Marginal V1

Abstract

Marginal is a decentralized, permissionless, peer-to-pool AMM for *physically* settled perpetuals on any pair of fungible tokens. One can think of the core mechanism of the protocol as analogous to overcollateralized short-selling with the interest paid by the trader dictated by a typical perpetual funding rate. Liquidity providers passively pool both X and Y tokens in liquidity pools to make the perpetual market for the pair. Traders take derivative positions by borrowing from the passively pooled liquidity, and subsequently swapping through the same pool only one side of the borrowed liquidity for more of the token the trader is looking to gain leverage on, overcollateralizing the result. At settlement, the trader delivers the debt needed to return the owed liquidity back to the pool receiving the originally longed token in return. Positions can be permissionlessly liquidated in the event the value of the margin token drops below maintenance requirements by sending the entirety of the collateral less fees back to the pool.

1 Introduction

The current landscape for DeFi derivatives is relatively permissioned when it comes to offering markets on pairs of any two fungible tokens. Instead, a decentralized derivatives protocol should take the principles of Uniswap to derivatives land: immutable, censorship-resistant, unstoppable code, with the ability for *anyone* to permissionlessly spin up new markets for any token pair. The only additional requirement would be that the associated decentralized spot market already exists on the network and has a manipulation-resistant oracle. Uniswap V2 and V3 [1] facilitate this spot market component via on-chain oracle functionality.

In order to safely provide perpetuals, a derivative protocol would likely need to *physically* settle the perpetual contracts, with liquidity providers acting as the counterparty securing future delivery of the token the trader wishes to long. Taking the approach of cash settlement, as most derivative protocols currently tend to do, leads to severe risk management issues as traders need to prepare for the likelihood of not being fully paid out for their position if the liquidity pool lacks the necessary funds in the cash token.

2 Overview

Marginal offers a solution in the following manner. Liquidity providers looking to earn fees off of leveraged positions passively pool capital in *both* token types. For a Marginal pool with existing token balances (x, y), a trader desiring leveraged exposure to the X token relative to Y can take out a position through the pool contract. In opening a position, the trader borrows *liquidity* from the pool in both tokens, which decreases available pool reserves. After borrowing liquidity, the trader swaps through the reduced reserve pool only the Y tokens borrowed for more X token the trader is looking to gain leverage on. The trader then sends initial margin in X token to the pool to overcollateralize the position, with the majority of the leverage provided by pool liquidity providers through the borrowed reserves.

In the same call to the pool:

1. $(\delta x, \delta y)$ of liquidity is removed from reserves. This is the liquidity fronted by liquidity providers for the trader to gain leveraged exposure.
2. (i_x, i_y) of the fronted liquidity is set aside as insurance against liquidity shortfall in the event of position liquidation.
3. The remaining $\delta y - i_y$ of Y token is swapped for $\delta x'$ more of X token through the reduced reserves. $\delta x'$ is determined by the choice of pool price curve.
4. c_x of X token is transferred in from the trader to overcollateralize the position. The position holds $(c_x + \delta x + \delta x' - i_x, 0)$ in collateral and owes (d_x, d_y) in debt to the pool reserves.

The debt the position owes (d_x, d_y) is set such that if the position were immediately settled, the transaction would replicate a simple swap on the spot market along the original constant liquidity curve L with s_x of X token out and d_y of Y token in. s_x is shorthand for the size less X token debt associated with the position.

