$-d(u) = 3$ $\quad d(p) = 2$
$v = 3$ $\quad q = 3$
$w = 2$ $\quad r = 3$
$x = 3$ $\quad s = 3$
$y = 1$ $\quad t = 1$.

(4). One-one correspondence on vertex & edge

vertex

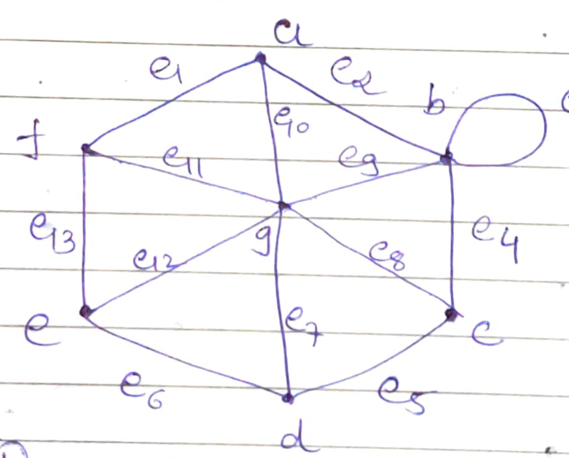
$y - t$
$w - p$
$v - s$
$u - q$
$x - r$

~~edges~~

q, r

## * Eulerian Graph :

* Walk : A walk is a finite alternating sequence $v_1 e_1 v_2 e_2 v_3 e_3 \ldots e_n v_n$ of vertices and edges, beginning and ending with same or different vertices.




then (1) $a e_1 f e_{11} g e_9 b e_4 c$ is a walk

(2) $a e_{10} g e_{11} f e_1 a e_2 b$. is a walk

(3) $a e_2 b e_9 g e_8 c e_4 b e_9 g e_{12} e$ is a walk

* **length of walk** : The number of edges is called length of walk

* **closed and Open walk** :

A walk is said to be closed if its origin and terminous vertex ($V_0 = V_n$) is equal.
- otherwise it is called open walk

Ex: $f e_{11} g e_{10} a e_1 f$ is closed walk

* **Trail** : Any walk having different edges is called Trail.

* **circuit** : A closed Trail is called circuit

Ex: $a e_1 f e_{11} g e_9 b e_2 a$.

* **Path** - : A walk is called Path if all vertices are not repeated.

Ex: $a e_2 b e_4 c e_8 g e_{11} f e_1 a$.

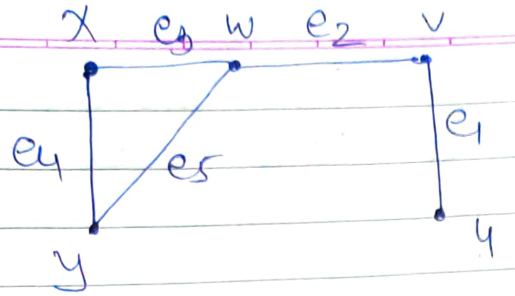
* **cycle** : A closed path is called cycle

Ex: $a e_{10} g e_7 d e_6 e e_{13} f e_1 a$.

* **Eulerian Path** :

A path is known as Eulerian path if every edge of the graph G appears exactly once in the path.

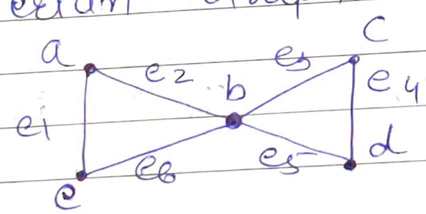
Ex



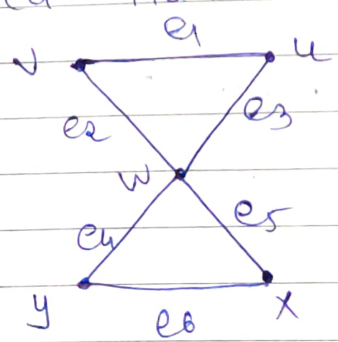

then $u e_1 v e_2 w e_3 x e_4 y e_5 w$ is Eulerian path.

