1 The 45th series graph

For three hysterons in series, we have obtained 44 realizable graphs, assuming a model where $c_{ij} = c_j, c_j \leq 0$. We construct the graphs using the base graph method, where we first construct all possible

1. When $c_{ij} = c_j$, there is no scrambling, so the only base graphs are the Preisach graphs.
2. When $c_j \leq$, there can only be antiferromagnetic avalanches,
3. As a result of restrictions 1 and 2, there can only be avalanches of length $l = 2$.
4. When $c_{ij} = c_j$ svalanches involving the same hysteron flips in the same directions must occur together.

After applying the restrictions, there are 45 candidate graphs for three hysterons in series. We find that 44 of these graphs are realizable, while a single graph is not (Fig. 1. We now ask why the 45th graph is not realizable.

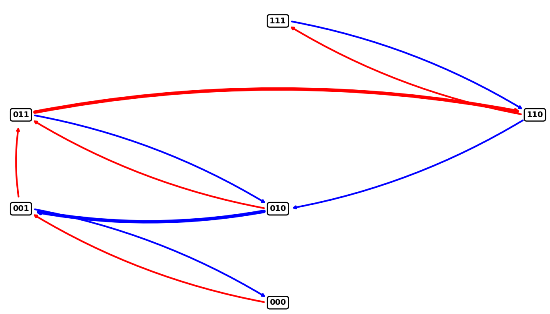

Figure 1: The single impossible candidate graph for three hysterons in series.

A graph is realizable if the underlying set of design inequalities has a solution. We reason that all four transitions 001 $\uparrow$ 011, 011 $\uparrow$ 111 $\downarrow$ 110, 110 $\downarrow$ 010 and 010 $\downarrow$ 000 $\uparrow$ 001 must be involved in the combination of inequalities that is incompatible, since more than one of the candidate graphs would otherwise be impossible.
To know whether a set of inequalities is solvable, one can use the Fourier-Motzkin method for elimination of variables. In the Fourier-Motzkin method, a variable x is eliminated by combining the inequalities to create a new set that does not contain x. A set of inequalities does not have a solution if, after all variables are eliminated, a contradictory statement remains such as $0 > 0$. By working backwards, we can identify which combinations of inequalities lead to

a contradictory statement and are therefore incompatible.
We find that for the 45th graph, there is a single combination of six inequalities that is incompatible, namely:

$$
\begin{aligned}
u_1^- - c_2 &> u_2^- - c_1 \\
u_3^+ &> u_1^- \\
u_2^- &\geq u_3^+ \\
u_2^+ - c_3 &> u_3^- - c_2 \\
u_1^+ &> u_2^+ \\
u_3^- - c_1 &\geq u_1^+ - c_3
\end{aligned}
\tag{1}
$$

We further note that the six inequalities arise from the four transitions 001 ↑ 011, 011 ↑ 111 ↓ 110, 110 ↓ 010 and 010 ↓ 000 ↑ 001 (Fig. 2), in agreement with our prediction.

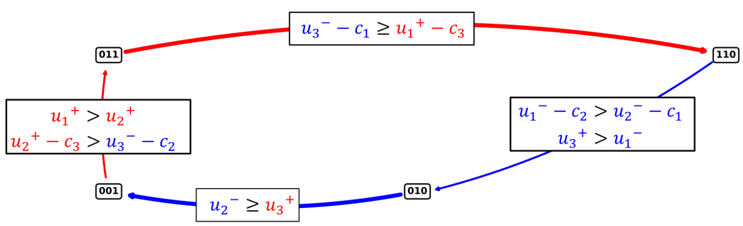

Figure 2: The four transitions of the 45th graph that are not compatible: 001 ↑ 011, 011 ↑ 111 ↓ 110, 110 ↓ 010 and 010 ↓ 000 ↑ 001.

The 45th graph thus illustrates that it is not trivial to see whether a combination of transitions of compatible, and that it therefore remains necessary to explicitly check whether candidate graphs are solvable by generating the design inequalities.