Adopting Uniswap's $xy = L^2$,

$$\delta x' = \delta x \cdot (1 - \delta x / x) \cdot \left(\frac{1 - i_y / \delta y}{1 - i_y / y} \right) \tag{1}$$

with debts

$$\begin{aligned} d_x &= \delta x \cdot \left(\frac{1 - \delta y / y}{1 - i_y / y} \right) - i_x && (2) \\ d_y &= \delta y \cdot \left(\frac{1 - i_y / \delta y}{1 - \delta y / y} \right) && (3) \end{aligned}$$

and size

$$\begin{aligned} s_x &= \delta x + \delta x' - (d_x + i_x) \\ &= \delta x \cdot \left(\frac{1 - i_y/\delta y}{1 - i_y/y} \right) \end{aligned} \tag{4}$$

constrained by

$$\delta y / \delta x = i_y / i_x = y / x = P \tag{5}$$

Price P is the ratio of Y to X reserves in the pool before the position is opened. The slippage experienced by the trader moves the pool price after opening the position to

$$P' = P \cdot \left(\frac{1 - i_y/y}{1 - \delta y/y} \right)^2 \tag{6}$$

The liquidity removed from reserves to open the position is

$$\begin{aligned} \delta L &= \sqrt{\delta x \cdot \delta y} \\ &= \sqrt{(d_x + i_x)(d_y + i_y)} \end{aligned} \tag{7}$$

Figure 1 diagrams the interaction between trader, pool, and insurance fund for a position opened then immediately settled. Figure 2 [2] shows the pool reserves transitioning between intermediate states for the same interaction. Figure 3 extends this to the general case of a position opened but settled at a future time.

To settle the trade, the trader delivers the outstanding debt balances (d_x, d_y) to the pool and receives the full collateral amount $c_x + s_x + d_x$ in X token originally reserved for the position. The insurance fund returns the insurance balances associated with the position (i_x, i_y) back to the pool. Pool contracts can avoid having the trader front the Y token to cover outstanding debts by implementing similar optimistic callback procedures to Uniswap V2 and V3. This would allow the trader to sell on the spot market the received collateral to cover debts at settlement.

The liquidity returned to the pool at settlement $\delta \tilde{L}$ will be at least the original liquidity borrowed δL, given the debt and insurance effectively replicate a hodl strategy. For a general state of the pool reserves $\tilde{x}\tilde{y} = \tilde{L}^2$, $\tilde{P} = \tilde{y}/\tilde{x}$ prior to position settlement, settling alters the reserve invariant to

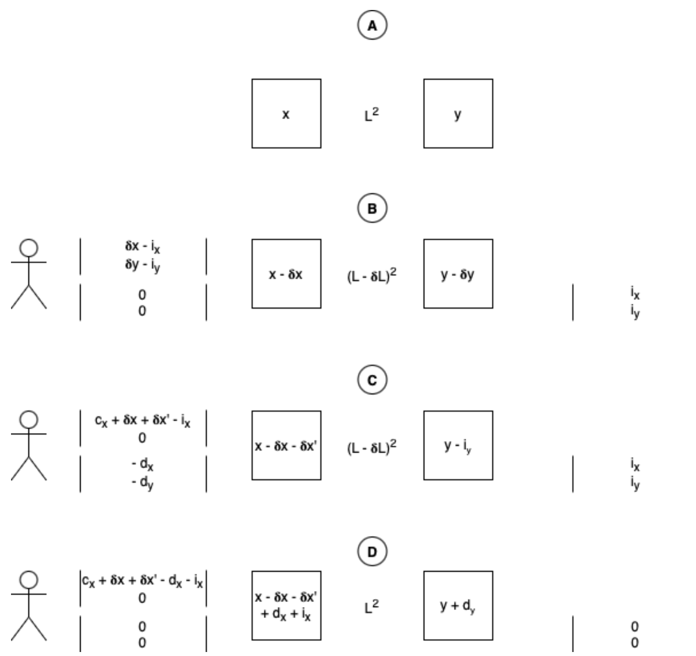

Figure 1: Flow of funds as a long position is opened then immediately settled. Trader collateral and debt values on the left, pool reserves in the middle, and insurance balances on the right. Opening a position transitions from $A \to B \to C$, with the trader's position being backed by collateral $(c_x + s_x + d_x, 0)$ and owing debt (d_x, d_y). Settlement transitions from state $C \to D$, with the trader sending (d_x, d_y) to the pool reserves in exchange for the X token collateral backing the position. The insurance funds (i_x, i_y) are also returned to the pool at settlement.