* <u>Eulerian Circuit</u> : The circuit which contains every edge of the graph exactly once is called Eulerian Circuit.

* <u>Eulerian Graph</u> : A connected graph which contains an Eulerian circuit is called Eulerian Graph.




* <u>Hamiltonian Path</u> : A path which contain every vertex of graph G exactly once is called Hamiltonian path.



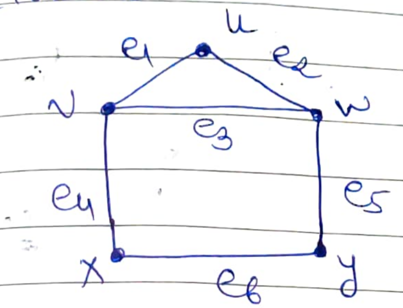

then $u e_1 v e_2 w e_5 x e_6 y$ is Hamiltonian Path.

* **Hamiltonian Circuit :**

A circuit that passes through each of the vertices in a group of exactly one vertex the starting vertex & end vertex is called Hamiltonian circuit.

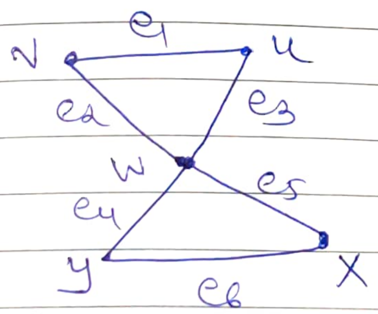
Ex

(1)




then
$u e_1 v e_4 x e_6 y e_5 w e_2 u$
is Hamiltonian circuit

(2)




then
$u e_1 v e_2 w e_5 x e_6 y e_4 w e_3 u$
is not Hamiltonian circuit.
- because w vertex repeat

* **Hamiltonian Graph :**

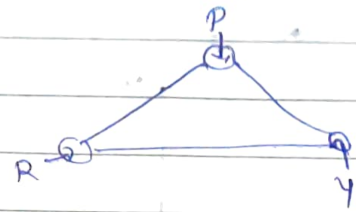
A connected graph which contain Hamiltonian circuit is called Hamiltonian graph

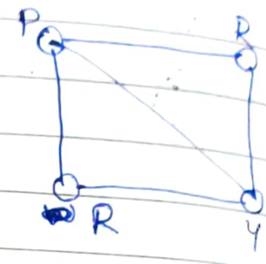
- Ex - 1 is Hamiltonian Graph
- Ex - 2 is not Hamiltonian Graph.

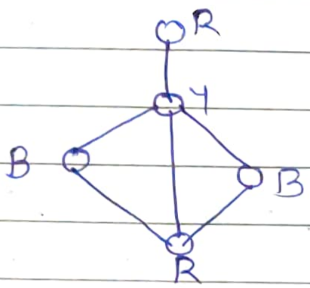
## * Graph Coloring :

Painting all the vertices of a graph with colors such that no two adjacent vertices have the same color is called coloring of Graph.

Ex





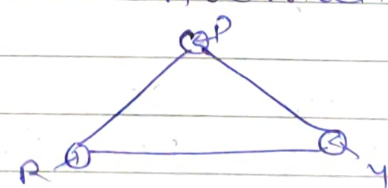


## * Chromatic Number :

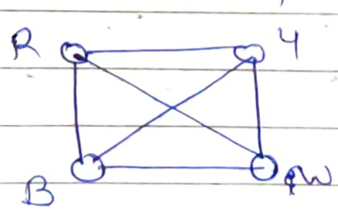
The least number of colors required for coloring of a graph G is called its chromatic number.

Ex

2-Chromatic Number ~~Graph~~



3-Chromatic ~~Graph.~~ Number



4-Chromatic Number

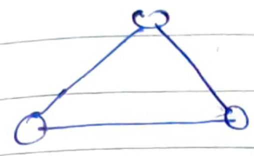
**Note :**

(1) The chromatic number of graph G is denoted by $\chi(G)$.

