⁿ using K-MAP.

(a) $F(x,y,z) = \Sigma(2,3,6,7)$

A \ BC	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{x}$ 0	0	1	1 (3)	1 (2)
x 1	4	5	1 (7)	1 (6)

$= \boxed{y}$

(b) $F(A,B,C,D) = \Sigma(7,13,14,15)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	3	2
$\bar{A}B$ 01	4	5	1 (7)	6
AB 11	12	1 (13)	1 (15)	1 (14)
$A\bar{B}$ 10	8	9	11	10

BCD

ABC

ABD

$= \boxed{BCD + ABC + ABD}$

(10) Use K-Map & obtain the simplified expⁿ in SOP for the following funⁿ.

(a) $xy + x'y'z' + x'yz' = xy(z+z') + x'y'z' + x'yz'$

$= xyz + xyz' + x'y'z' + x'yz'$

$= \Sigma(7,6,0,2)$

x \ yz	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{x}$ 0	1 (0)	1	3	1 (2)
x 1	4	5	1 (7)	1 (6)

$\bar{x}\bar{z}$

xy

$= \boxed{xy + \bar{x}\bar{z}}$

(b) $A'B + BC' + B'C'$

$= A'B(C+C') + BC'(A+A') + B'C'(A+A')$

$= A'BC + A'BC' + ABC' + A'BC' + AB'C' + A'B'C'$

$= \Sigma(3,2,6,4,0) = \Sigma(0,2,3,4,6)$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1 (0)	(1)	1 (3)	1 (2)
A 1	1 (4)	(5)	(7)	1 (6)

$\rightarrow \bar{A}B$

$\rightarrow \bar{C}$

$= \boxed{\bar{A}B + C}$.

(18) Obtain the simplified exⁿ in SOP for Boolean exⁿ using K-Map.

(a) $D(A'+B) + B'(C+AD)$

$= A'D + BD + B'C + B'AD$

$= A'D(C+C') + BD(A+A') + B'C(D+D') + B'AD$

$= A'DC + A'DC' + ABD + A'BD + B'CD + B'CD' + B'AD$

$= A'DC(B+B') + A'DC'(B+B') + ABD(C+C') + A'BD(C+C')$
$B'CD(A+A') + B'CD'(A+A') + B'AD(C+C')$

$= A'BCD + A'B'CD + A'BC'D + A'B'C'D + ABCD + ABC'D + A'BCD + A'BC'D + AB'CD + A'B'CD + AB'CD' + A'B'CD' + AB'CD + AB'C'D.$

$\Sigma(7,3,5,1,15,13,11,10,2,\cancel{3},11,9).$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	(0)	1 (1)	1 (3)	1 (2)
$\bar{A}B$ 01	(4)	1 (5)	1 (7)	(6)
AB 11	(12)	1 (13)	1 (15)	(14)
$A\bar{B}$ 10	(8)	1 (9)	1 (11)	1 (10)

$\rightarrow \bar{B}C$

$\rightarrow D$

$= \boxed{D + \bar{B}C}$

(b) $ABD + A'C'D' + A'B + A'CD' + AB'D$

$= ABD + A'C'D' + A'B(C+C') + A'CD' + AB'D.$

$= ABD + A'C'D' + A'BC + A'BC' + A'CD' + AB'D$

$= ABD(C+C') + A'C'D'(B+B') + A'BC(D+D') + A'BC'(D+D')$
$+ A'CD'(B+B') + AB'D(C+C')$

$= ABCD + ABC'D + A'BC'D' + A'B'C'D' + A'BCD + A'BCD'$
$+ A'BC'D + A'BC'D' + A'BCD' + A'B'CD'$
$+ AB'CD + AB'C'D.$

$\Sigma(0, 2, 4, 5, 6, 7, 9, 11, 13, 15)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1_0	$_1$	$_3$	1_2
$\bar{A}B$ 01	1_4	1_5	1_7	1_6
AB 11	$_{12}$	1_{13}	1_{15}	$_{14}$
$A\bar{B}$ 10	$_8$	1_9	1_{11}	$_{10}$

$\bar{A}\bar{D}$

$\bar{A}B$

AD

$= \boxed{\bar{A}\bar{D} + \bar{A}B + AD}$

(12) Use K-Map to obtain the simplified exp^n in POS.

(a) $F(x,y,z) = \Pi(0,1,4,5)$

x \ yz	yz 00	$y\bar{z}$ 01	$\bar{y}z$ 11	$\bar{y}z$ 10
x 0	0_0	0	$_3$	$_2$
$\bar{x}$ 1	0_4	0	$_7$	$_6$

$\boxed{y}$

(b) $F(A,B,C,D) = \Pi\,(0,1,2,3,4,10,11)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
AB 00	0	0	0	0
$A\bar{B}$ 01	0			
$\bar{A}\bar{B}$ 11				
$\bar{A}B$ 10			0	0

$\rightarrow \cancel{AB}\ (A+B)$

$\rightarrow \cancel{BC}\ (B+\bar{C})$

$\rightarrow \cancel{ACD}\ (A+C+D)$

$= \boxed{(A+B)(B+\bar{C})(A+C+D)}$

(13) Simplify the boolean function F in sum of products using the don't-care condition using KMAP.