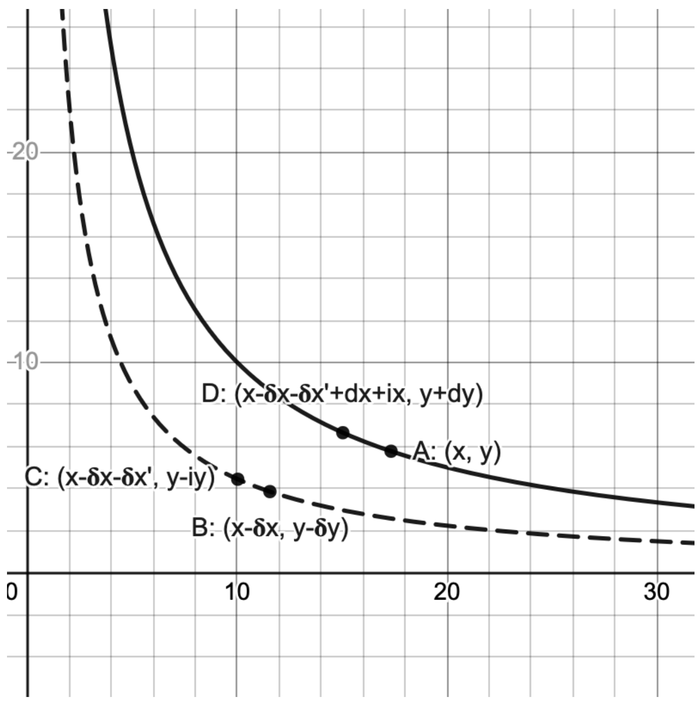

Figure 2: X, Y token pool reserves as a long position is opened then immediately settled. Trader opening a position leads to the reserves transitioning from $A \to B \to C$, decreasing pool liquidity and increasing price. Trader immediately settling the same position transitions the reserves from $C \to D$, maintaining the same price as at C but increasing liquidity back to the original state. $A \to D$ replicates a swap on the spot market.

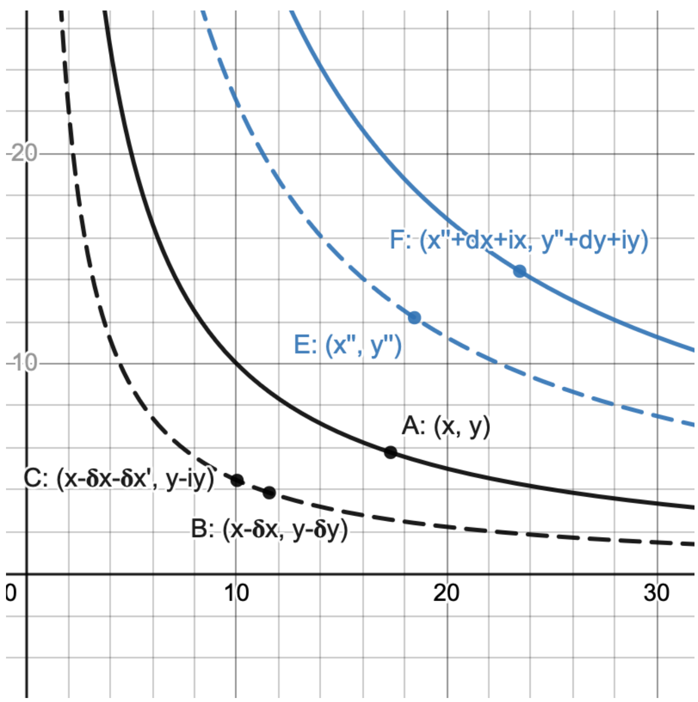

Figure 3: X, Y token pool reserves as a long position is opened. Position is settled at time t in the future, with liquidity and price conditions (x'', y'') changing due to other traders and liquidity providers interacting with the pool $C \to E$. Trader settling the position transitions the reserves from $E \to F$, altering price and increasing liquidity.

$$\begin{aligned}(\tilde{L} + \delta\tilde{L})^2 &= (\tilde{x} + d_x + i_x)(\tilde{y} + d_y + i_y) \\ &= \tilde{L}^2 \cdot [(\delta L/\tilde{L})^2 + (2\delta L/\tilde{L}) \cdot \cosh(p/2) + 1]\end{aligned}$$

such that $\delta\tilde{L} \geq \delta L$ for all possible values of $p = \ln(\tilde{P}/P')$.