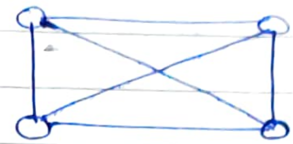
2) If $\chi(G) = K$ then the graph is called K-Chromatic.

3) Chromatic number of null graph is 1

4) chromatic number of complete graph $K_n$ of n-vertices is n.



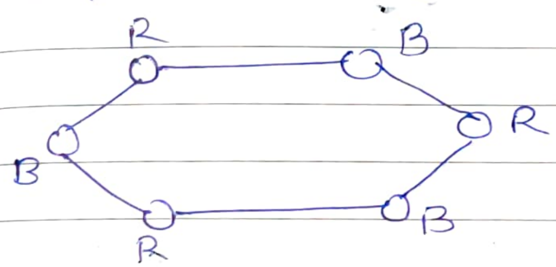
3-chromatic



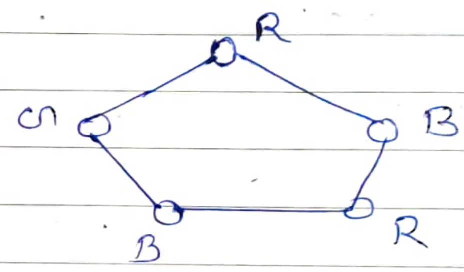
4-Chromatic

5) If a graph is circuit with n-vertices then

1) It is 2-chromatic if n is even
2) It is 3-chromatic if n is odd

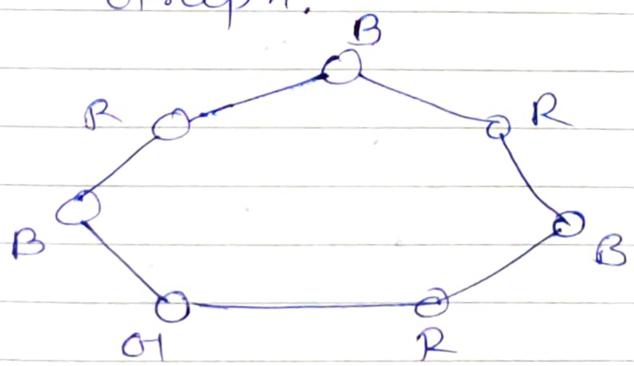



2-chromatic



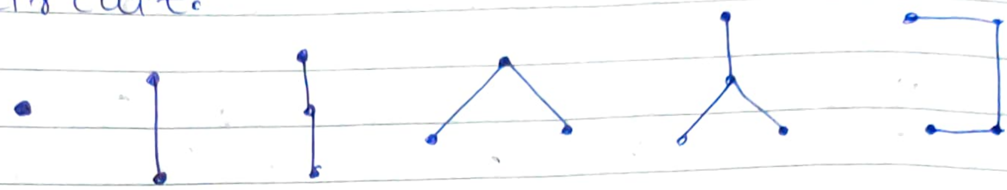

3-chromatic

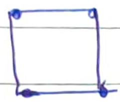
Ex-1 Determine the chromatic number of graph.




3 - Chromatic Number

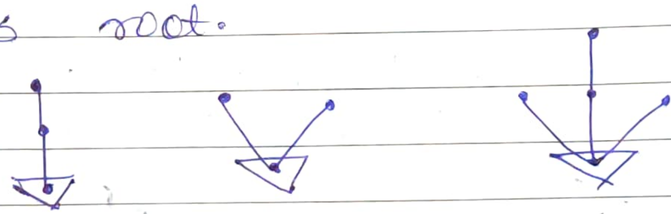
* **Tree** : A tree is a connected graph without any loop or circuit.



-  ← It is closed circuit
  - It is Graph but not Tree

* **Rooted Tree** : A rooted tree is a tree in which one vertex is root.