(a) $F = y' + x'z'$

$d = yz + xy$

$F = y'(x+x') + x'z'$

$= \cancel{y'x/x}$

$= xy' + x'y' + x'z'$

$= xy'(z+z') + x'y'(z+z') + x'z'(y+y')$

$= xy'z + xy'z' + x'y'z + x'y'z' + x'yz' + x'y'z'$

$= \Sigma(5, 4, 1, 0, 2)$

$d = yz + xy$

$= yz(x+x') + xy(z+z')$

$= xyz + x'yz + xyz + xyz'$

$\Sigma d(7, 3, 6)$

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$x \backslash yz$	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{x}$ 0	1_0	1_1	X_3	1_2
x 1	1_4	1_5	X_7	X_6

$\rightarrow \boxed{=1}$

$= \boxed{F = 1}$

(b) $F = B'C'D' + BCD' + ABCD'$
$d = B'CD + A'BC'D$

$F = B'C'D'(A+A') + BCD'(A+A') + ABCD'$
$= AB'C'D' + A'B'C'D' + ABCD' + A'BCD' + ABCD'$
$\Sigma(8, 0, 14, 6)$

$D = B'CD + A'BC'D$
$= B'CD(A+A') + A'BC'D$
$= AB'CD + A'B'CD + A'BC'D$
$\Sigma d(11, 3, 5)$

$AB \backslash CD$	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1_0	$_1$	X_3	$_2$
$\bar{A}B$ 01	$_4$	X_5	$_7$	1_6
AB 11	$_{12}$	$_{13}$	$_{15}$	1_{14}
$A\bar{B}$ 10	1_8	$_9$	X_{11}	$_{10}$

$\rightarrow BC\bar{D}$

$\rightarrow \bar{B}\bar{C}\bar{D}$

$Fd = \boxed{BC\bar{D} + \bar{B}\bar{C}\bar{D}}$

(14) Simplify the Boolean funⁿ using Tabulaⁿ method & determine Prime Implicants.

(a) $F(\omega, x, y, z) = \Sigma(0,1,2,8,10,11,14,15)$

	A	B	C	D
0 =	0	0	0	0
1 =	0	0	0	1 ✓
2 =	0	0	1	0 ✓
8 =	1	0	0	0 ✓
10 =	1	0	1	0 ✓
11 =	1	0	1	1
14 =	1	1	1	0
15 =	1	1	1	1

	ω (A)	x (B)	y	z
0,1	0	0	0	–
0,2	0	0	–	0 ✓
0,8	–	0	0	0 ✓
2,10	–	0	1	0 ✓
8,10	1	0	–	0 ✓
10,11	1	0	1	– ✓
10,14	1	–	1	0 ✓
11,15	1	–	1	1 ✓
14,15	1	1	1	– ✓

	ω	x	y	z
(0,2)(8,10) =	–	0	–	0
(0,8)(2,10) =	–	0	–	0
(10,11)(14,15) =	1	–	1	–
(10,14)(11,15) =	1	–	1	–

$$\boxed{F = \bar{\omega}\bar{x}\bar{y} + \bar{x}\bar{z} + \omega y}$$

(b) $F(\omega, x, y, z) = \Sigma(1,4,6,7,8,9,10,11,15)$

	ω	x	y	z
1 =	0	0	0	1
4 =	0	1	0	0
6 =	0	1	1	0
7 =	0	1	1	1
8 =	1	0	0	0
9 =	1	0	0	1
10 =	1	0	1	0
11 =	1	0	1	1
15 =	1	1	1	1

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ω	x	y	z
0	0	0	1
0	1	0	0
0	1	1	0

(b) $F = (w, x, y, z) = \Sigma(1, 4, 6, 7, 8, 9, 10, 11, 15)$

	w	x	y	z		w	x	y	z		w	x	y	z
1 =	0	0	0	1	(1,9)	–	0	0	1	8,10,9,11	1	0	–	–
4 =	0	1	0	0	(4,6)	0	1	–	0	8,9,10,11	1	0	–	–
8 =	1	0	0	0	(8,9)	1	0	0	– ✓					
6 =	0	1	1	0	8,10	1	0	–	0 ✓					
9 =	1	0	0	1	6,7	0	1	1	–					
10 =	1	0	1	0	9,11	1	0	–	1 ✓					
7 =	0	1	1	1	10,11	1	0	1	– ✓					
11 =	1	0	1	1	7,15	–	1	1	1					
15 =	1	1	1	1	11,15	1	–	1	1					

$$= \bar{x}\bar{y}z + \bar{w}x\bar{z} + \bar{w}xy + xyz + \bar{w}yz + w\bar{x}$$

	1	4	6	7	8	9	10	11	15
x'y'z	X					X			
w'xz'		X	X						
w'xy			X	X					
xyz				X					X
wyz								X	X
wx'					X	X	X	X	

$= \boxed{x'y'z + w'xz' + wx'}$