3 Maintenance requirements

The trader must manage the X collateral backing the position given the current debt owed. The Marginal pool uses the time-weighted oracle price fetched from the associated Uniswap spot market $\bar{P}_t$ to limit manipulation of the liquidation condition

$$(c_x + s_x) \cdot \bar{P}_t \geq (1 + M) \cdot d_y \tag{8}$$

If not satisfied at all times t, the position can be permissionlessly liquidated. The position is then overcollateralized by a factor of $1 + M$, where M is the maintenance margin requirement for the pool. The overcollateralization is provided by *liquidity providers*, allowing traders to gain leverage. The trader's leverage is the total position size in X token terms $c_x + s_x$ divided by the margin the trader provides c_x

$$l = 1 + \frac{s_x}{c_x} \tag{9}$$

Maintenance requirements restrict the maximum leverage traders can take. The trader has access to greater leverage for smaller amounts borrowed. In the limit $\delta x, \delta y \to 0$, Y token debts owed trend toward $d_y \to P \cdot s_x$ when the position is opened. For small deviations in the pool price v.s. the spot oracle, the liquidation condition reduces to

$$\lim_{\delta x, \delta y, t \to 0} c_x \geq M \cdot s_x \tag{10}$$

Arbitrage opportunities via immediate settlement with the Marginal replicated swap should constrain large deviations in price from the spot pool. Trader leverage will then be limited by

$$l \leq 1 + \frac{1}{M} \tag{11}$$

Table 1 provides maximum leverage values for various pool maintenance requirements, which should be set as an immutable on the pool by the Marginal factory contract.

$1+M$	l_{max}
1.25	5
1.5	3
2	2

Table 1: Maximum leverages offered for maintenance requirements.

Token pairs with different risk profiles will require different maintenance requirements. Liquidity providers should be careful when providing liquidity for pairs that are too volatile for the maintenance margin requirement M choice at initialization, as the pool experiences the shortfall in the event the position is not liquidated in time.

Liquidations are rather simple as the Marginal pool provides the backstop liquidity. If the position is liquidatable, liquidators can call a function on the pool that transfers the full X token balance of the position $c_x + s_x + d_x$ back to the pool reserves. This increases pool liquidity while lowering the price. The insurance fund simultaneously returns the insurance balances associated with the position (i_x, i_y) back to the pool to prevent the possibility of a liquidity shortfall in the event the pool has become illiquid.

Cascading liquidations will not happen directly, as the liquidation condition purposefully uses the time-weighted spot oracle price. Although spot will be influenced through subsequent arbitrage opportunities.

Figure 4 diagrams the interaction between trader, pool, and insurance fund for a position opened then immediately liquidated. Figure 5 shows the pool reserves transitioning between intermediate states for the same interaction. The pool gains reserves when the position has excess maintenance less liquidator incentives.

4 Insurance funds

When the pool becomes illiquid due to e.g. liquidity providers pulling funds or simply greater utilization of the pool for positions, the liquidity returned to the pool on liquidation from sending the full X token position collateral back may not be enough to recover the original liquidity removed, *regardless* of whether the position is liquidated before reaching the bankruptcy price. Insurance balances (i_x, i_y) are set to ensure at least the removed liquidity amount δL is returned to the pool regardless of the pool state prior.