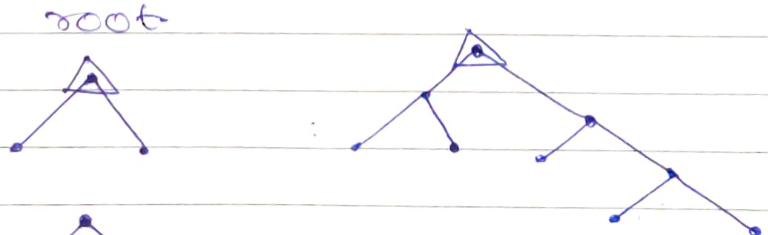
Ex



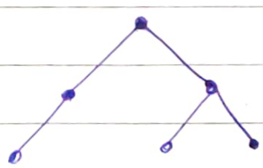
* **Binary Tree** :

A Binary tree is defined as a tree in which there is exactly one vertex of degree two and each of remaining vertices is of degree one or three and vertex of degree two is serves as a root

Ex



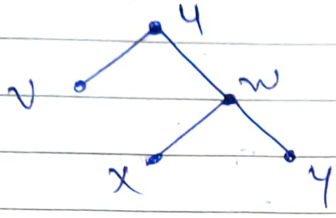
But



is not Binary Tree

## * Pendent vertex in Tree :

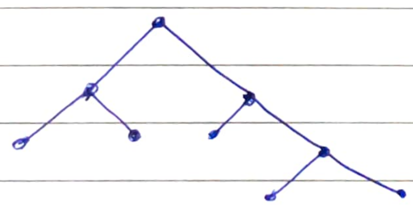
A vertex of degree one is called pendent vertex of tree.




## * Path Length of Tree :

A Path length of a tree is defined as a sum of edges from the root of all pendent vertices.

Ex

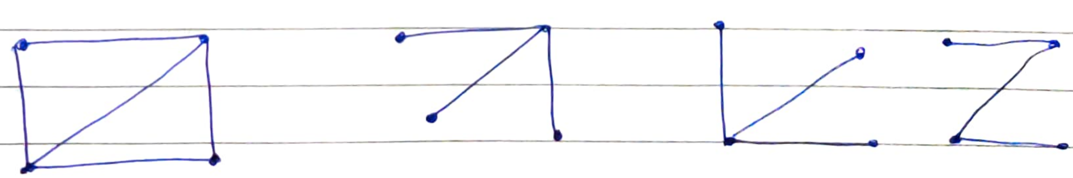


then path length
= 2 + 2 + 2 + 3 + 3
= 14.

## * Spanning Tree :

If G is any connected graph a spanning Tree in G is a sub~~group~~ graph T of G which is a Tree.

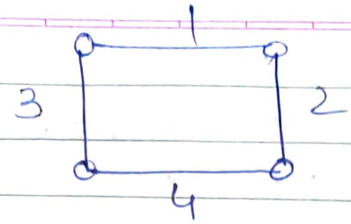
Ex



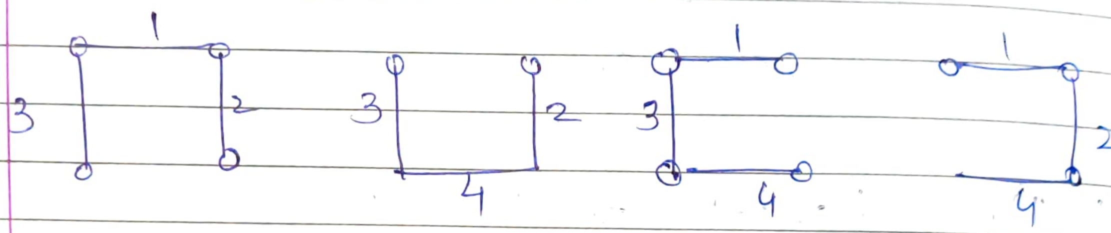
## * Minimal spanning Tree :

A G be a weighated graph. A minimal spanning tree of G is a spanning tree of G with minimal weight

Eg



⇒ spanning tree of G are



⇒ minimal spanning Tree are