For a general state of the pool reserves $\tilde{x}\tilde{y} = \tilde{L}^2$, $\tilde{P} = \tilde{y}/\tilde{x}$ prior to position liquidation, liquidating at the bankruptcy price

$$P_b = \frac{d_y}{c_x + s_x} \geq \frac{P}{1+M} \tag{12}$$

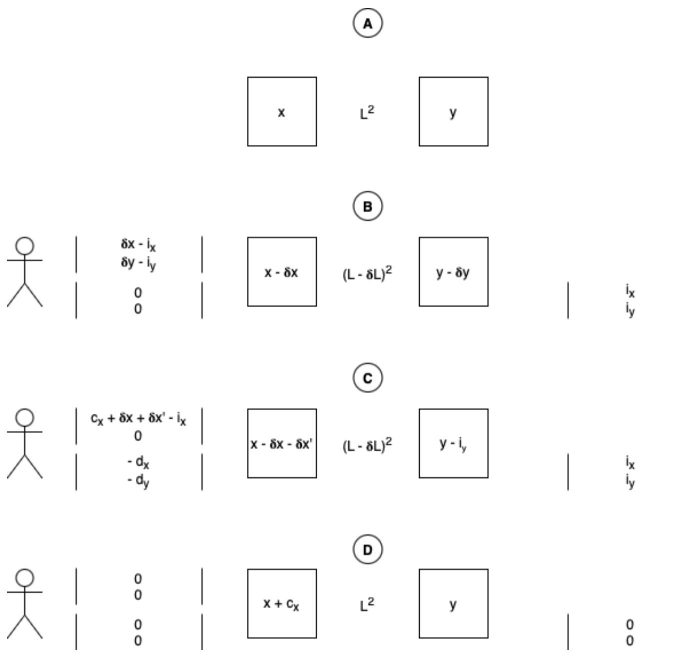

Figure 4: Flow of funds as a long position is opened then immediately liquidated. Trader collateral and debt values on the left, pool reserves in the middle, and insurance balances on the right. Liquidation transitions from state $C \to D$, with the X token collateral backing the position $c_x + s_x + d_x$ sent back to the pool reserves. The insurance funds (i_x, i_y) are simultaneously returned to the pool.

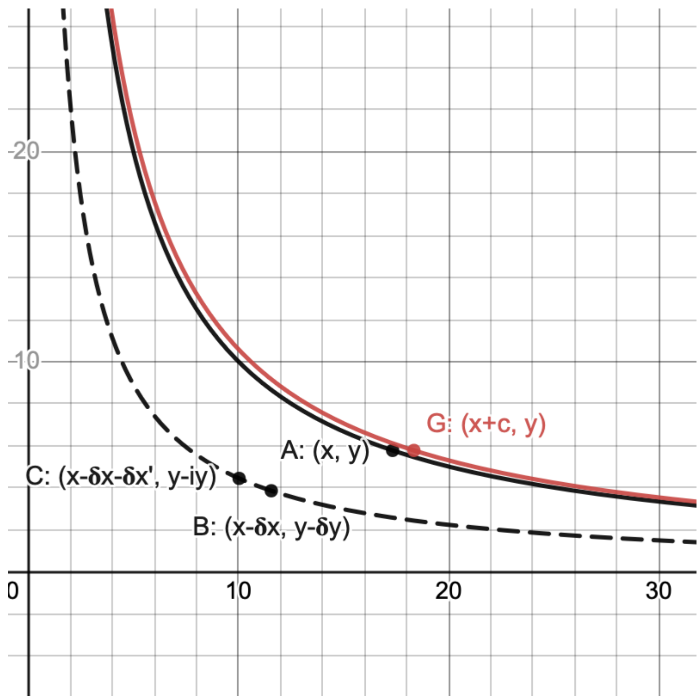

Figure 5: X, Y token pool reserves as a long position is opened then immediately liquidated. Liquidating the position transitions the reserves from $C \rightarrow G$, increasing liquidity past the original state and decreasing the price.

alters the reserve invariant to

$$
\begin{aligned}
(\tilde{L} + \delta\tilde{L})^2 &= (\tilde{x} + c_x + s_x + d_x + i_x)(\tilde{y} + i_y) \\
&= \tilde{L}^2 \cdot [(\delta L/\tilde{L})^2 + (2\delta L/\tilde{L}) \cdot \cosh(p/2) + 1] \\
&\quad +(c_x + s_x) \cdot [i_y - \tilde{P} \cdot (i_x + d_x)]
\end{aligned}
$$

which returns $\delta\tilde{L} \geq \delta L$ to the pool for all possible values of $p = \ln(\tilde{P}/P')$ and c_x only if

$$
i_y \cdot (1 - i_y/y) \geq \frac{\delta y}{1 + M} \cdot (1 - \delta y/y) \tag{13}
$$

Setting the insurance balance to the negative root

$$
i_y = \frac{y}{2} \cdot \left[1 - \sqrt{1 - \frac{4}{1+M} \frac{\delta y}{y} \left(1 - \frac{\delta y}{y}\right)}\right] \tag{14}
$$

satisfies the required relation and produces the necessary constraint on $i_y \leq \delta y$. This choice, however, imposes significant capital requirements on pools that offer high amounts of leverage, as the X token net position size

$$
\lim_{\delta x \to 0} s_x = \delta x \cdot \frac{M}{1 + M - \delta x/x}
$$

will be about a factor of M times the insurance balance for smaller liquidity amounts borrowed. The tradeoff is safety for liquidity providers against liquidity shortfall given an arbitrary pool state prior to liquidation, as long as the minimum margin requirement

$$
c_x \geq (1 + M) \cdot \frac{d_y}{P} - s_x \tag{15}
$$

is enforced.

Figure 6 illustrates this as pool reserves transition between states upon liquidation. Figure 7 plots liquidity shortfall as a function of pool price pre-liquidation to demonstrate liquidity providers always receive the originally borrowed liquidity back due to the insurance mechanism and minimum margin requirement.

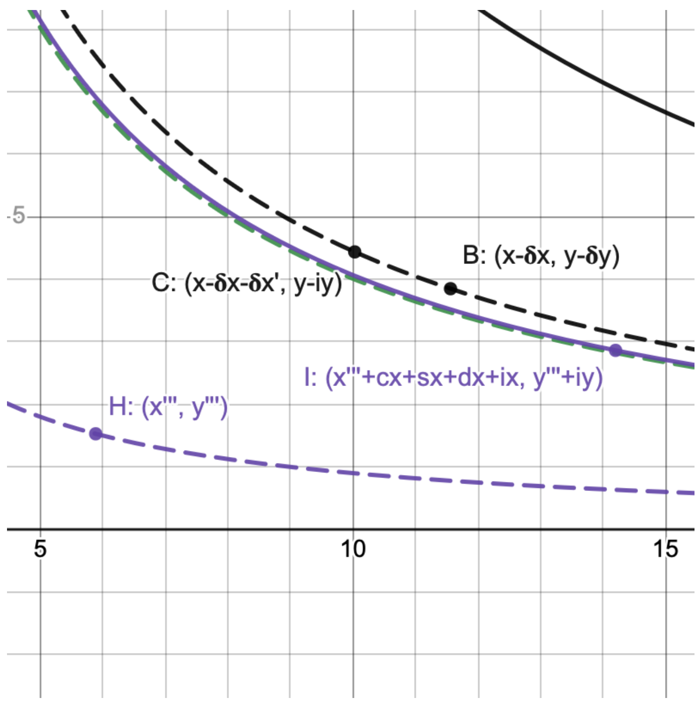

Figure 6: X, Y token pool reserves as a long position is opened then liquidated at time t in the future. H is the pre-liquidation state with price below position bankruptcy. I is the associated pool state post-liquidation after position collateral and insurance balances have been returned to the pool. Dotted green curve is the constant liquidity curve the pool must transition to in order to not experience a liquidity shortfall. The pool avoids a shortfall due to the insurance balances returned.

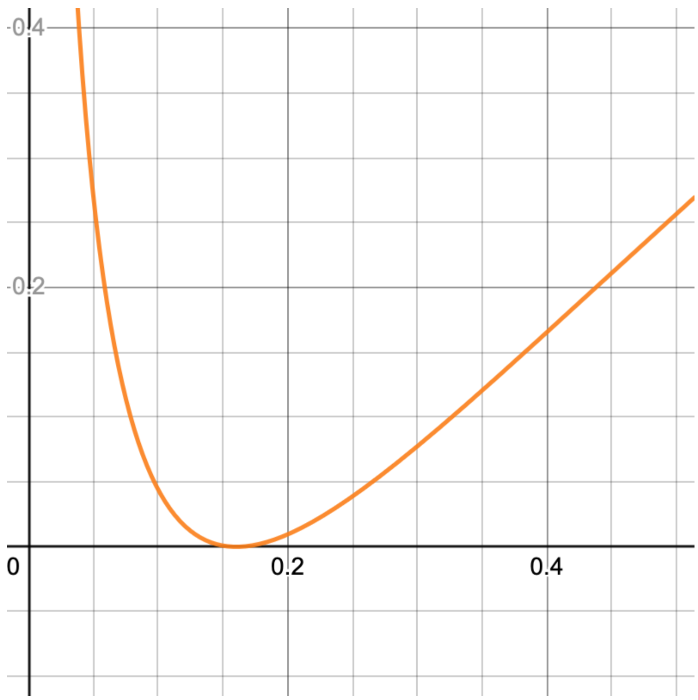

Figure 7: Net liquidity gained $(\tilde{L}+\delta\tilde{L})^2-(\tilde{L}+\delta L)^2$ plotted against pre-liquidation pool price $\tilde{P}$ as a long position is opened then liquidated at time t in the future. For the example, $P_b = d_y/(c_x + s_x) = 0.2664$. The pool avoids a shortfall at all possible pre-liquidation pool states due to the insurance balances returned and the requirements on c_x.

5 Funding payments

Greater utilization of the liquidity pool for positions leads to a less liquid Marginal market. This should decrease the profitability of arbitrage opportunities via Marginal replicated swaps and cause larger price deviations from the spot market. To further incentivize price convergence, the Marginal pool charges a typical perpetual funding rate on outstanding positions. Trader position debt changes based on the deviation in the time-weighted average of the Marginal pool price relative to the oracle price fetched from the associated Uniswap spot market. Positions charged funding through debt increases are pushed closer to liquidation, incentivizing earlier settlement.

The debt the position owes (d_{x_t}, d_{y_t}) becomes time dependent with funding. At position open

$$d_{x_0} = d_x \tag{16}$$

$$d_{y_0} = d_y \tag{17}$$

as before. However, for the same leveraged long X relative to Y, the position Y token debt is altered based on the relative difference in Marginal time-weighted average price versus the Uniswap spot time-weighted average price

$$d_{y_{t+\tau}} = d_{y_t} \cdot (P_{t,t+\tau} / \bar{P}_{t,t+\tau})^{\tau/T} \tag{18}$$

This produces continuous, in-kind funding [3] with period T an immutable on the pool. It has the effect of paying or receiving funding from the position's profit or loss. The X token debt remains unchanged.

6 Remarks

Swapping labels $X \rightleftharpoons Y$ in all of the above produces expressions for leveraged exposure to the Y token relative to X (i.e. leveraged short). These trades are executed through the same Marginal pool as the leveraged X relative to Y positions.

References

[1] Hayden Adams et al. *Uniswap v3 Core*. 2021. URL: https://uniswap.org/whitepaper-v3.pdf.

[2] *Marginal V1 Core – Desmos Calculator*. 2023. URL: https://www.desmos.com/calculator/poiqfdbq7p.

[3] Dave White et al. *Power Perpetuals*. 2021. URL: https://www.paradigm.xyz/2021/08/power-perpetuals.